

Determinants of banks' liquidity: a French perspective on market and regulatory ratio interactions*

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Abstract

The objective of the paper is to investigate how banks adjust the structure of their balance sheet as a response to a funding shock and to propose a methodology for projecting banks' liquidity ratios in a top-down stress test scenario. In line with a theoretical model assessing the effects of capital and liquidity constraints on banks' behaviour, we estimate the joint system of banks' solvency and liquidity ratios, using for proxy of the latter, the "liquidity coefficient" implemented in France before Basel III. We provide evidence of a positive effect of the solvency ratio on the liquidity coefficient: a high level of solvency enables the liquidity coefficient to improve due to a more stable funding structure. By contrast, we do not find firm evidence of an impact of the liquidity coefficient on the solvency ratio. We also show that financial variables capturing international markets' risk aversion and tensions in the interbank market have a significant impact during periods of stress only, confirming the evidence of strong interactions between market liquidity and bank funding liquidity during crisis periods.

Keywords: Bank Capital Regulation, Bank Liquidity Regulation, Basel III, stress tests

JEL classification: G28, G21

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1 Introduction

The 2008 global financial crisis has highlighted the systemic effects of banks' liquidity risks, a field which had not been addressed at the international regulatory level before. Indeed the crisis has shown that adequately-capitalized banks could suddenly default due to the loss of investors' confidence, preventing banks from meeting their financial commitments. Liquidity risks arise from different components and interactions. Banks experienced solvency and liquidity risks, through funding costs, fire sales and the balance sheet structure. Indeed, when well-informed investors start losing confidence in the solvency of an institution, they withdraw their short term deposits and raise margin calls, pushing the institution's funding costs up. The loss of funding might force the bank into fire sales, triggering a fall in their market prices. The rise in funding costs jointly with the decline in market prices, if the assets are marked-to-market, results in large losses for the institution, undermining its solvency.

Moreover, interactions between market liquidity and funding liquidity have also been questioned, the former being defined as the capacity to sell an asset without incurring a market price change, while the latter measures the ability of a financial institution to meet its own financial obligations by raising funds in the short term.

In the new Basel III regime, a liquidity regulatory framework has for the first time been agreed upon at the international level, with the introduction of two liquidity ratios. The Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR) pursue complementary objectives which are to promote the short-term resilience of banks' liquidity profile and to maintain a stable funding profile, respectively. While the latter has not yet entered into force, the former has already been implemented progressively since 2015. The LCR is designed to ensure that banks withstand a 30-day liquidity stress scenario. Within this context, this paper aims at assessing how banks adjust their liquidity and the structure of their balance sheet when facing a liquidity shock. With the objective of designing a liquidity stress test, the paper estimates the determinants of banks' liquidity ratios and how these ratios might get shocked in a stressed scenario.

Although supervisors have been paying increasing attention to stressed banking liquidity since the crisis, liquidity stress testing is still at its infancy, due to different factors. Data confidentiality constrains researchers to focus on proxies from publicly-available data. When available, data is limited to short time series due to the recent introduction of the global liquidity regulation. In this context, our study provides several contributions to the literature. First, we use data on a long-established regulatory liquidity ratio, close to the LCR, imposed on French banks since 1993, instead of proxies. Second, we shed light on the interactions between market and funding liquidity. More particularly, we estimate funding liquidity from a quantity perspective instead of a price perspective, as mostly seen in the literature. Indeed, data on internal transfer prices for the funding of individual transactions are most of the time not available. Third, we also capture the potential interactions between solvency and liquidity regulation, and assess banks' reactions to liquidity shocks. Finally, the paper develops a methodology to design a liquidity stress-test from a top-down perspective, *i.e.* run by supervisors or external analysts.

To this end, we develop a theoretical model in order to assess the effects of capital and liquidity

constraints on banks' behaviour. We maximize a representative bank's profit under solvency and liquidity constraints in order to highlight interactions between market liquidity, liquidity holdings and capital regulation. Precisely, the model concludes that liquidity and solvency are complementary (they reinforce each other) and shows that banks accumulate liquidity in order to face future liquidity shocks for precautionary motives: the lower the market liquidity, the more banks accumulate marketable securities rather than risky loans.

In line with the theoretical model, we present empirical evidence regarding interactions between liquidity and solvency ratios. We show that a higher level of solvency enables the liquidity ratio to improve. Even more interesting is that aggregate financial risk variables affect liquidity and solvency ratios only during periods of high stress, with a larger adverse effect on the liquidity than on the solvency ratio, confirming the evidence of strong interactions between market liquidity and bank funding liquidity during crisis periods. Consistently, when disentangling the impact of the financial variables on the different components of the regulatory liquidity ratio, we find that the effect of financial variables materializes mostly on the liability side of the liquidity coefficient, through net cash outflows. Given the non-linear relationship between financial variables and liquidity and solvency requirements, the implementation of contra-cyclical regulation for the liquidity ratio, similar to the capital regulation, could help prevent future crises. Surprisingly, the impact of the banking group membership only affects the relationship between financial risk variables and the solvency ratio, but we failed to find evidence of liquidity management at the group level. Likewise, we find that commercial banks are the most affected by the financial variables on their regulatory ratios. To a lesser extent, the solvency ratio of mutual banks and financial firms are impacted by the Vix variable and the interbank spread variable, respectively. Finally, our methodology could help supervisors to improve liquidity stress tests' framework. Our findings support the need to assess the combined effect of liquidity and capital regulation as both closely interact and have compounded effects.

The remainder of this paper is organized as follows. Section 2 reviews the literature on liquidity risks and their effects. Section 3 presents the theoretical model while Section 4 is devoted to the empirical analysis. Policy implications and the potential operational use of our methodology as a liquidity stress-testing tool are discussed in Section 5. Section 6 concludes.

2 Literature review

The global financial crisis highlighted the crucial role of liquidity in the outburst of destabilising confidence effects. Berger and Bouwman (2017) provide evidence that high levels of bank liquidity creation help predict future crises. Hanson et al. (2015) highlight the large synergies between the asset and liability sides of the balance sheet. The stable funding structure of traditional banks provides them a comparative advantage for holding assets potentially vulnerable to transitory price movements. Likewise, this is an application of Allen and Gale (2000) theoretical model that shows that negative externalities associated with liquidity transformation may occur via interregional cross holdings of deposits. Interbank contagion arises when banks tend to hoard liquidity by holding more liquid assets than usually. Despite this evidence, the determinants of banking liquidity

remain much less explored than those of banking capital, whose regulation has been implemented more recently. One exception is the paper by de Haan and van den End (2011), who use a panel VAR model to highlight fire sales and liquidity hoarding, by taking into account liquidity constraints. In this context of increasing liquidity issues, Hong et al. (2014) show that US banks' liquidity risks should be managed at both the individual level and the system level. It is thus important to assess the adjustments of banks' balance sheets related to the recent liquidity regulation.

Following the seminal paper by Diamond and Dybvig (1983) explaining how bank runs can affect healthy banks, liquidity regulation, through deposit insurance, received theoretical underpinnings. They point out the vulnerability arising from the liquidity transformation function performed by banks whereby they fund illiquid long-term assets with potentially unstable short-term liabilities. More recently, the impact of Basel III liquidity regulation has been assessed in terms of liquidity risk prevention as well as its overall macroeconomic impact. Most of studies focus on proxies of the regulatory liquidity ratios, such as *deposits over loans ratio* for Tabak et al. (2010), due to constraints on data confidentiality and availability. Among others, Roberts et al. (2018) use a *Liquidity Mismatch Index* to show evidence of reduced liquidity creation from banks that enforced the Liquidity Coverage Ratio. Similarly, Banerjee and Mio (2018) use the UK *Individual Liquidity Guidance* (ILG) ratio to study the effects of liquidity regulation on banks' balance sheet. Banks reacted to this liquidity regulation by increasing the share of high quality assets and non-financial deposits. They also reduced interbank/financial loans and short term wholesale funding, which is positive for the stability of the financial system. In contrast, we rely on a *Liquidity Coefficient* officially enforced in France, which shares some similarities with the *Liquidity Coverage Ratio* (see Section 4.1.3), over the 1993-2014 period, *i.e.* including the GFC.

Another strand of the literature, relevant to our paper, highlights how liquidity and capital regulations interact. So far, supervisors have considered liquidity and solvency risks but these risks were often viewed as independent. Regarding the solvency regulation, the consequences of increased bank capital are still uncertain and the literature shows mitigated effects on bank's liquidity creation. Berger and Bouwman (2009) find a negative relationship for small banks and a positive one for large banks. Recent papers tend to indicate a negative impact of higher capital requirements on credit distribution (see Fraisse et al (2017) for France, Aiyar et al. (2014) for the UK, Jimenez et al. (2017) for Spain or Behn et al. (2016) for Germany). While the impact of capital regulation is still a work in progress, studying the effect of the introduction of liquidity regulation, combined with solvency regulation, can help to determine the answer to this question.

Indeed, BCBS (2015) highlights four channels of transmission between liquidity and solvency risks: uncertainty about the quality of assets, fire sales, bank profitability and bank solvency. The authors estimate empirically the interactions between solvency and funding costs. More precisely, Kashyap et al. (2017) use a theoretical model in which credit risk and run risk endogenously interact, showing that capital regulation generates more lending while liquidity regulation deteriorates it. Conversely, Adrian and Boyarchenko (2018) recommend liquidity requirements as preferable prudential policy tool relative to capital requirements. Indeed, while liquidity requirements reduce

potential systemic distresses, without impairing consumption growth, capital requirements imply a trade off between consumption growth and distress probabilities. Finally, Kim and Sohn (2017) examine whether the effect of bank capital on lending differs depending upon the level of bank liquidity. Bank capital exerts a significantly positive effect on lending only when large banks retain sufficient liquid assets. While this interaction effect was more substantial for large banks during the recent financial crisis, the effect is found to be nonsignificant or negligibly negative for medium and small banks. Likewise, Acosta Smith et al. (2019), extending the Diamond and Dybvig (1983) model, highlight a tradeoff between a "skin in the game" effect that induces banks to accumulate more liquid assets in order to protect their capital and the impact of a more stable funding structure that may lead banks to shift their portfolio into more higher yielding illiquid assets. They show that the latter effect dominates the former in the UK so that the two regulations may appear as substitutes. In contrast, Faia (2018) and Kara and Ozsoy (2019) concludes that they are complementary. The former explain that equity requirements reduce banks' solvency region, while liquidity coverage ratios reduce the illiquidity region. The latter suggest that the enforcement of solvency requirements alone was ineffective in addressing systemic instability caused by fire sales. Given the existence of conflicting pieces of evidence, further work is needed to design an integrated stress testing framework including capital and liquidity interactions.

Moreover, liquidity risks strongly interact with other risks, giving rise to amplification mechanisms. In particular, Brunnermeier and Pedersen (2007) identify two types of liquidity risks and show how market and funding liquidity interact. The authors demonstrate that market liquidity is highly sensitive to further changes in funding conditions during liquidity crises and suggest that central banks can help mitigate market liquidity problems by controlling funding liquidity. For our part, we shed light on the relationship between market liquidity and funding liquidity by studying the impact of market liquidity indicators such as aggregate financial risk variables on banking liquidity via the liquidity coefficient. This interaction enables us to understand how liquidity mechanisms work and how contagion arises.

Against this background, this paper brings several contributions to the literature. To the best of our knowledge, this is one of the few papers using data on a long-established regulatory liquidity ratio, close to the LCR, imposed on French banks, instead of using a market- or balance sheet-based proxy. Moreover, our research focuses on interactions between liquidity and solvency as well as between market and funding liquidity. More particularly, this study estimates funding liquidity at the individual bank's level from a quantity perspective (a liquidity ratio), instead of an aggregate price perspective (funding costs), as mostly seen in the literature. Basing our estimations on a regulatory ratio rather than on market prices, we consider to be complementary and more robust as market prices might get distorted by market sentiment or other exogenous factors. Finally, the paper develops a methodology to design a liquidity stress-test from a top-down perspective.

3 Theoretical model

3.1 Set-up of the model and assumptions

The main objective of our model is to assess how banks react to liquidity shocks. We study the determinants of bank's liquidity and its interaction with market liquidity. Our model is based on a representative bank that maximizes its profit under balance sheet, capital and liquidity constraints.

In the model, the state variable s can take two values, h and l , denoting respectively high and low states of nature. They may be interpreted as normal (low risk aversion) and crisis periods (high risk aversion), respectively.

Two sources of financing are available to the bank: equity capital, denoted K ; and debt D , remunerated at the rate r^d . Depending on the state of the economy, a fraction $\alpha(s)$ of deposits is withdrawn.

There are two items on the asset side: loans L , with a long-term maturity and a return r^l ; and marketable securities G , whose return r^g is equal to the risk-free rate.

It can be illustrated by a look at the structure of a bank's balance sheet:

<i>Assets = TA</i>		<i>Liabilities =LBT</i>	
<i>L</i>	r^l	<i>D</i>	r^d
<i>G</i>	r^g	<i>K</i>	r^k
<i>Total = TA</i>		<i>Total = TA = LBT</i>	

We assume the following inequalities: $r^d < r^g < r^l$. Loans are considered to be riskier and, thus, provide a higher rate of return.

Bank's profit. The bank is assumed to behave as a mean-variance investor with risk aversion coefficient γ . The profit can be written as the following, with a risk-return arbitrage term as in Freixas and Rochet (2008) and in Fraisse, Lé, and Thesmar (*forthcoming*), among others:

$$\max_{G,L,D} \pi = r^l L + r^g G - r^d D - \frac{\gamma}{2} (\sigma_G^2 G^2 + 2\sigma_{GL} GL + \sigma_L^2 L^2) \quad (1)$$

with σ_G^2 and σ_L^2 being the variance of returns on securities and loans, respectively, and σ_{GL} the covariance between the returns on securities and loans. In the remainder of the paper, we will assume that $\sigma_{GL} = 0$ for simplification purposes.

Bank's constraints. The bank faces several constraints. The first one is a balance sheet constraint:

$$K + D = L + G \quad (2)$$

which can be rearranged in:

$$D = L + G - K \quad (3)$$

The bank is also subject to a solvency constraint that we assimilate to a leverage constraint for

simplicity purposes, in the form of :

$$K \geq \eta D \quad (4)$$

with $\frac{\eta}{1+\eta}$ the ratio of capital to total assets (and $0 < \eta < 1$). Assuming the solvency constraint holds with equality, the balance-sheet constraint becomes:

$$D(1 + \eta) = L + G \Leftrightarrow D = \frac{1}{1 + \eta}(L + G) \Leftrightarrow L + G = \left(\frac{\eta + 1}{\eta}\right)K \quad (5)$$

In addition, the bank faces a regulatory liquidity constraint which aims to ensure that it holds enough marketable securities to be able to cope with net cash outflows (deposit outflows, debt roll-off). To cope with deposit withdrawals at time t , the bank must sell marketable securities to get cash. However, depending on the market liquidity and the state of the economy, marketable securities are not necessarily sold at their book value. Therefore, the bank only gets an amount of cash given by the following formula:

$$C(s) = \beta G + (1 - \beta)\phi(s)G \quad (6)$$

with β being the share of marketable securities maturing, $\phi(s)$ being a fraction of the book value of the securities which were not maturing at t , i.e. a measure of the liquidity of the bank's marketable securities which is state-dependent.

As a result, the bank's liquidity constraint is the following:

$$\beta G + (1 - \beta)\phi(s)G \geq \alpha(s)D \quad (7)$$

with $\alpha(s)$ denotes the outflow rate on the liabilities.

After plugging the expression of D given by (5) into (7), the liquidity constraint gives the following inequality :

$$\beta G + (1 - \beta)\phi(s)G \geq \frac{\alpha(s)}{1 + \eta}(L + G) \Leftrightarrow \left[\frac{(\beta + (1 - \beta)\phi(s))(1 + \eta)}{\alpha} - 1\right]G \geq L \quad (8)$$

Another interpretation of (7) is that banks accumulate liquidity to meet depositors' withdrawals.

For a given level of total asset, higher capital K (or the existence of a capital buffer on top of regulatory requirements) implies lower deposits D , hence a softer liquidity constraint. However, solvency and liquidity requirements appear more complementary than substitutable.

3.2 The program of the bank

We are interested in identifying the determinants of the share of liquid assets G . The bank maximises its profit; its variables of choice are G , L and D :

$$\max_{G,L,D} \pi = r^l L + r^g G - r^d D - \frac{\gamma}{2}(\sigma_G^2 G^2 + \sigma_L^2 L^2) \quad (9)$$

subject to the balance sheet constraint (2), as well as:

$$L + G = \frac{\eta + 1}{\eta} K \quad (10)$$

and

$$\left[\frac{(\beta + (1 - \beta)\phi(s))(1 + \eta)}{\alpha} - 1 \right] G \geq L \quad (11)$$

as well as

$$G, L \geq 0 \quad (12)$$

After expressing the liquidity constraint as a condition on K , we can associate the following Lagrangian function, \mathcal{L} :

$$\begin{aligned} \mathcal{L}(G, L) = & r^l L + r^g G - r^d \left[\frac{1}{1 + \eta} (L + G) \right] \\ & - \frac{\gamma}{2} (\sigma_G^2 G^2 + \sigma_L^2 L^2) \\ & + \lambda \left[G \left(\frac{(\beta + (1 - \beta)\phi(s))(1 + \eta)}{\alpha(s)} - 1 \right) - L \right] \end{aligned} \quad (13)$$

with λ being the Lagrange multiplier of the liquidity constraint.

After solving the 2 first-order conditions on L and G , we get the following expression of G (See Proof in Appendix 10.1):

$$G = \frac{r^g - r^d \left(\frac{1}{1 + \eta} \right) + \lambda \left(\frac{(\beta + (1 - \beta)\phi(s))(1 + \eta)}{\alpha(s)} - 1 \right)}{\gamma \sigma_G^2} \quad (14)$$

and

$$L = \frac{r^l - r^d \left(\frac{1}{1 + \eta} \right) - \lambda}{\gamma \sigma_L^2} \quad (15)$$

with

$$\lambda = \frac{\left(\frac{r^l - r^d \left(\frac{1}{1 + \eta} \right)}{\gamma \sigma_L^2} \right) - \left(\frac{r^g - r^d \left(\frac{1}{1 + \eta} \right)}{\gamma \sigma_G^2} \right) \times \left(\frac{\beta + (1 - \beta)\phi(s)}{\alpha(s)} - 1 \right)}{\frac{(\beta + (1 - \beta)\phi(s) - 1)^2}{\gamma \sigma_G^2} + \frac{1}{\gamma \sigma_L^2}} \quad (16)$$

When the liquidity constraint is binding ($\lambda > 0$), the demand for liquid assets increases as λ is multiplied by a positive term ($A = (\frac{\beta + (1-\beta)\phi(s)}{\alpha(s)} - 1) > 0$, see Appendix 10.1).

Three cases are possible:

- case 1: α is small so that L and G are determined by the Markowitz portfolio as the liquidity constraint is not binding;
- case 2: α is larger, but $A > 0$ so that the liquidity constraint is binding, and the choice between L and G is twisted towards higher level of G , which is determined by the liquidity constraint and the solvency constraint;
- case 3: α is larger, but $A < 0$, so that $L = 0$.

Proposition: For α not too large, so that $A > 0$ (with $A = 1 - \frac{(\beta + (1-\beta)\phi(s))(1+\eta)}{\alpha(s)-1}$), the liquidity constraint is binding and $\frac{\partial G}{\partial \phi} \leq 0$ and $\frac{\partial G}{\partial \beta} \leq 0$.

(See Proof in Appendix 10.1.)

When the conditions set in the Proposition are met, the lower the market liquidity (the lower ϕ), the more banks accumulate marketable securities, such as sovereign debt, rather than risky loan assets. Cases where $A < 0$ may be associated with a crisis situation, where banks hoard liquidity and disinvest in L , but the banks forego investment opportunities by not investing in L . In case 2, when α is already high, the liquidity constraint is more likely to be binding. As in Allen and Gale (2000), any surprise on ϕ (the haircut on G when selling the asset is larger than anticipated) will induce banks to liquidate their position to meet depositors' withdrawals.

The three conclusions of the model are therefore that (i) liquidity and solvency are complementary, in the sense that they reinforce each other, and (ii) in normal times banks accumulate liquid assets in order to face liquidity shocks; (iii) in crisis times, banks are more likely to be constrained by their liquidity position and to sell liquid assets when hit by a shock, so that liquidity is overall reduced (if α high or ϕ low).¹

From model to data. The main variables of interest in our empirical model will be the bank's liquidity ratio, the bank's solvency ratio and a proxy for marketable securities' liquidity $\phi(s)$.

4 Empirical analysis

4.1 Data and descriptive statistics

4.1.1 Data

Our estimations use data from multiple sources and cover the period from 1993 to 2014, on a quarterly basis. Our two dependent variables are the *liquidity coefficient* and the *solvency*

¹A more formal way to derive such a result is to introduce an externality associated with firesales in a multiple bank system ($i = 1, N$), eg. if $\phi = \Phi(L^*)$ with L^* the aggregate demand for L : $L^* = \sum_{i=1}^N L(i)$. The micro-foundation of such an assumption can be found in a three-period Diamond and Dybvig (1983) model where banks investing in L need to face an interim liquidity need; however banks do not observe other banks' liquidity need, so that they fail to include firesales in their optimization. See Kara and Ozsoy (2019).

ratio, coming from the French Prudential Supervision and Resolution Authority (Banque de France/ACPR) databases. The Basel III Liquidity Coverage Ratio (LCR) is now the international standard for banking liquidity at the short-term horizon. Precisely, it is calculated as the ratio of the total amount of an institution’s holdings of High Quality Liquid Assets to the Total Net Expected Cash Outflows over a 30-day horizon in a stress scenario. The different components are granted different weights: in the numerator, the more liquid and higher quality an asset, the higher weight it gets; in the denominator, the more runnable a liability item, the higher weight it is assigned to. Wholesale funding receives a conservative treatment under the LCR in terms of assumed run-off rates. After a phase-in period that started in 2015, the minimum required level of the LCR reached 100 percent in 2018. Given the recent implementation and phasing-in as well as the limited time coverage of data, an analysis focusing on this ratio might not be relevant. Nevertheless, a binding Liquidity Coefficient was enforced in France from 1988 to 2014 for all banking institutions. This indicator provides a much larger set of observations both in terms of periods and cross sections than the LCR. The similarities and differences between both ratios are presented in Section 4.1.3. The LCR, like the French liquidity coefficient, has been implemented at the solo or legal entity level, meaning that each subsidiary of a banking group has to report and to abide by it. While liquidity management is often carried out at the consolidated level in banking groups, analyzing liquidity at the solo level might be more appropriate from an analytical point of view. Indeed, liquidity may not flow freely between the subsidiaries of a banking group and looking at liquidity on a purely consolidated level might bias the analysis by omitting particular behaviours (BCBS, 2013).

We also used the banks’ solvency ratio to capture the interactions between liquidity and solvency risks. However, the solvency ratio is available for the whole period only on a semi-annual basis. We therefore interpolated the series to obtain quarterly data for this variable. We can note that all the unit root tests implemented for the liquidity coefficient and the solvency ratio allowed us to reject the null hypothesis implying the presence of non-stationarity.² As a reminder, the regulatory liquidity coefficient must be above 100% while the solvency ratio must be no lower than 8%.

The liquidity coefficient and the solvency ratios are expected to have positive interactions: more capital means a larger share of stable funding, which is thus supposed to increase the liquidity coefficient. Conversely, in a liquidity crisis, a bank finds it more difficult and costly to get funding; the increase in its funding costs lowers its profits, meaning that a smaller amount of earnings can be retained to increase its own funds. Moreover, when facing a liquidity crisis, a bank may have to recourse to fire sales to get cash, which results in losses if the assets are marked-to-market, denting the bank’s solvency.

Our explanatory variables include the lagged liquidity and solvency ratios, aggregate financial risk indicators, macroeconomic variables, bank-specific control variables and a time dummy variable. The lagged dependent variables account for a possible autoregressive behaviour of the liquidity coefficient and the solvency ratio due to adjustments costs of liquid assets and capital. Here, we expect a positive sign.

Aggregate financial risk variables are taken from Bloomberg. These variables reflect the liquid-

²Tests are available upon request.

ity conditions in different markets (worldwide/European/national). They include:

- the Chicago Board Options Exchange SPX Volatility **VIX** Index, an indicator for worldwide risk aversion but also liquidity in international markets as liquidity is inversely correlated with volatility. We expect a negative sign on the coefficient of this variable in the liquidity equation as the higher the VIX index, the higher the investors' risk aversion, the lower market liquidity and thus the lower liquidity expected for banks;
- the **interbank spread** variable, taken as an indicator of the price of short-term debt, market sentiment in the short-term interbank market and bank default risk in the European markets. The choice of a market-wide spread instead of an individual spread allows us to mitigate endogeneity issues. Our spread is built as the spread between the 3-month interbank (Euribor) rate and the German sovereign 3-month bill rate, the latter being taken as the risk-free rate. We expect a negative sign on the coefficient of this variable as the larger the spread, the more expensive and difficult it is for banks to get funding, which is expected to result in deteriorated liquidity and solvency ratios.³

Macroeconomic variables are **GDP growth** and **inflation rate**, on a year-to-year basis, taken from INSEE (French National Statistical Institution). Both variables are expected to have a positive effect on solvency and liquidity ratios as credit and liquidity risks decline in good economic times. However, the literature has shown the impact of precautionary motives, which might induce banks to improve their ratios in bad times, by increasing their reserves.

Bank-specific control variables are taken from the SITUATION database (French Prudential Supervision and Resolution Authority/Banque de France), with a quarterly frequency. They are all lagged to avoid endogeneity issues:

- the **size** variable corresponds to the market share of the bank in terms of assets. The ratio of each bank's assets to the mean total assets is meant to avoid spurious correlation stemming from a time trend in banks' assets. A negative sign is expected on the coefficient of this variable, as big banks have less incentives to constitute capital or liquidity buffers due to a lower risk aversion, in line with the too-big-to-fail implicit assistance, and due to their higher ability to diversify risks and access funding;
- the **return on equity** ratio is used as a proxy for the cost of equity. In order to delete some reporting errors in the dataset, we dropped observations with a return on equity ratio above 100% or below -100%, which seems highly unlikely to occur. The expected sign of this variable is negative, at least for the solvency equation, as a higher return on equity means that banks will find it more expensive to raise more capital;
- the **retail** variable captures the bank's business model, built as the ratio of transactions with non-financial customers to total assets. The sign of this variable is uncertain. On the one hand, deposits from non-financials, in particular retail deposits, are supposed to be a

³We also ran all our estimations including the **bid-ask spread** on the French sovereign 10-year debt, taken as an indicator of market liquidity for an asset making up a large share of French banks' balance sheet. However, given the lack of significance of this variable in our regressions and its low volatility, we decided to not include it in the main specifications presented in this paper.

stable source of funding on the liability side, but on the other hand, loans to non-financial customers are not considered as liquid on the asset side.

We also included a dummy variable to deal with data characteristics: the d_{2010} time dummy variable takes the value 1 from 2010Q2 onward to capture the change in the definition of the liquidity coefficient variable. As the definition of liquid assets was made stricter and the coefficients on cash outflows were increased at that time, we expect a negative sign on the coefficient of this variable. It also corresponds to the period in which the new Basel 3 framework was announced.

Our models are estimated on a quarterly basis. Therefore, we calculated simple quarterly averages for series having a higher frequency, namely financial variables and the consumer price index.

4.1.2 Descriptive statistics

This subsection provides descriptive statistics about the dependent variables we used, namely the liquidity coefficient and the solvency ratio, as well as other financial and macroeconomic variables, described in Table 1. The French liquidity ratio (called "liquidity coefficient") is reported on a solo basis. Given the wide distribution of these variables, we decided to drop the 5th and the 95th percentiles of the sample for the liquidity coefficient and the solvency ratio, in order to address the misreporting issues and eliminate outliers. We also dropped observations equal to 0 that would reflect specific business models. We finally dropped banks with less than 5 observations (quarters) in the sample. We end up with more than 23,000 observations. In spite of this data cleansing, Table 1 shows a large dispersion in the liquidity ratio. In particular, the liquidity ratio displays a 90th percentile value of 1,741% while the 90th percentile value of the solvency ratio is at 54%. The solvency ratio thus displays a more concentrated distribution. Nevertheless, both the solvency and the liquidity ratios present a minimum value above the requirement threshold, which means that during the whole period, the banks composing our sample were compliant with regulatory ratios enforced in France.

Figure 1 displays the evolution of the average of the liquidity ratio and the solvency ratio. Overall, the liquidity and solvency ratios are usually not binding as the mean is always above the minimum requirements (dashed lines). In particular, the liquidity coefficient shows a continuous decline until 2010-2011, and a low level over the 2008-2011 period, characterized by a shortage of liquidity. Afterwards, liquidity picks up, with short run fluctuations until 2014. By contrast, the solvency ratio displays a rising trend from 2008.

Table 2 displays correlations between all the variables composing our models. A positive and significant correlation coefficient can already be observed between the liquidity and solvency ratios (0.29). Furthermore, the latter are negatively correlated with the VIX index, the risk aversion indicator, but positively correlated with the interbank spread. Given that our financial variables are related to different markets and different risks, we consider that the risk of colinearity is limited. In this context, the empirical analysis will enable us to better assess these interactions between market liquidity and banking regulatory ratios.

4.1.3 Liquidity coefficient, a good proxy for the LCR?

As mentioned above, the Liquidity Coverage Ratio has only been enforced since 2015. Although the LCR and the Liquidity Coefficient are both defined as ratios of liquid assets to net cash outflows over a 30-day period, there are some differences associated with the treatment of intragroup exposures and off-balance sheet items, as well as with the weights associated with the different components, with the LCR being stricter than the liquidity coefficient in terms of liquid asset definition. It is thus necessary to compare these ratios to assess to what extent our liquidity coefficient can be used as a proxy of the Liquidity Coverage Ratio in a regression.

The liquidity coefficient was implemented from 1988 to 2015 for all banking institutions, then interrupted and only reported by financial companies from 2015 to 2018. Although enforced from 2015, the LCR has been reported from 2010 to 2018. Thus there is some overlap in the reporting of both the LCR and the liquidity coefficient by the same institutions, which enables us to assess the relationship between the liquidity coefficient and the LCR. We first analyze correlation (see Table 3). We can see that the correlation between the LCR and the Liquidity Coefficient is positive and significant (0.19). When we disentangle the different components of the two ratios and consider their bilateral correlation, we can notice even higher correlation coefficients. This is the case with the numerators of the two ratios, namely the liquid assets, which display a correlation coefficient of 0.40, and with the denominators, namely the cash outflows, with a coefficient of 0.36. Both ratios are even more correlated when we consider them on a gross basis, i.e. before the application of regulatory weights to their different components, with a coefficient of 0.69. This means that the main differences between these ratios come from the application of different weights.

By regressing the LCR on the components of the liquidity coefficient (liquid assets and cash flows) (see Table 4), we find that the stock of liquid assets (numerator of the liquidity coefficient) taken in logarithm has a significant and positive impact on the LCR. Moreover, intragroup operations, which are not taken into account in the LCR calculation, are found to affect the LCR significantly and negatively. By contrast, net cash outflows (denominator of the liquidity coefficient) are found to impact the LCR negatively, but not significantly. These results are broadly in line with expectations. They reveal that there are some operations within banking groups that reduce the regulatory LCR. We will further explore in the paper to what extent the membership in the banking group affects the level of liquidity and solvency ratios.

All these results indicate a strong relationship between the liquidity coefficient and the Liquidity Coverage Ratio, which confirms the relevance of using the liquidity coefficient as a proxy of the LCR over an extended period of observations.

4.2 Simultaneous equations method

One of the objectives of this study is to assess the interactions between liquidity and solvency ratios. Therefore, we rely on the simultaneous equations regression using the Two Stage Least Squares (2SLS) estimator and fixed effects.⁴ This methodology enables us to run a system of

⁴One could suggest the use of Three Stages Least Squares (3SLS) method that also accounts for cross correlation in error terms. In our case, the result of the Hausman test supports the use of the 2SLS methodology at the usual confidence level (tests are available upon request).

equations which are endogenous, when the dependent variable's error terms are correlated with the independent variables. Indeed, in each equation, the Liquidity Coefficient and the Solvency Ratio are endogenous variables on both the left and right hand sides of the equation.

The reduced form of our simultaneous equations specification can be read as follows for bank i :

$$Y_{i,t} = \alpha_i + \phi Y_{i,t-1} + \beta X_t + \gamma Z_{i,t-1} + \epsilon_{i,t}$$

where Y is a vector of two endogenous variables (liquidity coefficient and solvency ratio); X is a vector of explanatory variables including aggregate financial risk variables (for example, the VIX index and the interbank spread), macroeconomic variables (GDP growth and inflation) and dummy variables; Z is a vector of bank-specific variables (size, retail, return on equity ratio); α_i is a vector of individual bank fixed effects and ϵ the vector of error terms, with i referring to bank i and t to time t .

4.3 Results

This section presents the results associated with the different specifications we used. Our baseline estimation analyzed the relationship between the liquidity coefficient, the solvency ratio and the set of explanatory variables previously defined. We then interacted some variables of this basic specification with specific dummies in order to capture non-linearities and to shed light on heterogeneous effects.

We first examine the baseline estimation, displayed in Table 5, showing a positive and significant interaction between the liquidity ratio and the solvency ratio. The first column refers to the liquidity coefficient equation, while the second one refers to the solvency ratio equation. Results indicate a positive and significant impact of the solvency ratio (5.22) on the liquidity coefficient, which provides evidence of positive interactions between solvency and liquidity. Precisely, when banks increase their solvency ratio by 1 percentage point in $t - 1$, this is associated with a 5.22 percentage point increase in the liquidity coefficient at the following period. In the solvency equation, we find a coefficient of the liquidity ratio close to zero, although significant. Therefore, both variables are found to move in the same direction, but with a more potent effect of solvency on liquidity. We also observe a high value of the autoregressive coefficients, particularly for the solvency ratio (0.89), and to a lesser extent for the liquidity coefficient (0.63), reflecting some inertia for these variables, although we did not find any evidence of a unit root process.⁵

Overall, the aggregate financial variables are found to have no significant effect on the liquidity and solvency ratios. The absence of significant effect of these variables might be due to the fact that on average the period of observation (1993-2014) corresponds to a time of "great moderation" (apart from a few crisis years), during which financial variables displayed little volatility. This explanation will be further investigated by breaking down the period under study between sub-

⁵Results for unit root tests are available upon request.

periods. By contrast, the macroeconomic variables (GDP growth and inflation rates) are found to have a significant and negative effect: the GDP growth rate negatively impacts both the liquidity coefficient and the solvency ratio. This negative relationship might reflect a precautionary behaviour on the banks' part. The latter tends to expand their balance sheet and take on more risks in good times by reducing the size of their liquidity and solvency ratios, while they build up some reserves in bad times. As for the effect of the inflation rate, it is found to be insignificant on the liquidity coefficient, but significant and negative on the solvency ratio (-0.12). The relationship between the inflation rate and the solvency ratio can reflect the lower profits banks make when inflation increases. Indeed, a higher inflation rate reduces banks' profits as interest rates on (mostly) fixed rate loans are not adjusted, while funding rates move upward.

Regarding the balance sheet variables, only the relative size variable does show a significant impact on the liquidity coefficient (-282.01). In other words, when banks' relative size increases, the liquidity ratio declines. This is in line with our expectations associated with the too-big-to-fail assumption, the greater ability of large banks to diversify their funding sources and their lesser incentive to build up liquidity buffers.

Finally, the coefficient on the dummy identifying the regulatory change in 2010 regarding the tighter definition of the liquidity coefficient is found to be significant and negative in the liquidity coefficient equation, as expected. The significant and positive effect of this dummy in the solvency ratio equation might be due to the rising trend in banks' solvency ratios since the 2008/2009 financial crisis.

Overall, these results confirm some interactions between the regulatory liquidity and solvency ratios. However, the liquidity coefficient does not seem to be impacted during periods of extremely high stress, which comes as a surprise. In order to analyze further this latter finding, the next subsection focuses on the impact of the financial variables during periods of high stress.

How do financial variables impact liquidity and solvency ratios during periods of stress?

This more specific analysis allows us to determine whether our variables of interest, the financial variables, have larger effects during certain periods or not. While Table 5 did not show any significant effects of the financial risk variables on the regulatory ratios during the whole observation period, the objective is here to capture possible nonlinear effects whereby the impact of the financial variables on the solvency and liquidity ratios would be larger and more significant when the value of these variables exceeds certain thresholds, meaning periods of stress. To that end, we add two interaction terms to the previous specification: *(i)* an interaction term between the level of the VIX index and the dummy variable denoting high VIX period, and similarly *(ii)* an interaction term between the level of the interbank spread and the dummy variable denoting high spread periods.

The two columns of Table 6 show the results of this new specification. When looking at the coefficients of the interaction terms, we can see that during periods of high VIX, reflecting high risk aversion, the VIX index has a negative and (although weakly) significant impact on the

liquidity coefficient (-7.54) but no significant effect on the solvency ratio. In contrast with our previous findings, we also find that during periods of large interbank spread, the latter impacts the liquidity coefficient very negatively and significantly (-154.74), implying a deterioration of the bank's liquidity coefficient. These results indicate that stricter financial conditions negatively impact the liquidity coefficient in periods of stress: in those periods, banks endure the financial environment more than they steer their liquidity ratio. However, this interbank spread variable positively affects the solvency ratio during high spreads periods (1.17). The deterioration of the interbank market sentiment has thus more negative consequences on the liquidity conditions of the bank, reflecting strong interactions between the interbank market situation and the funding bank liquidity. In turn, banks are likely to increase their level of capital buffers for precautionary motives.

This new analysis enables us to conclude that the relationship between financial variables and banks' liquidity and solvency ratios is non linear and stronger during high financial stress periods, which is in line with the literature findings on the determinants of capital ratios. This might call for designing a countercyclical regulation on banks' liquidity, as was done for solvency with the countercyclical capital buffer introduced by the Basel Committee in 2010.

How do financial risk variables impact liquidity and solvency ratios when banks are less liquid or capitalized?

The intuition we want to check now is whether the regulatory ratios of less liquid and less capitalized banks are more impacted by their financial environment. Indeed, given that these banks display smaller buffers, the regulatory minima may be more binding to them. Therefore, these banks might be facing a choice between targeting the level of their ratios or letting the external environment drive them. To that end, in Table 7 we introduce dummy variables identifying less liquid or less capitalized banks, on top of the previous variables used. The $d_lessliq_{i,t-1}$ dummy variable equals 1 when the bank is below the liquidity coefficient threshold of 120%, which leaves a margin to the 100% regulatory minimum. The $d_lesscap_{i,t-1}$ dummy variable equals 1 when the solvency ratio of the bank is below the 10% level, which is also close to the 8% regulatory minimum. The two columns of Table 7 include the interaction of these dummy variables $d_lessliq$ and $d_lesscap$ with the VIX variable and the interbank spread variable. As indicated by this table, none of these interactions is significant.⁶

Surprisingly, our latter results show that the financial variables, including global risk aversion and the interbank spread, do not have any significant impact on the regulatory ratios of banks that are less liquid or less capitalized. In this context, it is relevant to assess the effect of belonging to a larger banking group on the level of the solvency or liquidity ratios for a legal entity.

⁶A further analysis consists in introducing three kinds of interactions, to assess the impact of our financial variables on banks that are less liquid or capitalized, during the specific periods of stress, which combines the effects of the two previous specifications. However, we saw that even during stress periods, the financial variables do not show any significant impact on the liquidity coefficient and the solvency ratio of banks that are less liquid or less capitalized.

What is the contribution of a banking group membership?

A possible objection to our analysis is that we study the determinants of a liquidity ratio at the solo or legal entity level whereas liquidity management is usually carried out at a centralized or consolidated level within a banking group. To address this feature, in this section we include new variables to analyze the effect of belonging to a larger banking group on the level of the liquidity and solvency ratios. We thus create a dummy variable, d_group , to identify banks that belong to a larger group. However, the database containing this information has only been available since the second quarter of 1997. Therefore, we had to start our estimations in 1997Q2 for this specific estimation, instead of starting in 1993. The first two columns of Table 8 show the results associated with the interaction of this dummy d_group with our financial risk variables, VIX and interbank spreads, in order to see if the regulatory ratios of banks belonging to a larger group are more or less sensitive to the financial risk variables. We do not find any significant effect of these interaction terms on the liquidity coefficient. However, the solvency equation shows a positive coefficient of the interaction term between the group dummy variable and the interbank spread variable (7.79). In other words, the solvency ratio of banks that belong to a larger group reacts positively to a higher interbank spread, suggesting a reaction to the financial environment at the group level. Practically, when the interbank market sentiment deteriorates, these subsidiaries benefit from a capital management at the group level.

Second, we create a dummy equal to one when the banking group displays a large excess of its liquidity coefficient ($>150\%$) or a large excess of its solvency ratio (15%). In these cases, we assume that the sensitivity of the regulatory ratios to the financial risk variables is lower when the banking group to which the bank belongs shows an excess of liquidity or capital. Indeed, this excess of scarce resources enables the banking group to manage liquidity or solvency centrally and to allocate support to the subsidiaries if the financial environment deteriorates. The last two columns of Table 8 present the results. We can see that the VIX index has only a weakly significant and negative effect on the subsidiary's liquidity coefficient when there is an excess of liquidity at the group level (-4.37). Moreover, there is no sensitivity of the subsidiary's liquidity coefficient to the interbank spread when there is excess liquidity at the group level. By contrast, in column (4), an excess of capital at the group level makes the subsidiary more sensitive in a negative way to the interbank spread (-0.86), but not to the VIX index. Said differently, the solvency ratio of banks that belong to a larger group having an excess capital is more negatively affected by an increasing interbank spread. Given the assumed support of the group showing an excess of capital, the subsidiary may let fluctuate its solvency ratio in response to a riskier financial environment.

These outcomes show evidence of a stronger contribution of banking group membership to the level of solvency than to the level of liquidity of its subsidiaries. While this indicates that carrying out an estimation of a liquidity coefficient at the solo level is not too problematic, we show that the reaction of the solvency ratio to the financial environment strongly depends on the management at the group level.

Heterogeneous effects: the effect of banks' type

The aim of the following analysis is to assess to what extent financial variables affect more the regulatory ratios of some types of banks. To this end, we interact our financial variables (VIX and interbank spread) with a dummy referring to the type of the bank. Among the six types of banks available, we focused on the three main ones: d_Com for commercial banks, d_Mut for mutual banks and d_Fin for financial firms. As with group membership, this information has only been available since 1997. Therefore, our estimations are run over the 1997q2 - 2014q4 period. Table 9 presents the results on our two dependent variables. Breaking down by business models uncovers interesting dynamics regarding financial firms' and commercial banks' solvency ratios. We can see that the interbank spread variable has a significant and negative effect on the solvency ratios of both types of banks (-1.28 and -1.83 respectively). Higher interbank spreads may reduce funding and profitability, hence retained earnings and capital, and may decrease the solvency ratio. In contrast, the VIX variable has a positive and significant impact on the solvency ratios of commercial and mutual banks (0.08 and 0.03, respectively). Higher risk aversion may induce these banks to reduce risk taking, decreasing the denominator of the solvency ratio and thus increasing the ratio. By contrast, the liquidity coefficient does not seem to be strongly affected by our financial risk variables, whatever the type of the bank. These results highlight a strong heterogeneity between the different types of banks in terms of impact of external financial variables on their levels of solvency. The solvency ratios of financial firms and commercial banks seem to be the most sensitive to the external environment, due to their specific (more market-oriented) business model.

Disentangling the numerator and the denominator of the liquidity coefficient

To determine whether the impacts of financial variables on the regulatory ratios are predominant on the asset or the liability side of banks, this section disentangles the liquidity coefficient between liquid assets (the numerator of the liquidity coefficient) and the net cash outflows (denominator). To normalize the numerator and the denominator of the liquidity coefficient taken separately, we calculate the share of these two variables in the bank's total assets. We keep the solvency ratio as our third dependent variable. We thus run a system of simultaneous equations now including 3 equations whose dependent variables are liquid assets, net cash outflows, and the solvency ratio, respectively.

This new estimation (Table 10) indicates that among our aggregate financial risk variables, the most notable significant effect we capture is the impact of the interbank spread on the denominator of the liquidity coefficient, namely the share of net cash outflows, in high stress times (0.89). While the impact of the interbank spread variable on the net cash outflows is negative during normal times (0.30), it becomes positive during periods of large spreads, reflecting high stress. This means that when the interbank spread exceeds the 95th percentile of its distribution, a rise in the spread brings about a larger share of net cash outflows. This in turn entails a deterioration of the bank's liquidity ratio. This effect might reflect the mechanism whereby long-term debt markets shut down

for banks during periods of high spreads, compelling them to increase the share of their short-term funding.

Regarding interactions within the liquidity ratio or the solvency ratio, Table 10 provides results similar to the previous tables. In the first two columns, the numerator and the denominator of the liquidity coefficient display positive interactions: higher cash outflows lead to more liquid assets (0.20), which is expected with regard to the requirement for banks to display a liquidity coefficient higher than 1. More surprisingly, a larger share of liquid assets leads to larger cash outflows (0.004), to a lesser extent. More liquid assets lead banks to meet higher outflows in the next period. Moreover, while the solvency ratio has a positive impact on the share of liquid assets (0.11), its effect is found to be negative on the share of cash outflows (-0.009), which is expected as higher solvency means a more stable funding structure.

As for the solvency ratio (column 3), neither the share of liquid assets nor the share of cash outflows are found to have a significant impact on the ratio, confirming that the relationship between liquidity and solvency seems to be only a one-way causality.

These new results confirm the strong interactions occurring between the share of liquid assets, cash outflows, and the solvency ratio, which is in line with our previous findings. At the same time, they show that the effect of the financial variables in periods of stress on the liquidity coefficient mostly materializes on the liability side, through net cash outflows.

All these results allow a better understanding of the channels of liquidity stress transmission. The effect is only visible in periods of very high stress and is channelled mostly through unstable liabilities.

5 Supervisory liquidity stress-test

In our opinion, the methodology above-mentioned can be used to design a top-Down stress test for supervisors. As stress tests typically focus on developments in crisis times, we concentrate on the results of the specification presented in Table 6.

We run two experiments.

First, we can notice that the model in Table 6 is actually an Exogenous Structural VAR model (or SVAR-X) that can be inverted, yielding Impulse Response Functions (IRFs), or responses of the endogenous variables to a shock to exogenous variables (the aggregate financial risk variables). Therefore, we compute the IRFs to shocks on the interbank spread variable (jumping to 400bp) and the VIX variable (increasing to 80) and look at the impact on the liquidity coefficient and the solvency ratio, using the model displayed in Table 6. We consider successively the impacts of (i) a shock on interbank spreads in crisis time (measured by the high spread period) on the liquidity coefficient (see Figure 2) and on the solvency ratio (see Figure 3) and (ii) a shock on VIX in crisis time (measured by the high VIX period) on the liquidity coefficient (see Figure 4) and on the solvency ratio (see Figure 5). In both cases, the impacts are negative and very significant on the liquidity coefficient which is adversely affected. In contrast, the solvency ratio is negatively affected by the shock on the VIX, but positively affected by the shock on the spread. In all figures, the 90 % confidence bands are based on Monte Carlo simulations. They are drawn around the

median IRF.

In the second experiment, we use the same empirical results, based on Table 6. The latter includes the coefficients of the interaction term between our financial exogenous variables and the dummies identifying periods of high stress, taking the value of 1, allowing us to consider them as intercepts. As a reminder, we take the 95th percentile of the distributions of our exogenous variables (VIX index, interbank spread, GDP growth, inflation rate) as inputs to the stress scenarios. As pre-shock conditions, we assume that the values of the bank-specific variables (size, retail business, ROE) correspond to an average bank, except the fact that we want to focus on banks facing solvency and liquidity issues. That is why we assume starting liquidity coefficient and solvency ratio of 110 percent and 9 percent, respectively. Applying the coefficients presented in Table 6 to the stressed variables, we present the projection in a stress scenario in Table 11. We then end up with a large rise in the Liquidity Coefficient by 190 percentage points, from a liquidity coefficient value of 110 percent before stress to 300 percent afterwards. This large rise is explained by the very high values of the coefficients on the dummies identifying high VIX index and large interbank spread periods. This can be interpreted as a regime change in crisis times captured by the intercepts. This result provides evidence of a liquidity hoarding behaviour on the part of banks in stress times, due to the adverse financial environment and despite the negative effect of our aggregate financial variables.

6 Conclusion

This study aimed at estimating the determinants of banks' liquidity ratios, taking into account the interactions between solvency and liquidity as well as between market liquidity and funding liquidity risks. Indeed, our results show that a higher level of solvency enables the liquidity ratio to improve. By contrast, we do not find evidence that solvency ratios are affected the banks' liquidity. Likewise, financial risk variables affect liquidity and solvency ratios only during periods of high stress, with a larger adverse effect on liquidity than solvency, confirming the evidence of strong interactions between market liquidity and bank funding liquidity during crisis periods. The financial risk channel is found to materialize mostly on the liability side, through net cash outflows. Finally, the impact of the banking group membership affects the relationship between financial risk variables and the solvency ratio, but we failed to find evidence of liquidity management at the group level. Likewise, we find that financial firms and commercial banks are more affected by the financial risk variables on the solvency side than on the liquidity side.

Further extension of our analysis would be to add some dynamics in our model by including funding costs and modeling the price impact of banks' fire sales. A panel estimation based on Liquidity Coverage Ratio data once the series are long enough would allow supervisors to broaden the analysis and compare the effects of financial stress across countries.

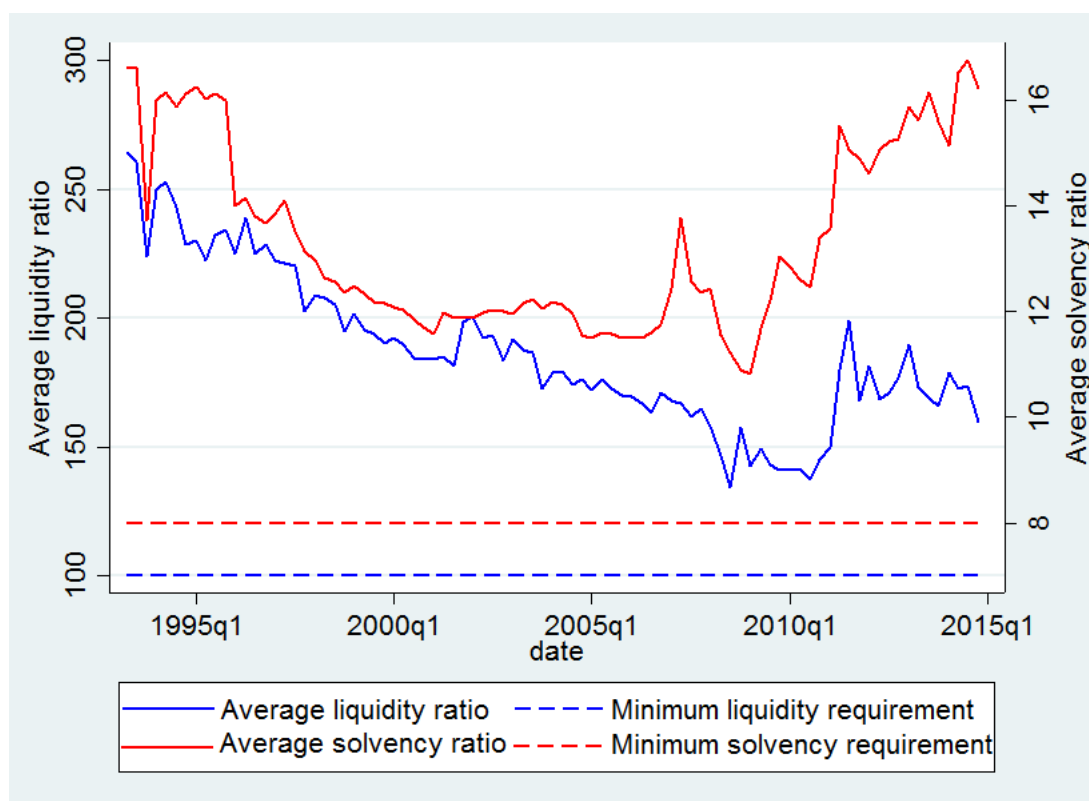
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8 Figures

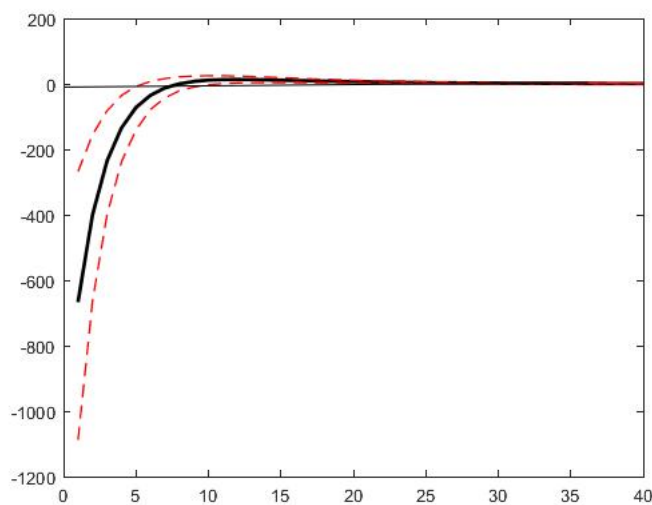
Figure 1: Liquidity Coefficient and Solvency Ratio over 1993-2015



Note: this figure displays the evolution of the Solvency Ratio in red on the right-hand axis and the Liquidity Coefficient in blue on the left-hand axis, on a weighted average basis. The sample includes all French banking institutions, reporting over the period 1993 to 2015.

Sources: ACPR, Authors' calculations.

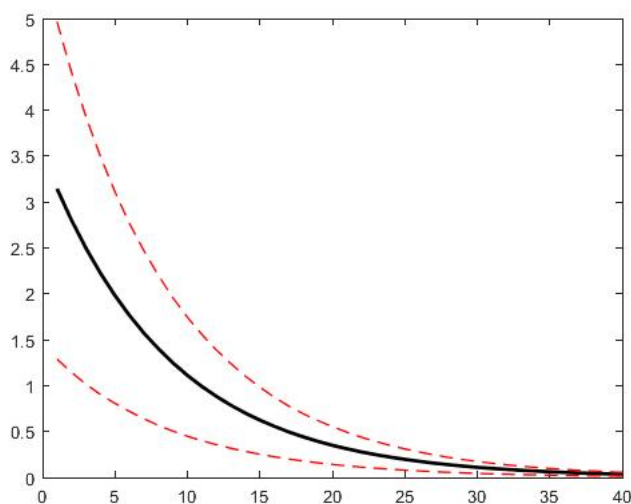
Figure 2: Impulse response Function : shock of interbank spread on Liquidity Coefficient



Note: this figure displays the response of the liquidity ratio to a shock of spread to 400bp, on the basis of the inversion of the equation in Table 6.

Sources: ACPR, Authors' calculations.

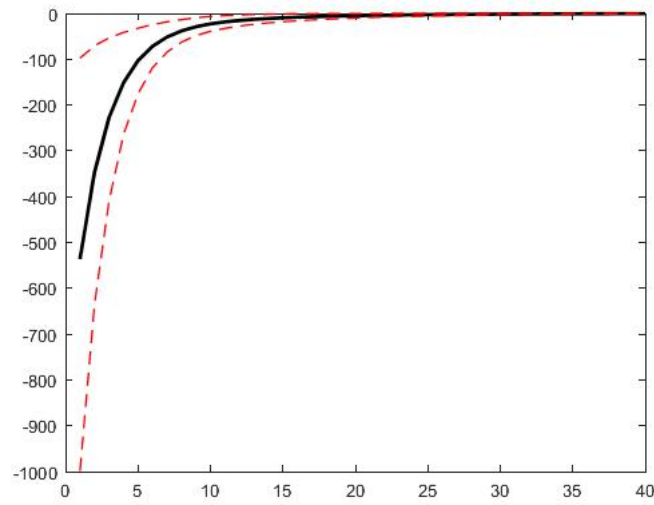
Figure 3: Impulse response Function : shock of interbank spread on Solvency Ratio



Note: this figure displays the response of the Solvency ratio to shock of spread to 400bp on the basis of the inversion of the equation in Table 6.

Sources: ACPR, Authors' calculations.

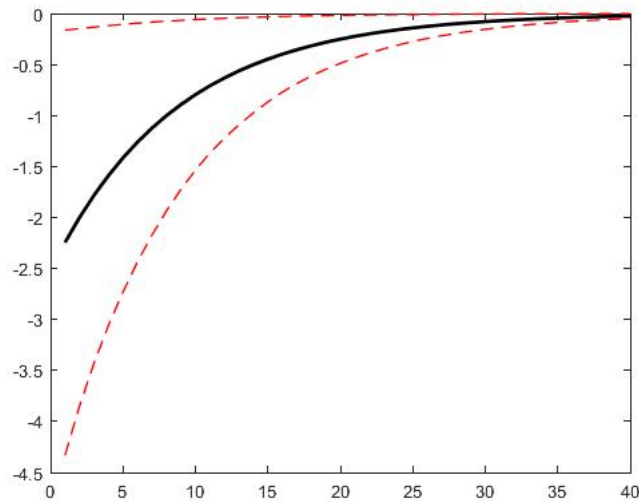
Figure 4: Impulse response Function : shock of VIX on Liquidity Coefficient



Note: this figure displays the response of the liquidity ratio to shock of VIX to 80 on the basis of the inversion of the equation in Table 6.

Source: ACPR, Authors' calculations.

Figure 5: Impulse response Function : shock of VIX on Solvency Ratio



Note: this figure displays the response of the solvency ratio to shock of VIX to 80 on the basis of the inversion of the equation in Table 6.

Sources: ACPR, Authors' calculations.

9 Tables

Table 1: **Descriptive statistics of the main variables**

VARIABLES	N	SD	P10	Median	Mean	P90
Liquidity ratio (in %)	25,611	2,306.57	127.18	225.87	907.23	1,740.87
Solvency ratio (in %)	25,611	21.19	9.82	15.58	24.64	53.97
Vix (in points)	25,611	7.48	12.44	18.53	19.89	29.30
Interbank spread (in %)	25,611	0.58	0.03	0.21	0.46	1.14
GDP growth (in %)	25,611	1.54	-0.11	1.88	1.71	3.37
Inflation (in %)	25,611	0.68	0.61	1.69	1.55	2.29

Table 1 presents descriptive statistics of the main variables used in the following estimations: the liquidity coefficient, the solvency ratio, the VIX index, the interbank spread, GDP growth and the inflation rate, on an unweighted average basis.

Sources: ACPR, INSEE and Bloomberg - Authors' calculations.

Table 2: **Correlations between the main variables**

VARIABLES	Liquidity coefficient	Solvency ratio	Vix	Interbank	GDP	Inflation
Liquidity ratio	1.0000					
Solvency ratio	0.2882*** (0.0000)	1.0000				
Vix	-0.0225*** (0.0003)	-0.0246*** (0.0001)	1.0000			
Interbank	0.0375*** (0.0000)	0.0144** (0.0211)	-0.0246*** (0.0001)	1.0000		
GDP growth	0.0140** (0.0247)	0.0056 (0.3741)	-0.2048*** (0.0000)	-0.2259*** (0.0000)	1.0000	
Inflation	0.0160** (0.0105)	0.0129** (0.0393)	-0.1645*** (0.0000)	0.2023*** (0.0000)	0.0379*** (0.0000)	1.0000

Table 2 presents correlation coefficients related to the main variables used in the following estimations: the liquidity coefficient, the solvency ratio, the VIX index, the interbank spread, GDP growth and the inflation rate. Standard errors are mentioned in brackets, as an indicator of confidence.

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Sources: ACPR, INSEE and Bloomberg - Authors' calculations.

Table 3: **Correlation between Liquidity Coefficient and Liquidity Coverage Ratio**

	LCR	gross LCR	(LCR) Liquid assets	(LCR) Cash outflows
LC	0.1851***			
gross LC		0.6946***		
(LC) Liquid assets			0.4063***	
(LC) Cash outflows				0.3587***

Table 3 shows the correlation between the French Liquidity Coefficient (LC) and the Basel III Liquidity Coverage Ratio (LCR). Variables are expressed in ratio and in gross level terms.

*** p<0.01, ** p<0.05, * p<0.1.

Sources: ACPR, Authors' calculations.

Table 4: **Liquidity Coefficient as a good proxy for the LCR?**

VARIABLES	(1) Liquidity Coverage Ratio
ln_Liquid assets (LC, gross)	75.74** (27.01)
ln_Cash outflows (LC, gross)	-17.31 (18.08)
ln_Intragroup operations (LC, gross)	-9.10** (3.80)
Constant	-655.83 (561.31)
Bank Fixed Effects	Yes
Observations	30
Number of cib	13
Adjusted R-squared	0.12

Table 4 reports estimates of the regression of the level of the Basel III Liquidity Coverage Ratio on the Liquidity Coefficient components (numerator and denominator) in gross terms. Standard errors are in brackets.

*** p<0.01, ** p<0.05, * p<0.1.

Sources: ACPR - Authors' calculations.

Table 5: **Simultaneous equations: Liquidity coefficient - Solvency ratio**

VARIABLES	(1)	(2)
	Liquidity ratio	Solvency ratio
Liquidity ratio (t-1)	0.625*** (0.005)	0.000*** (0.000)
Solvency ratio (t-1)	5.219*** (0.643)	0.891*** (0.003)
Vix	-0.125 (1.012)	-0.000 (0.005)
Interbank	-3.791 (13.563)	-0.064 (0.062)
GDP	-10.807** (5.113)	-0.050** (0.023)
Inflation	3.335 (10.876)	-0.119** (0.050)
Size (t-1)	-282.008** (129.363)	-0.163 (0.594)
Retail (t-1)	0.226 (0.710)	-0.003 (0.003)
RoE (t-1)	0.401 (0.577)	0.002 (0.003)
2010 Dummy	-84.067*** (22.301)	0.552*** (0.102)
Constant	No	No
Bank Fixed effects	Yes	Yes
Observations	23,264	23,264
Adjusted R-squared	0.798	0.978

Table 5 reports estimates of a system of 2 simultaneous equations with fixed effects. The two dependent variables are the liquidity coefficient and the solvency ratio, also used in lag among explanatory variables. The other explanatory variables include financial variables (VIX and interbank spread), macroeconomic variables (GDP and inflation rate), bank control variables (size, retail and return on equity ratio), a dummy variable (referring to 2010 as a change in definition of the liquidity coefficient, with the value 1 corresponding to the period from 2010 onwards) and individual bank fixed effects. Standard errors are in brackets.

*** p<0.01, ** p<0.05, * p<0.1.

Sources: ACPR, INSEE and Bloomberg - Authors' calculations.

Table 6: **High stress periods**

VARIABLES	(1)	(2)
	Liquidity ratio	Solvency ratio
Liquidity ratio (t-1)	0.625*** (0.005)	0.000*** (0.000)
Solvency ratio (t-1)	5.207*** (0.643)	0.891*** (0.003)
Vix	0.840 (1.381)	-0.003 (0.006)
Interbank	-13.017 (22.632)	-0.362*** (0.104)
d_high_vix	283.517* (162.508)	1.434* (0.746)
d_high_interbank	430.995** (171.363)	-1.903** (0.787)
Vix * d_high_vix	-7.535* (4.082)	-0.025 (0.019)
Interbank * d_high_interbank	-154.743** (75.841)	1.166*** (0.348)
GDP	-11.349** (5.757)	-0.050* (0.026)
Inflation	4.816 (11.231)	-0.061 (0.052)
Size (t-1)	-281.622** (129.394)	-0.216 (0.594)
Retail (t-1)	0.269 (0.710)	-0.003 (0.003)
RoE (t-1)	0.501 (0.579)	0.002 (0.003)
2010 Dummy	-80.785*** (22.776)	0.631*** (0.105)
Constant	No	No
Bank Fixed Effects	Yes	Yes
Observations	23,264	23,264
Adjusted R-squared	0.798	0.978

Table 6 reports estimates of a system of 2 simultaneous equations. The two dependent variables are the liquidity coefficient and the solvency ratio, also used in lag among explanatory variables. The other explanatory variables include financial variables (VIX and interbank spread), macroeconomic variables (GDP and inflation rate), bank control variables (size, retail and return on equity ratio), dummy variables (referring to the period from 2010 onwards, to the high VIX periods and to the high interbank spreads periods), interaction terms and individual bank fixed effects. Standard errors are in brackets.

*** p<0.01, ** p<0.05, * p<0.1.

Sources: ACPR, INSEE and Bloomberg - Authors' calculations.

Table 7: Less liquid/less capitalized banks

VARIABLES	(1)	(2)
	Liquidity ratio	Solvency ratio
Liquidity ratio (t-1)	0.625*** (0.005)	0.000*** (0.000)
Solvency ratio (t-1)	5.195*** (0.644)	0.890*** (0.003)
Vix	0.934 (1.403)	-0.003 (0.007)
Interbank	-10.650 (22.870)	-0.402*** (0.106)
Vix * d_high_vix	-7.626* (4.084)	-0.024 (0.019)
Interbank * d_high_interbank	-153.617** (75.890)	1.223*** (0.348)
d_high_vix	286.385* (162.554)	1.419* (0.746)
d_high_interbank	426.260** (171.518)	-1.999** (0.787)
d_lessliq	-23.749 (103.040)	
d_undercap		-0.806** (0.329)
Vix * d_lessliq	-0.886 (4.741)	
Vix * d_undercap		0.005 (0.014)
Interbank * d_lessliq	-27.323 (61.219)	
Interbank * d_undercap		-0.007 (0.202)
Macroeconomic variables	Yes	Yes
Bank controls	Yes	Yes
2010 Dummy	Yes	Yes
Constant	No	No
Bank Fixed Effects	Yes	Yes
Observations	23,264	23,264
Adjusted R-squared	0.799	0.978

Table 7 reports estimates of a system of 2 simultaneous equations. The two dependent variables are the liquidity coefficient and the solvency ratio, also used in lag among explanatory variables. The other explanatory variables include financial variables (VIX and interbank spread), macroeconomic variables (GDP and inflation rate), bank control variables (size, retail and return on equity ratio), dummy variables (referring to the period from 2010 onwards, to the least liquid banks, to the least capitalized banks, to the high vix periods and to the high interbank spreads periods) and individual bank fixed effects. Columns (1) and (2) present estimates of the impact of financial variables on regulatory ratios for banks that are less liquid (liquidity coefficient < 120%) or less capitalized (solvency ratio < 10%). Standard errors are in brackets. *** p < 0.01, ** p < 0.05, * p < 0.1.

Sources: ACPR, INSEE and Bloomberg - Authors' calculations.

Table 8: Impact of larger banking group membership

VARIABLES	(1)	(2)	(3)	(4)
	Liquidity ratio	Solvency ratio	Liquidity ratio	Solvency ratio
Liquidity ratio (t-1)	0.625*** (0.006)	0.000*** (0.000)	0.624*** (0.006)	0.000*** (0.000)
Solvency ratio (t-1)	2.586*** (0.672)	0.890*** (0.003)	2.491*** (0.673)	0.885*** (0.003)
Vix	48.204 (44.194)	0.173 (0.219)	48.745 (44.190)	0.193 (0.219)
Interbank	-532.704 (449.194)	-7.989*** (2.227)	-523.751 (449.158)	-7.978*** (2.222)
Vix * d_high_vix	-1.205 (6.929)	0.001 (0.034)	-1.554 (6.931)	-0.004 (0.034)
Interbank * d_high_interbank	-26.427 (71.300)	-0.018 (0.353)	-26.533 (71.304)	-0.021 (0.353)
d_group	843.721 (1,078.691)	-2.633 (5.347)	802.823 (1,078.787)	-2.472 (5.338)
Vix * d_group	-48.929 (44.212)	-0.188 (0.219)	-47.488 (44.222)	-0.210 (0.219)
Interbank * d_group	528.731 (449.733)	7.793*** (2.229)	504.955 (450.424)	8.113*** (2.228)
d_liq_excess			124.586** (49.038)	
Vix * d_liq_excess			-4.371* (2.347)	
Interbank * d_liq_excess			42.111 (51.605)	
d_cap_excess				0.944*** (0.253)
Vix * d_cap_excess				0.013 (0.012)
Interbank * d_cap_excess				-0.855*** (0.268)
Macroeconomic variables	Yes	Yes	Yes	Yes
Bank controls	Yes	Yes	Yes	Yes
Dummies	Yes	Yes	Yes	Yes
Constant	No	No	No	No
Bank Fixed Effects	Yes	Yes	Yes	Yes
Observations	18,114	18,114	18,114	18,114
Adjusted R-squared	0.817	0.980	0.818	0.980

Table 8 reports estimates of a system of 2 simultaneous equations. The two dependent variables are the liquidity coefficient and the solvency ratio, also used in lag among explanatory variables. The other explanatory variables include financial variables (VIX and interbank spread), macroeconomic variables (GDP and inflation rate), bank control variables (size, retail and return on equity ratio), dummy variables (referring to the period from 2010 onwards, to the high vix periods and to the high interbank spread periods) and individual bank fixed effects. Columns (1) and (2) present estimates including a dummy capturing larger banking group membership. Columns (3) and (4) also include dummies indicating an excess of liquidity (liquidity coefficient > 150%) or solvency (solvency ratio > 15%) at the banking group level. Standard errors are in brackets. *** p < 0.01, ** p < 0.05, * p < 0.1.

Sources: ACPR, INSEE and Bloomberg - Authors' calculations.

Table 9: **Heterogeneity between banks' types**

VARIABLES	(1)	(2)
	Liquidity ratio	Solvency ratio
Liquidity ratio (t-1)	0.624*** (0.006)	0.000*** (0.000)
Solvency ratio (t-1)	2.713*** (0.675)	0.888*** (0.003)
Vix	0.721 (2.605)	-0.051*** (0.013)
Interbank	-76.765 (56.551)	0.688** (0.280)
Vix * d_high_vix	-1.475 (6.929)	-0.003 (0.034)
Interbank * d_high_interbank	-25.352 (71.319)	0.019 (0.353)
d_fin	112.177 (187.432)	-0.114 (0.929)
Vix_d_fin	-0.044 (4.785)	-0.006 (0.024)
Interbank_d_fin	36.053 (113.954)	-1.276** (0.565)
d_com	193.505 (148.881)	-1.964*** (0.738)
Vix_d_com	-3.437 (3.110)	0.076*** (0.015)
Interbank_d_com	131.497* (67.174)	-1.830*** (0.333)
d_mut	68.605 (425.375)	-0.043 (2.108)
Vix_d_mut	-0.244 (3.199)	0.032** (0.016)
Interbank_d_mut	65.684 (71.189)	-0.423 (0.353)
Macroeconomic variables	Yes	Yes
Bank controls	Yes	Yes
Dummies	Yes	Yes
Constant	No	No
Bank Fixed Effects	Yes	Yes
Observations	18,114	18,114
Adjusted R-squared	0.818	0.980

Table 9 reports estimates of a system of 2 simultaneous equations. The two dependent variables are the liquidity coefficient and the solvency ratio, also used in lag among explanatory variables. The other explanatory variables include financial variables (VIX and interbank spread), macroeconomic variables (GDP and inflation rate), bank control variables (size, retail and return on equity ratio), dummy variables (referring to the period from 2010 onwards, to the high vix periods, to the high interbank spread periods and to the types of bank) and individual bank fixed effects. Columns (1) and (2) present estimates of the interaction between financial variables and regulatory ratios depending on the type of bank (commercial, mutual banks or financial firms). Standard errors are in brackets.

*** p<0.01, ** p<0.05, * p<0.1.

Sources: ACPR, INSEE and Bloomberg - Authors' calculations.

Table 10: **Simultaneous equations: Liquid assets - Cash outflows - Solvency ratio**

VARIABLES	(1)	(2)	(3)
	Liquid assets	Cash outflows	Solvency ratio
Liquid assets (t-1)	0.577*** (0.006)	0.004** (0.002)	0.003 (0.002)
Cash outflows (t-1)	0.200*** (0.014)	0.794*** (0.004)	0.004 (0.005)
Solvency ratio (t-1)	0.110*** (0.008)	-0.009*** (0.002)	0.892*** (0.003)
Vix	-0.010 (0.017)	0.000 (0.005)	-0.003 (0.006)
Interbank	-0.796*** (0.286)	-0.304*** (0.084)	-0.351*** (0.104)
Vix * d_high_vix	0.005 (0.052)	0.009 (0.015)	-0.026 (0.019)
Interbank * d_high_interbank	0.877 (0.958)	0.890*** (0.281)	1.168*** (0.348)
GDP	-0.251*** (0.073)	0.038* (0.021)	-0.051* (0.026)
Inflation	0.078 (0.142)	0.115*** (0.042)	-0.061 (0.052)
Size (t-1)	-0.384 (1.635)	0.659 (0.480)	-0.209 (0.594)
Retail (t-1)	-0.080*** (0.009)	-0.011*** (0.003)	-0.002 (0.003)
RoE (t-1)	-0.000 (0.007)	0.005** (0.002)	0.002 (0.003)
2010 Dummy	-3.956*** (0.294)	-0.873*** (0.086)	0.650*** (0.107)
Constant	No	No	No
Bank Fixed Effects	Yes	Yes	Yes
Observations	23,264	23,264	23,264
Adjusted R-squared	0.929	0.956	0.978

Table 10 reports estimates of a system of 3 simultaneous equations. The three dependent variables are the numerator of the liquidity coefficient (Liquid assets), the denominator of the liquidity coefficient (Cash outflows), both normalized, and the solvency ratio, also used in lag among explanatory variables. The other explanatory variables include financial variables (VIX and interbank spread), macroeconomic variables (GDP and inflation rate), bank control variables (size, retail and return on equity ratio), dummy variables (referring to the period from 2010 onwards, to the high vix periods and to the high interbank spreads periods) and individual bank fixed effects. Standard errors are in brackets.

*** p<0.01, ** p<0.05, * p<0.1.

Sources: ACPR, INSEE and Bloomberg - Authors' calculations.

Table 11: **Liquidity stress-test simulation**

Variables	(1) Bank's initial conditions or Stress parameters	(2) LC Stressed value
<i>Liqit</i> _{<i>t</i>-1} (in %)	110	300
<i>Capit</i> _{<i>t</i>-1} (in %)	9	
<i>Vix</i> _{<i>t</i>} (in points)	30.6	
<i>Vix</i> _{<i>t</i>} × <i>Dummy_high_vix</i> _{<i>t</i>} (in points)	30.6	
<i>Spread</i> _{<i>t</i>} (in %)	1.82	
<i>Spread</i> _{<i>t</i>} × <i>Dummy_high_spread</i> _{<i>t</i>} (in %)	1.82	
<i>GDP</i> _{<i>t</i>} (in %)	-0.79	
<i>Inflation</i> _{<i>t</i>} (in %)	0.21	
<i>Size</i> _{<i>it</i>-1} (in %)	8	
<i>Retail</i> _{<i>it</i>-1} (in %)	46	
<i>RoE</i> _{<i>it</i>-1} (in %)	13.2	
<i>D</i> ₂₀₁₀ _{<i>t</i>}	1	
<i>Dummy_high_vix</i> _{<i>t</i>}	1	
<i>Dummy_high_spread</i> _{<i>t</i>}	1	

Table 11 reports the simulation of a Liquidity Coefficient (LC) stressed value computed based on a bank's initial conditions and stressed parameters, using coefficients obtained in Table 6.

Sources: ACPR, INSEE and Bloomberg - Authors' calculations.

10 Annexes

10.1 Proof of the results of the theoretical model

The two first-order conditions of the Lagrangian are the following:

$$\frac{\partial \mathcal{L}}{\partial L} = 0 \Leftrightarrow r^l - r^d \left(\frac{1}{1+\eta} \right) - \gamma \sigma_L^2 L - \lambda = 0 \quad (17)$$

$$\Leftrightarrow L = \frac{r^l - r^d \left(\frac{1}{1+\eta} \right) - \lambda}{\gamma \sigma_L^2} \quad (18)$$

$$\frac{\partial \mathcal{L}}{\partial G} = 0 \Leftrightarrow r^g - r^d \left(\frac{1}{1+\eta} \right) - \gamma \sigma_G^2 G + \lambda \left(\frac{(\beta + (1-\beta)\phi(s))(1+\eta)}{\alpha(s)} - 1 \right) = 0 \quad (19)$$

Let's call A the expression $\left(\frac{(\beta + (1-\beta)\phi(s))(1+\eta)}{\alpha(s)} - 1 \right)$.

$$\Leftrightarrow G = \frac{r^g - r^d \left(\frac{1}{1+\eta} \right) + \lambda A}{\gamma \sigma_G^2} \quad (20)$$

If the liquidity constraint (11) holds with equality, then $AG = L$.

$$A \left(\frac{r^g - r^d \left(\frac{1}{1+\eta} \right) + \lambda A}{\gamma \sigma_G^2} \right) = \frac{r^l - r^d \left(\frac{1}{1+\eta} \right) - \lambda}{\gamma \sigma_L^2} \quad (21)$$

$$-A \left(\frac{r^g - r^d \left(\frac{1}{1+\eta} \right)}{\gamma \sigma_G^2} \right) + \frac{r^l - r^d \left(\frac{1}{1+\eta} \right)}{\gamma \sigma_L^2} = \frac{\lambda}{\gamma \sigma_L^2} + \frac{\lambda A^2}{\gamma \sigma_G^2} \quad (22)$$

This gives the expression of λ in (16).

- If the liquidity constraint is binding

$$(\beta + (1-\beta)\phi(s))G = \alpha D = \frac{\alpha}{\eta} K \Rightarrow G = \frac{\alpha K}{\eta[\beta + (1-\beta)\phi(s)]} \quad (23)$$

$$\frac{\partial G}{\partial \phi} \leq 0, \frac{\partial G}{\partial \beta} \leq 0 \text{ and } \frac{\partial G}{\partial \alpha} \geq 0$$

G increases if α increases or if β and ϕ decrease.

$$L = \left(\frac{\eta + 1}{\eta} - \frac{\alpha}{\eta[\beta + (1-\beta)\phi(s)]} \right) K \quad (24)$$

L decreases if α increases or if β and ϕ decrease.

- If the liquidity constraint is not binding

The programme of the the first boils down to the profit function and the balance sheet constraint (hence the Lagrangean is reduced to the first two parts of equation (13)). The FOCs lead to the following arbitrage conditions (as the terms in r^d cancel out):

$$r^l - \gamma \sigma_L^2 L = r^g - \gamma \sigma_G^2 G \quad (25)$$

Using the balance sheet constraint,

$$L + G = \left(\frac{\eta + 1}{\eta} \right) K \quad (26)$$

one gets the Markowitz portfolio:

$$G = \frac{(r^g - r^l) + \gamma \sigma_L^2 \left(\frac{1+\eta}{\eta} \right) K}{\gamma(\sigma_L^2 + \sigma_G^2)} \quad (27)$$

and

$$L = \frac{(r^l - r^g) + \gamma \sigma_G^2 \left(\frac{1+\eta}{\eta} \right) K}{\gamma(\sigma_G^2 + \sigma_L^2)} \quad (28)$$