Risk-based capital requirements and optimal liquidation in a stress scenario

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Abstract

We develop a simple yet realistic framework to analyze the impact of an exogenous shock on a bank's balance-sheet and its optimal response when it is constrained to maintain its risk-based capital ratio above a regulatory threshold. We show that in a stress scenario, capital requirements may force the bank to shrink the size of its assets and we exhibit the bank's optimal strategy as a function of regulatory risk-weights, asset market liquidity and shock size. When financial markets are perfectly competitive, we show that the bank is always able to restore its capital ratio above the required one. However, for banks constrained to sell their loans at a discount and/or with a positive price impact when selling their marketable assets (large banks) we exhibit situations in which the deleveraging process generates a *death spiral*. We then show how to calibrate our model using annual reports of banks and study in detail the case of the French bank BNP Paribas. Finally, we suggest how our simple framework can be used to design a systemic capital surcharge.

Keyworkds: Risk-weighted assets (RWA), stress-tests, fire sales, market impact, optimal liquidation, systemic capital surcharge

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1 Introduction

Over the last decades, micro-prudential regulation has developed several frameworks to assess the strength of each bank which consist essentially to monitor banks risk-based capital ratios (RBC), defined as the (regulatory) capital divided by the risk-weighted assets (RWA). The rationale to impose a RBC rather than a simple leverage ratio (e.g., regulatory capital divided by the total assets) is to force each bank to have capital which is commensurate to its risk. Typically, if two assets have the same value but different risk, the riskier security should have a larger risk-weight and require more capital than the less risky asset. While Basel I assigned risk-weights in an exogenous and transparent manner, since Basel II accords, the way each RWA is computed is complex and opaque for banks under the Advanced Internal Rating Based approach (AIRB) as they are allowed to make use of internal (proprietary-based) models to estimate the RWA, used in turn to determine the capital required.

Such opacity brought criticism on the use of RWAs, which can easily be manipulated and under-estimated by banks [Vallascas and Hagendorff, 2013]. The skepticism on RBC measures soared during the subprime crisis when some banks displayed high RBC ratios while showing pronounced weakness at that period. For instance, [Duffie, 2009] reports that Citibank, which received a significant bailout, had a Tier 1 capital ratio that never fell below 7% during the financial crisis. In the same vein, [Calomiris and Herring, 2013] report that weeks before the collapse of Northern Rock, regulators allowed the bank to adopt the AIRB approach, which reduced its capital requirement by 30%. Subsequent to the financial crisis, regulators have imposed additional measures to assess the strength of banks in the form of supervisory stress tests. Basically, a stress test is a hypothetical adverse shock which is translated into losses to assets on the balance-sheet of the bank [Acharya et al., 2014, Borio et al., 2014, Glasserman et al., 2015]. As long as these losses are fully absorbed by equity capital, i.e., by stockholders, the bank remains solvent and the other stakeholders (depositors, bondholders, deposits insurance fund ...) are not impacted. In the US, the Dodd-Frank Wall Street Reform and Consumer Protection Act (2010) requires the Federal Reserve to conduct an annual stress-test exercise, called the Comprehensive Capital Analysis and Review. In Europe, stress-tests have been conducted recently by the European Banking Authority in 2011 and 2014 and currently in 2016.

As designed and implemented by supervisors, stress tests have two major limitations. First, they are almost impossible to reproduce for outsiders as the stressed scenarios used by regulators involve dozens of variables and the computation of stressed RWAs and RBC is done by banks themsleves using proprietary models. As such, the results of various stress tests have been challenged. A remarkable example is the 2011 stress-test conducted by the EBA which found that “capital position of European banks has been strengthened significantly” while the outbreak of the European sovereign debt crisis showed that European banks were not as resilient as what was suggested by the stress test. These stress-test results were criticized because they were suspected to be essentially an attempt to calm financial markets. Second, a major limitation of current stress test exercises is that they do not take into account the potential endogenous reaction of the bank to the stress. Regulators acknowledge that regulatory capital requirements may lead a bank to engage in credit contraction and/or fire sales following a large shock ([BCBS, 2015b, p7]) but these supervisory stress tests do not take into account this feedback effect on the bank’s own balance-sheet.

In this paper, we develop a simple yet realistic framework which enables us to study the impact of an exogenous shock on the balance-sheet of a bank. When it is subject to regulatory capital requirements – typically RBC requirements – we show that the bank may have to shrink the size of assets following a large shock and engage in credit contraction and/or fire sales. We exhibit the optimal liquidation strategy for the bank as a function of the market (il)liquidity of assets[1], regulatory risk-weights and shock size. We also show how to calibrate our framework from publicly available data and we study in detail the case of BNP Paribas. Our paper hence provides a simple and transparent stress test toolkit which takes into account the endogenous behavior of the bank and the market liquidity of the bank’s assets.

Following the 2008 financial crisis, there is now a large body of academic and practitioners literature which typically analyzes resilience of banks to an adverse shock such as the loss on Sovereign bonds. An early interesting paper that makes use of the public information disclosed by the European Banking Authority is [Greenwood et al., 2015]. Assuming a 50% write-off on GIIPS sovereign debt, they provide a ranking of banks according to their systemicness, defined as a function (indeed the product) of three factors; the relative holding of a bank, the size of the fire sales and the linkage effect. Their analysis shows that the French bank BNP Paribas, considered in detail in this paper, belongs to the top-five of their systemicness ranking. In general, the resilience of banks critically depends on the way they interact, that is, on the way they are interconnected. [Glasserman and Young,] provides a comprehensive and up-to-date overview on the subject. While relevant and interesting, most “network papers” focus on systemic risk (e.g., [Amini et al., 2012, Chen et al., 2013, Eisenberg and Noe, 2001, Elsinger et al., 2006, Gourieroux et al., 2012]) but in general give

[1] We follow the terminology introduced in [Brunnermeier and Pedersen, 2009], see also [Tirole, 2011], in which market liquidity refers to the ease with which an asset can be transformed into cash. It can be more precisely defined as the degree to which an asset can be *quickly* bought or sold (i.e., turned into cash) in the market *without* affecting the asset’s price.

an oversimplified picture of the bank at the micro-prudential level. For instance, in some papers, the denominator of the capital ratio is the total assets rather than the risk-weighted assets while in others, each bank has only one asset and follows a simple mechanical rule to restore its capital ratio. An important aim of this paper is to bridge the gap. We offer a realistic micro-prudentially founded model in which the (aggregate) weights of the RWA is calibrated using the annual reports of banks and then used to simulate the endogenous behavior of the bank after the shock.

We consider a bank whose balance-sheet is composed of loans and reserves in the banking book and marketable assets in the trading book. When the bank suffers from a shock which brings its regulatory capital ratio below the minimum threshold authorized by regulators, we make the assumption that the bank is not in a position to issue new stocks typically because of the debt overhang problem (see [Hanson et al., 2011] for a detailed discussion). As done in [Brunnermeier and Oehmke, 2014, Greenwood et al., 2015], the unique possibility for the bank to (try to) restore its regulatory capital ratio is shrink the size of its balance sheet, that is, to rebalance its portfolio of assets (banking book and/or trading book). An important feature of our framework is that we explicitly consider the bank's reaction. More specifically, to use the terminology of the BCBS (2015), within our approach, the bank's response is driven by a rule for optimizing behaviour rather than a simple rule of thumb.

In our benchmark framework defined as the situation in which financial markets are perfectly competitive, i.e., there are no market frictions, we show that there always exists a unique optimal liquidation strategy for the bank to restore its capital ratio. This optimal strategy turns out to be very simple: the bank should sell first the asset which has the highest weight (as the weight plays a role similar to a tax) and, if this is not enough, the assets with lower risk-weights. This benchmark framework is interesting as it clearly shows that there always exists a solution for the bank to restore its regulatory capital ratio. It is unfortunately not very realistic as small banks may actually find it difficult to sell loans at fair value due to the so-called adverse selection problem (e.g., [Drucker and Puri, 2009, Gande and Saunders, 2012]) while large banks may have a price impact (e.g., [Shleifer and Vishny, 2011]) when selling marketable assets. For these reasons, we also analyze a more complex framework with adverse selection and price impact.

In the presence of market imperfection (or frictions), the existence of an optimal liquidation strategy to bring back the capital above the regulatory capital ratio is not guaranteed anymore. For instance, when the bank is only able to sell its originated loans at a discount (because of adverse selection), depending on the size of the shock, we show that the regulatory capital ratio may actually *decrease* with the amount of loans sold. As a result, depending on the bank's capital structure, liquidating all the assets of the trading book may not be enough to restore the capital ratio. In such a case, whether or not the bank liquidates its marketable assets, the bank remains insolvent. We also consider the situation in which liquidating the assets of the trading book generates a price impact. When this price impact is large, as one may expect, the bank may not be able to restore its capital ratio, especially if the shock is large. On the contrary, when both the price impact and the shock are moderate, an optimal solution may exist but is the result of a complex tradeoff in that it involves all the parameters of the model, the weight of each asset, the discount of the loans and the market impact. One of the main theoretical contribution of this paper is to explicitly exhibit the optimal behavior of the bank as a function of those various parameters. It thus enables to better anticipate the systematic reaction and potential consequences of a bank to a given shock. Another contribution is to show that the market (il)liquidity of an asset such as loans directly interacts with the situation of the bank after the deleveraging process. Since the market (il)liquidity of the loans directly impacts the numerator of the RBC, i.e., selling loans at a discount decreases the bank's equity, it obviously impacts the possibility for the bank to remain solvent after the deleveraging process. In that sense, as suggested by regulators in their recent document dedicated to supervisory stress tests ([BCBS, 2015b]), our model explicitly integrates the link between market (il)liquidity and (in)solvency.

The theoretical analysis of our framework assumes that the risk-weights are given. From an academic point of view, as recalled in [Le Leslé and Avramova, 2012], the literature on capital is vast but the empirical analysis on RWAs is more limited. To the best of our knowledge, empirical papers that explicitly focus on this analysis adopt an econometric point of view. For instance, in [Mariathasan and Merrouche, 2014, Vallascas and Hagendorff, 2013] to quote two recent articles on the subject, they consider a regression model in which the endogenous variable is the RWA divided by total assets and their results suggest a number of concerns with the current practice of risk-weighted approach: manipulation, underestimation of risk (see also [Barakova and Palvia, 2014]). While interesting to monitor general trends in the banking industry, the econometric approach is not suitable to study the case of a *given bank*. As a consequence, we adopt a *calibration* point of view similar in the spirit to the way market participants imply a volatility using the Black-Scholes model from option prices or a default probability using an intensity model from CDS spreads. From the publicly available registration reports of the banks, we use our theoretical model to *imply* the aggregate risk-weight of the banking book and the aggregate risk-weight of the trading book. Once calibrated, our model can be used to predict, as a function of the magnitude of the shock, the situation faced by that bank. We illustrate our framework using the example of BNP Paribas, since the registration document as of 2014 explicitly discloses the split of the balance-sheet by trading book and banking book. From a regulatory point of view, as this split can always be known, our model can be fine-tuned and used as an easy "stress-test toolkit" that complements the classical stress-tests.

As our model predicts that a large bank may be insolvent after the deleveraging process due to its positive price impact, it is natural from a regulatory point of view to require from that large bank a higher loss absorbency (HLA). In [Board, 2013], regulators presents a methodology to determine a systemic capital surcharge as a fraction of CET 1. Using our framework, we also suggest a way to compute this surcharge as a fraction of CET 1. The surcharge in capital is directly related to the price impact of a bank, i.e., to the “negative externality” it creates to the markets when it has to liquidates an important portion of assets with a positive price impact. We consider a realistic adverse scenario for BNP Paribas and suggest that insolvency could be avoided with a capital surcharge equal to 1% of CET 1.

The paper is organized as follows. In section 2, we provide a background on risk-based capital and shocks. Section 3 presents our framework and its main assumptions. In section 4, we present the results for our benchmark framework while section 5 develops a more complex framework with discount when selling loans and price impact when selling marketable assets. Section 6 is devoted to the calibration of our model and the study of BNP Paribas. Section 7 presents some policy implications for the determination of capital surcharge and section 8 concludes the paper.

2 Background on risk-based capital and shocks

From a micro prudential point of view, i.e., abstracting systemic risk issues, the capital of banks is regulated because they typically finance risky assets with government-insured deposits [Hanson et al., 2011], [McMillan, 2014][Chp3]. This gives an incentive to stockholders and/or managers acting on their behalf, to take on risk at the expense of the other stakeholders (depositors, debt holders, deposit insurance fund...). The aim of banking micro-prudential regulation is thus to force banks to hold a “safety cushion”, called capital, aimed at absorbing losses and which should be commensurate to the size and risk of the bank’s activities. Exactly how one should measure capital and the risk of banking activities is subject to a long debate [Berger et al., 1995].

Regulatory capital. Following Basel 3 accords [BCBS, 2011], regulation distinguishes between Tier 1 capital, defined as Common Equity Tier 1 plus additional Tier 1, and Tier 2 capital. Tier 1 capital is essentially composed of common shares issued by the bank and retained earnings and is considered by regulators as “going-concern” capital, that is, capital designed to absorb losses without impeding the continuation of the bank's activities. Tier 2 capital is a supplementary capital that is considered by regulators as gone-concern capital, i.e., designed to absorb losses in case the bank faces liquidation. [BCBS, 2011] details precisely the characteristics of securities which are eligible for inclusion in Tier 1 and Tier 2 capital. For instance, a perpetual deeply subordinated debt may be eligible for inclusion in Tier 2 capital (see [BCBS, 2011, p18]) as its principal is never repaid and, in case of liquidation, the holders of such a hybrid debt will be repaid *after* depositors, which means that this hybrid debt is able to absorb losses in a liquidation situation. Regulation imposes a minimum of going-concern capital and gone-concern capital. The larger the going-concern capital, the lower the default probability of the bank while the larger the gone-concern capital, the lower the loss-given default for the bank's debtholders (e.g., depositors and/or classic bondholders) in case of a liquidation of the bank.

Risk-weighted assets. As discussed above, regulation imposes that banks hold enough capital compared not only to the size but also the risk of the bank's activities. As a consequence, rather than using total assets, regulators use *risk-weighted assets* (RWA) in order to set capital requirements for banks. The *weights* are supposed to reflect the riskiness of the bank's assets (see for instance chapter 8 of [Schooner and Taylor, 2009] for a nice discussion). For instance, when the bank decides to invest a given amount in reserves (i.e., to put it on its bank account at the central bank), this investment is considered as riskless and thus has a weight of zero, i.e., no capital is required. On the other hand, when the bank decides to invest this amount in customers' loans or in corporate bonds, the bank is now subject to credit risk and possibly market risk. As a result, the risk-weight associated to this investment is positive and the bank is required to finance it with a minimum of capital. Naturally, the weight associated to an investment in a AAA corporate bond with maturity 5 years should be lower than that associated to an investment in a 10-year BBB corporate bond as the latter security is generally viewed as riskier than the former. According to [Le Leslé and Avramova, 2012], the use of risk-weighted assets is important as it provides a common measure for a bank's risk and ensures that capital allocated to assets is commensurate with the risks. In practice, total risk-weighted assets denoted RWA is defined as the sum of partial RWAs, where each partial RWA is related to a given risk, typically credit, market or operational risk. For banks with an important trading activity, they also now have a (partial) RWA for counterparty risk and also possibly for securitization.

Since Basel 2 and Basel 3, banks under the AIRB approach directly estimate the risk parameters used in turn to compute the weights and thus the capital required. Estimation of risk-parameters by banks is typically done using internal and proprietary models, which brought various criticism on the risk-based approach adopted by regulation, as risk-weights could be subject to manipulations by the banks [Mariathasan and Merrouche, 2014, Vallascas and Hagendorff, 2013]. In their somehow provocative but very interesting book, [Admati and Hellwig, 2014] argue that banks make use of various methods to optimize the risk of the assets so that they choose investments that are indeed riskier than the regulators believe and thus have a higher expected return. This practice is even acknowledged by regulators and officials: [BCBS, 2015a, p9] states that "internal models allow for more accurate risk measurement. But if they are used to set minimum capital requirements, banks have unintended incentives to underestimate risk" while [United States Senate, 2013] raises "systemic concerns about how many other financial institutions may be disregarding risk indicators and manipulating models to artificially lower risk results and capital requirements" in their investigation of JP Morgan Chase Whale trades abuses.

The debate is not new (see e.g., [Berger et al., 1995]) but quite surprisingly, the theoretical foundation of the risk-weighted assets (RWA) has been tackled only very recently by [Glasserman and Kang, 2014]. The starting point of their analysis might be the following observation. The RWA is a weighted average and is thus additive by definition. However, since risk is in general not additive, the exact objective which is implemented by regulators when they impose a constraint on the RWA is far from being clear. [Glasserman and Kang, 2014] confirm, somehow surprisingly, the interest of the RWA. They show that one can explicitly design risk weights such that the optimal solution of the bank's optimization problem without any capital constraint is *proportional* to the optimal solution with a RWA constraint. By design, the risk weights are chosen by the regulator such that they *do not change the relative mix of assets* since up to a positive scalar, the two portfolios (i.e., vectors) are equal. Quite interestingly, as remarked by the authors, their result shows that the regulatory ideal weights are virtually unrelated with risk but rather to expected (rate of) returns[2].

Bank capital requirements. In 2016, banks have to comply with the following capital ratios:

- Common Equity Tier 1 (CET 1) must be at least 4.5% of risk-weighted assets (RWA) at all times. Formally, at all dates t:

$$\frac{\mathrm{CET1_t}}{\mathrm{RWA}_t} \geq 4.5\% \tag{1}$$

[2]The derivation of the ideal weights requires however the regulator to explicitly know these expected returns. As a result, the authors also offer an iterative procedure which only assumes the regulator to observe the bank's portfolio. They show that this procedure converges to the (ideal) weights (and portfolio) that would hold under complete information.

- Tier 1 Capital must be at least 6.0% of risk-weighted assets at all times. Formally, at all dates t:

$$\frac{\mathrm{Tier1_t}}{\mathrm{RWA}_t} \geq 6\% \tag{2}$$

- Total Capital (Tier 1 Capital plus Tier 2 Capital) must be at least 8% of risk-weighted assets at all times. Formally, at all dates t:

$$\frac{\mathrm{Tier1_t} + \mathrm{Tier2_t}}{\mathrm{RWA}_t} \geq 8\% \tag{3}$$

[BCBS, 2015b] recalls that in 2018 banks will have to comply not only with a regulatory capital ratio, in which the denominator is the RWA, but also with a *leverage ratio*, in which the denominator is the total exposure of the bank, including the off-balance sheets items. Both approaches to measure the capital ratio have their strengths and weakness. This is why, according to [BCBS, 2015a], the regulatory framework should use "multiple constraints", i.e., various types of capital ratios, with weighted and unweighted assets. [BCBS, 2015a] reports that it has conducted a review of the risk-weighted asset approach, assumed to balance simplicity, comparability and risk sensitivity but it is currently an ongoing work. In addition, in Basel 3, Globally Systemically Important Banks will also have to comply with an additional capital buffer (surcharge in capital and a countercyclical capital) designed to mitigate systemic risk.

Stress-tests and behavioral reaction One way to examine the strength of each bank is to study its resilience to an exogenous shock, that is an adverse event which negatively impacts the bank's balance-sheet. Such adverse event may typically be an unexpected large number of defaults (loans, corporates, Sovereign...) or a decline in asset prices, as happened during the subprime crisis in 2007-2008. It may also be a loss related to fraud such as the Kerviel fraud in Société Générale in January 2008 (5 billion of euros) or a fine, as happened to BNP Paribas in 2014 (6 billion of euros).

Since the financial crisis of 2007-2008, supervisory stress-tests have been launched on a regular basis. Typically, the Federal Reserve and the European Banking Authority provide each year the precise methodology used to implement stress-tests respectively in the US and Europe[3]. In both cases, the aim of the stress-test is to assess the resilience of banks under supervisory (adverse) baseline scenarios provided by the competent Authority. For instance, the largest US bank holding companies that have important trading activities must consider

[3] For Europe, see the instructions (draft), *EU wide Stress Test* 2016 published by the European Banking Authorities (EBA) in Europen and, for USA, see the *Comprehensive Capital Analysis and Review* 2016 published by the Federal System Reserve.

global market shocks that adversely impact their trading book or their banking book. In Europe, banks are supposed to stress all their risk-weighted assets, i.e., essentially the RWA related to credit risk, market risk and operational risk, but also to counterparty risk and securitization, assuming that they maintain the same business model. As already discussed, replicating the stress-tests is impossible for a bank outsider and even almost impossible for a single insider due to the complexity of the task. In practice, the computation of the total RWA, as published in the annual report of a bank, is the result of many employees with various technical skills.

As recalled in [Borio et al., 2014], a central question for any stress test is the horizon over which the impact of the shock(s) on the balance sheet of the bank is assessed. In supervisory stress tests and in many models (e.g, [Elsinger et al., 2006]), the bank is not allowed to react, i.e., the bank's portfolio of assets is not restructured. The absence of reaction can be justified when the horizon is short, e.g., a month or a quarter, but there are various documented cases of banks that decided to react within few days. For instance, the fraudulent positions of Kerviel, which amounted to 50 billion of euros, were unwinded in three days when discovered by Société Générale[4] and the resulted loss was equal to approximately 5 billion of euros.

3 Model assumptions and preliminary results

We offer here a simple non-probabilistic framework of universal banking in which the bank is exposed to credit risk only in the banking book and to market risk only in the trading book.

3.1 Banking activities with pure risks : credit and market risks

We assume that the bank can invest in the three following types of assets:

- Loans, which are non-marketable assets (thus illiquid) subject to credit risk only.
- Marketable assets, i.e., tradable financial instruments and subject to market risk only.
- Cash, which constitutes the reserves of the bank at the central bank, i.e., European Central Bank for banks that belong to the countries of the Eurozone and Federal Reserve for US banks and is not subject to any risk.

[4] See the French document p. 54, Cour d'Appel de Paris - pôle 5 - chambre 12 - numéro rg 11/404 - arrêt rendu le 24 octobre 2012.

We thus make the assumption that loans are subject to credit risk only and marketable assets are subject to market risk only, in order to focus on pure risks. In practice, loans are also subject to interest rate risk while marketable assets may be subject to counterparty risk. The value of loans at date t is equal to $q \times v_t$, where v_t is the value of one loan (for instance at its amortized cost) and q is the number of loans granted by the bank. For the sake of simplicity only, we assume that the loans written by the bank all have the same characteristics. The mark-to-market value of the bank's positions at time t on tradable assets is equal to $Q \times V_t$ where V_t is the value of the financial instrument (i.e., ETF, stock index...) and Q is the number of shares bought by the bank. In our model, qv_t is thus the value of the *banking book* while QV_t is the value of the *trading book*. By construction, the banking book is subject to credit risk only, i.e., the risk of default of customer loans and the trading book is subject to market risk only, i.e., the risk of the price movement of the marketable asset. Without loss of generality, we also assume that the bank has no cash at time t. This assumption is reasonable – JP Morgan held only 1.6% of cash (as a fraction of total assets) in 2013 – and can easily be relaxed. The total value of assets of the bank at date t is thus equal to

$$A_t := qv_t + QV_t \tag{4}$$

On the liability side, let D be the sum of the value of deposits and debt securities, typically coupon bonds, that have been issued by the bank. The capital structure of the bank is here given exogenously but it is important to note at this stage that for many banks, the capital divided by the total assets is typically lower than 5%. Since equity holders' payment is subordinated to the full payment of depositors and bondholders, from limited liability of stockholders, the value of equity (or capital) at time t is given by:

$$E_t = \max\{A_t - D; 0\} = \max\{qv_t + QV_t - D; 0\} \tag{5}$$

As usual, if $E_t = 0$, the bank is said to be insolvent and can be closed by the competent regulator. From a theoretical point of view, one may assume that E_t is the sum of the regulatory capital, that is, Tier 1 and Tier 2. The balance-sheet of the bank at time t is hence given by:

Balance-sheet at time t

Assets	Liabilities
Banking book: qv_t	Debt: D
Trading book: QV_t	Equity: E_t
A_t	$E_t + D$

As discussed in the previous section, risky assets such as loans and marketable assets have positive weights and require the bank to have (i.e., issue) capital. On the contrary, cash, which is non-risky, is not subject to any capital requirement. Let α and β be the *risk weights* associated respectively to loans and marketable assets. These two weights are either estimated by the bank as a function of the risk parameters under the AIRB approach or they are provided by the regulator for banks under the standardized approach (e.g., from rating of credit rating agencies). The value of risk-weighted assets (RWA) is hence given by:

$$\mathrm{RWA}_t^{\alpha,\beta} := \mathrm{RWA}_t = \alpha q v_t + \beta Q V_t \tag{6}$$

and the regulatory capital ratio at date t is defined by:

$$\theta_t^{\alpha,\beta} := \theta_t = \frac{E_t}{\mathrm{RWA}_t} \tag{7}$$

In section 6, we show how these two weights α and β can be *implied* from the observed RWAs. Let θ_{min} be the minimum capital ratio for the bank imposed by regulatory institutions, typically 8% (see equation 3). From [BCBS, 2011], for all dates t, the bank must satisfy the following regulatory capital constraint:

$$\theta_t \geq \theta_{min} \tag{8}$$

that is, the regulatory capital ratio must be above a critical ratio defined by the regulator.

3.2 Adverse shocks and bank's reaction

Within our model, the bank is naturally exposed to credit and market shocks, which generate respectively losses in the banking book and the trading book. As discussed in section 1, it is the role of equity capital to absorb such asset losses. However, if losses are large enough, the bank's regulatory capital may drop below the minimum authorized threshold. In such a situation, the bank can either issue new equity – increase the numerator in equation (7) – or diminish the size of (risk-weighted) assets – decrease the denominator in equation (7) – in order to comply with the regulatory capital requirement given in equation (8). Generally, due to the so-called debt overhang problem [Myers, 1977], banks rule out new equity issuance and instead decide to shrink the size of assets in order to comply with regulatory capital requirements. As a consequence, in our model, as in [Greenwood et al., 2015] or [Brunnermeier and Oehmke, 2014], we assume that a bank which does not respect the regulatory capital requirement given in equation (8) – typically due to a large shock wiping out a significant portion of its equity –, will only shrink the size of assets, i.e. sell loans and/or marketable assets, in order to bring its capital ratio above θ_{min}.

We consider in what follows a credit shock in the banking book and examine the bank's response. We choose to examine a credit shock because the banking book generally represents a significant portion of the bank's balance-sheet, compared to the trading book – see for example section 6.2 – making banks naturally more exposed to credit shocks rather than market shocks. Note importantly that a similar analysis can be reproduced for a shock in the trading book. Considering two shocks simultaneously is also possible but only few scenarios are interesting and explains why we focus on one shock only[5].

We assume that at date t^+, the banking book suffers a (percentage) loss of Δ, i.e., at date t^+, the value of the banking book is equal to $qv_t(1-\Delta)$ and the value of the trading book is equal to QV_t. The bank may then react between dates t^+ and $t+1$. In practice (see the discussion on Société Générale) the lag between t^+ and $t+1$ is of the order of a few days so that it makes sense to assume that V_t remains constant.

The shock Δ in the banking book can be interpreted as follows. Let $LGD \in (0,1)$ be the loss given default of a given loan and let $\pi_{Def} \in (0,1)$ be the fraction of loans that have defaulted. Right after the loss, the total value of the banking book is thus equal to $(1-\pi_{Def})qv_t + \pi_{Def}(1-LGD)qv_t$. Since the value of the banking book is also equal to $qv_t(1-\Delta)$, it naturally follows that $\Delta = \pi_{Def}\ LGD$ which is simply the *average loss* per unit of exposure. Our approach could be of interest for regulators since they typically ask banks to stress the default probability and/or the loss given default at a given date. From equation (6), at time t^+, the value of risk-weighted assets is equal to $\mathrm{RWA}_{t^+}(\Delta) := \mathrm{RWA}_{t^+} = \alpha qv_t(1-\Delta) + \beta QV_t$. In case in which the bank remains solvent after the shock, i.e., $E_{t^+}(\Delta) > 0$, the capital ratio is equal to

$$\theta_{t^+}(\Delta) = \frac{E_{t^+}(\Delta)}{\mathrm{RWA}_{t^+}} = \frac{E_t - qv_t\Delta}{\mathrm{RWA}_t - \alpha qv_t\Delta} \tag{9}$$

and it is straightforward to show that $\theta_{t^+}(\Delta) < \theta_t$ as long as Δ is positive. If the shock is large enough, the bank's capital ratio may fall below the minimum regulatory capital ratio, i.e., $\theta_{t^+} < \theta_{min}$ and a response from the bank is expected. In our model, as already discussed, the bank will shrink the size of its assets in order to bring its capital ratio back above θ_{min}. We make here the realistic assumption that the bank liquidates the portfolio of assets so as to *minimize the total value of assets sold*, subject to the constraint that the new value of the capital ratio satisfies the regulatory constraint. Throughout the paper, we shall adopt the following notations:

- x and y are respectively the fraction of loans and tradable securities sold at time $t+1$.

[5] In [Glasserman et al., 2015], they make the same observation with risk factors, "many different combinations of movements of market factors can produce losses of similar magnitude (...)". Their approach is in some sense complementary to ours in that their aim is to identify the most likely scenario among all such combination.

- $L(x, y)$ and $\theta_{t+1}(\Delta, x, y)$ are respectively the total value of the sale and the regulatory capital *right after* the response of the bank, i.e., at time $t + 1$.

The bank is thus assumed to solve the following optimization problem:

$$\min_{(x,y)\in[0,1]^2} L(x, y) \tag{10}$$

$$\text{s/t} \quad \theta_{t+1}(\Delta, x, y) \geq \theta_{min} \tag{11}$$

As we shall see, the computation of the function $L(x, y)$ critically depends on the assumption of whether or not the assets can be sold at fair values. For instance, in our benchmark model without adverse selection and market impact, $L(x, y)$ will be a linear function of x and y. However, in the presence of adverse selection when selling loans and market impact when selling marketable assets, as we shall see, assuming even the simplest market impact function, $L(x, y)$ will be a *non-linear* function of y.

3.3 Situations after a shock

The following proposition details the three scenarios faced by the bank as a function the magnitude of the shock Δ.

Proposition 1 ***(Capital absorption, liquidation and fire sales thresholds)*** *Assume that $\theta_t > \theta_{min}$. There exist two critical thresholds Δ^*_{sale} and Δ^*_{liq} defined by*

$$\Delta^*_{liq} = \frac{E_t}{qv_t} > 0 \tag{12}$$

$$\Delta^{*(\alpha,\beta)}_{sale} := \Delta^*_{sale} = \frac{E_t - \theta_{min}\mathrm{RWA}_t}{qv_t(1 - \alpha\theta_{min})} > 0 \tag{13}$$

$$\text{where} \quad \Delta^*_{sale} < \Delta^*_{liq}$$

such that if:

1. *$\Delta \leq \Delta^*_{sale}$, the existing capital can absorb losses and the bank does not need to rebalance its positions as its capital ratio remains above θ_{min}.*

2. *$\Delta \in (\Delta^*_{sale}, \Delta^*_{liq})$, the bank's capital ratio is below θ_{min} and the bank shrinks the size of its assets in order to comply with regulatory capital requirements.*

3. *$\Delta \geq \Delta^*_{liq}$, the bank is insolvent and can be liquidated by the competent regulatory authority.*

Depending on the magnitude of losses Δ, the bank faces three different situations[6]. If the shock is small enough, the bank's capital ratio remains above θ_{min} and the bank does not need to rebalance its positions. When the shock is large enough, the bank's capital ratio may fall below the minimum value θ_{min} authorized by regulators. In this case, as discussed in the previous section, the bank decreases the size of assets (i.e., decreases the denominator in equation (7)), hence triggering fire sales and/or credit contraction in order to comply with regulatory capital. Finally, when the shock is extreme, the bank's equity is fully wiped out, leaving the bank insolvent.

In the rest of this paper, we analyze in detail the more interesting case in which $\Delta \in (\Delta^*_{sale}, \Delta^*_{liq})$ so that the bank must rebalance its portfolio of assets in order to comply with the regulatory capital ratio. When there is no ambiguity, we may sometimes denote the capital ratio as $\theta_{t+1}(x, y)$ instead of $\theta_{t+1}(\Delta, x, y)$.

4 The benchmark framework: liquidation at fair value

We consider in this section the simplest case in which the bank is able to liquidate its risky assets (i.e., loans and marketable assets) at *fair value* and *without price impact.* Since cash is not subject to any capital requirement, the bank can sell a dollar amount equal to xqv_t of loans in the banking book and an amount equal to yQV_t of assets in the trading book and put the proceeds of the sale in its bank account at the central bank. At time $t+1$, when this selling operation occurs, the bank's balance-sheet is given by:

Balance sheet at time $t+1$

Cash : $xqv_t(1-\Delta) + yQV_t$ Loans : $(1-x)qv_t(1-\Delta)$ Marketable assets : $(1-y)QV_t$	Debt: D Equity: E_{t+1}
$A_{t+1} = qv_t(1-\Delta) + QV_t$	$E_{t+1} + D$

which implies that the bank's equity at date $t+1$ is equal to:

$$E_{t+1} = A_{t+1} - D = E_t - qv_t\Delta \tag{14}$$

When there is no price impact when selling marketable assets and when the loans can be sold at fair value, the amount of regulatory capital E_{t+1} after the liquidation is *independent* of the way the bank liquidates its assets, i.e., this selling operation only affects the percentage

[6]Note that the positivity of Δ^*_{sale} follows directly from the assumption that $\theta_t > \theta_{min}$.

of risky assets and the percentage of non risky assets (cash). After liquidating a portion x of its banking book and y of its trading book, the bank's regulatory capital ratio is given by:

$$\theta_{t+1}(\Delta, x, y) = \frac{E_t - qv_t\Delta}{\alpha(1-x)qv_t(1-\Delta) + \beta(1-y)QV_t} \tag{15}$$

As a result, since the numerator of equation (15) remains invariant and its denominator goes to zero when x and y go to one, the bank is always able to bring its capital ratio back above θ_{min} by shrinking the size of its portfolio. In this benchmark case, (see equations (10) and (11)), the bank's optimization problem is simply given by:

$$\min_{(x,y)\in[0,1]^2} L(x,y) \quad := \quad xqv_t(1-\Delta) + yQV_t \tag{16}$$

$$\text{s/t} \quad \theta_{t+1}(\Delta, x, y) \quad \geq \quad \theta_{min} \tag{17}$$

which reduces to a simple linear programming problem as both the objective function and the constraint are linear functions of x and y. Note also that the constraint is obviously binding. The following proposition shows that the solution of the above linear programming problem critically depends on the risk weights α and β.

Proposition 2 *(**Sales at fair value**) Assume that the shock on loans is such that:*

$$\Delta^*_{sale} < \Delta < \Delta^*_{liq}$$

There is an optimal way for the bank to liquidate assets in order to comply with regulatory requirements and increase its capital ratio back above θ_{min}.

- *If $\alpha > \beta$, it is optimal for the bank to sell only the loans. If this is not enough to restore the regulatory capital ratio, it is optimal for the bank to sell 100% of the loans and a positive fraction $y^* < 1$ of the marketable assets such that $\theta_{t+1}(1, y^*) = \theta_{min}$.*

- *If $\alpha < \beta$, it is optimal for the bank to sell only the marketable assets. If this is not enough to restore the regulatory capital ratio, it is optimal for the bank to sell 100% of the marketable assets and a positive fraction $x^* < 1$ of loans $\theta_{t+1}(x^*, 1) = \theta_{min}$..*

- *If $\alpha = \beta$, all the solutions are indifferent for the bank, i.e. the bank may optimally liquidate loans or marketable assets first.*

In the absence of adverse selection when selling loans and price impact when selling marketable assets, the bank optimally liquidates assets with the largest risk weights. This result is similar to the conclusions of [Acharya et al., 2014] who claim that the existence of regulatory risk weights encourage banks to concentrate their positions in assets associated with low risk weights and high expected returns. Whether the bank should start by liquidating loans first or marketable assets first depends only on the sign of $\alpha - \beta$. It is thus robust to possible errors in the estimation of α and β. In addition, we are able to compute explicitly the optimal proportions x of the banking book and y of the trading book that need to be liquidated, as a function of the bank's balance-sheet parameters. When $\alpha > \beta$ (resp. $\alpha > \beta$) the optimal liquidation strategy for the bank is given by $x^* = \min(1 - \frac{E_t - qv_t\Delta - \beta QV_t\theta_{min}}{\alpha qv_t(1-\Delta)\theta_{min}}, 1)$ and $y^* = \max(0, 1 - \frac{E_t - qv_t\Delta}{\beta QV_t\theta_{min}})$ (resp. $y^* = \min(1 - \frac{E_t - qv_t\Delta - \alpha qv_t(1-\Delta)\theta_{min}}{\beta QV_t\theta_{min}}, 1)$ and $x^* = \max(0, 1 - \frac{E_t - qv_t\Delta}{\alpha qv_t(1-\Delta)\theta_{min}}))$. It is interesting to note that it is precisely due to the existence of the weights that the bank has a unique optimal strategy to liquidate its portfolio. Without weights, i.e., when $\alpha = \beta$, there is actually an infinite number of solutions to liquidate the portfolio of assets.

5 A more complex framework: adverse selection and price impact

In this section, we consider the more realistic case in which the bank is able to sell loans, but only at a discount, due to adverse selection. Moreover, we make an important distinction between a small bank, which is able to sell its marketable assets without price impact and a large bank – for example a Globally Systemically Important Bank – which naturally has a price impact when selling a significant fraction of its marketable assets.

5.1 The case of a small bank: adverse selection when selling loans

We assume now that, instead of being able to sell loans at fair value, the bank can only sell them at a discounted price due to adverse selection. As a result, when the bank sells a fraction x of its loans, the amount received in cash is equal to $x\lambda qv_t(1-\Delta)$ instead of $\lambda qv_t(1-\Delta)$, where the coefficient $\lambda \in [0, 1)$ is thus related to the severity of the adverse selection problem. The stronger the intensity of the adverse selection problem, the lower the λ and the lower the amount of cash received by the bank when liquidating loans. Since loans are not marked-to-market, loans which have not been sold remain accounted in the balance-sheet at (historical) value, that is $(1-x)qv_t(1-\Delta)$. As we shall see in the next sub section, a different accounting treatment will be applied for marketable assets when price impact will be considered. The balance-sheet of the bank after liquidating a portion x of loans and y of marketable securities is given by:

Balance-sheet at time $t+1$ after rebalancing

Assets	Liabilities
Cash: $\lambda xqv_t(1-\Delta)+yQV_t$ Loans: $(1-x)qv_t(1-\Delta)$ Marketable assets $(1-y)QV_t(1-\Delta)$	Debt: D Equity: E_{t+1}
$A_{t+1}=[1-x(1-\lambda)]qv_t(1-\Delta)+QV_t$	$E_{t+1}+D$

which implies that

$$E_{t+1}=E_t-\Delta qv_t-(1-\lambda)xqv_t(1-\Delta) \tag{18}$$

The above balance-sheet displays an important difference with the no discount case: the bank's equity after shrinking the size of assets explicitly depends on the proportion of loans sold x. Since it is assumed that $\Delta<\Delta^*_{liq}$, the bank's equity remains positive after the shock. However, because E_{t+1} now explicitly depends on x, when the discount, measured by $1-\lambda$, is large enough (i.e., low λ), the liquidation process (of loans) itself may generate a *death spiral* (in that it is doomed) and wipe out the bank's equity. Using equation (18), the optimization problem is given by:

$$\min_{(x,y)\in[0,1]^2} L(x,y):=\lambda qxv_t(1-\Delta)+yQV_t \tag{19}$$

$$\text{s/t} \quad \theta_{t+1}(\Delta,x,y):=\frac{E_t-\Delta qv_t-xqv_t(1-\Delta)(1-\lambda)}{\alpha(1-x)qv_t(1-\Delta)+\beta(1-y)QV_t}\geq\theta_{min} \tag{20}$$

and is still a linear programming problem. Let us define the following critical threshold:

$$\Delta^*_d(\lambda):=\Delta^*_d=1-\frac{1}{\lambda}\left(1-\frac{E_t}{qv_t}\right)=1-\frac{1}{\lambda}\left(1-\Delta^*_{liq}\right) \tag{21}$$

which is such that if $\Delta<\Delta^*_d$, the bank's equity given in equation (18) remains strictly positive *regardless* of the quantity $x\in[0,1]$ of loans sold. As a result, in the case where $\Delta<\Delta^*_d$, there always exists a solution to the above optimization problem since the denominator of (20) can be arbitrarily close to zero if the bank decides to liquidate almost 100% of its portfolio while the numerator remains strictly positive. The threshold Δ^*_d may however be negative. When $\lambda<1-\Delta^*_{liq}$, $\Delta^*_d<0$ and it is always the case that $\Delta>\Delta^*_d$. In such a case, a solution to the above optimization problem may not always exist. From equation (21), it is easy to see that the critical threshold Δ^*_d is an increasing and concave function of λ. This means that, when $\Delta^*_d>0$ (i.e., $\lambda>1-\Delta^*_{liq}$), the larger the λ (i.e., the lower the discount when selling loans) the larger Δ^*_d and hence the larger the likelihood that $\Delta<\Delta^*_d$.

This is intuitive as a large λ means that liquidating loans does not imply too large losses for the bank's equity. In the particular case in which λ is equal to one, Δ_d^* reduces to Δ_{liq}^* and we are back to the previous case without adverse selection. In order to solve the bank's optimization problem, we shall now distinguish whether the shock $\Delta < \Delta_d^*$ or $\Delta \geq \Delta_d^*$. In the next proposition, since it is assumed that $\Delta > \Delta_{sale}^*$, the conditions under which Δ_d^* is zero or negative are irrelevant (see Fig. 1).

Proposition 3 ***(Fire sales with discount on loans)***. *Assume that the shock Δ on loans is such that:*

$$\Delta_{sale}^* < \Delta < \Delta_d^*$$

There exists an optimal way for the bank to liquidate assets in order to comply with regulatory requirements and increase its capital ratio back above θ_{min}. This optimal liquidation strategy depends on the value of λ compared to the threshold

$$\lambda^* = \frac{1 - \alpha\theta_{min}}{1 - \beta\theta_{min}} \tag{22}$$

and is described as follows:

- *If $\lambda < \lambda^*$, it is optimal for the bank to sell only the marketable assets. If this is not enough to restore the regulatory capital ratio, it is optimal for the bank to sell 100% of the marketable assets and a positive fraction $x^* < 1$ of the loans such that $\theta_{t+1}(x^*, 1) = \theta_{min}$.*
- *If $\lambda > \lambda^*$, it is optimal for the bank to sell only the loans. If this is not enough to restore the regulatory capital ratio, it is optimal for the bank to sell 100% of the loans and a positive fraction $y^* < 1$ of the marketable assets such that $\theta_{t+1}(1, y^*) = \theta_{min}$.*
- *If $\lambda = \lambda^*$, all the solutions are indifferent for the bank.*

From the threshold provided in equation (22), when $\alpha < \beta$, it is easy to see that $\lambda^* > 1$ and it is always the case that $\lambda < \lambda^*$ so that the bank liquidates its marketable assets first. This result is intuitive: if $\alpha < \beta$, not only are loans associated with lower risk-weight but also they can be sold only at a discount, so this naturally makes it optimal to liquidate marketable assets first. When $\alpha > \beta$, $\lambda^* < 1$. In such a case, the bank is faced with a trade-off between liquidating loans, which are more "taxed" than marketable assets but subject to a discount and liquidating marketable assets, which are not sold at a discount. When λ

is close enough to one, the bank should sell loans first since we are back to proposition 2. However, when λ is close to zero, since loans are sold almost for free, the bank should sell its marketable assets first. Equation (22) provides the critical threshold λ^* below which it is no more optimal for the bank to liquidate loans first, although they are associated with larger risk-weights, because selling loans would be done at a too large discount.

We have already seen that Δ_d^* is defined such that if $\Delta < \Delta_d^*$, the bank's equity given in equation (18) remains strictly positive regardless of the quantity $x \in [0,1]$ of loans sold. In the appendix, we also show, somewhat surprisingly, that if $\Delta > \Delta_d^*$, then, for all $(x,y) \in [0,1[^2$, $\frac{\partial \theta_{t+1}}{\partial x}(x,y) < 0$. In such a case, liquidating loans would generate a *death spiral*. Since it is not difficult to show that for all $y \in [0,1]$, $\frac{\partial \theta_{t+1}}{\partial y}(x,y) > 0$, when $\Delta > \Delta_d^*$, the unique possibility for the bank to restore its capital ratio is to sell its marketable assets only. But selling 100% of the marketable assets might not be enough. In particular, if the shock Δ is too large, the maximum capital ratio $\theta_{t+1}(0,1)$ will remain below θ_{min} and the bankruptcy is thus unavoidable. Let Δ_i^* be the critical threshold such that $\theta_{t+1}(0,1) = \theta_{min}$. From equation (20), it is easy to show that $\theta_{t+1}(0,1) = \theta_{min}$ is equivalent to $\Delta_i^* := \frac{E_t - \theta_{min}\alpha q v_t}{q v_t(1-\alpha\theta_{min})} > 0$. Since it is assumed that $\theta_t > \theta_{min}$, it thus follows that Δ_i^* is strictly positive. It is only when the shock Δ lies between Δ_d^* and Δ_i^* that the bank is able to bring back its capital ratio to the minimum authorized. Otherwise, the bank is forced to go bankrupt. Proposition 4 gives the precise statements of these ideas and Fig. 1 provides a graphical illustration.

Proposition 4 ***(Only marketable assets are sold)****. Assume that the shock Δ on loans is such that*

$$\Delta_d^* \leq \Delta < \Delta_{liq}^*$$

In such a case, liquidating loans decreases the bank's capital ratio so that the only possibility for the bank to restore its capital ratio is to sell marketable assets only. Denote:

$$\Delta_i^* = \frac{E_t - \theta_{min}\alpha q v_t}{q v_t(1-\alpha\theta_{min})} = \frac{\Delta_{liq}^* - \alpha\theta_{min}}{1-\alpha\theta_{min}} > 0 \tag{23}$$

- *If $\lambda < 1 - \alpha\theta_{min}$, then, $\Delta_i^* > \Delta_d^*$ and there are two cases.*
  - *if $\Delta \leq \Delta_i^*$, liquidating marketable assets is enough to bring the bank's capital ratio above θ_{min}, i.e., there exists $y^* \leq 1$ such that $\theta_{t+1}(0,y^*) = \theta_{min}$.*
  - *if $\Delta > \Delta_i^*$, liquidating marketable assets is not enough to bring the bank's capital ratio above θ_{min}, i.e., $\theta_{t+1}(0,1) < \theta_{min}$ and the bank is forced to go bankrupt.*

- *If $\lambda \geq 1 - \alpha\theta_{min}$, then, $\Delta_i^* \leq \Delta_d^*$. Since $\Delta > \Delta_i^*$, liquidating marketable assets is not enough to bring the bank's capital ratio above θ_{min} and and the bank is forced to go bankrupt.*

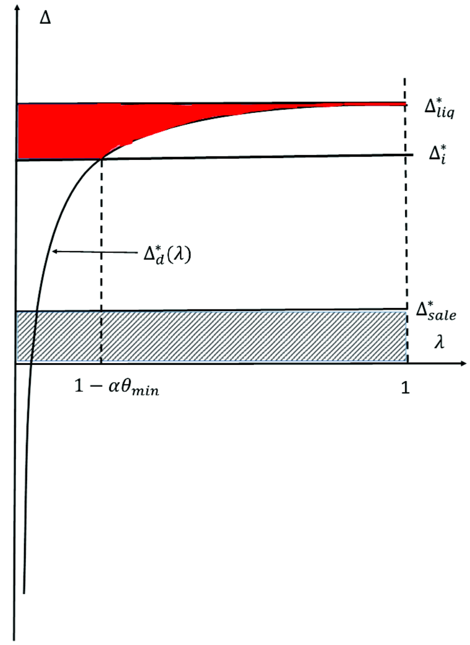

Figure 1: Region in red: Insolvency of the bank due to the existence of a discount when selling loans.

When the shock is too large, as we have seen, selling all the marketable assets is not sufficient to restore the regulatory capital ratio. In such a situation, the bank cannot meet its regulatory capital requirement and is forced to go bankrupt. It is however interesting to point out that the inability of the bank to restore its capital ratio is due to *market frictions* and not to "fundamentals", as the bank's equity was able to absorb the shock Δ in the first place, i.e., the bank was still solvent after the shock. Under such a bankruptcy due to market frictions only, regulators may step in and authorize temporarily the bank to have a capital ratio *lower* than θ_{min}. This kind of measure could however generate moral hazard. A better solution is to force banks to have more HLA (higher loss absorbency), related to the market imperfection. This idea is discussed in detail in section 7.

Fig. 1 provides an illustration of propositions 3 and 4. Since we restrict the analysis to the case in which $\Delta > \Delta^*_{sale}$, the region in "grey" (shaded) is irrelevant. The region in red depicts the situation of death spiral, that is, the situation of insolvency of the bank after the delevraging process.

5.2 The case of a large bank: adverse selection and price impact

Until now, we assumed that the bank could sell marketable assets at their market value V_t. This assumption is relevant as long as the quantities sold are not too large compared to the *liquidity* of the marketable assets. While this seems reasonable for small banks, it appears unrealistic for large banks which have large asset holdings and may impact prices when selling large blocks of assets.

In this section, we consider the case of a large bank which has a price impact when liquidating large blocks of marketable assets. We assume that the price impact is linear[7]: when the bank liquidates a portion y of its trading book, i.e., sells a quantity yQ of marketable assets, it generates a decrease in asset price from V_t to $V_t\left(1-\frac{yQ}{\Phi}\right)$ where Φ is the market depth of the marketable asset, which is a linear measure of the asset liquidity and is expressed in units or shares. The larger the market depth, the larger the asset liquidity and the lower the bank's price impact. As expected, when $\Phi=\infty$, i.e., $\frac{Q}{\Phi}=0$ the asset is infinitely liquid and there is no price impact. This corresponds to the classical perfect competition in which the bank acts as a *price taker*. When the quantity sold is negligible compared to market depth, price impact is also negligible. However, in practice, when a large bank sells large blocks of assets with finite liquidity, i.e., yQ is comparable to Φ, price impact may be significant. Empirical studies ([Gonnard et al., 2008, Tonello and Rabimov, 2010, Cont and Wagalath, 2016]) show that large institutional investors have asset holdings of the order of the asset market depth $\frac{Q}{\Phi}\sim 10\%$. We naturally assume that $\frac{Q}{\Phi}<1$, which means that the bank's holdings do not exceed the market depth.

While liquidating a portion y of its trading book, the bank makes the value of marketable assets decrease from V_t to $V_t\left(1-\frac{yQ}{\Phi}\right)$. In practice, the bank is not able to sell all yQ marketable assets at price V_t but rather sells smaller blocks of marketable assets at intermediate prices between V_t and $V_t\left(1-\frac{yQ}{\Phi}\right)$. Assuming that those sales are done in a *uniform and regular manner*, the average selling price is equal to $\frac{1}{2}\left(V_t+V_t\left(1-\frac{yQ}{\Phi}\right)\right)=V_t\left(1-\frac{yQ}{2\Phi}\right)$. As a consequence, the proceeds of the liquidation of a fraction y of the trading book is equal to $yQV_t\left(1-\frac{yQ}{2\Phi}\right)$. For assets that are *marked-to-market*, the fraction not sold has to be evaluated at fair value, i.e., at the new market value equal to $(1-y)QV_t\left(1-\frac{yQ}{\Phi}\right)$. As a consequence, the balance-sheet of the bank at $t+1$, after liquidation of a fraction x of the banking book and a fraction y of the trading book, is given as follows:

[7] We choose a linear price impact for clarity purpose only. Some empirical studies indeed show that price impact is linear at intraday ([Cont et al., 2014]) and daily ([Kyle and Obizhaeva, 2010]) frequencies. Other empirical studies find that price impact is not linear but concave ([Almgren et al., 2005, Moro et al., 2009, Bence et al., 2011]). The calculations could be adjusted to such price impact functions.

Balance-sheet at time $t+1$ after rebalancing

Assets	Liabilities
Cash: $\lambda x q v_t(1-\Delta) + yQV_t(1-\frac{yQ}{2\Phi})$ Loans: $(1-x)qv_t(1-\Delta)$ Marketable assets $(1-y)QV_t(1-\frac{yQ}{\Phi})$	Debt: D Equity: E_{t+1}
$A_{t+1} = [1-x(1-\lambda)]qv_t(1-\Delta) + QV_t[1-\frac{yQ}{\Phi}(1-\frac{y}{2})]$	$E_{t+1}+D$

The optimization problem is now given by:

$$\min_{(x,y)\in[0,1]^2} L(x,y) := x\lambda q v_t(1-\Delta) + yQV_t\left(1-\frac{yQ}{2\Phi}\right) \tag{24}$$

$$\text{s/t} \quad \frac{[1-x(1-\lambda)]qv_t(1-\Delta) + QV_t(1-\frac{yQ}{\Phi}(1-\frac{y}{2})) - D}{\alpha(1-x)qv_t(1-\Delta) + \beta(1-y)QV_t(1-\frac{yQ}{\Phi})} \geq \theta_{min} \tag{25}$$

and is more complex because both the objective function and the constraint are non-linear functions of y. After liquidation of a fraction x of loans and a fraction y of marketable assets, the bank's equity is equal to:

$$E_{t+1}(x,y) = \max\left\{[1-x(1-\lambda)]qv_t(1-\Delta) + QV_t\left[1-\frac{yQ}{\Phi}\left(1-\frac{y}{2}\right)\right] - D; 0\right\}$$

which can be written as:

$$E_{t+1}(x,y) = \max\left\{E_t - qv_t\Delta - xqv_t(1-\Delta)(1-\lambda) - \frac{yQ^2V_t}{\Phi}\left(1-\frac{y}{2}\right); 0\right\}$$

When equity is positive, it is a decreasing function of x and y, which means that the bank loses equity while liquidating, due to discount when selling loans and price impact when selling marketable assets. We define once again a critical threshold $\Delta^*_{d'}$ such that the bank's equity remains strictly positive even if it liquidates its whole portfolio ($x=y=1$). As a consequence, $\Delta^*_{d'}$ verifies:

$$E_t - qv_t\Delta^*_{d'} - xqv_t(1-\Delta^*_{d'})(1-\lambda) - \frac{yQ^2V_t}{\Phi}\left(1-\frac{y}{2}\right) = 0$$

which gives

$$\Delta^*_{d'}(\lambda,\Phi) := \Delta^*_{d'} = 1 - \frac{1}{\lambda}\left(1-\Delta^*_{liq}\right) - \frac{Q^2V_t}{2\Phi\lambda qv_t} = \Delta^*_d - \frac{1}{2}\,\frac{QV_t}{\lambda qv_t}\,\frac{Q}{\Phi} \tag{26}$$

where $\frac{Q}{\Phi} < 1$. From equation (26), it is easy to see that as long as $\frac{Q}{\Phi}$ is strictly positive, $\Delta^*_{d'} < \Delta^*_d$. By construction, when the positive shock $\Delta < \Delta^*_{d'}$, the bank's equity remains strictly positive, *regardless* of the proportion of loans and marketable assets it liquidates. The next proposition is the equivalent of proposition 3, that is, the bank is able to remain solvent after the liquidation process although it has a positive price impact.

Proposition 5 ***(Fire sales with discount on loans and price impact on marketable assets)*** *Assume that $\Delta^*_{d'} > 0$ and that the shock Δ on loans is such that*

$$\Delta^*_{sale} < \Delta < \Delta^*_{d'}$$

- *If $\lambda < 1 - \alpha\theta_{min}$, the bank should liquidate marketable assets only and is able to bring back its capital ratio to θ_{min}, i.e., there exists a positive $y^* < 1$ such that $\theta_{t+1}(0, y^*) = \theta_{min}$.*
- *If $\lambda \geq 1 - \alpha\theta_{min}$, there exists a critical threshold $\lambda^* = \frac{1-\alpha\theta_{min}}{1-\beta\theta_{min}}$ such that:*
  - *if $\lambda > \lambda^*$ or if $\frac{Q}{\Phi} > max\left\{\frac{\lambda^*}{\lambda} - 1; 2(1 - \frac{\lambda}{\lambda^*})\right\}$, it is optimal for the bank to sell only the loans. If this is not enough to restore the regulatory capital ratio, it is optimal for the bank to sell 100% of the loans and a positive fraction y^* of the marketable assets such that $\theta_{t+1}(1, y^*) = \theta_{min}$.*
  - *if $\frac{Q}{\Phi} < \frac{\lambda^*}{\lambda} - 1$ or if $\frac{Q}{\Phi} < 2(1 - \frac{\lambda}{\lambda^*})$ (which implies that $\lambda < \lambda^*$), it is optimal for the bank to sell only the marketable assets. If this is not enough to restore the regulatory capital ratio, it is optimal for the bank to sell 100% of the marketable assets and a positive fraction of loans x^* such that $\theta_{t+1}(x^*, 1) = \theta_{min}$.*
  - *if $\lambda = \lambda^*$ and $\frac{Q}{\Phi} = 0$, the bank is indifferent liquidating loans or marketable assets first.*

When shrinking the size of its portfolio in order to comply with regulatory capital requirements, the bank is faced with a trade-off: selling loans, with associated risk weight α and which can be done at a discount $1 - \lambda$ and selling a portion of its Q marketable assets, with associated risk weight β and which generates a price impact proportional to $\frac{1}{\Phi}$. Proposition 5 exhibits the optimal liquidation strategy for the bank, depending on the value of those parameters. It shows that the optimal strategy depends on the value of $\frac{Q}{\Phi}$ compared to $\frac{\lambda}{\lambda^*}$, where $\lambda^* = \frac{1-\alpha\theta_{min}}{1-\beta\theta_{min}}$ is the loan discount threshold introduced in section 5.1. The greater $\frac{Q}{\Phi}$, the greater the bank's price impact when selling marketable assets. Similarly, the lower $\frac{\lambda}{\lambda^*}$, the larger the discount when selling loans. Propositon 5 quantifies the threshold values of $\frac{Q}{\Phi}$ and $\frac{\lambda}{\lambda^*}$ which determine the bank's optimal liquidation strategy. As expected intuitively, when $\frac{\lambda}{\lambda^*}$ is large enough, it is optimal for the bank to sell loans first. On the contrary, when $\frac{Q}{\Phi}$ is low enough, it is optimal to start by selling marketable assets. More interestingly, we see that when $\lambda < \lambda^*$, i.e., liquidating loans is done at a significant discount, it is still optimal for the bank to liquidate loans first as long as $\frac{Q}{\Phi} > max\{\frac{\lambda^*}{\lambda} - 1; 2(1 - \frac{\lambda}{\lambda^*})\}$, because the price impact associated to selling marketable assets is too large. When $\frac{Q}{\Phi} = 0$, it is easy to see that proposition 5 reduces to proposition 3. When the shock on the banking book is such that $\Delta > \Delta^*_{d'}$, the bank's equity may be wiped out during the fire sales process, due to the existence of a discount when selling loans and a price impact when selling marketable assets. The following proposition quantifies the different situations faced by the bank.

Proposition 6 *Assume that the shock Δ on loans is such that*

$$\Delta^*_{d'} < \Delta < \Delta^*_{liq}$$

and denote

$$\Delta^*_{i'} := \Delta^*_i - \left(\frac{QV_t}{2qv_t(1-\alpha\theta_{min})}\right)\frac{Q}{\Phi} \tag{27}$$

- *if $\lambda < 1 - \alpha\theta_{min}$, then $\Delta^*_{i'} > \Delta^*_{d'}$ and there are two cases.*
  - *if $\Delta < \Delta^*_{i'}$, the bank can reach its target regulatory capital ratio θ_{min} by liquidating marketable assets only.*
  - *If $\Delta > \Delta^*_{i'}$, liquidating marketable assets is not enough to bring the bank's capital ratio above θ_{min}, i.e., $\theta_{t+1}(0,1) < \theta_{min}$ and the bank is forced to go bankrupt.*
- *if $\lambda > 1 - \alpha\theta_{min}$ and $\Delta < \Delta^*_d - \frac{1}{\lambda}\left(\frac{\beta\theta_{min}QV_t}{qv_t}\right)$, the bank can reach its target regulatory capital ratio θ_{min} by liquidating loans only. Otherwise, the bank cannot reach its target regulatory capital ratio by engaging in fire sales and is forced to go bankrupt.*

The first part of proposition 6 exhibits a new threshold $\Delta^*_{i'}$ which explicitly depends on the price impact $\frac{Q}{\Phi}$. Without price impact, (i.e., $\frac{Q}{\Phi} = 0$ because $\Phi = \infty$), it is always the case that $\Delta^*_{i'} = \Delta^*_i > 0$ and we know from proposition 4 that as long as $\Delta < \Delta^*_i$, the bank still has the possibility to liquidate its marketable assets to bring back its capital ratio. However, when the bank has a price impact when selling marketable assets, i.e., $\frac{Q}{\Phi} > 0$, $\Delta^*_{i'} < \Delta^*_i$. It turns out that there is a region, namely the interval $(\Delta^*_{i'}, \Delta^*_i)$ such that if $\Delta \in (\Delta^*_{i'}, \Delta^*_i)$, then the bank has to go bankrupt, see Fig. 2. It is important to point out that this situation of bankruptcy arises due to the existence of a positive price impact. Without price impact, the bank would have been able to bring back its capital ratio to the authorized level. This case is interesting as it clearly shows that the room for bankruptcy (or insolvency) *increases* with the severity of the price impact, measured by the parameter $\frac{Q}{\Phi}$. Everything else being the same, the higher the value of the price impact $\frac{Q}{\Phi}$, the more room there is for insolvency. This situation thus reveals the danger of the existence of a positive price impact for a large bank such as a G-SIB when it has to liquidate a large portion of its assets after a shock. This finding strongly supports the existence of a capital surcharge for such large banks, as in implemented Basel III.

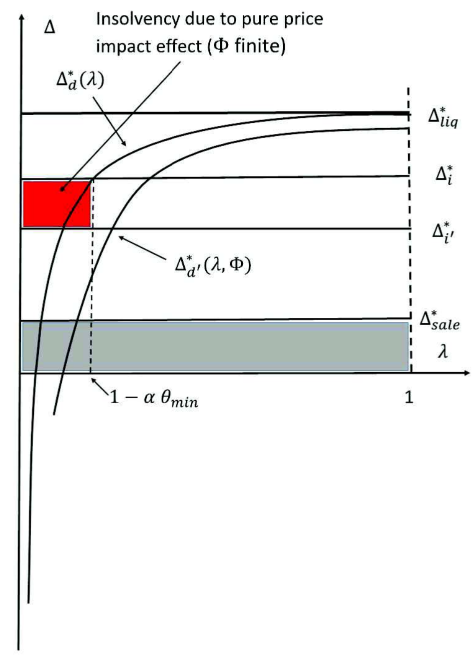

Figure 2: The bank can not restore its capital ratio due to the price impact.

The second part of the proposition is interesting and does not actually appear in proposition 4. When $\lambda > 1 - \alpha\theta_{min}$, if $\Delta < \Delta_d^* - \frac{1}{\lambda}\left(\frac{\beta\theta_{min}QV_t}{qv_t}\right)$ by liquidating loans only, the bank is indeed able to restore its capital ratio and this is why this threshold is independent of $\frac{Q}{\Phi}$. However, when $\frac{Q}{\Phi}$ tends to zero, $\Delta_{d'}^*$ converges toward Δ_d^* and as a result, the situation in which $\Delta < \Delta_d^* - \frac{1}{\lambda}\left(\frac{\beta\theta_{min}QV_t}{qv_t}\right)$ can not exist. This explains why this scenario is nonexistent in proposition 4.

The region in red on Fig 2 in which $\lambda < 1 - \theta_{min}$ illustrates the situation in which the bank is unable to bring back its capital ratio above the minimum authorized θ_{min} and is thus insolvent due to the existence of price impact.

6 Stress-test analysis

6.1 Model calibration : Implied risk-weights

Let t be a date of publication of a registration report and denote A_t^{Bank} and A_t^{Trad} the observed value of the banking and trading book respectively. Equations (4) and (6) can be re-written as follows:

$$A_t = A_t^{Bank} + A_t^{Trad} \tag{28}$$

$$\mathrm{RWA}_t = \alpha A_t^{Bank} + \beta A_t^{Trad} \tag{29}$$

In our model, there are only two types of risk, market risk (Mkt) and credit risk (Cdit). As a consequence, by construction,

$$\mathrm{RWA}_t = \mathrm{RWA}_t^{Cdit} + \mathrm{RWA}_t^{Mkt} \tag{30}$$

Since it is assumed that the banking book is subject to credit risk only and that the trading book is subject to market risk only, it naturally follows that the risk-weighted assets for credit risk and market risk can be written as:

$$\mathrm{RWA}_t^{Cdit} = \alpha A_t^{Bank} \quad \text{and} \quad \mathrm{RWA}_t^{Mkt} = \beta A_t^{Trad} \tag{31}$$

The two quantities RWA_t^{Cdit} and RWA_t^{Mkt} are always publicly disclosed in the annual report of the bank. Moreover, when A_t^{Bank} and A_t^{Trad} are disclosed – this is the case for example for BNP in 2014 – one can thus imply α and β as follows:

$$\alpha = \frac{\mathrm{RWA}_t^{Cdit}}{A_t^{Bank}} \quad \text{and} \quad \beta = \frac{\mathrm{RWA}_t^{Mkt}}{A_t^{Trad}} \tag{32}$$

which are an average aggregate measure of the risk-weights implied from publicly available data and that enable to calibrate our model and hence conduct a stress-test analysis.

6.2 The case of BNP Paribas

In this subsection, we examine the case of BNP Paribas using registration report as of 2014[8], which explicitly provides the split between banking book and trading book, enabling us to calibrate our model. When this split is not explicitly available, an outsider needs to make additional assumptions to obtain it although the assignments of the most important items should be fairly clear. In their consultative document on the fundamental review of the trading book, [BCBS, 2013] notes p.8 that banks must "*disclose their policies for the assignment of instruments to the trading book or banking book and make available such documentation to supervisors*". As a result, supervisors are in a position to obtain more information than outsiders on each bank. All quantities involved in what follows are expressed in euros.

[8]The document can be downloaded at the following address https://invest.bnpparibas.com/en/annual-reports

6.2.1 Balance-sheet split by trading and banking books

The annual report (p.326) details the assets of BNP, which can be decomposed into the banking book $A_t^{Bank} = 1170,99$ billion and the trading book $A_t^{Trad} = 726.86$ billion. Total assets are hence equal to $A_t := A_t^{Bank} + A_t^{trad} = 1897.856$ billion. On the liability side, we use the information provided on the regulatory capital (Basel 3, phased in) given in table 3 p.262, where it is reported that the value of Tier 1 is equal to 70.378 billion and the value of Tier 2 is equal to 6.79 billion. Since the total value of the assets must be equal to the total value of the liabilities, we can imply[9] the value of BNP's (other) liabilities. Before calibrating our model using the data contained in the registration report of BNP Paribas as of 2014, let us examine its balance-sheet given below more in detail.

Prudential balance-sheet split by trading and banking books

Assets	Liabilities and equity
Banking book : 1170,99	**Total liabilities: 1820.688**
Cash : 117.663	
Due from credit institutions: 38.02	**Tier 1: 70.378**
Loans and receivables due from customers: 664.769	Retained earning: 50.182
Available-For-Sale : 150.522	Capital instruments: 20.196
Derivatives used for hedging purposes: 19.7	
Other assets : 180.3	
Trading book: 726.86	**Tier 2: 6.790**
Trading securities: 145.902	
Loans and repurchase agreements: 171.101	
Derivatives financial instruments: 409.863	**Total capital: 77.168**
$A_t = 1897.856$	1897.856

Banking book. As expected, the most important item is customer loans, equal to 665 billion, which shows that the lending activity is the main business activity of BNP Paribas. Two other important items are 1) the cash and amount due from central banks equal to 118 billion euros, which constitute highly liquid assets and 2) the Available-for-sale financial

[9] This situation arises because the total value of equity is equal to 93.5 billion of euros while the regulator obtains a smaller amount, equal to 77.16 billion of euros, due the application of a more stringent definition of capital. The implied value of liabilities equal to 1820.68 is thus obtained by subtracting 77.16 to 1897.85.

assets (AFS), for a value equal to 150 billion[10]. From the registration report p.174, we know that more than 90% of the AFS are T-bills and governments bonds, which are market instruments with low volatility. It is further reported in table 65 p.343 that almost 100% of the Sovereign exposure of the banking book are Central governments securities (Sovereign bonds), including a 70% exposure to euro-zone sovereign debt. As expected, the group holds sovereign bonds as part of its liquidity management strategy, i.e., it holds securities that are eligible as collateral for (possible) refinancing at ECB. Regarding now the position (on the asset side) on derivatives used for hedging purpose, it is approximately equal to 1% of the total value of the assets. For instance, p.341, it is reported that interest rate risk in the banking book is hedged using instruments such as swaps and options and a discussion is provided about the current environment, characterized by low interest rates.

Trading book. The most important item is derivatives financial instruments. According to the group p.172, "*the majority of derivatives financial instruments held for trading are related to transactions initiated for trading purposes. They may result from market-making or arbitrage activities*". In the registration report p.173, interest rate derivatives are accounted for an amount equal to 295.65 billion of euros, that is, approximately 70% of the total mark-to-market value of the derivatives financial instruments of the trading book[11]. It is further reported that foreign exchange derivatives and equity derivatives are accounted for 57.2 and 33.11 billion respectively. No information is unfortunately provided regarding the type of derivatives and their values. However, p.181, the group reports some information about the type of products and the unobservable inputs for the product under consideration. For instance, for interest rate derivatives, unobservable inputs are forward volatility of interest rate or the volatility of the cumulative inflation and range of parameters are provided, e.g., from 0.8% to 10% for the volatility of the cumulative inflation.

6.2.2 Implied values for α and β

We here follow the methodology developed in section 6.1 to imply the two (aggregate) risk-weights α and β for BNP Paribas. In practice, things are naturally more complex than our model assumptions because banks typically have RWA for other types of risk than market and credit risk. As a result, the way those other RWAs are assigned to credit or market must be explained. Let RWA_t^{Risk} be the RWA computed for a specific risk and let $\mathcal{R}$ be the set of risk types. From the annual report p. 265, the RWA_t^{Risk} is reported

[10] The total value of the AFS in accounting scope is equal to 250 billion of euros. No clear explanation is given for this important difference between the prudential scope and the accounting scope.

[11] Note that 0.6% of these derivatives financial instruments are assigned to the banking book. Note also that most interest rate derivatives are level 2.

for credit, securitization, counterparty, equity, market and operational risk and the values, in billion euros, are respectively $\text{RWA}_t^{Credit} = 442.358, \text{RWA}_t^{Secur.} = 13.988, \text{RWA}_t^{Count.} = 29.995, \text{RWA}_t^{Equity} = 58.693, \text{RWA}_t^{Market} = 20.357, \text{RWA}_t^{Oper.} = 54.433$. The (total) RWA is thus equal to

$$\text{RWA}_t := \sum_{Risk \in \mathcal{R}} \text{RWA}_t^{Risk} = 619.827 \tag{33}$$

$$= \text{RWA}_t^{Cdit} + \text{RWA}_t^{Mkt} \tag{34}$$

Since they are more than two partial risk-weighted assets, we now explain how, for each risk, whether RWA_t^{Risk} contributes to RWA_t^{Cdit}, the risk-weighted assets for credit risk, to RWA_t^{Mkt}, the risk-weighted assets for market risk, or both. It is clear that the RWA for credit risk and the RWA for securitization, called indeed banking book securitization positions, are related to the banking book so that RWA_t^{Credit} and $\text{RWA}_t^{Secur.}$ contribute to RWA_t^{Cdit}. In the same way, RWA_t^{Equity} and RWA_t^{Market} contribute to RWA_t^{Mkt} as they are related to the trading activity of the bank. Things are not so clear for operational risk and counterparty risk. Since counterparty risk is defined by the Group p. 319 as the "*credit risk embedded in financial transactions between counterparties*", this RWA is related to the trading activity of the bank and thus should count as market risk. For operational risk, we use the information provided p. 264 of the registration report, in which the risk-weighted assets is distributed by risk type and business. From this table, one can see that RWA_t^{Oper} is related to the retail banking activity of the bank for an amount equal to 24 billion and the rest is related to the corporate and investment banking activity and to investment solutions. We thus add these 24 billion to the RWA for credit and the rest 54-24=30 billion to the RWA for market. As a result, we find that $\text{RWA}_t^{Cdit} = 442.358 + 13.988 + 24 \approx 481$ billion of euros and that $\text{RWA}_t^{Mkt} = 30 + 59 + 20 + 30 = 139$ billion. Using now (32), we thus obtain:

$$\alpha = \frac{\text{RWA}_t^{Cdit}}{A_t^{Bank}} = \frac{481}{1170,99} \approx 41\% \tag{35}$$

$$\beta = \frac{\text{RWA}_t^{Mkt}}{A_t^{trad}} = \frac{139}{726.866} \approx 19\% \tag{36}$$

6.2.3 Stress-testing BNP Paribas

Common Equity Tier 1 is reported p. 262 to be equal to $\text{CET1}_t = 64.47$ billion It thus follows that:

$$\frac{\text{CET1}_\text{t}}{\text{RWA}_t} = \frac{64.47}{619.82} \approx 10.4\% \quad \text{(required : 4\%)} \tag{37}$$

$$\frac{\text{Tier1}_\text{t}}{\text{RWA}_t} = \frac{70.378}{619.82} \approx 11.35\% \quad \text{(required : 5.5\%)} \tag{38}$$

$$\frac{E_t}{\text{RWA}_t} = \frac{77.168}{619.82} \approx 12.5\% \quad \text{(required : 8\%)} \tag{39}$$

From the above equations, it appears that the regulatory constraint that BNP Paribas is the most likely to violate after a shock is the third one, given formally in equation 3. We hence choose $\theta_{min} = 8\%$ for our empirical study. Given the parameters of BNP Paribas described above and following proposition 1, we find:

$$\Delta_{sale} = 2.44\% \quad \text{and} \quad \Delta_{liq} = 6.6\%$$

This means that a shock greater than 6.6% in the banking book is sufficient to wipe out all the equity of the French bank and make it insolvent. When the banking book suffers a percentage loss lower than 2.44%, the cushion of equity built up by BNP Paribas is enough not only to absorb the shock and keep the bank solvent but also to maintain its regulatory capital ratio above 8%. The interesting case is when the shock in the banking book lies between 2.44% and 6.6%. As described throughout the paper, such a shock is absorbed by the bank's equity but leaves the bank with a regulatory capital ratio which is lower than 8%, forcing the bank to shrink the size of its assets. If BNP Paribas is able to sell its assets at fair value, since $\alpha > \beta$, our benchmark model described in section 4 thus predicts that it is optimal for the bank to first liquidate assets in the banking book in order to restore its regulatory capital ratio. If this is not enough, BNP Paribas should liquidate the banking book in full and part of the trading book. This happens when the shock in the banking book is greater than 5.6%. This situation is illustrated in Figure 3, which displays the volume (in billion euros) of asset sales as a function of the magnitude of the losses in the banking book. It is important to point out that liquidating 100% of the banking when Δ is higher than 5.6% is obviously not a realistic solution. In practice, the maximal amount which is possible to liquidate is unknown but is much lower than 100%. This is an interesting point as this clearly suggests that BNP Paribas could be in trouble with an adverse shock which is far from being an extreme one.

In the presence of market frictions, the consequences of an exogenous shock may differ: depending on the magnitude of the discount (when selling assets in the banking book) and

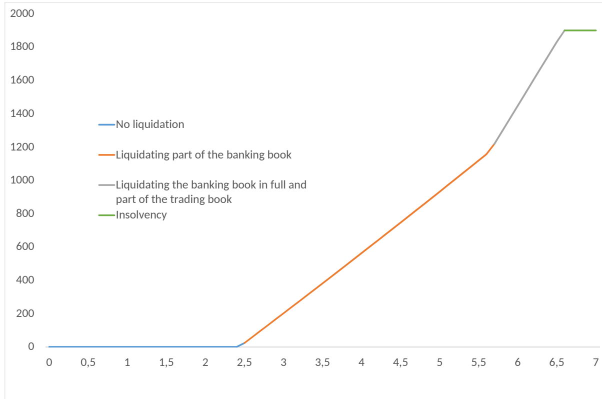

Figure 3: Volume (in billion euros) of liquidation for BNP (Y axis) as a function of a shock Δ in the banking book (X axis) in the benchmark model (no adverse selection, no price impact)

the price impact (when liquidating assets of the trading book), it may be optimal for BNP Paribas to start by liquidating the trading book rather than the banking book in order to comply with regulatory capital requirements. In some cases, the deleveraging process itself may lead to the insolvency of the bank, i.e., to a death spiral. Tables 1 and 2 illustrate, for a given shock, respectively of 2.5% and 5%, whether the bank is able to readjust its portfolio without being led to insolvency. Note that, in those tables, when the bank is trapped in a spiral to insolvency, this is only due to market frictions and the fact that the bank deleverages its portfolio in order to restore its capital ratio.

Consider the case where the shock on the banking book is equal to 2.5% (table 1). In that situation, when there is no discount when selling loans ($\lambda = 1$), the bank is able to "survive" the deleveraging process even if the assets in the trading book are not infinitely liquid ($\frac{Q}{\Phi}$ =5% or 10%). Similarly, when there is no price impact when selling marketable assets ($\frac{Q}{\Phi} = 0$), the bank remains solvent after the liquidation process even if loans are sold at a 5% (i.e. $\lambda = 0.95$) or 10% (i.e. $\lambda = 0.90$) discount. However, in this particular case of discount when selling loans, it becomes optimal for the bank to start by liquidating the trading book first. When $\lambda = 0.95$ or 0.90 and $\frac{Q}{\Phi} = 5\%$ or 10%, market frictions generate a spiral to insolvency, i.e., a death spiral. In the case when $\lambda = 0.975$, the bank survives the deleveraging process, even in the case of positive price impact, thanks to an acceptable discount when selling loans. In addition, we see that when the shock is larger (equal 5%, as displayed in table 2), even if there is no price impact ($\frac{Q}{\Phi} = 0$), the bank is trapped in a spiral to insolvency in the presence of a 2.5%, 5% or 10% discount when liquidating the banking book, which is not the case when the shock is equal to 2.5%.

$\frac{Q}{\Phi}$ \ λ	1	0.975	0.95	0.90
0	Survival	Survival	Survival	Survival
0.05	Survival	Survival	Spiral to default	Spiral to default
0.10	Survival	Survival	Spiral to default	Spiral to default

Table 1: Consequences of a shock $\Delta = 2.5\%$

$\frac{Q}{\Phi}$ \ λ	1	0.975	0.95	0.90
0	Survival	Spiral to default	Spiral to default	Spiral to default
0.05	Survival	Spiral to default	Spiral to default	Spiral to default
0.10	Survival	Spiral to default	Spiral to default	Spiral to default

Table 2: Consequences of a shock $\Delta = 5\%$

The (il)liquidity parameter $\frac{Q}{\Phi}$ represents the size of the trading book as a fraction of market depth (or liquidity). The greater this parameter, the greater the price impact when liquidating assets in the trading book. In our empirical study, we use values of $\frac{Q}{\Phi}$ equal to 0, 5% and 10%. This is consistent with empirical studies [Gonnard et al., 2008, Cont and Wagalath, 2016] which show that the asset holdings of large institutional investors typically range from 5% to 20% of the market depth. This choice is also in line with the liquidity parameter used in [Greenwood et al., 2015] and equal to $\frac{1}{\Phi V_t} = 10^{-13}$ which would give, in the case of BNP Paribas, a liquidity parameter equal to approximately 5%.

7 Policy implication : capital surcharge for large banks

Within our framework, the bank's equity may remain positive after a shock although the bank is unable to bring back its capital above the authorized level because of the interaction between market (il)liquidity and solvency. As this inability to restore the capital ratio is due to a pure market imperfection, say a positive price impact, it seems natural from a regulatory point of view to require from that bank a higher loss absorbency (HLA). More generally, the HLA requirement should be a function of the "systemicness" of the bank. As already said in the introduction of this paper, in [Greenwood et al., 2015], the systemicness of a bank is measured as the product of three factors, the impact of one bank on the rest of the banking sector, the relative size of that bank and finally the size of the fire sales. In their table 3, [Greenwood et al., 2015] propose a ranking of banks according to their systemicness and BNP Paribas is in the top five. Regulators also share the view that (see [Board, 2013]) large financial institutions must have (and indeed will have) HLA capacity to reflect the greater risks that they pose to the financial system. The systemicness of a bank, as defined in [Board, 2013], is measured using five unweighted indicators – size, interconnectedness, complexity, substituability and cross-jurisdictional activity – which leads to the computation of a final score in basis points[12]. The regulator considers five tranches (called buckets), each with a range of 100 bps and the capital surcharge is function of the tranche where the final score is. The first tranche (in bps) is 130-229 and requires a capital surcharge of 1% of CET 1 while the last tranche is 530-629 and requires a capital surcharge of 3.5% of CET 1. We shall now use the first part of our proposition 6 to illustrate how it could be used to determine the capital surcharge due to a positive price impact for the bank BNP Paribas. In proposition 6, we have shown that when $\lambda < 1-\alpha\theta_{min}$ and $\Delta > \Delta^*_{i'}$, then it is optimal to sell marketable assets only, but the bank is unable to restore its capital ratio. For BNP Paribas, we have seen that $\alpha = 41\%$ and $\beta = 19\%$ so that $1-\alpha\theta_{min} = 96.72\%$. Let us assume that $\lambda = 95\%$, which implies that $\lambda < 1-\alpha\theta_{min}$ and a price impact parameter $\frac{Q}{\Phi} = 3\%$, following the discussion of section 6.2.3. Recall that $qv_t = 1171$, $E_t = 77.16$ so that $\Delta_{liq} = \frac{E_t}{qv_t} = 6.6\%$. Using the numerical values, $\Delta^*_i = \frac{\Delta^*_{liq}-\alpha\theta_{min}}{1-\alpha\theta_{min}} = 3.43\%$ and $\left(\frac{QV_t}{2qv_t(1-\alpha\theta_{min})}\right)\frac{Q}{\Phi} = 0.9626\%$ so that the threshold $\Delta^*_{i'} = 3.43\% - 0.9626\% = 2.467\%$. Let us assume that $\Delta = 2.5\%$ so that $\Delta > \Delta^*_{i'}$. The scenario we consider here thus is

$$\Delta = 2.5\%,\ \lambda = 95\%,\ \frac{Q}{\Phi} = 3\%$$

In such a case, from proposition 6, the optimal liquidation strategy is $(x^* = 0, y^* = 1)$ and the regulatory capital given in equation (25) reduces after rebalancing to the following formula

$$\theta_{t+1}(0,1,\Delta) = \frac{E_t - qv_t\Delta - (\frac{Q}{\Phi})\frac{QV_t}{2}}{\alpha qv_t(1-\Delta)}$$

Using the numerical values, this gives $\theta_{t+1}(0,1,2.5\%) = \frac{37}{468.11} = 7.9\%$, which means that 0.1% of capital is missing. Since regulators define the capital surcharge in terms of CET1, we denote by κ the percentage surcharge of CET1 such that:

$$\theta_{t+1}(0,1,\Delta,\kappa) = \frac{E_t + \kappa\ \mathrm{CET1}_t - qv_t\Delta - (\frac{Q}{\Phi})\frac{QV_t}{2}}{\alpha qv_t(1-\Delta)} \geq \theta_{min}$$

We know that $\mathrm{CET1}_t$ is equal to 64.47 bn, that Tier 1 is equal to 70.378 bn and finally that the total capital is equal to 77.168 bn. It is easy to see that if $\kappa\ \mathrm{CET1}_t \geq 0.45$ bn, then

[12]See BCBS (2014), "*The G-SIB assessment methodology-score calculation*", Bank for international Settlements.

$\theta_{t+1}(0, 1, \Delta, \kappa) \geq \theta_{min}$ which means that $\kappa \geq 0.70\%$. By requiring a capital surcharge at time t equal say to 1% of CET1_t, i.e. the minimum capital surcharge planned by regulators for GSIBs, in principle, this would avoid the insolvency situation of BNP Paribas in our scenario in which $\lambda = 95\%$, $\frac{Q}{\Phi} = 3\%$ and $\Delta = 2.5\%$.

To put this result in a more practical perspective, it is important to recall that we made the assumption that BNP Paribas is able to sell 100% of the trading book which is far from being a realistic assumption. The total value of the derivatives financial instruments is equal to 410 bn for the asset side but is equal to 408 bn on the liability side. As a consequence, liquidating an important portion of the derivatives on the asset side would clearly generate a higher market risk due to the important difference between the asset side and the liability side. If we take this liquidation constraint into consideration, the capital surcharge should be much higher...

8 Conclusion

We have presented in this paper a simple stress-test framework which is well-founded from a micro prudential point of view in that we explicitly consider a risk-based capital. Yet, our framework is able to capture some macro effects on the bank's situation such as its price impact when it liquidates marketable assets. We have shown that due to the strong interaction between (market) liquidity and solvency, the deleveraging process of the bank after an adverse shock may generate a death spiral due only to market imperfections (e.g., price impact). We thus suggested a possible way to determine, as a fraction of CET1 of the bank, the capital surcharge of that bank. For BNP Paribas, this leads to a capital surcharge of 1% under the unrealistic assumption that BNP Paribas is able to liquidate all the assets of the trading book. The case of BNP Paribas is interesting since we explicitly make use of the balance-sheet split between trading and banking books. As the ratio CET 1 divided by RWA is much higher than the required threshold, it was only relevant to focus on the global RBC, i.e., total capital divided by RWA. It would however be interesting to analyze the case of a bank such that the ratio CET1 divided by RWA is lower than the required one after a shock. More generally, as regulators will now make use of various ratios, i.e., RBC, leverage ratio but also two liquidity ratios, it would be of interest to analyze, as suggested in [BCBS, 2015b] p.9, the optimal response of the bank as a function of the ratio which is not satisfied.

9 Appendix

Proof of proposition 1.

To obtain the expression of the thresholds, consider first the liquidation scenario. By definition, Δ^*_{liq} is the smallest value of Δ such that the equity is wiped out, that is, $E_{t+1} = qv_t(1-\Delta^*_{liq}) + QV_t - D = 0$. Solving this equation yields Δ^*_{liq}. Since $\theta_t > \theta_{min}$, the threshold Δ^*_{sale} is defined as the smallest value of Δ such that $\frac{qv_t(1-\Delta^*_{sale}(\alpha,\beta))+QV_t-D}{\alpha qv_t(1-\Delta^*_{sale}(\alpha,\beta))+\beta QV_t} = \theta_{min}$. Solving this equation yields $\Delta^*_{sale}(\alpha,\beta)$ $\square$

Proof of proposition 2.

Existence of a solution to the bank's optimization problem (see paragraph before proposition 2) has already been discussed. For the optimal (x,y), it is clear that the constraint is binding, that is $\theta_{t+1}(x,y) = \theta_{min}$. Equation 15 can be written $\theta_{t+1}(x,y) = \frac{E_t - qv_t\Delta}{\mathrm{RWA}_t - \alpha qv_t\Delta - \alpha xqv_t(1-\Delta) - y\beta QV_t}$ and so the optimal (x,y) verify:

$$\frac{E_t - qv_t\Delta}{\mathrm{RWA}_t - \alpha qv_t\Delta - \alpha xqv_t(1-\Delta) - y\beta QV_t} = \theta_{min} \tag{40}$$

which implies that:

$$xqv_t(1-\Delta) = \frac{-\beta}{\alpha} yQV_t + \frac{1}{\alpha}\left(\mathrm{RWA}_t - \alpha qv_t\Delta - \frac{E_t - qv_t\Delta}{\theta_{min}}\right) \tag{41}$$

and so the optimization problem can be written:

$$\min_{y\in[0,1]} g(y) := yQV_t\left(1 - \frac{\beta}{\alpha}\right) + \frac{1}{\alpha}\left(\mathrm{RWA}_t - \alpha qv_t\Delta - \frac{E_t - qv_t\Delta}{\theta_{min}}\right) \tag{42}$$

Since $g'(y) = QV_t\left(1 - \frac{\beta}{\alpha}\right)$, this naturally means that if $\left(1 - \frac{\beta}{\alpha}\right) > 0$ (resp. <0), the bank should minimize (resp. maximize) y. When $\left(1 - \frac{\beta}{\alpha}\right) = 0$, the bank is indifferent minimizing (or maximizing) x or y first. It thus follows that if $\alpha > \beta$ (resp. $\alpha < \beta$), the bank should liquidate loans (resp. marketable assets) in priority while it is indifferent in liquidating loans or marketable assets when $\alpha = \beta$.

Assume now that $\alpha > \beta$. We know that the bank should optimally liquidate loans first and, if this is not enough to restore its capital ratio above θ_{min}, should liquidate also marketable assets. If the bank can restore its capital ratio only by liquidating loans, the optimal proportion x of loans liquidated should hence verify:

$$\theta_{t+1}(x,y) := \frac{E_t - qv_t\Delta}{\mathrm{RWA}_t - \alpha qv_t\Delta - \alpha xqv_t(1-\Delta)} = \theta_{min} \tag{43}$$

which means that, in this case, the optimal proportion of loans to liquidate is equal to $x = 1 - \frac{E_t - qv_t\Delta - \beta QV_t\theta_{min}}{\alpha qv_t(1-\Delta)\theta_{min}}$. If the bank cannot restore its capital ratio only by liquidating loans, then we have $\frac{E_t - qv_t\Delta}{\text{RWA}_t - \alpha qv_t\Delta - \alpha qv_t(1-\Delta)} < \theta_{min}$, which implies that $1 - \frac{E_t - qv_t\Delta - \beta QV_t\theta_{min}}{\alpha qv_t(1-\Delta)\theta_{min}} > 1$ and that $x = 1$. As a consequence, we can write that the optimal x as given by $x = \min(1 - \frac{E_t - qv_t\Delta - \beta QV_t\theta_{min}}{\alpha qv_t(1-\Delta)\theta_{min}}, 1)$. If $x = 1$ is optimal, then, the optimal y verifies:

$$\theta_{t+1}(1, y) := \frac{E_t - qv_t\Delta}{\beta(1-y)QV_t} = \theta_{min} \tag{44}$$

which means that $y = 1 - \frac{E_t - qv_t\Delta}{\beta QV_t\theta_{min}} > 0$. When $x < 1$ is optimal, this means that $\theta_{t+1}(1, 0) := \frac{E_t - qv_t\Delta}{\text{RWA}_t - \alpha qv_t\Delta - \alpha qv_t(1-\Delta)} > \theta_{min}$, which implies that $1 - \frac{E_t - qv_t\Delta}{\beta QV_t\theta_{min}} < 0$ and $y = 0$. So this means that we can write $y = \max(0, 1 - \frac{E_t - qv_t\Delta}{\beta QV_t\theta_{min}})$. This concludes the proof for the explicit expression for the optimal (x, y) when $\alpha > \beta$. The case where $\alpha < \beta$ is solved in a symmetric manner $\square$

Proof of proposition 3.

After liquidating a portion x of loans and y of marketable assets, the regulatory capital ratio of the bank can be written:

$$\theta_{t+1}(x, y) = \frac{E_t - \Delta qv_t - xqv_t(1-\Delta)(1-\lambda)}{\text{RWA}_t - \alpha\Delta qv_t - \alpha xqv_t(1-\Delta) - \beta yQV_t}$$

We already explained in the text why, when $\Delta < \Delta_d^*$, there exists at least one (x, y) such that $\theta_{t+1}(x, y) \geq \theta_{min}$. Obviously, for the optimal (x, y), the constraint is binding and $\theta_{t+1}(x, y) = \theta_{min}$. This means that;

$$yQV_t = \frac{1}{\beta}\left(\text{RWA}_t - \alpha qv_t\Delta - \frac{1}{\theta_{min}}(E_t - \Delta qv_t)\right) + xqv_t(1-\Delta)\left(\frac{1-\lambda}{\beta\theta_{min}} - \frac{\alpha}{\beta}\right)$$

and so minimizing $\lambda xqv_t(1-\Delta) + yQV_t$ is equivalent to minimizing:

$$xqv_t(1-\Delta)\left(\lambda + \frac{1-\lambda}{\beta\theta_{min}} - \frac{\alpha}{\beta}\right)$$

and we have

$$\lambda + \frac{1-\lambda}{\beta\theta_{min}} - \frac{\alpha}{\beta} > 0 \Leftrightarrow \lambda < \lambda^* = \frac{1-\alpha\theta_{min}}{1-\beta\theta_{min}}$$

As a consequence, if $\lambda < \lambda^*$, the bank should start by liquidating marketable assets first (ie: minimize x). If $\lambda > \lambda^*$, the bank should start by loans first (ie: maximize x). Finally, if $\lambda = \lambda^*$, the bank is indifferent liquidating loans or marketable assets first. The explicit expressions for the optimal x and y are found in the exact same way as done in the proof of proposition 2. This concludes the proof of Proposition 3.

Proof of proposition 4

$\theta_{t+1}(x,y) = \frac{E_t - \Delta qv_t - xqv_t(1-\Delta)(1-\lambda)}{\mathrm{RWA}_t - \alpha\Delta qv_t - \alpha xqv_t(1-\Delta) - \beta yQV_t}$ is clearly an increasing function of y. Let us calculate the partial derivative of the capital ratio with respect to x:

$$\frac{\partial\theta_{t+1}}{\partial x}(x,y) = qv_t(1-\Delta)\frac{\alpha(E_t - \Delta qv_t) - (1-\lambda)(\mathrm{RWA}_t - \alpha qv_t\Delta - \beta yQV_t)}{\left[RWA_t - \alpha\Delta qv_t - \alpha xqv_t(1-\Delta) - \beta yQV_t\right]^2}$$

$$\frac{\partial\theta_{t+1}}{\partial x} = qv_t(1-\Delta)\frac{\alpha E_t - (1-\lambda)(\alpha qv_t + \beta QV_t - \beta yQV_t) - \Delta qv_t\lambda\alpha}{\left[RWA_t - \alpha\Delta qv_t - \alpha xqv_t(1-\Delta) - \beta yQV_t\right]^2} \tag{45}$$

For all $y \in [0,1]$ and for all $\lambda \in [0,1[$, since $qv_t(1-\Delta)$ is positive, the following inequality is true regarding the numerator of equation (45).

$$\alpha E_t - (1-\lambda)(\alpha qv_t + \beta QV_t - \beta yQV_t) - \Delta qv_t\lambda\alpha \leq \underbrace{\alpha E_t - (1-\lambda)\alpha qv_t - \Delta qv_t\lambda\alpha}_{A(\Delta)}$$

and note that the inequality is strict when $y < 1$. Let $\Delta_d^* = 1 - \frac{1}{\lambda}\left(1 - \frac{E_t}{qv_t}\right)$ be defined such that $A(\Delta_d^*) = 0$. Since $A(\Delta)$ is a decreasing function of Δ, if $\Delta > \Delta_d^*$, for all $y \in [0,1]$ and for all $\lambda \in [0,1[$:

$$\alpha E_t - (1-\lambda)(\alpha qv_t + \beta QV_t - \beta yQV_t) - \Delta qv_t\lambda\alpha \leq A(\Delta_d^*)$$

which means that:

$$\alpha E_t - (1-\lambda)(\alpha qv_t + \beta QV_t - \beta yQV_t) - \Delta qv_t\lambda\alpha \leq 0$$

As a consequence, for all $(x,y) \in [0,1]^2$ and for all $\lambda \in [0,1[$:

$$\frac{\partial\theta_{t+1}}{\partial x}(x,y) \leq 0$$

which implies that liquidating loans will actually *decrease* the bank's capital ratio. As a consequence, in order to bring its capital ratio back above θ_{min}, the bank's only possibility is to liquidate marketable assets. The maximum capital ratio for the bank is equal to:

$$\theta_{t+1}(0,1) := \frac{E_t - \Delta qv_t}{\mathrm{RWA}_t - \alpha\Delta qv_t - \beta QV_t} = \frac{E_t - \Delta qv_t}{\alpha qv_t(1-\Delta)}$$

and the capital ratio is larger than θ_{min} if and only if

$$\Delta \leq \Delta_i^* = \frac{E_t - \theta_{min}\alpha qv_t}{qv_t(1-\alpha\theta_{min})}$$

- If $\lambda < 1 - \alpha\theta_{min}$, then, $\Delta_i^* > \Delta_d^*$. When $\Delta \in (\Delta_d^*; \Delta_i^*]$, there exists a unique solution given by $\theta_{t+1}(0,y^*) = \theta_{min}$. When $\Delta \in (\Delta_i^*; \Delta_{liq}^*)$, $\theta_{t+1}(0,1) < \theta_{min}$ so that the bankruptcy can not be avoided.

- If $\lambda > 1 - \alpha\theta_{min}$, then, $\Delta_i^* < \Delta_d^*$. Since $\Delta > \Delta_d^*$ by assumption, it thus follows that $\Delta > \Delta_i^*$. As a result, $\theta_{t+1}(0,1) < \theta_{min}$ so that the bankruptcy can not be avoided $\square$

Proof of proposition 5

When $\Delta < \Delta_{d'}^*$, the bank's equity remains strictly positive and is given by:

$$E_{t+1}(x,y) = E_t - qv_t\Delta - xqv_t(1-\Delta)(1-\lambda) - \frac{yQ^2V_t}{\Phi}(1-\frac{y}{2})$$

As a consequence, the regulatory capital ratio after rebalancing is given by

$$\theta_{t+1}(x,y) = \frac{E_t - qv_t\Delta - xqv_t(1-\Delta)(1-\lambda) - \frac{yQ^2V_t}{\Phi}(1-\frac{y}{2})}{\alpha(1-x)qv_t(1-\Delta) + \beta(1-y)QV_t(1-\frac{yQ}{\Phi})}$$

As x and y go to 1, the numerator of θ_{t+1} remains strictly positive while the denominator goes to zero, hence θ_{t+1} goes to $+\infty$ and there exist (x,y) such that $\theta_{t+1}(x,y) \geq \theta_{min}$. For the optimal (x,y), the constraint is binding and $\theta_{t+1}(x,y) = \theta_{min}$. Note that the risk-weighted assets after rebalancing can be written

$$\text{RWA}_{t+1}(x,y) = \text{RWA}_t - \alpha qv_t\Delta - \alpha xqv_t(1-\Delta) - \beta yQV_t(1+\frac{(1-y)Q}{\Phi})$$

and so

$$\theta_{t+1}(x,y) = \frac{E_t - qv_t\Delta - xqv_t(1-\Delta)(1-\lambda) - \frac{yQ^2V_t}{\Phi}(1-\frac{y}{2})}{\text{RWA}_t - \alpha qv_t\Delta - \alpha xqv_t(1-\Delta) - \beta yQV_t(1+\frac{(1-y)Q}{\Phi})}$$

We define

$$f(x,y) = E_{t+1}(x,y) - \theta_{min}\text{RWA}_{t+1}(x,y)$$

and note that $\theta_{t+1}(x,y) \geq \theta_{min} \Leftrightarrow f(x,y) \geq 0$. Since by assumption $\Delta > \Delta_{sale}^*$, we have that $f(0,0) < 0$. Furthermore, when $\Delta < \Delta_{d'}^*$, as $\lim_{x\to1,y\to1} E_{t+1}(x,y) := E_{t+1}(1,1) > 0$, we thus have that $\lim_{x\to1,y\to1} f(x,y) := f(1,1) > 0$. Given the expressions for $E_{t+1}(x,y)$ and $\text{RWA}_{t+1}(x,y)$, we can write

$$\begin{aligned} f(x,y) = {} & [E_t - qv_t\Delta - \theta_{min}(\text{RWA}_t - \alpha qv_t\Delta)] + xqv_t(1-\Delta)(\lambda - 1 + \alpha\theta_{min}) \\ & + yQV_t\left(\beta\theta_{min}(1+\frac{(1-y)Q}{\Phi}) - \frac{Q}{\Phi}(1-\frac{y}{2})\right) \end{aligned}$$

Consider the case in which $\lambda \leq 1 - \alpha\theta_{min}$. In such a case, since f is decreasing in x, it thus follows that $f(0,1) > f(1,1) > 0$. Since $f(0,0) < 0$, by continuity, there exists $y^* < 1$ such that $f(0,y^*) = 0$.

Consider now the case in which $\lambda > 1 - \alpha\theta_{min}$. For the optimal (x,y), as the constraint is binding, we have $f(x,y) = 0$ which means that

$$xqv_t(1-\Delta)(\lambda - 1 + \alpha\theta_{min}) = yQV_t\left(y(-\frac{Q}{2\Phi} + \frac{Q}{\Phi}\beta\theta_{min}) + \frac{Q}{\Phi} - \beta\theta_{min}(1+\frac{Q}{\Phi})\right)$$

$$+\theta_{min}(\text{RWA}_t - \alpha q v_t \Delta) - E_t + q v_t \Delta$$

Minimizing $L(x,y) = \lambda x q v_t(1-\Delta) + yQV_t(1-\frac{yQ}{2\Phi})$ is hence equivalent to minimizing

$$\psi(y) = \frac{y\lambda}{\lambda - 1 + \alpha\theta_{min}}\left[y(-\frac{Q}{2\Phi} + \frac{Q}{\Phi}\beta\theta_{min}) + \frac{Q}{\Phi} - \beta\theta_{min}(1+\frac{Q}{\Phi})\right] + y\left(1 - \frac{yQ}{2\Phi}\right)$$

which is quadratic in y. We find that

$$\psi'(0) = \frac{\lambda}{\lambda - (1-\alpha\theta_{min})}\left(\frac{Q}{\Phi} - \beta\theta_{min}(1+\frac{Q}{\Phi}\right) + 1$$

which means that

$$\psi'(0) > 0 \Leftrightarrow \frac{Q}{\Phi} > \frac{\lambda^*}{\lambda} - 1$$

where $\lambda^* = \frac{1-\alpha\theta_{min}}{1-\beta\theta_{min}}$. In addition, we have:

$$\psi'(1) = \left(1 - \frac{\lambda\beta\theta_{min}}{\lambda - (1-\alpha\theta_{min})}\right)\left(1 - \frac{Q}{\Phi}\right)$$

which means that

$$\psi'(1) > 0 \Leftrightarrow \lambda > \lambda^*$$

because $\frac{Q}{\Phi} < 1$. As a consequence, we find that:

$$\psi'(0) > 0 \text{ and } \psi'(1) > 0 \Leftrightarrow \frac{Q}{\Phi} > \frac{\lambda^*}{\lambda} - 1 \text{ and } \lambda > \lambda^* \Leftrightarrow \lambda > \lambda^*$$

as $\frac{Q}{\Phi} \geq 0$. In this case, ψ is increasing for $y \in [0,1]$ which means that minimizing ψ is equivalent to minimizing y. As a consequence, when $\lambda > \lambda^*$, the bank should start by liquidating loans first. Similarly, we find that:

$$\psi'(0) < 0 \text{ and } \psi'(1) < 0 \Leftrightarrow \frac{Q}{\Phi} < \frac{\lambda^*}{\lambda} - 1 \text{ and } \lambda < \lambda^* \Leftrightarrow \frac{Q}{\Phi} < \frac{\lambda^*}{\lambda} - 1$$

In this case, ψ is decreasing for $y \in [0,1]$ which means that minimizing ψ is equivalent to maximizing y. As a consequence, when $\frac{Q}{\Phi} < \frac{\lambda^*}{\lambda} - 1$, the bank should start by liquidating marketable assets first.

We now examine the case where $\psi'(0)$ and $\psi'(1)$ do not have the same sign. We observe that $\psi'(1) > 0$ implies that $\psi'(0) > 0$ and that $\psi'(0) < 0$ implies that $\psi'(1) < 0$. As a consequence, we cannot encounter the case where $\psi'(0) < 0$ and $\psi'(1) > 0$. Let us now examine the case where $\psi'(0) > 0$ and $\psi'(1) < 0$ which means that $\frac{Q}{\Phi} > \frac{\lambda^*}{\lambda} - 1$ and $\lambda < \lambda^*$. In this case, ψ is increasing and then decreasing on $[0,1]$, so it attains its minimum for $y = 0$ or $y = 1$. We have $\psi(0) = 0$ and we find that:

$$\psi(1) = 1 - \frac{Q}{2\Phi} + \frac{\lambda}{\lambda - 1 + \alpha\theta_{min}}\left(\frac{Q}{2\Phi} - \beta\theta_{min}\right)$$

and so

$$\psi(1) > \psi(0) \Leftrightarrow \frac{Q}{\Phi} > 2\left(1 - \frac{\lambda}{\lambda^*}\right)$$

and so when $\frac{Q}{\Phi} > 2\left(1 - \frac{\lambda}{\lambda^*}\right)$, ψ reaches its minimum for $y = 0$ and so the bank should start liquidating loans first. On the contrary, when $\frac{Q}{\Phi} < 2\left(1 - \frac{\lambda}{\lambda^*}\right)$, ψ reaches its minimum for $y = 1$ and so the bank should start liquidating marketable assets first.

Finally, when $\psi'(0) = \psi'(1) = 0$, ie when $\lambda = \lambda^*$ and $\frac{Q}{\Phi} = \frac{\lambda^*}{\lambda} - 1 = 0$, ψ is constant and the bank is naturally indifferent liquidating loans or marketable assets first. This concludes the proof of proposition 5.

Proof of proposition 6

We examine the case where: $\Delta^*_{d'} < \Delta < \Delta^*_{liq}$. Now, the function f defined in the proof of proposition 5 is such that $f(1,1) < 0$. Furthermore, as $\Delta^*_{sale} < \Delta$, we still have $f(0,0) < 0$. Since f is linear in x, its maximum is reached for $x = 0$ or $x = 1$, for all y. When $\lambda > 1 - \alpha\theta_{min}$ (resp. < 0), f is increasing (resp. decreasing) in x and its maximum is reached for $x = 1$ (resp. $x = 0$). Furthermore, f is quadratic in y and we have:

$$\frac{1}{QV_t}\frac{\partial f}{\partial y}(x,y) = y\frac{Q}{\Phi}(1 - 2\beta\theta_{min}) + \beta\theta_{min}(1 + \frac{Q}{\Phi}) - \frac{Q}{\Phi}$$

which implies (when evaluated at $(x, 1)$) that

$$\frac{1}{QV_t}\frac{\partial f}{\partial y}(x,1) = \beta\theta_{min}\left(1 - \frac{Q}{\Phi}\right)$$

and thus is always strictly positive as $\frac{Q}{\Phi} < 1$. As a consequence, this means that f is either increasing in $y \in [0,1]$ or decreasing and then increasing on $[0,1]$.

- If $\lambda < 1-\alpha\theta_{min}$, since f is decreasing with x, the maximum for f is $f(0,0)$ or $f(0,1)$. As $f(0,0) < 0$, there exists a liquidation strategy such that the bank reaches a capital ratio of θ_{min} if and only if $f(0,1) > 0$, which is equivalent to $\frac{Q}{\Phi} < \frac{2}{QV_t}\left(E_t - qv_t\Delta - \alpha qv_t(1-\Delta)\theta_{min}\right)$ which is in turn equivalent to $\Delta \leq \Delta^*_{i'}$. Otherwise, i.e., if $\Delta > \Delta^*_{i'}$, $f(0,1) < 0$ and the bank has to go bankrupt.

- If $\lambda > 1 - \alpha\theta_{min}$, since f is increasing with x, the maximum for f is hence $f(1,0)$ or $f(1,1)$. As $f(1,1) < 0$, there exists a liquidation strategy such that the bank reaches a capital ratio of θ_{min} if and only if $f(1,0) > 0$, which is equivalent to $\Delta < 1 - \frac{1}{\lambda}\left(1 - \frac{E_t}{qv_t} + \frac{\beta\theta_{min}QV_t}{qv_t}\right)$, which is equivalent to $\Delta < \Delta^*_d - \frac{1}{\lambda}\left(\frac{\beta\theta_{min}QV_t}{qv_t}\right)$.

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