

INSURANCE SUPERVISION UNDER CLIMATE CHANGE: A PIONEERS DETECTION METHOD *

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Abstract

This research introduces a novel supervisory tool, the Pioneers Detection Method, aimed at enhancing resilience in insurance markets dealing with the uncertainties of climate change. The paper builds on a theoretical model of the insurance market, where independent experts set premiums based on their individual risk evaluations. The segmented nature of the private insurance market slows the estimations of the tail parameter of the loss distribution, and there's no direct way to eliminate bias, as extreme events are infrequent. The proposed supervisory tool uses temporal changes to consolidate expert opinions, pinpointing those who rapidly and accurately identify extreme climate-related events. The effectiveness of the Pioneers Detection Method is affirmed through a series of simulations, where it surpasses traditional pooling methods within a Bayesian framework. This supervisory approach also proves to be the most beneficial in improving welfare in a fragmented insurance market comprised of a few private insurance companies.

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1 INTRODUCTION

Climate change, characterized by long-term shifts in temperatures and weather patterns, leads to both direct (e.g., heatwaves, droughts) and indirect (e.g., desert expansion, wildfires, storms) extreme events with increasing frequency and intensity (Stott et al., 2016). Although a well-established phenomenon¹, there is no consensus on whether climate change will be continuous or whether there will be unexpected shocks that will pose significant challenges for financial systemic risks, which fall within the mandate of financial supervisors (Svartzman et al., 2021). One such challenge is determining the insurability of risks in a constantly changing environment² and one of the supervisory tools for this challenge is to leverage insurance company expertise by pooling their opinions. In the foundational work by Stone (1961), opinion pooling is presented as the method of merging experts' subjective probability distributions to form a collective judgment. However, traditional pooling methods were not specifically devised for situations involving radical uncertainty, as defined by Knight (1921), and a loss distribution tail parameter that can never be exactly estimated. As a result, they encounter two primary hurdles: potential inconsistencies in expert opinions over time due to model shortcomings, and slow decision-making processes in accepting or rejecting insurability hypotheses. In this study, an insurance market is modeled to design, validate, and estimate the welfare benefits of introducing a novel opinion pooling strategy specifically designed for this context.

I start by designing an insurance market model derived from Raviv (1979) and following the insurability problem presented by Charpentier (2008). I derive this model and study its equilibrium to determine the optimal insurance contracts and supervision actions. Climate change is impacting both the frequencies and magnitudes of events and, in the model, yearly aggregated³ insurance losses for an asset class follow a Pareto distribution with an unobservable tail index. This paper models non-cooperative⁴ private insurance experts with

¹Early works on climate change include those by Fourier (1824), Foote (1857), Arrhenius (1896), Callendar (1938). Recent contributions are reviewed in Hegerl et al. (2007) and (Change, IPCC Climate and others, 2014).

²For insurability criteria, see Berliner (1985).

³The aggregate or magnitude component of losses can be understood as yearly aggregates of losses faced by an insurance company for a given asset class. The paper assumes that insurance companies cannot completely discriminate policyholders, so premium pricing is equivalent to modeling the magnitude components.

⁴The European Court of Justice established in 1987 (Judgement of 27.1.198) that EU competition law is fully applicable to the insurance sector. Some exemptions were allowed by the Insurance BER" (Regulation (EEC) No. 3932/1992) as Collaboration between insurance undertakings [...] in the compilation of information (which may also involve some statistical calculations) allowing the calculation of the average cost of covering a specified risk [...] makes it possible to improve the knowledge of risks [...]. This can in turn

proprietary information sets and modeling expertise. Insurance buyers are slower at updating their loss distributions, for simplicity I consider that in the time frame of the model they do not update their estimations. Therefore after a tipping point⁵ there is a situation of heterogeneous beliefs among participants. After this distinct and unanticipated shock, insurance experts update their risk models to determine whether, and under what conditions, they provide insurance coverage for the affected asset class⁶. This paper doesn't study the strategic behavior of insurance companies, and market shares are considered as fixed and exogenous⁷. An insurance supervisor with a financial stability mandate completes the model and decides whether it should give incentives for experts to continue providing insurance coverage for a given class of assets. To do so and for the supervisor to estimate the risk after a tipping point, the context calls for a new insurance supervision tool.

In this paper, I introduce a potential innovative supervisory tool, a Pioneers Detection Method, tailored for the dynamic climate context, where the unavailability of ground truth data hinders the evaluation of expertise based on historical estimates. The method aims to allocate substantial weight to reliable pioneers, who are defined as experts⁸ who consistently caution against the escalating severity of climate-related risks, using transparent models and datasets, and whose opinions are being implicitly validated by their peers. The weights of the opinion pooling are conceptualized as implicit inter-temporal votes among experts.

When an asset class's insurability suddenly becomes uncertain, two problematic scenarios can arise from a financial stability perspective. The first concern is that some insurance companies might underestimate the impacts of the change and be exposed to ruin. This paper primarily focuses on the second scenario, where non-cooperative insurance companies can exit the market or offer premiums close to the coverage limit⁹. To determine whether

facilitate market entry and thus benefit consumers." The "Insurance BER" was stopped on April 1, 2017. Automatic exemptions were discarded to avoid misapplications, considering that guidelines were sufficient (Stancke, 2017).

⁵This concept refers to a critical threshold in a complex system, where minor perturbations can result in substantial, often irreversible, changes in the system's overall behavior or state (Gladwell (2006) and Lenton et al. (2008)).

⁶A class can be understood here as either a region or a business line.

⁷An extension is to use the supervisory data to estimate the demand sensitivity to premiums to make the market shares endogenous to the model, resulting in heterogeneous private information and incentives to invest in modeling each class's idiosyncratic risk. This would allow introducing situations where an insurer with low exposure to an asset class could offer loss-making premiums to grow its share, introducing ruin gamble not present in this model.

⁸Examples include Meadows et al. (1972) and Jean-Marc Jancovici (Jancovici, 2004), whose diagnostics have been based on transparent data and utilized for making predictions and recommendations that are increasingly becoming mainstream over time.

⁹Winter (1994) acknowledges that supply will always be positive at some price but mentions premiums reaching 90% of the coverage limit for firms removing asbestos.

the reduction in insurance supply is justified by the increase in risks or the inability of insurance experts to accurately estimate risks, the supervisor can decide to collect expertise using ad-hoc on-site inspections of insurance companies and pool expertise. Although there is a rich literature on stress tests (Battiston et al., 2017) and their use in learning about the insurance market and guiding supervisory actions¹⁰, little research has been conducted on recommending ad-hoc inspections for pooling opinions for the same purpose¹¹. This paper aims to fill this gap by demonstrating how supervisors can use their regular supervision or ad-hoc inspection activities to reinforce their knowledge and understanding of climate change’s disruptive potential for the insurance markets.

More broadly, this paper contributes to the continuing discourse on insurance supervision in the context of climate change. In countries like France, debates exist on whether housing insurance state guarantees for clay soil risks should be scrapped and how a supervisor can act once the market is fully private. Currently, 40% of the state natural catastrophe scheme (CatNat) is used to cover losses for houses exposed to the risk of Withdrawal-Swelling of Clay Soils (WSCS) under the effect of drought¹². The Court of Auditors is advising the exclusion of WSCS from the CatNat regime, given that the unpredictable nature of the risk is no longer being addressed due to the effects of climate change (Cour des comptes, 2022). The novel methodology enhances supervision by aggregating expertise at the insurance group level, a common practice, instead of initiating procedures to obtain granular underwriting data for evaluating climate-related financial risks post-shock. While the United States Department of the Treasury’s Federal Insurance Office (FIO) has considered collecting granular data from property and casualty insurers regarding homeowners’ insurance¹³, it has faced resistance. One policy recommendation of this paper is to allow the use of this new tool to help supervisors determine if an asset class is insurable. Upon determining an asset class’s insurability, the supervisor can then advise the regulator on whether to utilize public or private insurance mechanisms, such as syndication or reinsurance, for coverage.

Literature This paper contributes to the literature on opinion pooling and combination forecasting, building upon previous reviews by Genest and Zidek (1986), Clemen (1989), Timmermann (2006), and Wang et al. (2022). This literature concludes that in a Gaussian

¹⁰For example, the ECB Banking Supervision launched a 2022 climate risk stress test using macroeconomic scenarios from the NGFS.

¹¹The closest strategy is NAIC’s Climate Risk Disclosure Survey, a voluntary risk management tool through which state regulators can request annual, non-confidential disclosure of insurers’ assessment and management of their climate-related risks. Source: Moody’s report on Insurance Conference 2022.

¹²Fleureau et al. (1993) characterizes the phenomenon, and Babaeian et al. (2019) describes the modeling challenges compounded by limitations of remote sensing of soil moisture.

¹³The FIO issued a request for comment on a proposed data call on October 18, 2022.

context it is almost impossible to beat a mean method potentially excluding outliers. In a context with fat tails, extreme events do occur but are rare, hence learning is slow and I design a new method that relies on inter-temporal opinion changes to weight experts that are the most likely to learn faster about extreme events.

This paper adds to the rich literature and ongoing debate surrounding the impacts of climate change on insurance losses. There are mixed conclusions on the effects of climate change on loss trends, as discussed by [Pielke Jr et al. \(2008\)](#), [Kousky \(2014\)](#), [Hsiang \(2016\)](#), [Botzen, Deschenes and Sanders \(2020\)](#), and [Mills \(2005\)](#). I use publicly available data to model the evolution of losses over time in [Appendix A](#). I find that the combined influences of climate and macroeconomic changes lead to increases in both average expectations and tail fatness of losses. The modeling addresses heavy-tailed distributions, drawing on the Extreme Value Theory (EVT) literature and its applications in insurance, as seen in the seminal works of [Frechet \(1927\)](#), [Fisher and Tippett \(1928\)](#), [Gnedenko \(1943\)](#), [de Haan \(1970\)](#), [Balkema and De Haan \(1974\)](#). EVT has been applied to various aspects of climate change, including the analysis of extreme weather events ([Coles et al., 2001](#); [Palutikof, Subak and Agnew, 1997](#)) and the estimation of extreme financial losses ([McNeil, Frey and Embrechts, 2015](#)). The primary challenge, which remains unresolved, is that loss tails must be estimated with limited observations. Consequently, after an unforeseen tipping point, expert model uncertainty increases. This paper adds to the literature on modeling loss tails under uncertainty, building upon the works of [Danielsson et al. \(2001\)](#), [Scarrott and MacDonald \(2012\)](#), and [Raftery et al. \(2017\)](#), by exploring the combination of EVT and expert opinion.

This paper builds upon the literature on insurability and the challenges climate change poses to insurability. Climate change is testing the insurability of various business lines and geographic regions, as demonstrated by [Kunreuther \(1996\)](#), [Charpentier \(2008\)](#), [Kousky and Cooke \(2012\)](#), [Mills \(2007\)](#), and [Surminski \(2014\)](#). Both pandemics and climate change risks are difficult to mutualize in a cross-sectional manner due to their complex nature and widespread impacts. While pandemics can be insured intertemporally due to their episodic nature, climate change might not be, as it is expected to be an irreversible trend with potential tipping points. This paper demonstrates that, although preventive actions against climate change are primarily the responsibility of regulators, a supervisor operating further down the line for insurance market stability can utilize this paper to direct his prudential activities to avoid disturbances caused by uncertainty. My approach differs from the work of [Jaffee and Russell \(1997\)](#) as I do not focus solely on catastrophes. [Jaffee and Russell](#)

(1997) consider that catastrophes are actuarially insurable¹⁴ as they are infrequent, local and uncorrelated. In my approach, I consider yearly cumulative losses each insurance company face and how climate change fatten the subjectively estimated tail of this distribution up to uninsurability.

Section 2 presents a model of the insurance market in the context of climate change. In Section 3, I introduce a novel insurance supervision instrument designed to aggregate expert knowledge under extreme uncertainty, the Pioneers Detection Method. Section 4 describes simulation-based validations of the new tool. Section 5 provides policy recommendations based on the model and simulations. Finally, Section 6 offers concluding remarks.

2 A MODEL OF INSURANCE MARKET UNDER CLIMATE CHANGE

A risk-averse insurance buyer (IB) with a utility function denoted by U , with $U'(\cdot) > 0$ and $U''(\cdot) < 0$, faces a risk of loss of x (α^t) to his asset, where x is a random variable following a Pareto¹⁵ distribution with an unknown tail parameter α^t . An insurance contract for an insurance company (IC) i is defined as a pair $(I(x), \Pi_i(I(x)))$, with an indemnity schedule $I(x)$ and an insurance premium $\Pi_i(I(x))$. I consider risk-neutral IC who act on a perfectly competitive insurance market. Raviv (1979) extends Arrow (1963) and demonstrates that under these conditions an optimal contract is full insurance above a deductible. Ghossoub (2017) extends Marshall (1992) and demonstrates that under heterogeneous beliefs with a non-decreasing ratio of subjective probability density the deductible result holds. Without loss of generality, I normalize this deductible to unity even when it is found non-constant. This simplifies the insurance contract to a one-dimension parameter¹⁶ problem where the indemnity follows a Pareto with a tail parameter α^t and a unity threshold. Pareto optimal

¹⁴Jaffee and Russell (1997) that considers that uninsurability is mainly due to the lack of (tax) incentives for insurance companies to accumulate a pool of liquid assets to meet catastrophe losses.

¹⁵Hogg and Klugman (1983), Hogg and Klugman (1984) on grouped data, Embrechts, Klüppelberg and Mikosch (2013). Balkema and De Haan (1974) introduces the class of functions for which exceedances over a large threshold converge to the Generalized Pareto Distribution (GPD).

¹⁶The tail parameter is sufficient to characterize both the magnitude of the expected indemnity and the tail of the distribution: $\mathbb{E}^i [I(x^t)] = \frac{\hat{\alpha}_i^t}{\hat{\alpha}_i^t - 1}$.

contracts are solution of the simplified program 1.

$$\begin{cases} \max_{I(\alpha^t), \Pi} \mathbb{E} [U(w - x(\alpha^t) + I(\alpha^t) - \Pi_i [I(\alpha^t)])] \\ \text{subject to } \Pi_i(\alpha^t) = \mathbb{E} [I(\alpha^t) + \gamma_i(I(\alpha^t))] \\ \text{and } 0 \leq I(\alpha^t) \leq x(\alpha^t) \end{cases} \quad (1)$$

where w is the IB initial wealth and γ_i each IC has his private cost policy. I make the usual assumption that $\gamma'_i(\cdot) \geq 0$, to reflect the monitoring and auditing effort when claims are processed.

In a hypothetical steady state, IB and IC would agree on an estimation of α , α^- and each IC would have his own estimates $\hat{\alpha}_i^-$ which aggregate to α^- . Climate change, combined with macroeconomic factors such as inflation¹⁷, impacts the tail parameter over time t with potentially disturbing tipping points, but the realization of α^t is never observable. IB do not internalize the effect of climate change on their estimates and keep a subjective tail parameter estimation of α^- . IC have Bayesian or Frequentist experts to estimate the tail parameter and calibrate their insurance contracts.

An insurance supervisor¹⁸ S , with financial stability mandate completes the model. S is a social planner which program is to maximize the welfare of the IB and IC, which is equivalent to maximizing the IB welfare¹⁹, solving the program 1. S can evaluate his own tail parameter $\hat{\alpha}_S^t$ and decide if he can improve the welfare by sharing his opinion with the IC.

3 A PIONEERS DETECTION METHOD

Two main assumptions from the model lead to the design of a new method for weighing expert estimates. First, estimations can never be compared to a realization of the tail parameter for weight calculation based on historical performance or forecast errors (Genest and Zidek, 1986; Stock and Watson, 2004). As a result, the tool must rely on cross-sectional and temporal comparisons of expert model outcomes. Second, the distributions exhibit fat

¹⁷In this paper, both climate and economic effects on risk are referred to as “climate change” for simplicity.

¹⁸Real life examples will be the International Association of Insurance Supervisors (IAIS)’s members: European Insurance and Occupational Pensions Authority (EIOPA), France’s Autorité de Contrôle Prudentiel et de Résolution (ACPR), Germany’s Bundesanstalt für Finanzdienstleistungsaufsicht (BAFIN), UK’s Prudential Regulation Authority (PRA), etc.

¹⁹This is a “benevolent supervisor assumption” which is equivalent to consider that S dislike leaving a rent to the IC or considering S maximizes IB and IC surplus when the IC is risk-neutral in a competitive insurance market.

tails, indicating that extreme events have low probability of occurrence but do happen. Experts learn from their claims²⁰, hence experts already exposed to extreme event at time t will have a faster learning rates compared with experts not yet exposed to tail events. This is a significant departure from the traditional literature on expert combination, where only a minority exhibit a bias (Min and Zellner, 1993). In a climate change environment, a subset of experts might accurately internalize the change before a majority converges toward this initial subset. The Pioneers Detection Method (PDM) aims to identify this subset of pioneers as early as possible to increase the learning of tail parameters.

Pioneers are experts who deviate from the majority opinion but towards which other experts' opinions converge over time, although experts do not cooperate²¹. This can also be thought of as implicit inter-temporal voting among experts to identify pioneers.

The main benefit of the PDM is that the supervisor recognizes the convergence as soon as it begins and does not wait for the differences in estimates to narrow; a convergence in direction is enough to trigger a shift in weights. This PDM could also be envisaged for other context where non-cooperating experts are learning on rare events. I introduce four steps for the PDM.

Step 1: identify trends

First, I smooth the time series to identify underlying trends in pioneers and followers. This can be achieved using moving averages or filtering techniques, depending on the specific phenomenon being studied.

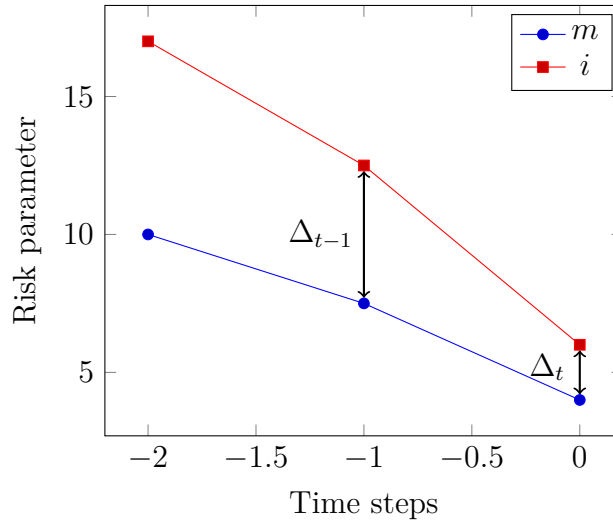
Step 2: distance reduction

Second, I determine if the distance between an expert's estimate and the average of the other experts' estimates has decreased between $t - 1$ and t . This is represented by a dummy variable, $\delta_{\text{distance}}^t$, which serves as a necessary but not sufficient condition for an expert to be considered a pioneer.

²⁰Equation 36 provides an expression of their posterior when experts estimations are Bayesian

²¹Liefferink and Wurzel (2017) introduce a clear distinction between pioneers as being 'ahead of the troops or the pack' and leaders which have 'the explicit aim of leading others, and, if necessary, to push others in a follower position'.

FIGURE 1
Distance reduction dummy to identify Pioneers



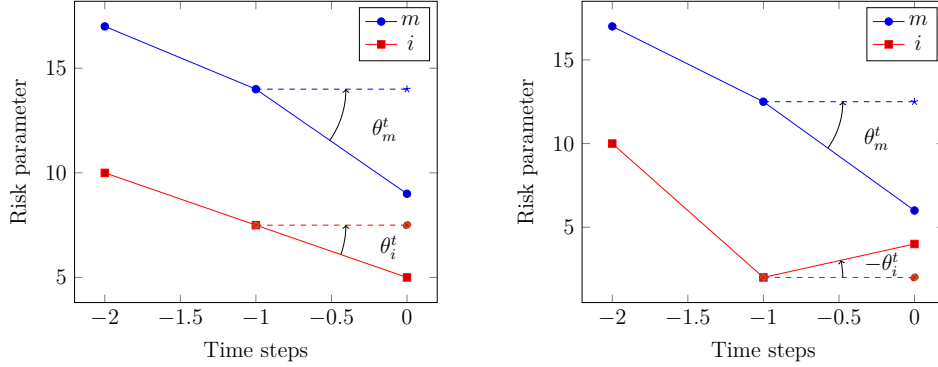
Notes: i represents the expert of interest and m the average estimate of his competitors (i excluded). $\delta_{\text{distance}}^t = \mathbb{1}_{\Delta_t < \Delta_{t-1}}$

Step 3: orientation change for convergence

Third, I identify if the orientation of the segments between an expert's estimate and the average of the other experts' estimates has decreased between $t-1$ and t . This is represented by a dummy variable, $\delta_{\text{orientation}}^t$. If the second step results in a positive dummy, this step helps determine whether the significant convergence of trends is due to the other experts agreeing²² with the expert (who is then considered a pioneer) or if the expert is the one converging toward the average of the other expert judgments (and is thus a follower). This distinction is made by attributing the change in direction primarily to either the expert or the average of his peers. If the change in direction is mainly due to the average of the expert's peers, then the expert is considered a pioneer.

²²Agreeing here means: Their estimates change direction over time towards the expert's estimate, and both time series converge (or are cointegrated).

FIGURE 2
Orientation change dummy to identify Pioneers



Notes: i represents the expert of interest and m the average estimate of his competitors (i excluded). The measure is $\delta_{\text{orientation}}^t = \mathbb{1}_{\theta_m^t > \theta_i^t}$. In both panels, i can be considered a pioneer. The right panel is interesting because i is not consistent, an aspect that is taken care of both in the smoothing of the time series and in the second characteristic of the measure.

Step 4: proportion of the convergence attributed to a Pioneer

Fourth, I calculate the proportion of the orientation decrease attributable to the average of the other estimates, m , with respect to the considered expert's estimate, i . The pioneer score for each expert, indexed by i , is then derived from a combination of the four steps, as shown in Equation 2 and is used as a weighing for his opinion in the final estimate of S : $\hat{\alpha}_S^t = \sum_i w_i^t \hat{\alpha}_i^t$.

$$w_i^t = \delta_{\text{distance}}^t \times \delta_{\text{orientation}}^t \times \frac{|\theta_m^t|}{|\theta_m^t| + |\theta_i^t|} \quad (2)$$

where m is the mean of all expert estimates, i excluded.

3.1 Pioneers Detection Method convergence properties

I show that the new supervision has some desired convergence properties.

Property 1. When Bayesian experts' claims are independent and identically distributed (iid), as $m \rightarrow \infty$, the PDM converges to the mean of the experts' estimations.

Writting S 's subjective opinion o_t , it is constructed with the PDM assigning a vector w_t to the experts judgment \hat{A}_t , $o_t = \hat{A}_t w_t$. As each expert posterior of α follows a Gamma distribution with all the same t and α , then as demonstrated in Mathai (1982), an omniscient expert opinion will follow a Gamma distribution with mt as a shape parameter, hence the mean of the experts' opinions. As $m \rightarrow \infty$, the Bayes omniscient expert opinion will converge

to the true value of α , [de Zea Bermudez and Turkman \(2003\)](#) find this Bayesian approach robust and having lower variance than Maximum Likelihood Estimation. Then as $m \rightarrow \infty$, it becomes impossible for any expert to lead the omniscient expert, which is also the average of all other experts, hence the PDM applies a weight of 1 to the omniscient expert and 0 on all experts, which is equivalent to assigning a weight $\frac{1}{m}$ to each expert’s opinion.

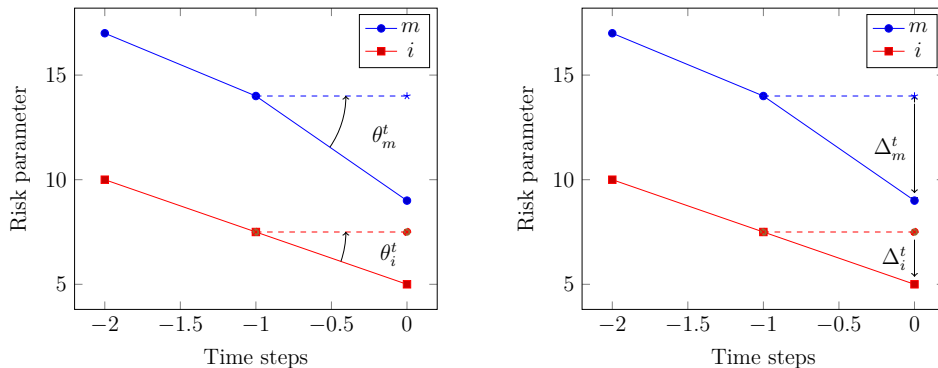
Property 2. When Bayesian experts’ claims are iid, as $t \rightarrow \infty$, the PDM converges to the mean of the experts’ estimations.

For an expert, his posterior’s rate parameter is an exponential autoregressive process as defined in [Gaver and Lewis \(1980\)](#), with a unit root: $r_i^{t+1} = r_i^t + \epsilon_i^t$ with $\epsilon_i \sim \text{Exp}(\alpha), \forall i$. As such, $\text{cov}(r_i^{t+1}, r_i^t) = \frac{1}{\alpha^2}t$ hence as $t \rightarrow \infty$ the covariance between two experts estimates cannot be distinguished as leading any other and all experts will be treated equivalently by the PDM, hence $w_i^t \xrightarrow[t \rightarrow \infty]{} \frac{1}{m}$.

3.2 Discussion and alternative novel approaches

The last two steps of the PDM rely on angles characterizing the direction change between the previous estimate and the latest. These last two steps could be envisaged with a distance ratio as illustrated in the right panel [Figure 3](#). The main difference between the two approaches lies in the speed of convergence which is only taken into account when weights are defined with angles as detailed in [Appendix B](#). This weighing with distance is found to be non-robust to non-linear transformation of estimates, [Table 2, 3 and 4](#).

FIGURE 3
Alternative to the orientation change dummy to identify Pioneers



Notes: i represents the expert of interest and m the average estimate of his competitors (i excluded). The weight in the left panel is $\frac{|\theta_m^t|}{|\theta_m^t| + |\theta_i^t|}$. The weight in the right panel is $\frac{|\Delta_m^t|}{|\Delta_m^t| + |\Delta_i^t|}$

Alternative inter-temporal pioneers detection methods could also be implemented with

more traditional time series methods. I list the five candidate methods and demonstrate how it boils down to implementing the Granger Causality, lagged-correlation and probabilistic combinations methods as alternatives to the PDM.

3.2.1 Granger Causality

The core principle of the PDM entails assigning indirect votes based on expert estimates. While experts are non-cooperative and do not exert influence on one another, this approach exhibits certain similarities with the identification of Granger Causality, which tests whether a given time series is beneficial in predicting another. It examines whether an expert's opinion change appears to precede a similar opinion change from his competitors. I implement the test introduced by [Granger \(1969\)](#). [Toda and Yamamoto \(1995\)](#) take into account potential integration and cointegration between time series. [Hasbrouck \(1995\)](#) also introduces cointegration to determine the information share of each random variable. In the context of climate change and small time series (maximum lag of 2), the integration or cointegration orders cannot be tested robustly. Therefore, I assume that the time series are locally stationary and apply a Granger Causality test with a lag of one.

3.2.2 Lagged Correlation

An alternative approach involves measuring correlations between lagged estimates from each expert and the estimates from his competitors. This can be measured using the Pearson coefficient ([Pearson, 1895](#)), as in [Sakurai, Papadimitriou and Faloutsos \(2005\)](#) and [Forbes and Rigobon \(2002\)](#) for financial applications.

3.2.3 Probabilistic combinations

When estimates are not points but probabilistic distributions, on top of the above methods, two additional methods are considered. The first is the Bayesian Model Averaging (BMA, [Draper \(1995\)](#)) which presents three challenges ([Wang et al., 2022](#)), a blocking one for my approach is to elicit a prior which comes back to doing a combination choice. Quantile combinations ([Vincent, 1912](#)) present the advantage to keep location-scale family after the transformation. I apply the vincentization as a candidate in a probabilistic set up, results are available from the author.

3.2.4 Multivariate Linear Regressions

A related approach to the Granger causality test involves using multivariate linear regressions of each expert’s estimates on the average estimate from his competitors, as in [Yi et al. \(2000\)](#). If significant, the coefficients can be considered as voting weights for each expert. This approach with limited history returns to searching for correlation and Granger causality between time series.

3.2.5 Information Transfer

I also explore the information transfer literature and measures ([Schreiber, 2000](#)). The idea is to measure whether the time series act as if there were transfer entropy, that is, information transport from one series to another. For financial applications with non-Gaussian random variables, this approach requires discretizing continuous time series with bins. [Dimpfl and Peter \(2014\)](#) limit their analysis to three bins and divide the return data along the 5% and 95% quantiles, as they “assume that extreme (tail) events are more informative than the median observation.”

The information transfer method tests whether an expert’s opinion change appears to be informative to his competitors. With non-cooperative experts, no information is exchanged, and therefore, if an apparent information transfer is detected, it is as if one expert learnt from the Data Generating Process (DGP) before his competitors, and then the DGP informs the competitors, which is similar to the expert directly informing his competitors if the expert learns faster. [Barnett, Barrett and Seth \(2009\)](#) demonstrate that this method is similar to Granger causality if the random variables are Gaussian. When experts are Bayesian, the posteriors are the random variables of interest and can be approximated as Gaussian, hence I implement the Granger causality method.

4 TESTING AND VALIDATING THE PIONEERS DETECTION METHOD

I test and validate the PDM against alternative opinion pooling methods using the Root Mean Square Error (RMSE), employing a known fat-tailed distribution with Monte Carlo simulations.

4.1 Comparison the Pioneers Detection Method with Existing Combination Methods

I compare the PDM with existing combination forecast methods. Following the work of [Stock and Watson \(2004\)](#), which relies on the dataset from [Bessler and Brandt \(1981\)](#), all methods are evaluated using their RMSE relative to the autoregressive model. As with the original paper, I find that combination forecasts improve upon autoregressive forecasts (Table 1). The PDM performs best, although with only three forecasters, there are some situations where no expert can be identified as a pioneer (e.g., when all three experts are diverging from one another). This is a limitation in the application of this method for combining forecasts. Note that the three novel methods introduced—Granger Causality, Lagged Correlation, and Pioneers—do not rely on past performance and do not include the actual true time series to be estimated.

4.2 Validation of the Pioneers Detection Method with Fragmented Insurance Market Shares

I evaluate the PDM on a fragmented insurance market through the use of Monte Carlo simulations. By fixing a tail parameter, I can calculate the RMSE for each expert estimation. The unknown tail parameter represents a new value following a tipping point and I simulate that after this tipping point the climate has stabilized to an unobservable state. I benchmark the performance of the PDM against existing opinion pooling techniques as well as other novel approaches introduced within this study.

As IC experts can be either Bayesian or Frequentist, I test two extreme configurations, a full Bayesian and a full Frequentist.

4.2.1 A full Bayesian configuration

I model Bayesian insurance experts who learn losses generated from a Pareto type I distribution with a tail parameter close to unity, meaning a fat tail environment. Each expert's observations are independent from one another both in cross-section and over time. In such an environment, experts will observe extreme values over time, but might have to wait for some time before observing an extreme event and being able to refine their calibration of the tail parameter. The PDM starts with at least one set of past data points, and it weights pioneers as soon as a change of direction to their estimates occurs. The supervisor needs at least one past period of estimations to form his own subjective opinion. Once the estimation

can be formed, the PDM outperforms the other opinion pooling methods (Table 2). This performance is robust to scaling (Table 3) and non-linear transformations such as logarithmic (Table 4), the method is not scale but ordinaly invariant. The performance of the new method is significant in early stages, especially at the first period where an estimate can be formed ($t = 2$). The performance is also found to be more stable over time, using mean, median, and the standard deviation over the first ten periods (Table 5).

I test the new method’s capacity to identify linear relationships between time series (Section Appendix C). As expected from the literature, in a Gaussian context, the linear opinion pooling performs best, and the new PDM does not improve performance, but its performance converges in the long term with the linear method (Table 6).

Table 7 reports robustness checks for the new supervision tool where the tail parameter α and the number of Bayesian experts are varied. The PDM minimizes the RMSE for all configurations.

4.2.2 Frequentist experts

I follow the same philosophy as section 4.2.1 except that I now consider Frequentist insurance experts. When I assume that all experts are Frequentist, Table 8 reports robustness checks for the new supervision tool where the tail parameter α and the number of Frequentist experts are varied. The PDM minimizes the RMSE for configurations with the least expert counts.

I therefore recommend using the PDM in a Bayesian context, but this conclusion is more nuanced when some experts can be considered as Frequentist.

5 POLICY RECOMMENDATIONS

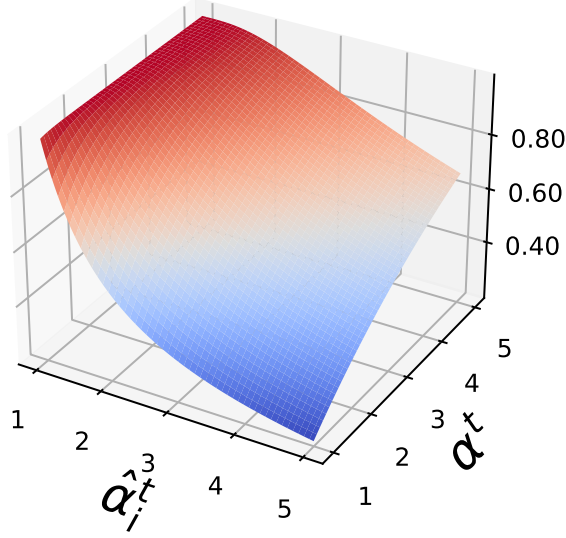
5.1 Insurability testing when the IB has a reserve premium

An asset is uninsurable if the optimal premium is above what the IB can afford to pay for the coverage. I model a simple situation where the IB has a reservation premium²³ Π^* that for simplicity is constant²⁴ over the observation period. S tests the insurability hypothesis, that

²³The reservation premium is dependent on the IB outside option and ability to pay premiums based on their income and asset value.

²⁴This inelasticity can be interpreted as buyers having limited capacity to estimate climate change impacts on losses and the reservation premium representing their ability to pay premiums based on their income and asset value. In practice this reservation premium would need to be corrected at least with inflation and wage changes.

FIGURE 4
IC profit probability as a function of his estimate and the tail parameter



Source: author's computation.

in for this test is simplified leaving out the IC cost function ($H_S^t : \Pi^t = \mathbb{E}[I(\alpha^t)] = \frac{\hat{\alpha}_i^t}{\hat{\alpha}_i^t - 1} \leq \Pi^*$), becomes ($H_S^t : \alpha^t > \alpha^*$, if $\alpha^* > 1$).

When the insurability condition is met, then the probability for an IC i to actually make a profit is given by a Pareto cdf

$$z(\hat{\alpha}_i^t, \alpha) := P\left(\text{loss} \leq \frac{\hat{\alpha}_i^t}{\hat{\alpha}_i^t - 1}\right) = 1 - \left(\frac{\hat{\alpha}_i^t}{\hat{\alpha}_i^t - 1}\right)^{-\alpha}$$

$$\frac{\partial z}{\partial \alpha} = (1 - z) \ln\left(\frac{\hat{\alpha}_i^t - 1}{\hat{\alpha}_i^t}\right) > 0, \forall \alpha > 1, \hat{\alpha}_i^t > 1$$

$$\frac{\partial z}{\partial \hat{\alpha}_i^t} = -\frac{\alpha}{(\hat{\alpha}_i^t)^2} \left(\frac{\hat{\alpha}_i^t - 1}{\hat{\alpha}_i^t}\right)^{\alpha-1} < 0, \forall \alpha > 1, \hat{\alpha}_i^t > 1$$

z is increasing in α and decreasing in $\hat{\alpha}_i^t$, illustrated Figure 4. When $\hat{\alpha}_i^t$ decreases the profit likelihood of IC increases as the IB is willing to pay more, hence an expert would want to set his offered premium based on the IB reserve α^* . As α increases, the likelihood of losses decreases so the expected profit of insurance companies increases.

5.1.1 Case of interest: reserve premium above the true insurable risk

When $1 < \alpha^* < \alpha^t$, this means that the IB reserve premium is above the true cost of the risk exposure and the asset is certainly insurable. When the expert estimate respects $\alpha^* \leq \hat{\alpha}_i^t$, then the expert advise to offer protection and the insurance company is likely to make profit. When at time t the expert estimation is such that $\hat{\alpha}_i^t < \alpha^*$ then she advise her IC to exit the market, which is a missed opportunity for her company. Hence, the situation of interest for S is when $\hat{\alpha}_i^1 < \alpha^* < \alpha$, meaning that after the first year, one expert i wants to exit the insurance market and hence stop learning. [Appendix D](#) provides the stopping condition for S based on the risk estimate and time spent. As there is no closed form formula for the PDM, I adopt a welfare consideration framework.

5.2 Climate change, welfare and supervision actions

The previous section explored how S can test the insurability hypothesis in a Bayesian context. I next explore S 's possible actions and evaluate their welfare implications.

When S believes an IC is optimist ($\hat{\alpha}_i^t > \hat{\alpha}_S^t$) and subject to ruin, the traditional approach is for S to ask the regulator to modify the IC solvency constraint. When S believes that an IC is pessimist and decide to reduce insurance coverage, I show that my approach with a PDM and a communication delivers the best welfare improvement among the actions considered. First, it would not be efficient for S to ask the regulator to modify the IC solvency constraint. In a model à la [Kleindorfer and Klein \(2003\)](#), the IC exposure is impacted by the decision on the solvency constraint, but in a context of climate change where risk and uncertainty increase, only a more stringent rule seems credible. Hence, in a configuration where IC are pessimist, an action from S on the solvency constraint cannot alleviate a reduction in insurance coverage. A second approach is to regulate the rate-of-return of IC. I adapt [Averch and Johnson \(1962\)](#) to this model and show [Appendix E](#) that such a regulation cannot meet both the IC participation constraint and the aim to reduce an IC pessimist bias. A third approach, following [Lee \(2017\)](#), is to impose a maximal margin on IC premium. I show [Appendix F](#) that the effect can be ambiguous. The fourth approach, put forward in this paper, is linked to the third and focuses on reducing the IC bias. As part of his supervision activities, S already collects internal model reporting from IC and thus has access to their individual estimates $\hat{\alpha}_i^t$. S observe the insurance market contracts and can decide 1) to exploit the estimates $\hat{\alpha}_i^t$ at a fix cost c_S (dedicate a team to this task on available data) or 2) to audit IC (e.g. with on-site ad-hoc inspections) to get detailed claim history at a cost

proportional to the information size. S functioning costs are embedded in the costs of the IC, as IC pay fees to the supervisor, those fees as passed on to the IB as part of the IC cost function. As the realization of α^t is never observable, S has to form ex ante his own estimate to decide whether it would be beneficial to engage the costs and influence expert estimations at the estimated costs. S can decide to pool information using the new tool and impact the premium if

$$\mathbb{E} \left[U \left(w - x \left(\alpha^t \right) + I \left(\alpha^t \right) - \left[\Pi \left(\hat{\alpha}_{S,tool}^- \right) + c_S \right] \right) \right] \geq \mathbb{E} \left[U \left(w - x \left(\alpha^t \right) + I \left(\alpha^t \right) - \Pi \left(\bar{\alpha}^t \right) \right) \right] \quad (3)$$

where $\bar{\alpha}^t$ is the aggregate of offered premium based on the IC estimates.

S can decide to acquire granular information for supervision at a higher cost with the fix cost of gathering a team plus a term linear in the size of the information to collect $c_S + \lambda \#x$. If S acquire this information, then it can estimate Π based on a more precise estimation of the tail parameter $\hat{\alpha}_{S,full}^-$, the precision depends on the risk itself (α^t) and on the size of the available sample. S can decide to invest in this audit if:

$$\mathbb{E} \left[U \left(w - x \left(\alpha^t \right) + I \left(\alpha^t \right) - \left[\Pi \left(\hat{\alpha}_{S,full}^- \right) + c_S + \lambda \#x \right] \right) \right] \geq \mathbb{E} \left[U \left(w - x \left(\alpha^t \right) + I \left(\alpha^t \right) - \left[\Pi \left(\hat{\alpha}_{S,tool}^- \right) + c_S \right] \right) \right] \quad (4)$$

In the decision equation 3 and 4, I make the implicit assumption that a supervisor announcement is fully trusted by an IC, hence I plug $\Pi \left(\hat{\alpha}_{S,tool}^- \right)$ and $\Pi \left(\hat{\alpha}_{S,full}^- \right)$. For completeness, once calibrated, S should integrate in the simulation how his announcement, $\hat{\alpha}_{S,tool}^-$, can be integrated by a Bayesian or Frequentist expert depending on his evaluation of his credibility and alignment with his objectives.

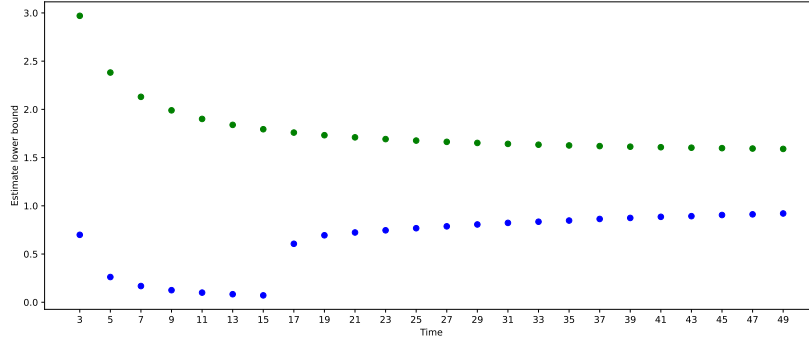
5.3 Bayesian versus Frequentist estimations and improvement with observations count

5.3.1 Bayesian experts

Following [Arnold and Press \(1989\)](#), each Bayesian insurance expert's natural conjugate prior family for the Pareto exponent is a Gamma distribution of shape s_i^t and rate r_i^t , detailed in [Appendix G](#). The posterior of the tail parameter of each expert is unimodal and roughly symmetric, as demonstrated by [Le Cam \(1953\)](#), and can be approximated by a normal distribution.

$$\pi \left(\alpha^t \mid \cup_t x_i^t \right) \sim \mathcal{N} \left(\tilde{\alpha}_i^t, \frac{[\tilde{\alpha}_i^t]^2}{s-1} \right) \quad (5)$$

FIGURE 5
Bayesian versus Frequentist lower confidence interval bound



Source: author's computation. In green the Bayesian lower bound evolution with the observations count and in blue the Frequentist. With $\alpha = 1.5$ and 10^5 Monte Carlo runs.

where $\tilde{\alpha}_i^t$ is the mode of the Gamma posterior.

5.3.2 Frequentist experts

Frequentist experts will use maximum likelihood estimation. Zajdenweber (1996) demonstrates that above a unit threshold, the tail parameter can be estimated²⁵ with t claims as

$$\frac{1}{\hat{\alpha}_i} = \frac{1}{t} \sum_{k=1}^t \log x_i^k \quad (6)$$

As x_i^1, \dots, x_i^t are independent and identically Pareto distributed, then $\ln(x_i^k)$ are independent and identically (exponentially) distributed, as demonstrated in (Rytgaard, 1990),

$$\sqrt{t}(\hat{\alpha}_i - \alpha) \rightarrow \mathcal{N}(0, \alpha^2) \quad (7)$$

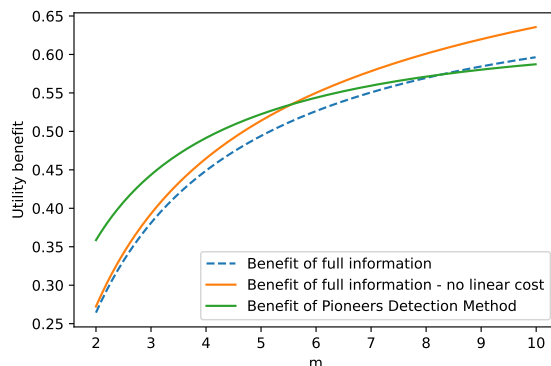
Therefore, asymptotically the Frequentist and Bayesian methods estimate follow the same normal distribution. Nevertheless, this is not true with few yearly observations as in the insurance market model, therefore I compare the bias that each approach can introduce in the estimation of the tail parameter.

5.3.3 Comparison of Bayesian and Frequentist expert approaches

To compare Bayesian with Frequentist approaches, I run Monte Carlo simulations with the same tail parameter at 1.5 and observation counts from a minimum of 3 to 50. I use the

²⁵This is called the Hill estimator, (Hill, 1975).

FIGURE 6
Cost versus utility benefit of S actions



Source: author's computation.

asymptotic distribution of the estimate to compute the 95% confidence interval lower bound, $\hat{\alpha} - \Phi(.025)\frac{\hat{\alpha}^2}{t}$ where $\Phi(.)$ is the percentage point function of the normal distribution. Figure 5, the frequentist lower bound evolution is non linear. When estimating the tail parameter, the lowest observation is taken as the Pareto threshold and when the sample size is small, this initially limits the capacity to improve the tail estimate. As the sample size grows, the threshold is more likely to be close to the actual Pareto threshold and the likelihood to observe tail events increases, hence the tail parameter estimation improves with the sample size. Depending on the IC approach, Bayesian or Frequentist, in the model the IC can be either optimist or pessimist. S can elicit the IC method to determine the most likely sign of the bias to evaluate the opportunity to intervene.

5.4 Policy recommendation to use the Pioneers Detection Method

The recommendation to use the PDM will depend on the insurance market configuration and the evolution of the tail parameter. I use a logarithm utility as in Mossin (1968) and apply the Delta method to estimate the gain or loss in utility²⁶ at the 95% lower confidence interval bound. The loss of utility is proportional to the variance of the estimate of α , I normalize the variance with the Bayesian estimate variance in the case of a market with a unique expert, then I can express the variance for the fully informed S as $\frac{1}{mt}$.

As there is no closed form for the variance of the estimate outcome with the PDM, I apply Monte Carlo simulations with a tail parameter $\alpha = 1.5$ and vary the number of IC,

²⁶It simplifies to applying the Delta method to $Var\left(\log\left[\text{constant} + \frac{\alpha}{\alpha-1}\right]\right)$

m , and find a strong linear relationship between m and the ratio of the estimate standard deviations, $\frac{\sigma_{\hat{\alpha}}^{\text{tool}}}{\sigma_{\hat{\alpha}}^{\text{full}}} = a + bm$. Figure 6 illustrates the utility improvement versus the cost S has to spend. Figure 6, I display the normalized benefit of full information without the linear cost and find that S would never find it beneficial to spend the effort to collect the full information set from IC using on-site inspection when there are less than five IC. When S incurs a linear cost to collect granular information, this threshold shifts to the right, I illustrate this with a dashed line.

With my model and set up, in a configuration with more than five ICs, it depends on the cost to collect granular information whether it is welfare improving for S to collect the full set of information to reduce the uncertainty in the premium estimation. In a configuration with less than five ICs, it is always more beneficial for S to use the PDM, regardless of the cost function, to improve the welfare.

6 CONCLUSION

This paper contributes to the literature on opinion pooling, climate change impacts on losses, insurability, and insurance supervision in the context of climate change. By introducing a Pioneers Detection Method that pools expertise under radical uncertainty, the paper offers a practical approach to assess the insurability of asset classes and enhance insurance market supervision amid the challenges of climate change to anticipate changes in risk parameters. It tests the policy recommendation for insurance supervisors to focus on-site inspections on uncertainty in the insurance market and stabilize coverage. In the event of a sudden shock or panic in the insurance market for a given asset class, a supervisor must determine as quickly as possible whether the asset class is insurable to fulfill their mandate. The new tool identifies experts who are quick at understanding the implications of climate change to tail losses. I demonstrate that, compared to traditional opinion pooling methods, the Pioneers Detection Method is the fastest and has an advantage when there are limited observations available in the context of extreme events. By making forward-looking announcements and influencing expert opinions, supervisors can reduce uncertainty and help avoid crises and equilibrium shifts in insurance supply.

In terms of policy implications, a supervisor can use the Pioneers Detection Method to monitor the insurability of asset classes and advise a regulator to design appropriate public or private insurance schemes. Additionally, it promotes transparency and collaboration within the insurance sector, contributing to a more resilient market in the face of climate change.

Further research into the behavior of insurance experts should follow, especially to test if they have a tendency for optimism or pessimism bias due to the lack of extreme observations in their private datasets or due to modelling choices. I recommend setting up experiments to determine how actuaries would behave in this context, particularly to test if their approaches are Bayesian or Frequentist. The paper also highlights the need for further research in the area of insurance supervision and climate change, exploring topics such as the endogeneity of market shares, demand sensitivity to premiums, and ruin gambles in the context of a changing climate. Future studies could investigate the effectiveness of the proposed tool in different regulatory environments and under varying degrees of climate uncertainty, refining the tool and providing valuable insights for insurance supervisors and regulators worldwide.

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TABLE 1
New supervision tool compared with combination methods

	RMSE	Constant	Econometric	ARIMA	Expert
Expert	1.35				
Econometric	1.22				
Method C out-sample	1.00	-22.21	0.93	0.22	0.31
AR	1.00				
Method A out-sample	0.79	0	-1.11	4.63	-2.62
Method B out-sample	0.69	0	-1.01	4.63	0.28
ARIMA	0.64				
Median	0.64				
Minimum MSE adaptive	0.61		0.32	0.41	0.27
Discounted MSFE	0.57		0.19	0.46	0.35
Minimum MSE adaptive, expert excluded	0.57		0.41	0.59	
Correlation	0.56		0.30	0.35	0.36
Two forecast Simple average	0.55		0.50	0.50	
GC	0.55		0.50	0.50	0.00
Minimum Variance, expert excluded	0.55		0.48	0.52	
Three forecast Simple average	0.52		0.33	0.33	0.33
Minimum Variance	0.48		0.53	0.25	0.22
Method B	0.47	0	0.54	0.25	0.22
Method A	0.47	0	0.53	0.25	0.22
Method C	0.44	15.19	0.10	0.40	0.18
Pioneers	0.42		0.70	0.17	0.12

NOTE.—The data set is taken from [Bessler and Brandt \(1981\)](#). The reported weights are the average when they are varying over time for the method. Simple averages, Minimum MSE adaptive and Minimum Variance are implemented as in [Bessler and Brandt \(1981\)](#). Methods A, B and C are from [Granger and Ramanathan \(1984\)](#), weights were computed on past performances for the out-sample. Discounted MSFE is the first method in [Bates and Granger \(1969\)](#). Principal Component forecast combination was not added as this is mainly to tackle situations with a large number of forecast. I add three novel methods, Granger Causality (GC), (lagged) Correlation and the Pioneers Detection Method.

TABLE 2

New supervision tool validation - full Bayesian

	Pioneers	Linear	Median	Pioneers distance-weighted	Correlation	GC
2	1.00	4.05	1.67	1.56	13.61	4.63
3	1.00	3.80	1.46	1.53	104.72	4.01
4	1.00	2.09	1.30	1.25	39.53	2.39
5	1.00	1.75	1.26	1.19	20.17	2.02
6	1.00	1.61	1.24	1.15	3.66	1.92
7	1.00	1.53	1.23	1.13	11.60	1.86
8	1.00	1.48	1.23	1.11	2.84	1.82
9	1.00	1.44	1.23	1.10	2.47	1.79

NOTE.—The tail parameter is taken from a Pareto type one distribution with $\alpha = 1.5$. Five non-cooperative bayesian experts are modeled with independent observations from the loss distribution. 10^5 Monte Carlo simulations are run. The new Pioneers Detection Method outperform established opinion pooling methods (linear and median) as well as alternative methods introduced in this paper: pioneer with weight based on distance rather than angle, lagged correlation and Granger Causality.

TABLE 3

New supervision tool validation, robustness to scaling - full Bayesian

	Pioneers	Linear	Median	Pioneers distance-weighted	Correlation	GC
2	1.00	2.62	1.34	1.29	7.89	2.88
3	1.00	2.33	1.22	1.25	54.36	2.34
4	1.00	1.48	1.14	1.12	17.42	1.50
5	1.00	1.31	1.12	1.09	7.63	1.32
6	1.00	1.24	1.11	1.08	1.64	1.26
7	1.00	1.20	1.10	1.07	3.55	1.22
8	1.00	1.18	1.10	1.06	1.34	1.19
9	1.00	1.17	1.09	1.06	1.26	1.18

NOTE.—The tail parameter is taken from a Pareto type one distribution with $\alpha = 1.5$. Five non-cooperative Bayesian experts are modeled with independent observations from the loss distribution. 10^5 Monte Carlo simulations are run. The estimates have been scaled by 100.

TABLE 4

New supervision tool validation, robustness to non-linear transformation - full Bayesian

	Pioneers	Linear	Median	Pioneers distance-weighted	Correlation	GC
2	1.00	1.68	1.49	1.34	5.38	1.88
3	1.00	1.16	1.05	0.94	20.32	1.38
4	1.00	0.74	0.80	0.80	390.63	1.05
5	1.00	0.60	0.73	0.79	12.75	0.90
6	1.00	0.59	0.72	0.80	3.47	0.82
7	1.00	0.61	0.72	0.81	8.30	0.79
8	1.00	0.63	0.74	0.82	2.22	0.77
9	1.00	0.65	0.74	0.84	1.85	0.77

NOTE.—The tail parameter is taken from a Pareto type one distribution with $\alpha = 1.5$. Five non-cooperative Bayesian experts are modeled with independent observations from the loss distribution. 10^5 Monte Carlo simulations are run. The estimates have been transformed with a logarithm $\log(c + \alpha)$ with $c = 2$ a constant to avoid issues when $\hat{\alpha}$ are close to 0.

TABLE 5

New supervision tool validation - full Bayesian						
	Pioneers	Linear	Median	Pioneers distance-weighted	Correlation	GC
mean	1.00	2.74	1.40	1.34	33.65	3.10
median	1.00	1.69	1.25	1.17	15.53	1.98
std	1.00	5.06	1.77	1.73	96.71	5.60

NOTE.—The tail parameter α is fixed at 1.5 and a Monte Carlo simulation is run with 10^5 runs. Five non-cooperative Bayesian experts are modeled with independent observations from the loss distribution. The mean, median and standard deviation of the Root Mean Square Errors are reported over the first 10 estimation period.

TABLE 6

New supervision tool validation, relevance in a Gaussian context						
	Pioneers	Linear	Median	Pioneers distance-weighted	Correlation	GC
2	1.00	0.58	0.70	0.79	2.38	0.63
3	1.00	0.74	0.88	0.91	13.43	0.87
4	1.00	0.65	0.77	0.89	2.94	0.71
5	1.00	0.73	0.87	0.98	0.82	0.80
6	1.00	0.80	0.96	1.04	0.86	0.89
7	1.00	0.87	1.04	1.09	0.93	0.96
8	1.00	0.91	1.09	1.12	1.02	1.01
9	1.00	0.95	1.14	1.14	1.01	1.07

NOTE.—The loss samples are taken from a standard normal law. Five non-cooperative Bayesian experts are modeled with independent observations from the loss distribution. 10^5 Monte Carlo simulations are run.

TABLE 7

New supervision tool robustness checks - full Bayesian					
α	experts	Linear RMSE	Median RMSE	Pioneers RMSE	
1.1	2	3.29	3.29	1.00	
1.1	5	3.51	1.54	1.00	
1.1	20	3.25	1.26	1.00	
2.0	2	3.19	3.19	1.00	
2.0	5	3.70	1.56	1.00	
2.0	20	3.73	1.30	1.00	
3.0	2	3.03	3.03	1.00	
3.0	5	3.57	1.41	1.00	
3.0	20	4.52	1.34	1.00	

NOTE.—The tail parameter α and the number of Bayesian experts are varied. The last three columns report the average Root Mean Square Errors for the competing opinion pooling tools over the three initial estimation periods.

TABLE 8
New supervision tool robustness checks - full Frequentist

α	experts	Linear RMSE	Median RMSE	Pioneers RMSE
1.1	2	2.32	2.32	1.00
1.1	5	1.48	0.85	1.00
1.1	20	0.99	0.21	1.00
2.0	2	2.60	2.60	1.00
2.0	5	1.62	0.73	1.00
2.0	20	0.91	0.11	1.00
3.0	2	2.88	2.88	1.00
3.0	5	2.01	0.72	1.00
3.0	20	1.23	0.06	1.00

NOTE.—The tail parameter α and the number of Frequentist experts are varied. The last three columns report the average Root Mean Square Errors for the competing opinion pooling tools over the three initial estimation periods.

TABLE 9
New supervision tool validation

	x	y	z
Pioneers	0.60	0.22	0.19
Correlation	0.46	0.48	0.06
GC	0.62	0.38	0.00

NOTE.—The time series are random sample of length 10. $b = d = e = .9$ and $a = c = .1$. 10^5 Monte Carlo simulations are run and the average of each weights are reported. The weights per method are expected to be ranked such that on average $w_x > w_y > w_z$.

TABLE 10
New supervision tool validation

	x	y	z
Pioneers	0.31	0.21	0.48
Correlation	1.28	0.09	-0.37
GC	0.50	0.44	0.06

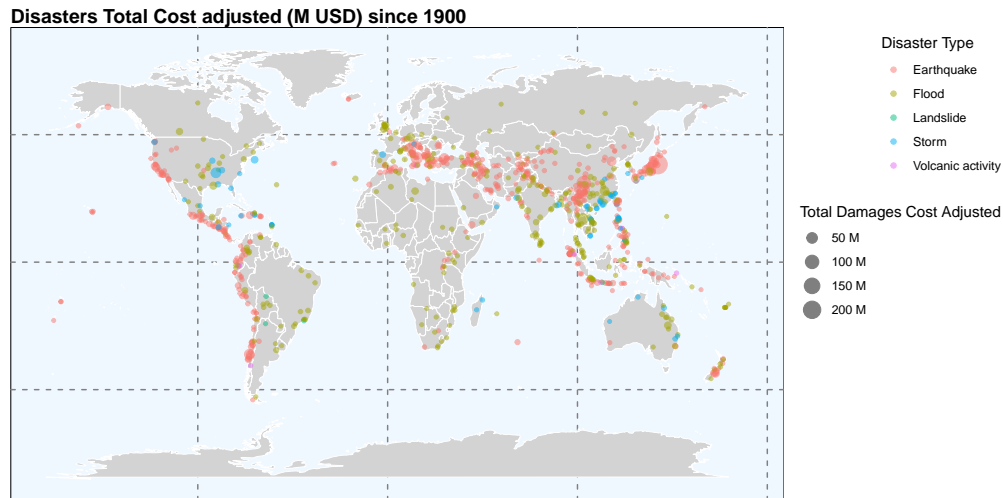
NOTE.—The time series are random sample of length 10. $b = d = e = .1$ and $a = c = .9$. 10^5 Monte Carlo simulations are run and the average of each weights are reported. The weights per method are expected to be ranked such that on average $w_x > w_y > w_z$.

I APPENDIX

Appendix A Climate change impact on losses distribution

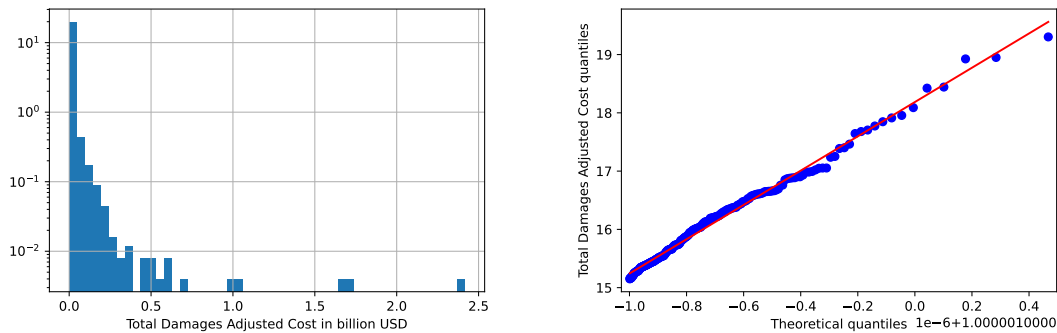
I play the role of an insurance expert using a public data set, the Emergency Events Database (EM-DAT) ²⁷. Figure A.1 displays the information with total damage adjusted cost for each disaster from 1990 to 2021.

FIGURE A.1
Disaster Total Cost Adjusted (M USD) - map



Source: EM-DAT, CRED / UCLouvain, Brussels, Belgium – www.emdat.be (D. Guha-Sapir). Total damages in adjusted billion USD.

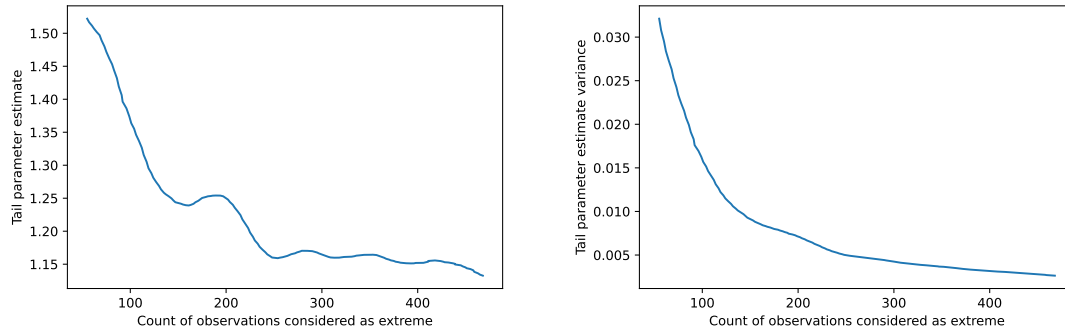
FIGURE A.2
Disaster Total Cost Adjusted (M USD) - histogram and Q-Q plot



Source: EM-DAT, CRED / UCLouvain, Brussels, Belgium – www.emdat.be (D. Guha-Sapir). Total damages in adjusted billion USD. Author's computation. The quantile-quantile plot is done against a Pareto type 1 distribution.

²⁷EM-DAT: The Emergency Events Database - Université catholique de Louvain (UCL) - CRED, D. Guha-Sapir - www.emdat.be, Brussels, Belgium.

FIGURE A.3
Heavy-tail shape parameter: the observation count trade-off



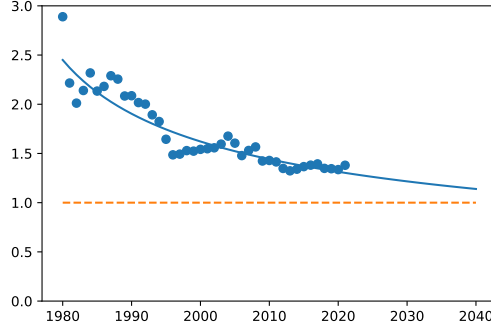
Source: Author’s computation on EM-DAT.

To simulate a learning from an evolving environment, I apply a 80-year rolling-window estimation exercise without the possibility to change the model calibration methods as the window was rolled. Hence, I design a model and calibration approaches that I apply to each 41 year from 1980 to 2021 on the natural disaster losses. Figure A.2 left panel, the distribution of total damaged adjusted costs displays heavy-tail property, leading to use EVT to model the distributions. Hence, it is current practice for insurance experts to test extreme events quantiles against parametric heavy tail distributions quantiles. Figure A.2 right panel displays a quantile-quantile plot of the 95-th percentile of the damages against a Pareto type I distribution, which can be accepted or rejected depending on the expert experience for some more elaborated distributions. There is no one-size-fit-all approach in EVT to define a threshold above which values can be considered as part of a tail²⁸, I face the trade-off each expert is facing when dealing with fat tailed loss distributions. Figure A.3 illustrates for a simple Pareto type I law the trade off between limiting the fit on ”relatively” few observations and the increase variance in the parameter estimates as the observations left in the sample decrease. As the observations count kept to fit a Pareto distribution increases, the value of the parameter estimated decreases toward one, which is not an indication of increased risk but rather of including observations that do not belong to the tail. As the observations increase, the parameter estimate variance improves. Hence, each expert faces a trade-off between limiting the sample to extremes and improving his estimations variance with more observations.

I report Figure A.4 the time series of estimated tail parameters (α^t). I also project Figure A.4 how the Pareto exponent estimates of climate change damages decrease over time for the upcoming 20 years with a linear fit post Box-Cox transformation. Table 1 two linear regressions, pre (1) and post (2) Box-Cox transformation, of the Pareto coefficient over time are applied and a significant downward trend indicate that threshold of 1 could be reached within the next century. Therefore, my model and this data set leads me to conclude on an increase in the insurance losses over time attributed to climate change up to becoming uninsurable for the considered natural catastrophes (unbounded expectation).

²⁸Rolski et al. (1999) demonstrated the difficulty to verify the heavy-tail property statistically

FIGURE A.4
Pareto estimates over time



Source: Author's experiment and computation.

TABLE 1
Pareto exponent evaluation

	(1)	(2)
Intercept	2.269*** (0.052)	0.386*** (0.006)
t	-0.027*** (0.002)	-0.005*** (0.000)
Observations	42	42
R^2	0.794	0.888
Adjusted R^2	0.789	0.885
Residual Std. Error	0.172(df = 40)	0.020(df = 40)
F Statistic	154.184*** (df = 1.0; 40.0)	318.005*** (df = 1.0; 40.0)

Note: *p<0.1; **p<0.05; ***p<0.01
NOTE.—The first column is estimated over α and the second after a Box-Cox transformation.

Appendix B Weighing convergence with distances or angles

In the last two steps of the PDM, the weights can be defined with angles or distances. Angles are the preferred approach because they allow the supervisor to take into account the speed of convergence between time series. If θ is the angle between vector \vec{u} and \vec{v} , it can be computed as

$$\begin{aligned} \theta &= \cos^{-1} \left(\frac{u_x v_x + u_y v_y}{\sqrt{u_x^2 + u_y^2} \sqrt{v_x^2 + v_y^2}} \right) \\ &= \cos^{-1} \left(\frac{s^2 + u_y v_y}{\sqrt{s^2 + u_y^2} \sqrt{s^2 + v_y^2}} \right) \end{aligned} \tag{8}$$

the distance relevant to the measure are taken from the y-axis u_y and v_y and the weight is computed as $\frac{|v_y|}{|u_y|+|v_y|}$ which do not include the x-axis $u_x = v_x = s$ the step between two observation points.

Appendix C Pioneers Detection Method and linearly related time series

I test the Pioneers Detection Method introduced in this paper to identify linearly related time series.

I define three time series as the experts x , y and z in the spirit of (Granger and Newbold, 1974):

$$\begin{aligned}x_t &= x_{t-1} + \epsilon_t \\y_t &= ay_{t-1} + bx_{t-1} + \nu_t \\z_t &= cz_{t-1} + dy_{t-1} + ex_{t-1} + \xi_t\end{aligned}\tag{9}$$

with ϵ , ν and ξ white noises, I set $x_0 = y_0 = z_0 = 0$. The results are sensitive to the coefficients, I set auto-regressive coefficients significantly lower than cross linear relationships so the cross-relationships can be captured by the method. x is the main pioneer of the group as the innovations x faces are passed on to other time series as x has a unit root. On the contrary, y has an auto-regressive coefficient below unity and his innovations are transient. The main aim of this test is to confirm that all methods can identify the pioneership of x and measure to which extend the effect of y on z can be identified. Table 9 report the average weight each of the three novel Pioneers Detection Method assigns to each time series. Table 10 reports the detection capacity when the coefficient has a less dampening effect on noise.

Appendix D Stopping condition for S depending on the tail parameter and the time spent

The probability that at time t , $\hat{\alpha}_i^t > \alpha^*$ for an expert i is given by the cumulative distribution function (cdf) of a Gamma distribution, this is the probability that expert i exit the insurance market:

$$P\left(\hat{\alpha}_i^t = \frac{t}{\sum_t \ln x_i^t} \leq \alpha^*\right) = \frac{1}{\Gamma(t)} \gamma\left(t, \alpha^* \sum_t \ln x_i^t\right)\tag{10}$$

Taking the expectation

$$E\left[P\left(\hat{\alpha}_i^t \leq \alpha^*\right)\right] = \frac{1}{\Gamma(t)} \gamma\left(t, \frac{t\alpha^*}{\alpha}\right)\tag{11}$$

If at time T , $\hat{\alpha}_i^T < \alpha^*$ and α^* is outside of the acceptable confidence region (e.g. 95%) of the posterior density $p_i(\alpha^t|x)$, then there won't be any time t such that $t > T$ and α^* will move back inside the same confidence region criteria of the posterior density. So when this is the case, the only rational decision for S is to advise insurance experts to stop providing coverage. Now, looking at the claim level. Let's imagine that $\exists t, \hat{\alpha}_i^t = \frac{t}{\sum_t \ln x_i^t} < \alpha^*$, then it is worth covering the asset class if there is some non-zero probability that $\hat{\alpha}_i^{t+1} = \frac{t}{\sum_t \ln x_i^{t+1}} \geq \alpha^*$,

this condition is equivalent to

$$x_i^{t+1} \leq \exp \left[\frac{t+1}{\alpha^*} - \sum_t \ln x_i^t \right] = \tilde{x} \quad (12)$$

As with only one expert, S has only as much information as the expert, then both their believes are

$$P \left(x_i^{t+1} \leq \tilde{x} \mid \hat{\alpha}_i^t \leq \alpha^* \right) = 1 - (\tilde{x})^{-\hat{\alpha}_i^t} \leq 1 - (\tilde{x})^{-\alpha^*} \quad (13)$$

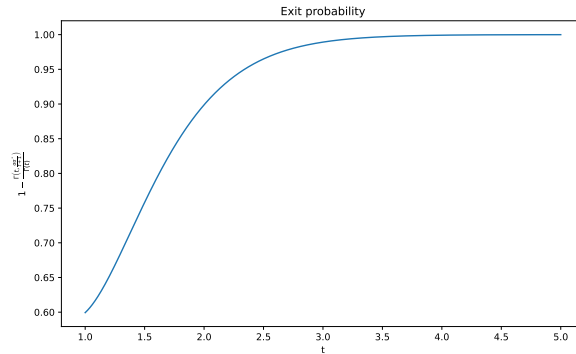
As there is a lower bound on x , the process stops whenever

$$\exp \left[\frac{t+1}{\alpha^*} - \sum_t \ln x_i^t \right] \leq 1 \quad (14)$$

that is whenever $\sum_t \ln x_i^t \geq \frac{t+1}{\alpha^*}$. As $\sum_t \ln x_i^t$ follows an inverted Gamma distribution and taking realist tail parameters, the process is likely to stop after 3 iterations, illustrated Figure A.5.

$$P \left(\sum_t \ln x_i^t > \frac{t+1}{\alpha^*} \right) = 1 - \frac{\Gamma \left(t, \frac{\alpha \alpha^*}{t+1} \right)}{\Gamma(t)} \quad (15)$$

FIGURE A.5
Stopping probability when there is a unique IC



Source: Author's computation, $\alpha = 1.5$ and $\alpha^* = 1.6$.

In a multiple IC configuration, when at least one expert estimate the risk to be higher than what policy holders are ready to pay ($\exists i, \hat{\alpha}_i^t < \alpha^*$), the decision maker S might want to intervene in case he has enough evidence to believe that the willingness to pay leave enough room for an insurance market to exists, that is even after the tipping point the true risk is insurable without any public intervention ($\alpha^* < \alpha$). This writes

$$P \left(\alpha^* \leq \hat{\alpha}_S^t \mid \hat{\alpha}_i^t \leq \alpha^* \right) \quad (16)$$

S can form $\hat{\alpha}_S^t$ with the Pioneers Detection Method introduced in this paper, but for which there is no closed form formula. Hence I test this method in a welfare consideration framework Section 5.2.

Appendix E Rate-of-return regulation for IC

I follow assumptions 2, 3 and 9 of [Laffont and Tirole \(1993\)](#), S can observe $\gamma_i(I(x^t))$, but need to invest to observe $I(x^t)$ hence the profit of the IC. The IC can refuse to provide insurance cover for x^t if its profit is strictly negative. The IC profit writes:

$$z(\Pi(x^t), I(x^t)) = \Pi(x^t) - I(x^t) - \gamma_i(I(x^t)) \quad (17)$$

I consider a regulator that can put some constraint on the rate of return of the IC, r . I consider that the acquisition of IB is at a cost c_1 of premium and normalize this cost to one and the program becomes for the IC:

$$\begin{cases} \max_{\Pi(I), I} \Pi(I) - I - \gamma_i(I) \\ \text{subject to } \Pi(I) - \gamma_i(I) - r\Pi(I) \leq 0 \end{cases} \quad (18)$$

The Lagrange function writes:

$$\Lambda = \Pi(I) - I - \gamma_i(I) - \lambda [\Pi(I) - \gamma_i(I) - r\Pi(I)] \quad (19)$$

The Kuhn-Tucker conditions are:

$$\Lambda_{\Pi} = 1 - \lambda(1 - r) \leq 0 \quad (20)$$

$$\Lambda_I = \Pi'(I) - 1 - \gamma'_i(I) - \lambda [\Pi'(I) - r\Pi'(I) - \gamma'_i(I)] \leq 0 \quad (21)$$

$$\Lambda_{\lambda} = \Pi(I) - \gamma_i(I) - r\Pi(I) \leq 0 \quad (22)$$

We end up with a constraint on the cost function

$$\gamma'_i(I) \geq \frac{1-r}{r} \quad (23)$$

Hence a condition on the premium becomes $\Pi[I(x)] \leq \left(I(x) + \frac{1-r}{r}I(x)\right) = \frac{1}{r}I(x)$ which is incompatible with the IC participation constraint as long as $r < 1$ and this regulatory approach cannot alleviate IC pessimist bias embedded in a premium.

Appendix F Imperfect information and uncertain probability following [Lee \(2017\)](#)

I follow [Lee \(2017\)](#), but rather consider that IB are optimist, meaning that they don't internalize the increased risk with climate change. I consider that IC are biased and pessimist, meaning that they internalize the increased risk with climate change and overestimate this risk. In this set up, I implicitly consider that the risk is still insurable.

The IB is subject to a loss x which used to have the probability of occurrence p . Now with climate change, it can take two values each with probability of occurring $\frac{1}{2}$, $p_1 = p$ and

$p_2 = p + \delta$, with $\delta \in (p, 1 - p)$.

The IC is pessimist and consider that the probability p_2 is B with $1 > B > \frac{1}{2}$. The IB is optimist as he doesn't internalize the increase in risk due to climate change and consider that the probability stayed p . The expected utility of the optimist IB depends on his wealth w , deductible D and premium Π , with the implicit assumption that the loss $x > D$ (in this simplified model, x is not a rv which also simplifies the expression of the premium equation 25):

$$\mathbb{E}W = pU(w - \Pi(D) - D) + (1 - p)U(w - \Pi(D)) \quad (24)$$

If I consider for simplicity that the cost is linear, then the premium for a risk-neutral insurer in perfect competition is set to cover the expected loss and administrative costs and profits, so λ is a constant.

$$\Pi(D) = \Phi(x - D) \quad (25)$$

where $\Phi = (1 + \lambda)(p + B\delta)$. Then the first-order condition to find the optimal deductible that maximizes the IB's expected utility yields:

$$\frac{\partial W}{\partial D} = -p(1 - \Phi)U'_1 + (1 - p)\Phi U'_2 = 0 \quad (26)$$

where in equation 26 subscript 1 denotes the state with the loss and 2 without the loss. The second-order condition for the maximization problem is

$$\frac{\partial^2 W}{\partial D^2} = A = p(1 - \Phi)^2 U''_1 + (1 - p)\Phi^2 U''_2 < 0 \quad (27)$$

I totally differentiate the first-order:

$$\frac{\partial^2 W}{\partial D \partial \Phi} = pU'_1 + (1 - p)U'_2 - p(1 - \Phi) \left[-(x - D) + (\Phi - 1) \frac{\partial D}{\partial \Phi} \right] U''_1 + (1 - p)\Phi \left[-(x - D) + \Phi \frac{\partial D}{\partial \Phi} \right] U''_2 \quad (28)$$

rearranging

$$\frac{\partial D^*}{\partial \Phi} = -\frac{1}{A} [pU'_1 + (1 - p)U'_2 - (1 - p)(x - D)\Phi U''_2 + p(1 - \Phi)(x - D)U''_1] \quad (29)$$

contrary to Lee (2017), the sign in equation 29 is ambiguous as the sign of $p(1 - \Phi)(x - D)U''_1$ is ambiguous, unless $\Phi > 1$ which is equivalent to saying that the premium loading λ must be above a minimum threshold such that $(1 + \lambda) > (p + B\delta)^{-1}$. If this is the case, then the utility-maximizing choice of deductible increases in the extent of actual risk and IC bias B .

Then the realized utility for the IB is

$$\bar{W} = (p + \delta)U_1 + (1 - p - \delta)U_2 \quad (30)$$

then the sign of the utility variation of the IB with the increase in the risk uncertainty is ambiguous:

$$\frac{\partial \bar{W}}{\partial \delta} = -\delta(1 + \lambda)B [(1 - \Phi)U'_1 + \phi U'_2] \frac{\partial D^*}{\partial \Phi} \quad (31)$$

indeed, if $\Phi > 1$ then the sign of $\frac{\partial D^*}{\partial \Phi}$ is unambiguous but then the sign inside the bracket is ambiguous. Under this circumstances, an increase in risk uncertainty is taken into account by the IC and the effect on the IB welfare can be positive and the need for regulation is not clear. The same can be said about an increase in IC pessimism, as long as the risk is deemed insurable.

Appendix G Bayesian prior and posterior distribution of the Pareto exponent

In this paper, I consider that the insurance deductible (the Pareto threshold) is known ²⁹. Following [Arnold and Press \(1989\)](#), each Bayesian insurance expert's natural conjugate prior family for the tail parameter is a Gamma distribution. Each expert i receives n_i^t observations (claims) per year t . i updates his posterior shape parameter as $s_i^t = s_i^{t-1} + n_i^t$ and rate parameter as $r_i^t = r_i^{t-1} + \sum_{k=1}^{n_i^t} \ln(x_k^t)$. If I consider yearly claim aggregates, then $\forall i, t, n_i^t = 1$, so the evolution of the shape parameter is deterministic, $s_t = t$, and the evolution of the rate parameter is a random variable following an inverted gamma distribution $r_i^t \sim \text{invGamma}(t, \alpha)$.

I follow [Meyers \(1996\)](#), the prior for the α of the Pareto distribution follows a Gamma distribution:

$$\pi_A(\alpha) = \frac{r^s}{\Gamma(s)} \alpha^{s-1} e^{-r\alpha} \quad (32)$$

The model pdf is denoted $f_{X|A}(x|\alpha)$, with x a sample from X and α our parameter from the parameter space A . For a Pareto distribution, this pdf is:

$$f_{X|A}(x|\alpha) = \frac{\alpha^n}{\prod_{i=1}^n x_i^{\alpha+1}} \quad (33)$$

The posterior distribution is the conditional probability distribution of the parameters given the observed data. According to Bayes' theorem it is:

$$\pi_{A|X}(\alpha|x) = \frac{f_{X|A}(x|\alpha) \pi(\alpha)}{\int f_{X|A}(x|\alpha) \pi(\alpha) d\alpha} \quad (34)$$

which yields a Gamma distribution with shape $n + s$ and rate $r + \sum_i \ln(x_i)$:

$$\pi_{A|X}(\alpha|x) = \frac{\alpha^{n+s-1} \exp[-\alpha(\sum_i \ln(x_i) + r)]}{(n+s-1)! \left(\frac{1}{\sum_i \ln(x_i) + r} \right)^{n+s}} \quad (35)$$

Furthermore, Equation 5 can be developed for each Bayesian expert as the mode can be expressed with the Pareto distributed loss realizations.

$$\pi_i(\alpha^t | \cup_t x_i^t) \sim \mathcal{N} \left(\frac{t-1}{\sum_{k=1}^t \ln x_i^k}, \frac{t-1}{\left(\sum_{k=1}^t \ln x_i^k \right)^2} \right) \quad (36)$$

²⁹[Arnold and Press \(1989\)](#) demonstrate that if this threshold is unknown, a modified Lwin prior should be implemented.

Following Meyers (1996), the Bayesian posterior will follow a normal distribution:

$$\pi(\alpha|\hat{\alpha}) \sim \mathcal{N}\left(\hat{\alpha}, \frac{\hat{\alpha}^2}{t}\right) \quad (37)$$