The Collateral Risk of ETFs

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Abstract

As most Exchange-Traded Funds (ETFs) engage in securities lending or are based on total return swaps, they expose their investors to counterparty risk. To mitigate the funds' exposure, their counterparties must pledge collateral. In this paper, we present a framework to study collateral risk and provide empirical estimates for the $40.9 billion collateral portfolios of 164 funds managed by a leading ETF issuer. Overall, our findings contradict the allegations made by international agencies about the high collateral risk of ETFs. Finally, we theoretically show how to construct an optimal collateral portfolio for an ETF.

Keywords: Asset management, passive investment, derivatives, optimal collateral portfolio, systemic risk

JEL classification: G20, G23

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1 Introduction

With their low fees and ability to provide exposure to a variety of asset classes, exchange-traded funds (ETFs) have become popular investment vehicles among individual and institutional investors alike. The global ETF industry reaches a total of $2,400 billion in assets under management (AUM) in 2014-Q1 and has experienced an average growth of 30% per year for the past ten years (Blackrock, 2014).

ETFs come in two types. In a physical ETF, investors' money is directly invested in the index constituents in order to replicate the index return. Differently in a synthetic ETF, the fund issuer enters into a total return swap with a financial institution which promises to pay the performance of the index (Ramaswamy, 2011). A recent report by Vanguard (2013) indicates that 17% of the ETFs in the US are synthetic compared to 69% in Europe.[1] Furthermore, all leveraged and inverse ETFs traded in the world are based on synthetic replications.[2]

Besides investment risk, tracking risk (Buetow and Henderson, 2012), and liquidity risk (Hassine and Roncalli, 2013), ETF investors can also be exposed to counterparty risk. Indeed, physical ETF issuers can generate extra revenues by engaging in securities lending (Amenc et al., 2012; Bloomberg, 2013). Hence, there is a possibility that the securities will not be returned in due time. Furthermore, for synthetic ETFs, there is a risk that the total return swap counterparty will fail to deliver the index return. In order to mitigate counterparty risk, both securities lending positions and swaps must be collateralized. Recently, the Financial Stability Board (2011), the International Monetary Fund (2011), and Ramaswamy (2011) raised concerns about the poor quality of the collateral of ETFs and warned about potential financial stability issues that may arise from ETFs. As shown in Figure 1, these allegations attracted wide attention from the public

[1] The disparity in popularity of synthetic and physical ETFs between Europe and the US stems from regulation limiting derivatives usage and tax treatment (Vanguard, 2013). Other key differences include much higher penetration of the retail market in the US and higher fragmentation and lower liquidity for ETFs in Europe (International Monetary Fund, 2011).

[2] Leveraged ETFs provide exposure that is a multiple ($2\times$, $3\times$) of the performance of the index, whereas inverse ETFs generate the inverse performance of the index.

as well as from the press and triggered severe outflows from synthetic ETFs.[3]

< Insert Figure 1 >

In this paper, we assess the collateral risk of ETFs, which we define as the risk that the value of the collateral falls below the value of the Net Asset Value (NAV) of the fund. To do so, we study the composition of the $40.9 billion collateral portfolio of 164 ETFs managed by the fifth largest ETF provider in the world, db X-Trackers. These funds track a variety of asset classes: equities (74.5% of AUM, which is around industry's average) and, in decreasing order, Governments bonds, money markets, commodities, hedge fund strategies, credit, corporate bonds, currencies, and multi-assets. Furthermore, the sample ETFs provide different geographic exposures: Europe (58.9% of AUM), World, Asia-Pacific, North America, and the rest of the World.

For each fund, we know the exact composition of the collateral portfolio every week between July 5, 2012 and November 29, 2012. This is, to the best of our knowledge, the first time that such a dataset is used in an academic study. The high granularity of our data allows us to study empirically the collateral risk of ETFs for various asset exposures, regional exposures, and types of replication. In particular, we are able (1) to measure the level of collateralization, (2) to assess the quality and the liquidity of the pledged collateral, and (3) to test whether the value of the collateral and the ETF are positively correlated.

We find that the collateral portfolios of the 164 sample ETFs are populated with 3,299 different collateral securities. On average, there are 81 collateral securities per fund. However, the number of securities varies a lot across funds: from 10 to 20 for fixed-income funds to more than 100 for funds on equity or commodity. Furthermore, the value of the collateral portfolio often exceeds the NAV of the fund. Indeed, the average level of collateralization in our sample is 108.4%. However, the level of overcollateralization very much depends on the technology used to construct the ETF. Indeed, the level of collateralization is much higher for ETFs based on a funded swap (114.6%),

[3] A survey by Morningstar (2012b) shows that 89% of investors in the UK stating a specific preference for physical ETFs over synthetic ETFs.

in which invertors' money is transferred to the swap counterparty and haircuts applied, than for ETFs based on an unfunded swap (101.3%), in which investors' money is used to purchase a basket of securities from the swap counterparty.[4]

While ETFs are *passive* investment vehicles, they engage in *active* collateral management. Indeed, when studying the dynamics of the collateral, we report significant time-variation in the composition of the collateral portfolios of ETFs. On average, more than a third of the pledged collateral in a given fund changes from one week to the next. Changes through time are particularly strong for the collateral portfolios of equity and commodity ETFs (43.9% and 72.7% per week, respectively), which mainly include equities, than those of fixed income ETFs (around 2% per week), which are predominantly made of bonds.

In our sample, there is a good fit between the asset exposure of the fund (e.g. equity or fixed income) and the collateral used to secure the swap. This feature is extremely important given the fact that in the case of a default of the swap counterparty, the asset manager would need to sell the collateral in order to meet redemptions from investors. Hence, having the tracked index and the collateral moving in unison mitigates the risk exposure of investors. We uncover that 92.5% of the collateral securities in an equity ETF are made of equities and 96.5% of the collateral securities in a Government bond ETF are Government bonds. The match between exposure and collateral turns out to be lower for geographic exposures. For instance, 71.8% of the collateral securities of European index-tracking ETFs are issued in Europe. The matching score between exposure and issuers drops to around 20% for ETFs tracking Asia-Pacific or North-American indices. We interpret the greater popularity of European collateral as a form of home bias in collateral management. Furthermore, we show that the correlation between the returns of the ETF and of its collateral is on average positive. However, the correlation is particularly low for funds that track commodities, currencies, and money market funds and the correlation is strongly negative for inverse funds.

[4]More information on ETF structuring is provided in Section 2.

The equities used as collateral mainly come from large, European, non-financial firms that exhibit good level of liquidity and positive correlation with the index tracked by the fund. Reassuringly, collateralized equities correlate less with the stock return of the swap counterparty than with the ETF. Turning to fixed-income securities, we notice that bonds predominantly have European issuers (88.3%) and a AAA rating (65.5%). We find that the fraction of bonds with at least a AA rating is 84.5% for Government bonds and 64.9% for corporate bonds. Speculative-grade and undefined ratings account collectively for 1.4% of the bonds used as collateral. Furthermore, we find that ETF issuers tend to match the duration of the collateral with the duration of the fixed-income index tracked by the fund, which is sound risk-management practice.

In an attempt to quantify the collateral risk of ETFs, we propose two original collateral risk metrics: (1) the probability for a fund of not having enough collateral on the following day and (2) the magnitude of the collateral shortfall conditional on not having enough collateral. Using these two risk metrics, we can plot all ETFs on a risk map called "Torino scale" and identify funds with higher exposure to collateral risk, which may be of interest for both regulators and investors. In a final step, we show theoretically how to design an optimal collateral portfolio that aims to minimize collateral risk. The composition of the optimal collateral portfolio can be obtained by minimizing the variance of the collateral shortfall under the constraint that the fund will be sufficiently collateralized on average. We show that the optimal weights can be expressed as a function of the weights of the Markowitz's mean-variance portfolio but they also reflect the correlations between the collateral assets and the NAV.

This paper contributes to the growing literature on the potential financial stability issues arising from ETFs.[5] Ben-David, Franzoni and Moussawi (2014) show that ETF ownership increases stock volatility through the arbitrage trade between the ETF and the underlying stocks and through inflows and outflows. In the same vein, Krause, Ehsani and Lien (2013) document that volatility-

[5] For additional evidence on the link between asset management and financial stability, see Coval and Stafford (2007), Mitchell, Pedersen and Pulvino (2007), Boyson, Stahel and Stulz (2010), Chen, Goldstein and Jiang (2010), Jotikasthira, Lundblad and Ramadorai (2012), Manconia, Massa and Yasuda (2012), Kacperczyk and Schnabl (2013), and Schmidt, Timmermann and Wermers (2014).

spillover from ETF to the index constituents depends on the ETF liquidity and on the proportion of the stock held in the ETF. During the flash crash of May 6, 2010, Borkovec et al. (2010) provide evidence that price discovery failed dramatically for ETFs, and that it was primarily due to an extreme deterioration in liquidity to individual securities in the baskets tracked by the funds. Using a large sample of US equity ETF holdings, Da and Shive (2013) find a strong relation between measures of ETF activity and return comovement at both the fund and the stock levels (see also Sullivan and Xiong, 2012).

Other studies focus on the real effects of ETF rebalancing activities. Bessembinder et al. (2014) study trading activity and market price around the time of the Crude Oil ETFs rolls of crude oil futures. Consistent with the idea that these scheduled trades are not perceived as informed, they find that market liquidity improves on roll versus non-roll days, and report no evidence of predatory trading. Focusing on leveraged and inverse funds, Bai, Bond and Hatch (2012) find that late-day leveraged ETF rebalancing activity significantly moves the price of component stocks. Moreover, Shum et al. (2014) shows that predatory traders could profit on days of large market swings by "front-running" the potential rebalancing trades.

Unlike previous academic studies, we do not focus on the interplay between the ETFs and the assets they track. Instead, our study considers a source of risk for ETF investors that attracted significant attention from regulators and the media but, so far, little academic research: the collateral risk of ETFs. Using the holdings of a sample of equity ETFs, Cheng, Massa and Zhang (2013) show that ETFs strategically deviate from their indices and overweigh promising stocks for which the ETF affiliated bank has superior (lending-related) information. However for physical ETFs (95% of their sample), the holdings are typically lent to short sellers who need to post collateral with the fund. This is not investigated by Cheng, Massa and Zhang (2013). Looking at one MSCI Emerging Markets ETF in January 2011, Ramaswamy (2011) shows that the collateral composition of the fund has very little overlap with the composition of the MSCI Emerging Market index itself. Half of the equity collateral is made of Japanese stocks and the collateral also includes bonds, threequarters of which are from US issuers. Because it is based on one fund only and remains at the aggregated level, Ramaswamy' study does not tell us whether collateral mismatch is a widespread phenomenon. Furthermore, a rigorous study of the collateral of ETFs requires granular data about all collateral securities and, as acknowledged by Ramaswamy (2011) on page 9, "*Extracting this information using the International Security Identifying Number (ISIN) provided for each of the collateral assets would be a cumbersome process*". We take on this task in the present paper and reach different conclusions.

We make several contributions to the existing literature. To the best of our knowledge, our study is the first attempt to assess empirically the quality of the collateral for a large and representative sample of ETFs. Overall, our analysis of the quality of the securities pledged by the swap counterparties do not support the allegations made about the overall poor quality of ETF collateral. This, of course, does not mean that the agencies were wrong at the time but we find no support for this claim using more recent data. Our second contribution is methodological: we develop several risk management tools that can be used by asset managers, regulators, and investors to quantify the collateral risk of an ETF. Our framework allows one to compare the collateral risk of several ETFs or, alternatively, to build a collateral portfolio with the lowest possible level of collateral risk. We believe that this is the first attempt to derive optimal allocation rules for collateral portfolios, which is a topic of growing importance given the emphasis put on collateral by the recent financial regulatory reform (Dodd Franck in the US and EMIR in Europe).

The rest of our study is structured as follows. Section 2 introduces the different types of ETF structures in a common framework and defines collateral risk. Section 3 presents our dataset and discusses our main empirical findings. In Section 4, we show how to build an optimal collateral portfolio that aims to minimize collateral risk. We conclude our study in Section 5.

2 ETF Structures and Collateral Risk

2.1 Unfunded Swap Model

The most commonly used structure for synthetic replications is the unfunded swap model. In this model, the ETF issuer enters into a total return swap with a counterparty, which can either be an affiliated bank from the same banking group or another bank (see Figure 2, right panel). The swap counterparty commits to deliver the return of the reference index and sells a substitute basket of securities to the ETF issuer. The second leg of the swap consists of the performance of the basket of securities paid by the issuer to the swap counterparty. An important feature of this model is that the ETF issuer becomes the legal owner of the assets and enjoys direct access to them. This means that if the swap counterparty defaults, the ETF issuer can immediately liquidate the assets.

< Insert Figure 2 >

One key reason for the great popularity of the unfunded swap model in Europe is the fact that it allows ETFs to be eligible to the tax-friendly share savings plans, such as the French *Plan d'Epargne en Actions* (PEA). In theory, eligible mutual funds must be made up of at least 75% of shares of companies headquartered in a European member state. As a result, an ETF based on an unfunded swap and that tracks an index of Asian or US equities is eligible as long as the substitute basket is made of enough European equities.

At any point in time, the counterparty exposure of the issuer, or swap value, is measured as the difference between the ETF's NAV, denoted I_t, and the value of the substitute basket used as collateral, denoted C_t. The swap is marked to market at the end of each day and reset whenever the counterparty exposure exceeds a given threshold, $\theta \in [0, 1[$, expressed as a percentage of the NAV:

$$I_t - C_t > \theta \; I_t. \tag{1}$$

In the event of a reset, the swap counterparty delivers additional securities for a value of $I_t - C_t$ to top up the substitute basket. A comprehensive industry survey conducted by Morningstar (2012a) indicates that most ETF issuers use a θ coefficient of less than 10% and some even maintain full collateralization, $\theta = 0$.

2.2 Funded Swap Model

In the funded swap model, the ETF issuer transfers investors' cash to a swap counterparty in exchange for the index performance plus the principal at a future date (see Figure 2, left panel). The swap counterparty pledges collateral assets in a segregated account with a third party custodian. The posted collateral basket is made of securities which come from the counterparty's inventory and meet certain conditions in terms of asset type, liquidity, and diversification. In practice, appropriate haircuts apply to the assets posted as collateral to account for the risk of value fluctuations and for imperfect correlation between the index and the collateral value. As a consequence, funded-swap based ETFs are expected to be overcollateralized, $C_t > I_t$.

The swap counterparty exposure is measured by the difference between the NAV and the collateral value, once properly adjusted for haircut. If we denote by h the haircut, it means that if:

$$I_t - C_t\,(1 - h) > 0 \tag{2}$$

additional collateral corresponding to $I_t - C_t\,(1 - h)$ is requested in order to maintain appropriate collateralization.

2.3 Collateral Risk Metrics

We now propose a global representation for synthetic ETFs which includes both funded and unfunded swap models as special cases. For all ETFs, the counterparty exposure expressed in percentage of the NAV must be smaller than a given threshold θ:

$$I_t - C_t\,(1 - h) \leq \theta\; I_t \tag{3}$$

with $h = 0$ and $\theta \geq 0$ for unfunded-swap based ETFs and $h > 0$ and $\theta = 0$ for funded ones.[6] If condition (3) is not satisfied, additional collateral or reinvestment in the substitute basket is

[6]We assume that the swap is not reset if the fund is overcollateralized ($I_t - C_t < 0$) since this does not induce any counterparty risk.

required to move to $C_t(1-h) = I_t$ (Morningstar, 2012a).

Within this framework, we can capture different facets of collateral risk. We denote by Δ_{t+1} the collateral shortfall of a given fund at time $t+1$:

$$\Delta_{t+1} = I_{t+1}(1-\theta) - C_{t+1}(1-h) \tag{4}$$

Given the information available at time t, this quantity is stochastic since the future values of the collateral portfolio and the NAV are unknown. In this context, we propose two collateral risk metrics. The first risk metrics we define is the probability for a fund of being undercollateralized:

Definition 1 (Probability of collateral shortfall) *The probability for a fund of being undercollateralized at time $t+1$ is defined as:*

$$p_{t+1} = \Pr\left(\Delta_{t+1} > 0 \mid \mathcal{F}_t\right) \tag{5}$$

where $\mathcal{F}_t$ denotes the set of information available at time t.

A second important dimension of collateral risk is the magnitude of the collateral shortfall.

Definition 2 (Expected collateral shortfall) *The expected collateral shortfall at time $t+1$ is:*

$$S_{t+1} = \mathbb{E}_t\left(\Delta^*_{t+1} \;\middle|\; \Delta_{t+1} > 0\right) \tag{6}$$

where $\mathbb{E}_t$ denotes the expectation conditional on $\mathcal{F}_t$ and $\Delta^*_{t+1} = I_{t+1} - C_{t+1}(1-h)$.

Notice that in the case of a funded-swap based ETF ($\theta = 0$, $h > 0$), the expected collateral shortfall can be viewed as a standard expectation of a truncated random variable, defined as $\mathbb{E}(Y \mid Y > 0)$. Indeed, for a funded-swap based ETF, a collateral shortfall appears when the NAV exceeds the net value of the collateral portfolio, i.e., when $I_{t+1} > C_{t+1}(1-h)$, and the required additional collateral is then equal to $I_{t+1} - C_{t+1}(1-h)$. On the contrary, in the case of an unfunded-swap based ETF ($\theta > 0$, $h = 0$), when the counterparty exposure exceeds $\theta\%$ of the NAV, i.e., when

$I_{t+1}(1-\theta) > C_{t+1}$, the swap is reset to 0. In this case, the required additional collateral is then equal to $I_{t+1} - C_{t+1}$.[7]

We estimate the collateral risk metrics using a simple nonparametric method. To do so, we define $\omega_t = (\omega_{1,t}, ..., \omega_{K,t})$, the vector of the weights associated with the K assets that enter into the collateral portfolio of a fund at time t, with $\sum_{k=1}^{K} \omega_{k,t} = 1$. The collateral shortfall at time $t+1$ can be written as:

$$\begin{aligned}
\Delta_{t+1} &= I_{t+1}(1-\theta) - C_{t+1}(1-h) && (7) \\
&= I_t(1-\theta)(1+r_{I,t+1}) - C_t(1-h)(1+r_{C,t+1}) && (8)
\end{aligned}$$

where $r_{I,t+1}$ and $r_{C,t+1}$ denote the return of the NAV and the return of the collateral portfolio, respectively. Given the information set $\mathcal{F}_t$, the current NAV I_t, the value of collateral portfolio C_t and its composition ω_t can be considered as constant. Thus, the potential collateral shortfall at time $t+1$ only depends on two random variables, $r_{I,t+1}$ and $r_{C,t+1}$.[8]

Similar to what is done in the Value-at-Risk literature (Berkowitz and O'Brien, 2002), we define a series of hypothetical collateral shortfalls:

$$\Delta_s = I_t(1-\theta)(1+r_{I,s}) - C_t(1-h)(1+r_{C,s}) \tag{9}$$

where $r_{I,s}$ is the historical daily return of the NAV, for $s = 1, ..., t$, $r_{C,s} = \sum_{k=1}^{K} \omega_{k,t} r_{k,s}$ is the daily return of the collateral portfolio with weights ω_t, and $r_{k,s}$ is the daily return of the k-th collateral security at time s. The hypothetical collateral shortfall Δ_s measures the shortfall that

[7] The two risk metrics can also be defined conditionally:

$$\begin{aligned}
p^c_{t+1} &= \Pr(\Delta_{t+1} > 0 \mid Crisis_{t+1}) \\
S^c_{t+1} &= \mathbb{E}_t\left(\Delta^*_{t+1} \mid \Delta_{t+1} > 0 \;;\; Crisis_{t+1}\right)
\end{aligned}$$

where *Crisis* refers to a situation of global financial stress (e.g. the year 2008). Alternatively, the risk metrics can be estimated conditional on the swap counterparty being in financial distress:

$$\begin{aligned}
p^d_{t+1} &= \Pr(\Delta_{t+1} > 0 \mid Distress_{t+1}) \\
S^d_{t+1} &= \mathbb{E}_t\left(\Delta^*_{t+1} \mid \Delta_{t+1} > 0 \;;\; Distress_{t+1}\right)
\end{aligned}$$

where the *Distress* of the swap counterparty can be proxied by its CDS exceeding a given threshold.

[8] One way to estimate the shortfall probability and the expected collateral shortfall is to assume a given distribution for the returns of the NAV and for the collateral portfolio. Differently in the following, we implement a simple nonparametric estimation method.

would have arisen in the past with the current values of I_t, C_t, and ω_t and past returns on the NAV and on the collateral securities. Then, we can define two nonparametric estimators for the probability and the expected collateral shortfall:

$$\widehat{p}_{t+1} = \frac{1}{t} \sum_{s=1}^{t} \mathbb{I}\left(\Delta_s > 0\right) \tag{10}$$

$$\widehat{S}_{t+1} = \frac{\sum_{s=1}^{t} \Delta_s^* \times \mathbb{I}\left(\Delta_s > 0\right)}{\sum_{s=1}^{t} \mathbb{I}\left(\Delta_s > 0\right)} \tag{11}$$

where $\mathbb{I}(.)$ denotes the indicator function and $\Delta_s^* = I_t\left(1 + r_{I,s}\right) - C_t\left(1 - h\right)\left(1 + r_{C,s}\right)$. Note that if Δ_s is a stationary process, these estimators are consistent and asymptotically normally distributed (Chen, 2008). One advantage of this nonparametric method is that it is model-free since it avoids biases caused by using a misspecified loss distribution.

Finally, we propose an original approach to graphically summarize the collateral risk of ETFs in a two-dimensional graph that is called a Torino scale. The latter is a method developed in the mid 1990's by MIT Earth, Atmospheric, and Planetary Sciences Professor Richard P. Binzel for categorizing asteroids. The scale aims to measure asteroid-specific collision probabilities with the earth and the energy generated by such collisions. Torino scales are useful whenever one needs to jointly display the probability of an event ("the impact") and the effects of this event ("the energy"). In our context, the "impact" corresponds to the occurrence of a collateral shortfall in a fund and the "energy" to the magnitude of the shortfall.

Definition 3 (Torino Scale for collateral risk) *A Torino scale for collateral risk is a two-dimensional graph in which each fund is represented by its probability of being undercollateralized (y-axis, capped at 100%) and its expected collateral shortfall in percentage of the NAV (x-axis, open-ended).*

The funds that are located in the North-East corner of the Torino scale are the ones that are the most exposed to collateral risk, as shown in Figure 3. The key advantage of this graphic representation is that it permits to easily compare the collateral risk for different ETFs or to monitor the collateral risk of a given fund through time. Hence, it provides a global view of the collateral risk in the ETF market, which may be of interest for both regulators and investors.

< Insert Figure 3 >

In a Torino scale, the collateral risk of the fund increases with the distance between the origin of the graph and the data point. This distance could be estimated by weighting differently the probability and magnitude components. Alternatively, we use the Euclidean distance that gives the same importance to both dimensions of the collateral risk. This distance, which we call Aggregate Collateral Risk (ACR), is measured by:

$$ACR_{t+1} = \sqrt{p_{t+1}^2 + S_{t+1}^2}. \tag{12}$$

These three measures of collateral risk, p, S, and ACR, will be estimated with actual data for all sample ETFs in the following section.

3 An Empirical Analysis of ETF Collateral

We study the collateral of a sample of ETFs issued by db X-Trackers, the fifth largest ETF issuer in the World and the second in Europe (by AUM, as of December 2012). We see in Table 1 that the 164 sample ETFs have a combined AUM of \$37.927 billion, which corresponds to more than 10% (respectively 30%) of the total AUM of all (synthetic) ETFs in Europe (Blackrock, 2012; Vanguard, 2013).[9] All the funds in our study are synthetic and most of them are based on a funded swap (112 funds vs. 52 funds based on an unfunded swap). However, both types of funds account for comparable AUM (\$20.1 billion vs. \$17.8 billion). It is also important to notice that a significant fraction of our funds (30 funds and 5.1% of AUM) are inverse funds that deliver the inverse performance of an index.

[9]The ETF data have been retrieved from the db X-trackers website (www.etf.db.com). Our original sample includes 207 ETFs with combined asset of \$40.2 billion. In our analysis, we only consider the 164 funds for which we have been able to retrieve from Datastream and Bloomberg at least one year of data for the ETF price and its index.

In terms of asset exposure, the majority of the funds in our sample track a stock index as equity funds account for 74.5% of AUM. Our sample is representative of the European ETF industry as most ETFs are synthetics and the share of equity ETFs is around 70% (Vanguard, 2013). Besides equity, the other funds allow investors to be exposed to a variety of asset classes including Government bonds (11% of AUM), treasuries and commercial papers (6.6%), commodities (3.8%), hedge funds (2.2%), credit (0.7%), corporate bonds (0.6%), and currencies (0.3%). Furthermore, half of the sample funds track European indices (79 funds out of 164 and 58.9% of AUM) whereas the remaining funds replicate the returns of some World indices (22.3%), Asia-Pacific indices (9.4%), North-American indices (7.2%), or indices from the rest of the World (2.2%).

< Insert Table 1 >

3.1 A First Look at the Collateral Portfolios

Allegations were recently made about the overall poor quality of ETF collateral. For instance, the Financial Stability Board (2011, page 4) states: "*As there is no requirement for the collateral composition to match the assets of the tracked index, the synthetic ETF creation process may be driven by the possibility for the bank to raise funding against an illiquid portfolio [...] the collateral basket for a S&P 500 synthetic ETF could be less liquid equities or low or unrated corporate bonds in an unrelated market.*"

To formally test the validity of these allegations, we collect for each sample fund the composition and the value of its collateral portfolio with a weekly frequency between July 5, 2012 and November 29, 2012. The collateral data have been retrieved from the db X-Trackers website but because the website keeps no historical data, we had to download the collateral data for each fund, every week over our sample period. Then for each security used as collateral, we obtain its historical daily prices from Datastream.[10]

In Table 2, we see that the aggregate size of all collateral portfolios is equal to $40.9 billion, which

[10]For bonds, we use the time series of the bond index return that best matches the attributes of the bonds: its type (sovereign vs. corporate), country, rating, and maturity.

indicates that, on average, the funds included in our analysis are overcollateralized (AUM = \$37.9 billion). For a given fund, the value-weighted average level of collateralization is 108.4%. However, the level of overcollateralization is higher for funded-swap based ETFs (114.6%) and for inverse funds (115.4%). The result for funded swaps comes from the fact that haircuts are applied to the value of the pledged collateral according to the following weighting scheme: 0% for Government bonds, 10% for corporate bonds, and between 7.5% and 20% for equities (Deutsche Bank, 2012). Differently, no haircut applies for unfunded swaps as, in this case, the asset managers purchase some securities, i.e., the substitute basket, from its swap counterparty.

< **Insert Table 2** >

In total, there are 3,299 different securities that are used as collateral in our sample, which leads to 81 collateral securities per fund on average. We notice that the number of securities is much higher for equity or commodity funds (around 100 securities per fund) than for fixed-income funds tracking Government or corporate bond indices, money market funds, or CDS indices (10 to 20 securities per fund). In our sample, most of the collateral portfolio is made of equities. Out of the 3,299 securities, 2,591 are equities. When measured in value, equities account for around 75% of the collateral vs. 20% for Government bonds and 5% for corporate bonds.

We also report significant time-variation in the composition of the collateral portfolios of ETFs. For a given fund, we define the turnover of the collateral portfolio as the ratio between the number of different securities, as defined by their ISIN, that enter (n^+) or exit (n^-) the collateral portfolio between t and $t+1$ divided by the total number of collateral securities (N) on both dates:

$$turnover_{t,t+1} = \frac{n^+_{t,t+1} + n^-_{t,t+1}}{N_t + N_{t+1}}. \quad (13)$$

On average in our sample, more than a third of the pledged collateral in a given fund changes from one week to the next (see Table 2). Changes through time are particularly strong for unfunded-swap based ETFs, as almost half of their collateral is replaced from one week to the next. Furthermore, the managers of equity and commodity ETFs rebalance their collateral to a much greater extent (43.9% and 72.7%, respectively) than those of fixed income ETFs (around 2%).

One of the most persistent criticisms addressed to ETFs is the fact that the collateral may not be positively correlated with the index tracked by the ETF. Indeed, when the correlation is negative, the hedge provided by the collateral is less efficient: if the index return is large and positive and the swap counterparty defaults, the value of the collateral shrinks and a collateral shortfall mechanically arises. To look at this issue empirically, we compare the index tracked by the ETF and the securities included in the collateral portfolio. In Panel A of Table 3, we notice that there is a good match between the two as 92.5% of equity ETFs are backed with equity and 96.5 of Government bonds ETFs are collateralized with Government bonds. Differently, the collateral of Corporate bonds ETFs is not made of corporate bonds but, instead, exclusively of Government bonds. In Panel B, we see that the match between exposure and collateral turns out to be lower for geographic exposures. While 66% of the collateral are made of European securities, we find that this percentage rises to 71.8% for ETFs tracking European indices.[11] The matching score between exposure and issuers drops to 24.1% for ETFs tracking Asia-Pacific indices and to 19.5% for ETFs tracking North-American indices.

< Insert Table 3 >

We present in Figure 4 the distribution of the correlation between the return of the ETF and the return of its collateral portfolio.[12] We clearly see that most correlations are positive (the average correlation $\bar{\rho}$ is 0.305), which confirms the close connection that exists between the pledged collateral and the index tracked by the ETF. However, the distribution is bimodal with significant

[11] To understand the predominant role played by European collateral, which we dub "collateral home bias", one needs to understand the origin of the pledged collateral. Indeed, these securities come from the books of the swap counterparty, typically a large financial institution. In our sample, the swap counterparty is Deutsche Bank and as a result, its books predominantly include securities issued by local firms held for investment purposes, market making, or other intermediation activities.

[12] When the returns of one or several securities included in a collateral portfolio are missing, we apply the following rule: (1) when the cumulative weight of the missing securities exceeds 5%, we do not compute the portfolio return on this particular date and (2) when the cumulative weights of the missing securities is equal or less than 5%, we compute the portfolio return using all available returns (with weights properly rescaled to sum to 100%). When computing historical correlations or betas between a collateral security and another security, we impose a minimum of 50 days during which both returns are available.

mass associated with negative correlations. Looking more closely at our sample of funds reveals that most negative correlations are associated with inverse ETFs ($\bar{\rho}_{INV} = -0.595$).[13]

We regress the correlation between the return of the ETF and its collateral portfolio on a series of firm-specific variables using a panel linear specification with individual effects.[14] We show in Table 4 that the level of collateralization of the fund, the number of securities in the collateral portfolios, and the fraction of European equities in the collateral portfolio are positively and significantly associated with the ETF-collateral correlation. Differently, inverse funds and funds that track commodities, currencies, and money market funds tend to have a lower correlation with their collateral. This latter result is due to the fact that these asset classes are typically more difficult to include in the collateral portfolio.

< Insert Figure 4 and Table 4 >

To get a better sense of the type of securities used as collateral, we conduct an in-depth analysis of all equities, and then of all bonds. We start with equities in Table 5 and show that most equities used as collateral are issued by large, European, non-financial firms. Furthermore, collateralized equities exhibit good liquidity on average, with an average bid-ask spread of 0.21% and an average daily trading volume that corresponds to 4.67% of the market capitalization. We also find that 92.5% of the equities correlate positively with the index tracked by the ETF. Another reassuring finding is the fact that, on average, the pledged equities have a higher beta, or conditional beta, with respect to the ETF than with respect to the stock return of Deutsche Bank, which is the swap counterparty for all ETFs. The average beta is 60% higher when computed with the index than with the swap counterparty and its distribution has more mass on any value greater than 0.4. Using collateral securities that correlate strongly with the counterparty or even worse, are issued

[13] When computing the ETF-collateral correlations for inverse ETFs we systematically use the short index and not the regular index. For instance, for the db X-tracker on the S&P 500 SHORT index, we use the latter and not the S&P 500 index.

[14] In order to guarantee that the estimated correlation remains within the $[-1, 1]$ range, the dependent variable is defined as $\ln\left(\left(1 + corr_{i,t}\right) / \left(1 - corr_{i,t}\right)\right)$. We also estimated a binary specification (panel logit model with random effects) for the sign of the correlation and obtained qualitatively similar (unreported) results.

by the counterparty, would be inefficient as their value would go to zero in the case of a default of the counterparty.

< Insert Table 5 >

When we look at the bond part of the collateral portfolio in Table 6, we notice that bonds predominantly have European issuers (88.3%), which is again consistent with the existence of a home bias (see Panel A). The fraction of European bonds is higher for corporate bonds (96.6%) than for Government bonds (86.0%). Note that the lower fraction of European Sovereign bonds may be due to the fact that our sample period corresponds to the end of the Eurozone crisis of 2012. Turning to bond ratings in Panel B, we see that 65.5% of the bonds have a AAA rating, which is significantly lower than the 74.7% mentioned by Deutsche Bank (2012) at the end of the year 2011. This difference could be due to the fact that many downgrades occurred in 2012, including many large banks and several European sovereign issuers. In our sample, the fraction of bonds with at least a AA rating is 84.5% for Government bonds and 64.9% for corporate bonds. Speculative-grade ratings account for 0.4% of the bonds used as collateral and undefined ratings for 1.3%. Our results drastically contrast with those of Ramaswamy (2011) that were based on a single equity ETF. He reports that 8.7% of the bonds are rated AAA, 13.1% at least AA, and 38% are unrated (vs. 65.5%, 80.3%, and 1.3%, respectively, in our sample), which seems to indicate that the fund selected by Ramaswamy is not representative of the entire ETF industry.

< Insert Table 6 >

The maturity spectrum of the bonds within the collateral portfolios covers a wide range of maturities from less than a year to more than 10 years (see Panel C). Overall, 40.8% of the bonds used as collateral have a maturity of less than 3 years and 23.1% have a maturity of more than 10 years. Government bonds and North-American bonds are more on the long side whereas the maturity of corporate bonds remains almost exclusively below 5 years. Interestingly, we find that the duration of the collateral matches well with the duration of the fixed-income index tracked by the fund. Indeed, ETFs that track an index with a maturity below 3 years mainly have collateralized bonds with a maturity less than 3 years (71.1%). The match is even stronger for funds that track medium maturity indices (3-10 years), with 89.3% of the collateralized bonds within the 3-10 year maturity bucket, and for funds that track long term indices (>10 years), for which 84.9% of the collateralized bonds also have a maturity of more than 10 years.

3.2 Collateral Risk Analysis

We estimate for each of the 164 ETFs its probability of being undercollateralized and its expected collateral shortfall. Both risk measures are computed with a one-day horizon and using the non-parametric estimators presented in Equations (10) and (11). Following Deutsche Bank (2012), we consider a counterparty risk exposure of $\theta = 5\%$ and haircuts that depend on the type of securities: equities, Government bonds, corporate bonds, and others.

In Figure 5, we display for all sample funds both the average shortfall probability and expected collateral shortfall, expressed as a percentage of the NAV, in a Torino scale. For each fund, the risk metrics are averaged across time. The main result in this figure is that collateral risk remains moderate. On average, the shortfall probability is 4.30% and the average expected shortfall is 0.94%. Furthermore, most observations concentrate in the region close to the origin of the graph. We find that 79.62% of the funds have a probability of less than 5% of being undercollateralized and that 73.25% of the funds have an expected collateral shortfall lower than 1%. We also find that ETFs based on unfunded swaps have higher expected shortfalls on average (1.11%) than those based on funded swaps (0.86%). This result is due to the current practice of reseting unfunded swaps when their collateral value fall below 95% of the NAV. In such a case, the swap counterparty has to provide extra collateral to go back to full collateralization, and not to 95% of the NAV.

Furthermore, we contrast the collateral risk exposure of inverse and long ETFs. On the one hand, as inverse ETFs tend to be more collateralized than long ETFs, as shown in Table 2, we expect inverse funds to have lower exposure to collateral risk. On the other hand, the negative relationship between the returns of inverse ETFs and its collateral documented in Figure 4 is expected to increase collateral risk exposure. While the net effect is a priori unknown, the univariate empirical evidence in the bottom part of Figure 5 suggest that inverse funds exhibit a lower probability of being undercollateralized (1.92% vs. 4.86%) but higher expected shortfall (1.63% vs. 0.78%).

< Insert Figure 5 >

We complement our analysis by running multivariate regressions in which we regress different dimensions of collateral risk (p, S, and ACR) on the characteristics of the funds. Overall, we see in the last three columns of Table 4 that funds based on funded swaps (p and ACR) and inverse funds (p and S) exhibit on average a higher level of collateral risk. Furthermore, larger funds, as well as funds that track commodities or currencies exhibit a higher level of collateral risk. Finally, and as expected, the level of collateralization of a fund strongly decreases its exposure to collateral risk, regardless of the risk metrics considered.

4 Optimal Collateral Portfolio

4.1 Definition

We show in this section how to construct an optimal collateral portfolio that aims to protect the ETF issuer, as well as its investors, against collateral risk. The collateral portfolio shall be mutually agreed upon by both the collateral provider (the swap counterparty, typically a large financial institution) and the collateral receiver (the ETF issuer). The process that leads to the optimal collateral portfolio can be divided into three steps.

First, the swap counterparty and the ETF issuer have to determine a set of eligible securities. In practice, the securities pledged as collateral directly come from the inventory of the swap counterparty, which includes securities held for investment purposes, market making, underwriting, or other intermediation activities. As a result, there is no need for the collateral provider to purchase any new securities to meet collateral requirements. In general, when choosing the securities to be pledged, the collateral provider primarily transfers the ones that minimize the opportunity cost of holding collateral. Such securities include those with relatively low fees on the securities lending market; those with relatively low collateral value in the repo market (Bartolini et al., 2011); those that are not eligible as collateral for central-bank credit operations; and securities for which the demand is low on the secondary market (Brandt and Kavajecz, 2004). On the receiver side, only collateral with sufficient tradability will be admitted. For instance, securities that are not listed, issued by small firms, with wide bid-ask spreads, or low market depth may not be accepted as collateral. Similarly, the collateral receiver may prevent unrated bonds from being included in the collateral portfolio. This interaction between the provider and the receiver of collateral leads to the determination of a set Θ of K eligible securities that need to be allocated.

Second, both parties have to determine the level of collateralization. At the end of day t, the value of the collateral portfolio C_t is determined by the NAV of the fund:[15]

$$C_t = I_t \left(\frac{1-\theta}{1-h} \right). \tag{14}$$

Third, given the eligible securities and the level of collateralization on day t, the composition of the collateral portfolio is set to minimize collateral risk on day $t+1$, which is defined as the risk of being undercollateralized given the potential changes in the fund's NAV and in the value of the collateral securities. The collateral shortfall in the next day is defined as:

$$\Delta_{t+1} = (1-\theta)\, r_{i,t+1} - (1-h) \sum_{k=1}^{K} \omega_{k,t} r_{k,t+1} \tag{15}$$

where $\omega_{k,t}$ denotes the weight of the k^{th} security in the collateral portfolio, with $\omega_{k,t} \geq 0$ and $\sum_{k=1}^{K} \omega_{k,t} = 1$, $r_{k,t}$ the corresponding return, and $r_{i,t}$ the return of the fund's NAV. Note that if $\Delta_{t+1} > 0$, additional collateral is required at the end of day $t+1$. The collateral shortfall then depends on the return of the NAV and the returns of the collateral securities, which are unknown given the information available at the end of day t.

We define the optimal collateral portfolio as follows:

[15] Alternatively, we may consider the case in which the fund is overcollateralized, $C_t = \alpha I_t \left(1-\theta\right) / \left(1-h\right)$ with $\alpha > 1$.

Definition 4 (Optimal Collateral Portfolio) *The optimal collateral portfolio of an ETF is the portfolio that minimizes both the probability of having a collateral shortfall and the expected collateral shortfall.*

Among all portfolios that can be generated by combining the securities included in Θ, the optimal portfolio is the closest one from the origin of the Torino scale (see Figure 3). In order to minimize the expected collateral shortfall (S_{t+1}), a solution consists in minimizing the volatility of the collateral shortfall (Δ_{t+1}). Indeed, the expected collateral shortfall corresponds to the truncated expectation of Δ_{t+1}, where the truncation parameter is set to 0. Then minimizing the variance of Δ_{t+1} necessarily reduces the average of the positive values of Δ_{t+1}, as can be seen in Figure 6.

However, minimizing the variance of the shortfall is not sufficient to minimize the probability of being undercollateralized (p_{t+1}). Indeed, when the variance of Δ_{t+1} tends to 0, the collateral shortfall converges to a constant $\mathbb{E}(\Delta_{t+1})$. If $\mathbb{E}(\Delta_{t+1}) > 0$, like in the bottom part of Figure 6, the fund will necessarily be undercollateralized on day $t+1$. As a result, in order to jointly minimize p_{t+1} and S_{t+1}, $\mathbb{E}(\Delta_{t+1})$ must be constrained to be negative. The economic interpretation of this constraint is that the expected return of the collateral portfolio must be larger than the expected return of the NAV.

< Insert Figure 6 >

This argument can be formalized under the normality assumption. In this case, p_{t+1} and S_{t+1} can be defined as functions of $\mathbb{E}\left(\Delta_{t+1}\right) = \mu_\Delta$ and $\mathbb{V}\left(\Delta_{t+1}\right) = \sigma^2_\Delta$:

$$p_{t+1} = \Phi\left(\frac{\mu_\Delta}{\sigma_\Delta}\right) \tag{16}$$

$$S_{t+1} = \mu_\Delta + \sigma_\Delta \lambda\left(\frac{\mu_\Delta}{\sigma_\Delta}\right) \tag{17}$$

where $\lambda\left(z\right) = \phi\left(z\right)/\Phi\left(z\right)$ denotes the inverse Mills ratio, $\Phi\left(z\right)$ denotes the cdf of the standard normal distribution, and $\phi\left(z\right)$ the pdf of the standard normal distribution. We show in Appendix

A that, when $\mu_\Delta \leq 0$, the two collateral risk metrics are increasing functions of the variance of the collateral shortfall, σ^2_Δ.

4.2 The Optimization Program

We have shown that the composition of the optimal collateral portfolio can be obtained by minimizing the variance of the collateral shortfall under the constraint that the fund will be sufficiently collateralized on average. In turn, the weights of the optimal collateral portfolio, denoted $\boldsymbol{\omega} = (\omega_{1,t}, ..., \omega_{K,t})^\intercal$, with $k \in \Theta$, for $k = 1, ..., K$, are the solutions of the following program:

$$\min_{\boldsymbol{\omega}} \mathbb{V}(\Delta_{t+1}) \tag{18}$$
$$\text{subject to} \begin{cases} \mathbb{E}(\Delta_{t+1}) \leq 0 \\ \boldsymbol{\omega} \geq 0 \\ \boldsymbol{e}^\intercal \boldsymbol{\omega} = 1 \end{cases}$$

where $\boldsymbol{e}$ is the $K \times 1$ unit vector. Note that we prevent the weights from being negative since short positions in collateral would be nonsensical.

Both $\mathbb{E}(\Delta_{t+1})$ and $\mathbb{V}(\Delta_{t+1})$ can be expressed as functions of the moments of the returns of the collateral securities and of the NAV. If we denote by $\boldsymbol{r}_t = (r_{1,t}, ..., r_{K,t})^\intercal$ the $K \times 1$ vector of returns of the collateral securities, the moments are:[16]

$$\mathbb{E}(\boldsymbol{r}_{t+1}) = \frac{1}{1-h}\boldsymbol{\mu} \tag{19}$$

$$\mathbb{E}\left((\boldsymbol{r}_{t+1} - \mathbb{E}(\boldsymbol{r}_{t+1}))(\boldsymbol{r}_{t+1} - \mathbb{E}(\boldsymbol{r}_{t+1}))^\intercal\right) = \frac{1}{(1-h)^2}\boldsymbol{\Sigma} \tag{20}$$

Define $\boldsymbol{s}_t = (r_{i,t}(1-\theta), \boldsymbol{r}_t^\intercal (1-h))^\intercal$ the $(K+1) \times 1$ vector of transformed returns for the NAV and the collateral securities:

$$\mathbb{E}(\boldsymbol{s}_{t+1}) = \left(\begin{array}{c} \underset{(1,1)}{\mu_i} \\ \hline \underset{(K,1)}{\boldsymbol{\mu}} \end{array} \right) \tag{21}$$

$$\mathbb{E}\left((\boldsymbol{s}_{t+1} - \mathbb{E}(\boldsymbol{s}_{t+1}))(\boldsymbol{s}_{t+1} - \mathbb{E}(\boldsymbol{s}_{t+1}))^\intercal\right) = \left(\begin{array}{c|c} \underset{(1,1)}{\sigma_i^2} & \underset{(1,K)}{\boldsymbol{\Sigma}_i^\intercal} \\ \hline \underset{(K,1)}{\boldsymbol{\Sigma}_i} & \underset{(K,K)}{\boldsymbol{\Sigma}} \end{array} \right). \tag{22}$$

[16] These moments may be unconditional or conditional on the information available at time t. In the latter case, the moments are time-varying and must be indexed by time.

where μ_i and σ_i^2 denote the mean and variance of $r_{i,t}(1-\theta)$. The program in Equation (18) becomes:

$$\begin{array}{ll} \min\limits_{\boldsymbol{\omega}} & \frac{1}{2}\boldsymbol{\omega}^{\intercal}\boldsymbol{\Sigma}\boldsymbol{\omega} + \frac{1}{2}\sigma_i^2 - \boldsymbol{\omega}^{\intercal}\boldsymbol{\Sigma}_i \\ \text{subject to} & \left\{ \begin{array}{l} \mu_i - \boldsymbol{\omega}^{\intercal}\boldsymbol{\mu} \leq 0 \\ \boldsymbol{\omega} \geq 0 \\ \boldsymbol{e}^{\intercal}\boldsymbol{\omega} = 1 \end{array} \right. \end{array} \tag{23}$$

If we neglect the positivity constraints, the Lagrange function $f(\boldsymbol{\omega},\lambda_1,\lambda_2)$ is:

$$f(\boldsymbol{\omega},\lambda_1,\lambda_2) = \frac{1}{2}\boldsymbol{\omega}^{\intercal}\boldsymbol{\Sigma}\boldsymbol{\omega} + \frac{1}{2}\sigma_i^2 - \boldsymbol{\omega}^{\intercal}\boldsymbol{\Sigma}_i - \lambda_1(\boldsymbol{e}^{\intercal}\boldsymbol{\omega} - 1) - \lambda_2(\boldsymbol{\omega}^{\intercal}\boldsymbol{\mu} - \mu_i) \tag{24}$$

with $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$. To solve this problem, we need to distinguish two cases: one in which the $\mu_i - \boldsymbol{\omega}^{\intercal}\boldsymbol{\mu} \leq 0$ constraint is not binding and another one when it is. First, if the expected return of the collateral portfolio is larger than the expected return of the NAV ($\mu_i - \boldsymbol{\omega}^{\intercal}\boldsymbol{\mu} < 0$ and $\lambda_2 = 0$), the optimal weights of the collateral portfolio can be expressed as a function of the weights of the Global Minimum Variance Portfolio (GMVP).

Proposition 1 *If $\mu_i - \boldsymbol{\omega}^{\intercal}\boldsymbol{\mu} < 0$, the optimal weights of the collateral portfolio are:*

$$\boldsymbol{\omega}^* = \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i + \left(1 - \boldsymbol{e}^{\intercal}\boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i\right)^{-1}\widetilde{\boldsymbol{\omega}}_{GMVP} \tag{25}$$

where $\widetilde{\boldsymbol{\omega}}_{GMVP}$ corresponds to the weights of the GMVP:

$$\widetilde{\boldsymbol{\omega}}_{GMVP} = \frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{e}}{\boldsymbol{e}^{\intercal}\boldsymbol{\Sigma}^{-1}\boldsymbol{e}}. \tag{26}$$

The proof of Proposition 1 is provided in Appendix B. The optimal weights depend (i) on the variance covariance matrix, $\boldsymbol{\Sigma}$, of the returns of the collateral securities and (ii) on the vector of covariances, $\boldsymbol{\Sigma}_i$, between the returns of these securities and the return of the NAV. Notice that if the returns of the collateral securities and the NAV are independent ($\boldsymbol{\Sigma}_i = \mathbf{0}_{K\times 1}$), the optimal weights simply correspond to those obtained by minimizing the variance of the collateral portfolio, that is to $\widetilde{\boldsymbol{\omega}}_{GMVP}$.

Second, when the constraint on the expected level of collateralization is binding ($\mu_i - \boldsymbol{\omega}^{\intercal}\boldsymbol{\mu} = 0$ and $\lambda_2 > 0$), the optimal weights can be expressed as a function of the weights of the Markowitz's mean variance portfolio. We define three scalar terms a, b, and c such that:

$$a = \boldsymbol{e}^\intercal \boldsymbol{\Sigma} \boldsymbol{e} \qquad b = \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} \qquad c = \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} \tag{27}$$

Proposition 2 *If $\boldsymbol{\omega}^\intercal \boldsymbol{\mu} - \mu_i = 0$, the optimal weights of the collateral portfolio are:*

$$\begin{aligned} \boldsymbol{\omega}^* &= \widetilde{\boldsymbol{\omega}}_{MV} + \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i + \left(\frac{c\boldsymbol{e}^\intercal - b\boldsymbol{\mu}^\intercal}{b^2 - ac} \right) \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i \boldsymbol{\Sigma}^{-1} \boldsymbol{e} \\ &\quad + \left(\frac{a\boldsymbol{\mu}^\intercal - b\boldsymbol{e}^\intercal}{b^2 - ac} \right) \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} \end{aligned} \tag{28}$$

where $\widetilde{\boldsymbol{\omega}}_{MV}$ corresponds to the weights of the Markowitz's mean variance portfolio.

The proof of Proposition 2 is provided in Appendix C. In this case, the optimal weights depend on $\boldsymbol{\Sigma}$ and $\boldsymbol{\Sigma}_i$ as in the previous case, but they also depend on the vector of expected returns of the collateral securities $\boldsymbol{\mu}$ and on the expected return of the NAV μ_i. The lattest can be viewed as a target for the expected return of the collateral portfolio, $\boldsymbol{\omega}^\intercal \boldsymbol{\mu}$. Notice that if the collateral securities and the NAV are independent, $\boldsymbol{\Sigma}_i = \mathbf{0}_{K \times 1}$, then the optimal weights simply correspond to the weights $\widetilde{\boldsymbol{\omega}}_{MV}$ of the mean variance portfolio with a target mean μ_i.

In practice, more constraints can be taken into account in the program. For instance, one can impose a positivity constraint on all weights. Alternatively, it is possible to prevent any issuer to account for more than a certain fraction of the collateral portfolio as it is the case under UCITS regulation. Although it is not possible to obtain an analytic solution in the latter case, we can numerically solve the optimization problem using a quadratic programming algorithm.

Finally, the results presented in this section can be extended by taking into account the correlations between the returns of the collateral assets and the constituents of the tracked index, rather than the index defined as a whole. Indeed, if we neglect the tracking error, the return of the NAV can be expressed as $r_{i,t} = \sum_{j=1}^{J} \delta_{j,t} r_{j,t}^i$ where $\delta_{j,t}$ denotes the weight of the j^{th} asset in the index, with $\delta_{j,t} \geq 0$ and $r_{j,t}^i$ is the return of the j^{th} index constituent. Under these assumptions, it is possible to define the weights of the optimal portfolio as a function of the covariances between $r_{k,t+1}$ and $r_{j,t+1}^i$ for $k \neq j$ by using exactly the same methodology as the one presented in Section 4.2.

4.3 Illustration

As an illustration, we consider a simple example with two collateral securities ($K = 2$). Define the vector of transformed returns:

$$\boldsymbol{s}_t = \left(r_{i,t}\left(1-\theta\right), r_{1,t}\left(1-h\right), r_{2,t}\left(1-h\right)\right)^{\intercal} \tag{29}$$

where $r_{i,t}$ is the return of the fund's NAV, $r_{1,t}$ and $r_{2,t}$ are the returns of the two securities. We assume that the vector $\boldsymbol{s}_t$ has a normal distribution with:

$$\mathbb{E}\left(\boldsymbol{s}_{t+1}\right) = \left(\begin{array}{c} 0.1 \\ \hline 0.5 \\ 0.5 \end{array}\right) \tag{30}$$

$$\mathbb{E}\left(\left(\boldsymbol{s}_{t+1} - \mathbb{E}\left(\boldsymbol{s}_{t+1}\right)\right)\left(\boldsymbol{s}_{t+1} - \mathbb{E}\left(\boldsymbol{s}_{t+1}\right)\right)^{\intercal}\right) = \left(\begin{array}{c|cc} 2 & 1 & 0.2 \\ \hline 1 & 2 & 0.2 \\ 0.2 & 0.2 & 1 \end{array}\right). \tag{31}$$

In this example, the two collateral securities are positively correlated, but the first security is more volatile than the second one. Furthermore, both securities are positively correlated with the NAV but the correlation with the first security is larger (0.5) than with the second one (0.1414). In this case, increasing the weight of the first security has two opposite effects on collateral risk: (1) since the first security is more risky, it increases the volatility of the shortfall but (2) since the first security is more strongly correlated with the NAV, it decreases the volatility of the shortfall.

Consider a collateral portfolio with a vector of weights $\boldsymbol{\omega} = (\omega_1, 1-\omega_1)^{\intercal}$ where $\omega_1 \geq 0$ corresponds to the weight of the first security. The Torino scale in Figure 7 displays the values of the probabilities of facing a collateral shortfall and the corresponding expected collateral shortfalls for all portfolios ranging from $\omega_1 = 0$ to $\omega_1 = 1$. We see that the optimal portfolio $\boldsymbol{\omega}^* = (0.6154, 0.3846)^{\intercal}$, given by Equation (25), corresponds to the lowest values for the two collateral risk metrics.

< Insert Figure 7 >

4.4 Hybrid ETF: Both Synthetic and Physical

One special case of the above optimization program is worthwhile mentioning. When the set of eligible securities for the collateral portfolio includes all the index constituents, the optimal collateral portfolio mirrors the index. In this case, the return of the NAV is $r_{i,t} = \sum_{j=1}^{J} \delta_{j,t} r_{j,t}$ where $\delta_{j,t}$ denotes the weight of the j^{th} asset in the index. In this particular case, an obvious way to set the collateral shortfall to zero is to maintaining a perfect match between the collateral portfolio and the index, $\omega_{k,t}^{*} = \delta_{k,t}$ for $k = 1, ..., J$, and $\omega_{k,t}^{*} = 0$ for $k > J$ (see Appendix D). This situation corresponds to a hybrid ETF combining some of the features of both synthetic and physical ETFs. While the fund is based on a swap, it benefits from a physical replication of the index.

5 Conclusion

How safe is the backup parachute of ETFs? To answer this question, we measure the collateral risk of ETFs using a \$40.9 billion collateral portfolios. Overall, our results do not support the allegations made by the Financial Stability Board and other international agencies about the poor quality of the collateral used to produce ETFs and about the disconnect between the index tracked and the collateral. Funds in our sample tend to be overcollateralized and the collateral is mainly made of European securities and, for the most part, equities issued by large firms or highly-rated bonds. Furthermore, the collateral of equity (respectively bond) funds are mainly made of equities (respectively bonds) and the duration of the collateral matches well with the one of the bond index tracked by the ETF. We also provide evidence that collateral portfolio are actively managed with more than one third of the collateral of a given fund changing from one week to the next.

Our results concur with Louis Brandeis' saying that "*sunlight is said to be the best of disinfectants*". Indeed, scrutiny by the media, regulators, and investors following the criticisms of the ETF industry by international agencies lead to improved disclosure by ETF providers about their replication technology and collateral holdings (Morningstar, 2012). We show that increased transparency forced the industry to improve standards and practice on collateral management.

We find some heterogeneity in the collateral risk exposure of funds. We show that exposure to collateral risk is higher for funds that track commodities or currencies and for inverse ETFs which deliver the inverse performance of the underlying security. In order to facilitate collateral management, we provide closed-form solutions for minimum collateral-risk portfolios.

Future research could generalize our framework to study other sources of collateral risk. One may for instance relax the assumption that the pledged securities are not lent out and study this additional layer of collateral risk due to rehypothecation.

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Appendix

Appendix A: Proof of $\frac{\partial p}{\partial \sigma_{\Delta}} > 0$ and $\frac{\partial S}{\partial \sigma_{\Delta}} > 0$

Proof. The sensitivity of the probability of collateral shortfall to the variance of the collateral shortfall is:

$$\frac{\partial p}{\partial \sigma_{\Delta}} = \frac{\partial}{\partial \sigma_{\Delta}} \Phi\left(\frac{\mu_{\Delta}}{\sigma_{\Delta}}\right) = -\frac{\mu_{\Delta}}{\sigma_{\Delta}^{2}} \phi\left(\frac{\mu_{\Delta}}{\sigma_{\Delta}}\right) \tag{A1}$$

Since the pdf is always positive, the probability of collateral shortfall is an increasing function of σ_{Δ} as soon as the ETF is sufficiently collateralized on average, $\mu_{\Delta} < 0$. ■

Proof. A similar argument can be raised for the expected shortfall:

$$\begin{aligned} \frac{\partial S}{\partial \sigma_{\Delta}} &= \frac{\partial}{\partial \sigma_{\Delta}}\left(\mu_{\Delta} + \sigma_{\Delta}\lambda\left(\frac{\mu_{\Delta}}{\Delta_{\Delta}}\right)\right) \\ &= \lambda(u) - \sigma_{\Delta}\lambda(u)(u+\lambda(u))\frac{\partial u}{\partial \sigma_{\Delta}} \\ &= \lambda(u) + \sigma_{\Delta}\frac{u}{\sigma_{\Delta}^{2}}\lambda(u)(u+\lambda(u)) \\ &= \lambda(u)\left(1+u^{2}+u\lambda(u)\right) \end{aligned} \tag{A2}$$

with $u = \mu_{\Delta}/\sigma_{\Delta}$. Since the inverse Mills ratio is always positive, the sign of $\partial S/\partial \sigma_{\Delta}$ corresponds to that of $1 + u^{2} + u\lambda(u)$. So, the expected shortfall is an increasing function of σ_{Δ} as soon as:

$$1 + u^{2} + u\frac{\phi(u)}{\Phi(u)} > 0 \tag{A3}$$

or equivalently when:

$$\Phi(u) + u^{2}\Phi(u) + u\phi(u) > 0 \tag{A4}$$

Since that, for a standard normal distribution, we have $\phi(u) = -\Phi(u)\,u$, this condition becomes:

$$\Phi(u) > 0 \tag{A5}$$

Whatever the value of μ_{Δ} and σ_{Δ}, this condition is always satisfied. As a consequence, the expected shortfall is always an increasing function of the variance of the collateral portfolio σ_{Δ}^{2}, whatever the value of its expectation μ_{Δ}. ■

Appendix B: Proof of Proposition 1

Proof. The Lagrange function $f(\boldsymbol{\omega},\lambda_1,\lambda_2)$ is defined as to be:

$$f(\boldsymbol{\omega},\lambda_1,\lambda_2) = \frac{1}{2}\boldsymbol{\omega}^{\intercal}\boldsymbol{\Sigma}\boldsymbol{\omega} + \frac{1}{2}\sigma_i^{2} - \boldsymbol{\omega}^{\intercal}\boldsymbol{\Sigma}_i - \lambda_1\left(\boldsymbol{e}^{\intercal}\boldsymbol{\omega} - 1\right) - \lambda_2\left(\boldsymbol{\omega}^{\intercal}\boldsymbol{\mu} - \mu_i\right) \tag{B1}$$

with $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$. The corresponding Kuhn-Tucker conditions are:

$$\boldsymbol{\Sigma}\boldsymbol{\omega} - \boldsymbol{\Sigma}_i - \lambda_1\boldsymbol{e} - \lambda_2\boldsymbol{\mu} = 0 \tag{B2}$$

$$\boldsymbol{e}^{\intercal}\boldsymbol{\omega} - 1 = 0 \tag{B3}$$

$$\min\left(\boldsymbol{\omega}^{\intercal}\boldsymbol{\mu} - \mu_i, \lambda_2\right) = 0 \tag{B4}$$

If $\boldsymbol{\omega}^\intercal \boldsymbol{\mu} - \mu_i > 0$ and $\lambda_2 = 0$, the Kuhn-Tucker conditions become:

$$\begin{aligned} \boldsymbol{\Sigma\omega} - \boldsymbol{\Sigma}_i - \lambda_1 \boldsymbol{e} &= 0 \qquad \text{(B5)} \\ \boldsymbol{e}^\intercal \boldsymbol{\omega} - 1 &= 0 \qquad \text{(B6)} \end{aligned}$$

From Equation (32), we have:

$$\boldsymbol{\omega} = \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i + \lambda_1 \boldsymbol{\Sigma}^{-1}\boldsymbol{e} \qquad \text{(B7)}$$

Next, multiply both sides by $\boldsymbol{e}^\intercal$ and use second equation to solve for λ_1 :

$$\begin{aligned} \boldsymbol{e}^\intercal \boldsymbol{\omega} &= \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i + \lambda_1 \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{e} = 1 \qquad \text{(B8)} \\ \lambda_1 &= \frac{1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i}{\boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{e}} \qquad \text{(B9)} \end{aligned}$$

and the optimal weights are equal to:

$$\boldsymbol{\omega}^* = \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i + \left(1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i\right) \frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{e}}{\boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{e}} \qquad \text{(B10)}$$

These weights can be expressed as a linear function of the weights of the Global Minimum Variance Portfolio (GMVP). The GMVP corresponds to the solution of the following optimization problem:

$$\begin{aligned} \min_{\widetilde{\boldsymbol{\omega}}} \quad & \frac{1}{2}\widetilde{\boldsymbol{\omega}}^\intercal \boldsymbol{\Sigma} \widetilde{\boldsymbol{\omega}} \qquad \text{(B11)} \\ \text{subject to} \quad & \boldsymbol{e}^\intercal \widetilde{\boldsymbol{\omega}} = 1 \qquad \text{(B12)} \end{aligned}$$

The corresponding optimal solution is:

$$\widetilde{\boldsymbol{\omega}}_{GMVP} = \frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{e}}{\boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{e}} \qquad \text{(B13)}$$

As a consequence, we have:

$$\boldsymbol{\omega}^* = \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i + \left(1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i\right)^{-1} \widetilde{\boldsymbol{\omega}}_{GMVP} \qquad \text{(B14)}$$

■

Appendix C: Proof of Proposition 2

Proof. If $\boldsymbol{\omega}^\intercal \boldsymbol{\mu} - \mu_i = 0$ and $\lambda_2 > 0$, the Kuhn-Tucker conditions become:

$$\begin{aligned} \boldsymbol{\Sigma\omega} - \boldsymbol{\Sigma}_i - \lambda_1 \boldsymbol{e} - \lambda_2 \boldsymbol{\mu} &= 0 \qquad \text{(C1)} \\ \boldsymbol{e}^\intercal \boldsymbol{\omega} - 1 &= 0 \qquad \text{(C2)} \\ \boldsymbol{\omega}^\intercal \boldsymbol{\mu} - \mu_i &= 0 \qquad \text{(C3)} \end{aligned}$$

From the first equation, we have:

$$\boldsymbol{\omega} = \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i + \lambda_1 \boldsymbol{\Sigma}^{-1}\boldsymbol{e} + \lambda_2 \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} \qquad \text{(C4)}$$

Given this expression for $\boldsymbol{\omega}$, the two constraints can be rewritten as:

$$\begin{aligned} 1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i &= \lambda_1 \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{e} + \lambda_2 \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} \qquad \text{(C5)} \\ \mu_i - \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i &= \lambda_1 \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{e} + \lambda_2 \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} \qquad \text{(C6)} \end{aligned}$$

Define three three scalar terms a, b, and c such that:

$$a = \boldsymbol{e}^\intercal \boldsymbol{\Sigma} \boldsymbol{e} \qquad b = \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} \qquad c = \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} \tag{C7}$$

The constraints can be expressed as:

$$1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i = \lambda_1 a + \lambda_2 b \tag{C8}$$

$$\mu_i - \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i = \lambda_1 b + \lambda_2 c \tag{C9}$$

Solving for λ_1 and λ_2, we have:

$$\lambda_1^* = \frac{b\left(\mu_i - \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i\right) - c\left(1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i\right)}{b^2 - ac} \tag{C10}$$

$$\lambda_2^* = \frac{b\left(1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i\right) - a\left(\mu_i - \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i\right)}{b^2 - ac} \tag{C11}$$

By substituting λ_1^* and λ_2^* in the expression $\boldsymbol{\omega}$, we have:

$$\begin{aligned} \boldsymbol{\omega}^* &= \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i + \left(\frac{b\left(\mu_i - \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i\right) - c\left(1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i\right)}{b^2 - ac}\right) \boldsymbol{\Sigma}^{-1} \boldsymbol{e} \\ &\quad + \left(\frac{b\left(1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i\right) - a\left(\mu_i - \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i\right)}{b^2 - ac}\right) \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} \end{aligned} \tag{C12}$$

The standard mean variance portfolio with a target mean μ_i is the solution of the following program:

$$\min_{\boldsymbol{\omega}} \frac{1}{2} \boldsymbol{\omega}^\intercal \boldsymbol{\Sigma} \boldsymbol{\omega} \tag{C13}$$

$$\text{subject to} \quad \begin{cases} \boldsymbol{\omega}^\intercal \boldsymbol{\mu} = \mu_i \\ \boldsymbol{e}^\intercal \boldsymbol{\omega} = 1 \in \mathbb{N}^K \end{cases}$$

The corresponding optimal solution is:

$$\widetilde{\boldsymbol{\omega}}_{MV} = \left(\frac{b\mu_i - c}{b^2 - ac}\right) \boldsymbol{\Sigma}^{-1} \boldsymbol{e} + \left(\frac{b - a\mu_i}{b^2 - ac}\right) \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} \tag{C14}$$

As a consequence, we have:

$$\begin{aligned} \boldsymbol{\omega}^* &= \widetilde{\boldsymbol{\omega}}_{MV} + \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i + \left(\frac{c\boldsymbol{e}^\intercal - b\boldsymbol{\mu}^\intercal}{b^2 - ac}\right) \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i \boldsymbol{\Sigma}^{-1} \boldsymbol{e} \\ &\quad + \left(\frac{a\boldsymbol{\mu}^\intercal - b\boldsymbol{e}^\intercal}{b^2 - ac}\right) \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} \end{aligned} \tag{C15}$$

■

Appendix D: Hybrid ETF

Proof. Let us assume that $J \leq K$, and the indices j and k are ordered in the same way, *i.e.*, the first asset in the collateral portfolio corresponds to the first asset of the index, and so on. Under these assumptions, the collateral shortfall defined in Equation (15) can be rewritten as:

$$\Delta_{t+1} = (1 - \theta) \sum_{j=1}^{J} \delta_{j,t} r_{j,t+1}^{i} - (1 - h) \sum_{k=1}^{K} \omega_{k,t} r_{k,t+1} \tag{D1}$$

where $r^{i}_{j,t+1} = r_{j,t+1}$ for $j = 1, ..., J$. An obvious way to set this shortfall at zero consists in choosing the weights ω_k such that:

$$(1-\theta)\,\delta_{k,t} = (1-h)\,\omega_{k,t} \quad \text{for } k = 1, ..., J$$

$$\omega_{k,t} = 0 \quad \text{for } k > J \tag{D2}$$

However, the weights ω_k do not sum to one. Indeed:

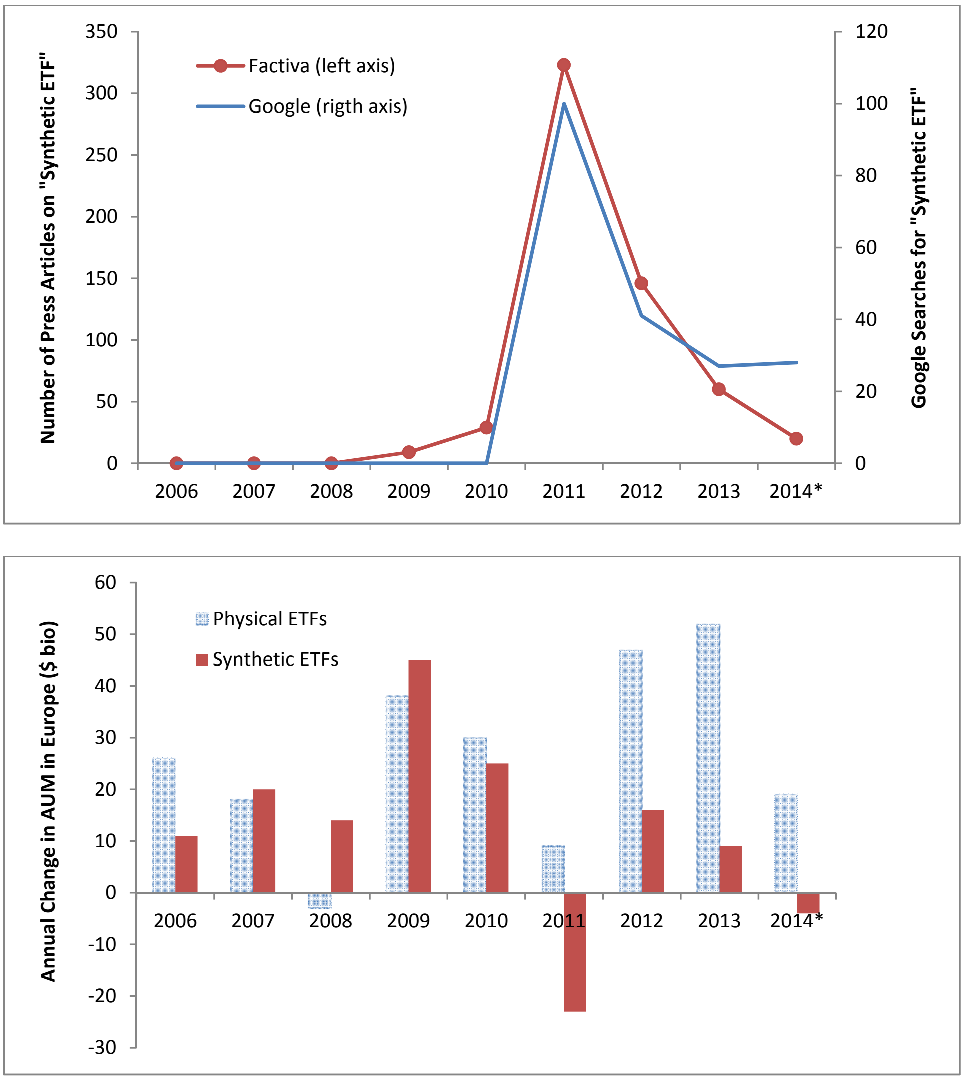
$$\sum_{k=1}^{K} \omega_{k,t} = \sum_{k=1}^{J} \left(\frac{1-\theta}{1-h}\right) \delta_{k,t} = \frac{1-\theta}{1-h} \tag{D3}$$

since $\sum_{j=1}^{J} \delta_{j,t} = 1$. So, the normalized optimal weights are:

$$\omega^{*}_{k,t} = \frac{\omega_{k,t}}{\sum_{k=1}^{K} \omega_{k,t}} = \delta_{k,t} \quad \text{for } k = 1, ..., J \tag{D4}$$

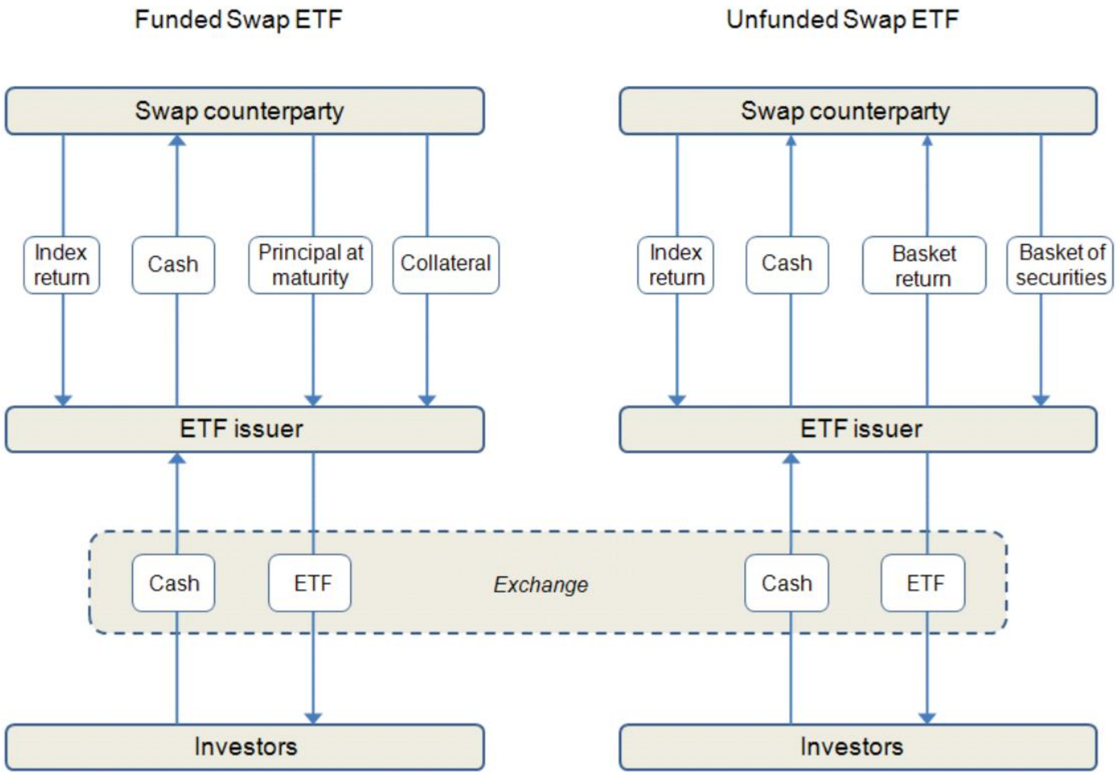
and zero otherwise. The composition of the index and the collateral are strictly identical whatever the value of h and θ. ■

Figure 1 – Investors' Attention and ETF Flows



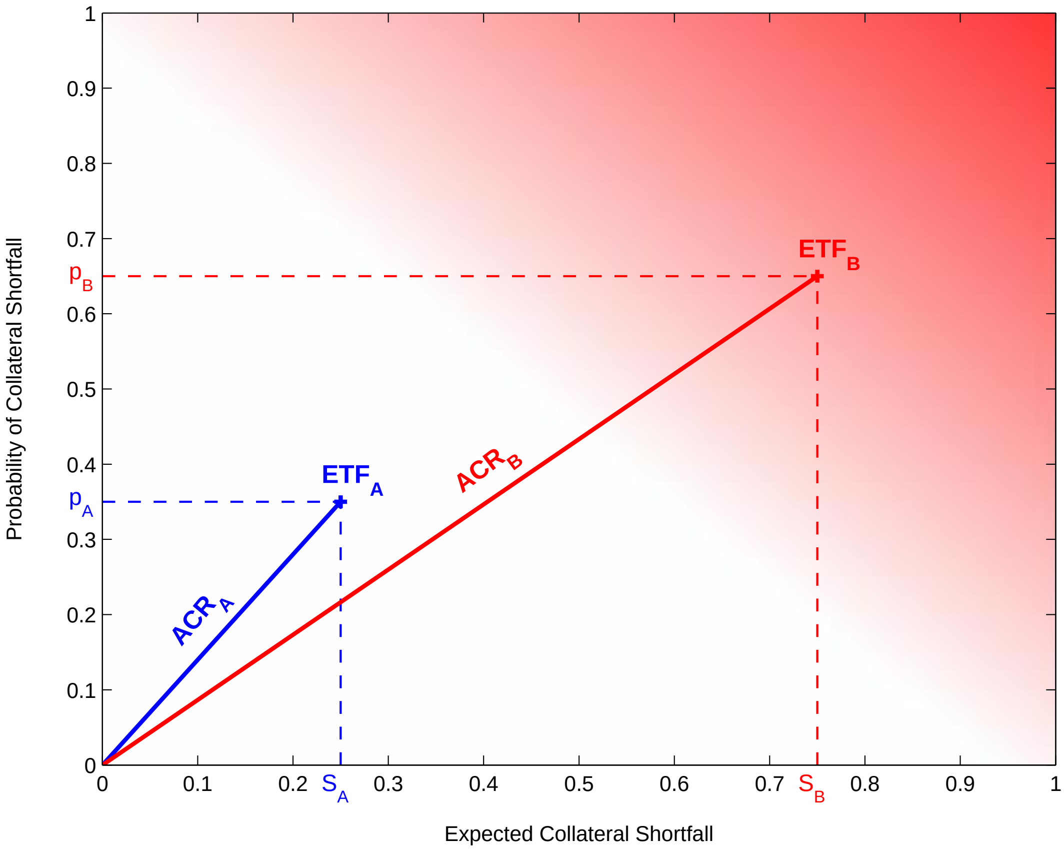
Notes: The upper panel displays the annual number of articles that include the words "synthetic ETF" in the financial press (left axis) and the relative number of queries on Google including the keywords "synthetic ETF" (right axis). The amount of global media coverage is assessed using the Factiva database and the worldwide Google search figures are from Google Trends. The lower panel presents the annual change in aggregate Asset Under Management (AUM) of all European ETFs based on physical replication, respectively synthetic replication. The AUM figures are from ETFGI. The asterisk in 2014 indicates that the values are as of the end of 2014-Q1.

Figure 2 – The Structures of Synthetic ETFs



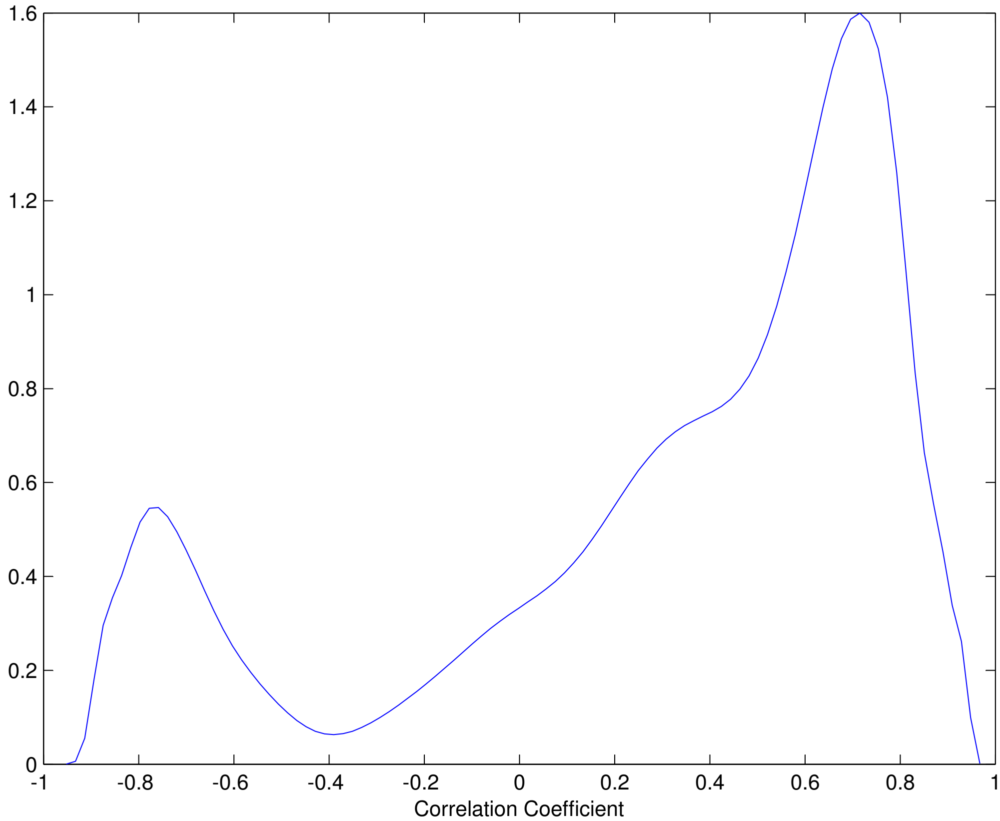
Notes: This figure describes the different cash-flows and asset transfers for two types of synthetic ETFs: the funded-swap based ETFs (left part) and the unfunded-swap based ETFs (right part).

Figure 3 – Displaying Collateral Risk on a Torino Scale



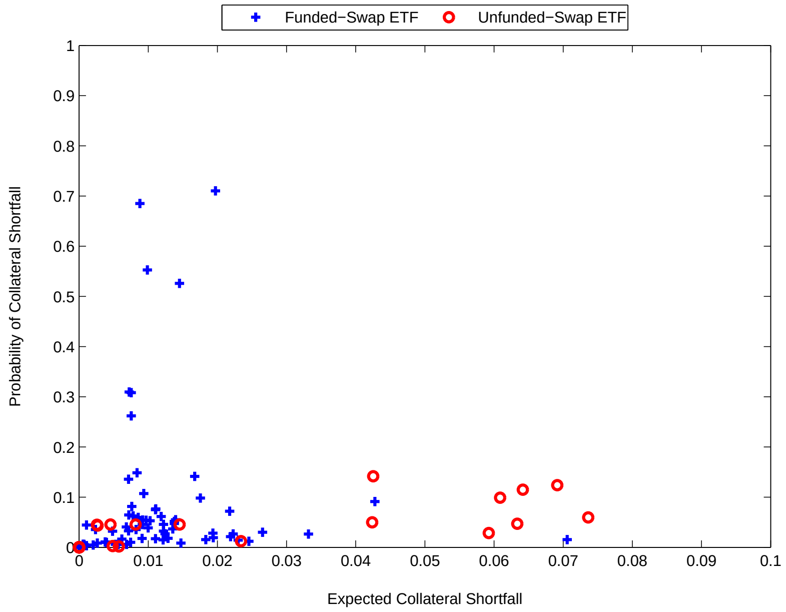
Notes: This figure presents a Torino scale that displays, for two hypothetical ETFs (A and B), their probability p of having a collateral shortfall (y-axis), their expected collateral shortfall S (x-axis), and their Aggregate Collateral Risk $ACR = (p^2 + S^2)^{\frac{1}{2}}$. The funds located in the North-East corner of the Torino scale are the most exposed to collateral risk.

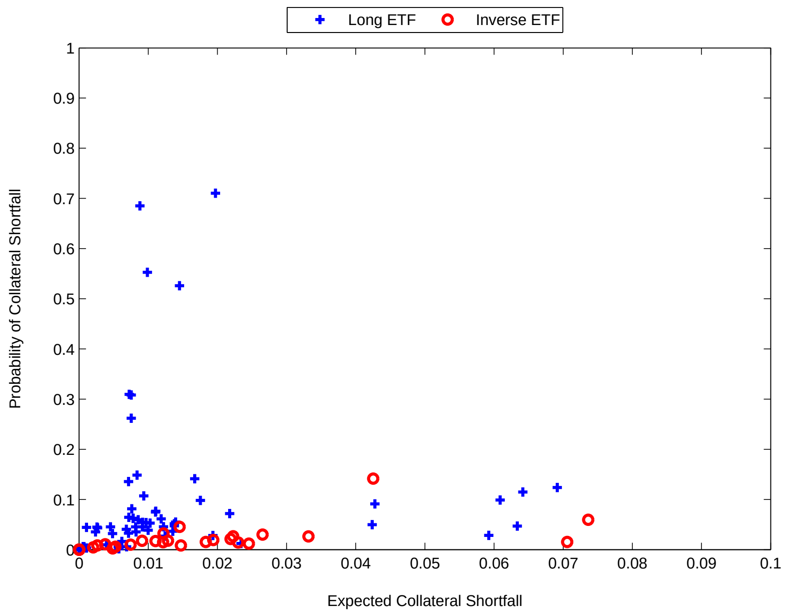
Figure 4 – Correlation between the ETF Return and the Collateral Return



Notes: This figure presents the nonparametric kernel smoothing probability density function of the correlation coefficient between the ETF return and the return of the collateral portfolio. The estimation is based on all weekly combinations of ETF and collateral returns.

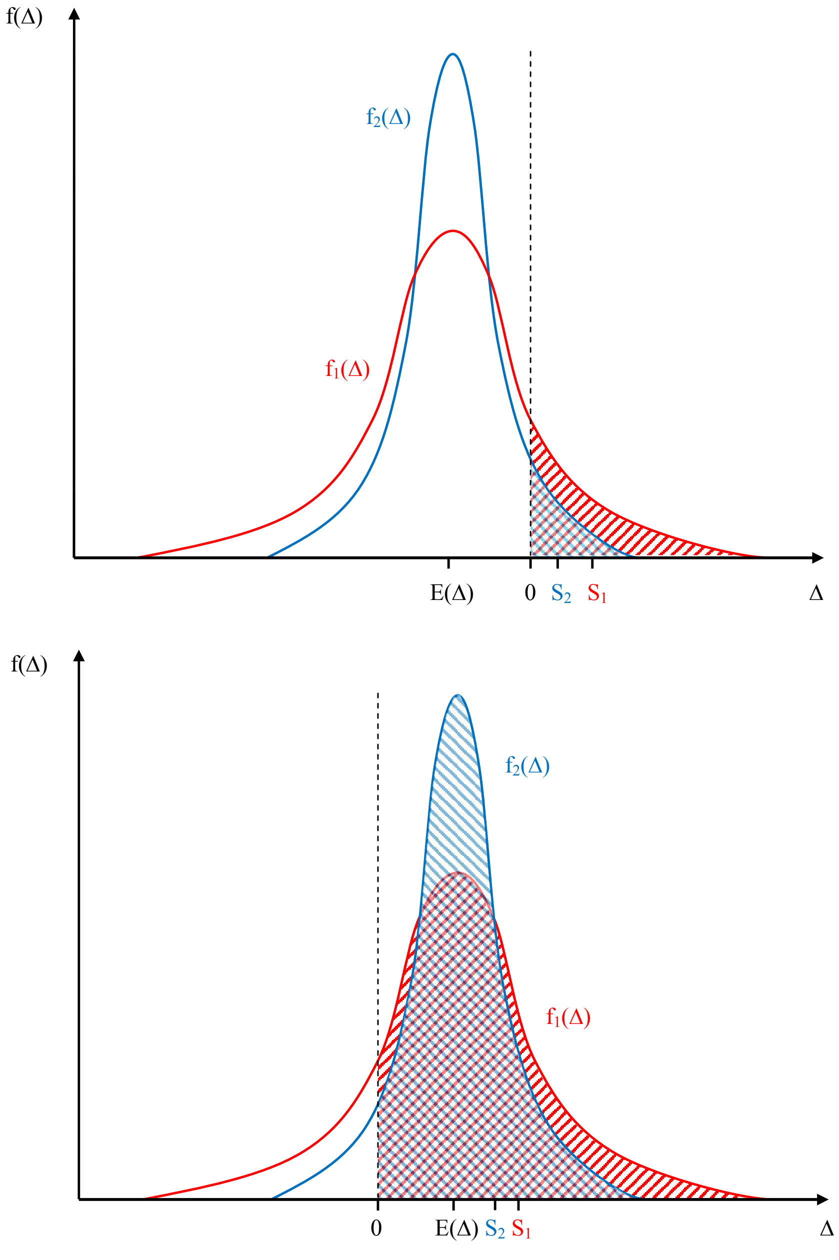
Figure 5 – Estimated Collateral Risk of all Sample ETFs





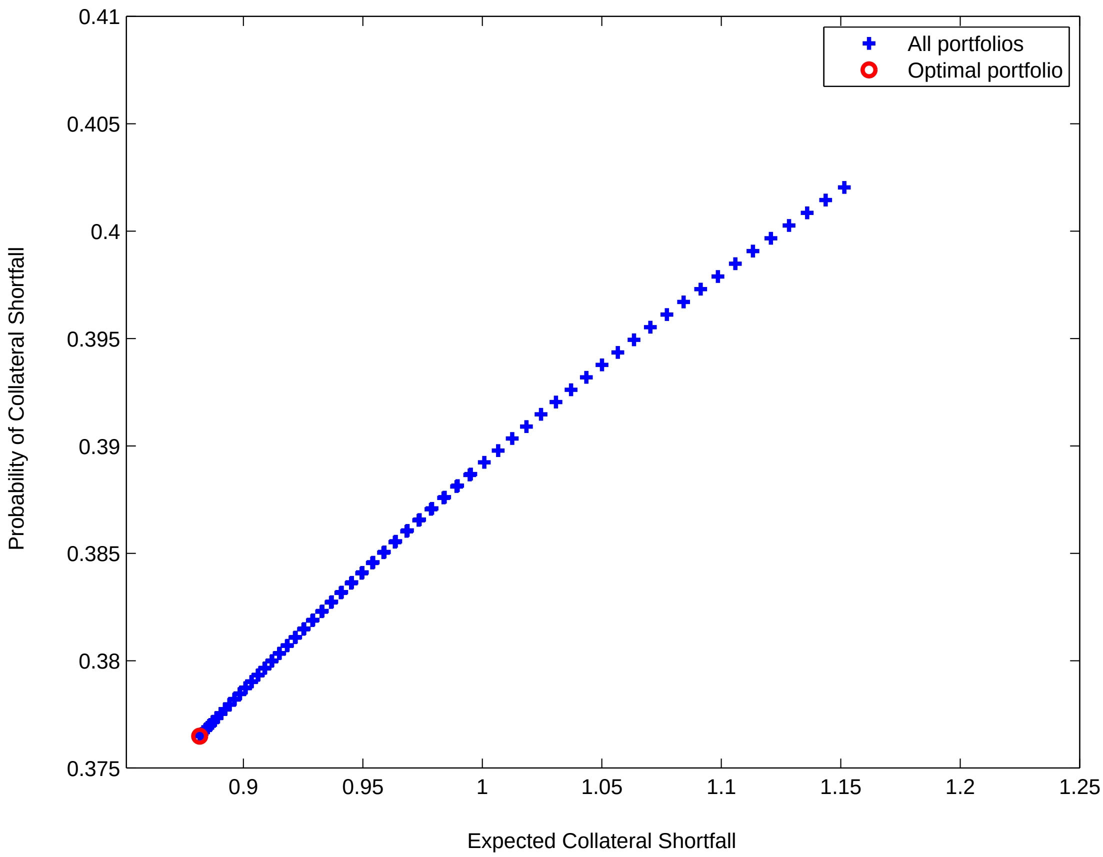
Notes: These figures present two Torino scales that display, for all sample ETFs, their probability p of having a collateral shortfall (y-axis, range = 0-100%), their expected collateral shortfall S (x-axis, range = 0-10%), and their Aggregate Collateral Risk $ACR = (p^2 + S^2)^{\frac{1}{2}}$. The funds located in the North-East corner of the Torino scale are the most exposed to collateral risk. In the top figure, we contrast ETFs that are based on funded swaps to those based on unfunded swaps. In the lower figure, we contrast long ETFs to inverse ETFs which deliver the inverse performance of the index.

Figure 6 – Some Visual Intuition about the Optimization Program



Notes: This figure presents two probability density functions f(Δ) that have the same expected values E(Δ) but different volatilities (high volatility for the red curve, $f_1(\Delta)$, and low volatility for the blue curve, $f_2(\Delta)$). The dashed areas correspond to the probabilities of having positive values. The S values correspond to the expected values of the truncated distribution.

Figure 7 – Illustration of the Optimal Collateral Portfolio



Notes: This Torino scale displays the probability of facing a collateral shortfall and the expected collateral shortfalls for all portfolios obtained by combining two collateral securities described in Section 4.2. The portfolios are obtained by varying the weight of the first security from $w_1 = 0$ to $w_1 = 1$ using a 0.01 increment.

Table 1 – Summary Statistics on the ETFs

		Total	Funded	Unfunded	Long	Inverse
Number of ETF Funds		164	112	52	134	30
AUM ($ Mio)	Total	37,927	20,122	17,805	36,011	1,916
Asset Exposure	Equities	74.5% (111)	85.5% (100)	61.9% (11)	74.0% (90)	82.2% (21)
	Government Bonds	11.0% (24)	2.2% (2)	20.8% (22)	10.8% (21)	13.1% (3)
	Money Markets	6.6% (4)	-	14.0% (4)	7.0% (4)	-
	Commodities	3.8% (2)	7.2% (2)	-	4.0% (2)	-
	Hedge Funds Strategies	2.2% (6)	3.9% (3)	0.3% (3)	2.3% (5)	0.4% (1)
	Credits	0.7% (9)	-	1.6% (9)	0.5% (4)	4.3% (5)
	Corporate Bonds	0.6% (3)	-	1.4% (3)	0.7% (3)	-
	Currencies	0.3% (4)	0.6% (4)	-	0.4% (4)	-
	Multi Assets	0.3% (1)	0.6% (1)	-	0.3% (1)	-
Geographic Exposure	Europe	58.9% (79)	29.4% (41)	92.4% (38)	57.4% (54)	84.1% (25)
	World	22.3% (41)	37.0% (38)	5.5% (3)	23.6% (40)	1.1% (1)
	Asia-Pacific	9.4% (28)	17.6% (24)	0.2% (4)	9.9% (27)	1.3% (1)
	North America	7.2% (14)	11.9% (7)	1.9% (7)	6.8% (11)	13.5% (3)
	Rest of the World	2.2% (2)	4.1% (2)	-	2.3% (2)	-

Notes: This table presents some summary statistics for all sample ETFs, as well as separately for funded-swap based ETFs vs. unfunded-swap based ETFs, and long ETFs vs. inverse ETFs. The table displays the total number of funds, the combined assets under management (AUM) in USD million, the value-weighted averages of asset and geographic exposures, along with the number of funds in parentheses. The sample period is July 5, 2012 - November 29, 2012.

Table 2 – Size and Turnover of Collateral Portfolios

		Total	Funded	Unfunded	Long	Inverse
Collateral Value ($ Mio)	All	40,939	23,083	17,856	38,706	2,233
Collateralization	All	108.4%	114.6%	101.3%	107.9%	115.4%
	Equities	109.6%	115.7%	99.9%	109.0%	117.8%
	Government Bonds	102.8%	100.7%	103.1%	102.8%	102.8%
Number of Collateral Securities	All	3,299	3,014	1,141	3,253	2,511
Average Number of Collateral Securities per ETF Fund	All	81	110	18	90	43
	Equities	109	117	35	120	58
	Government Bonds	14	13	16	15	9
	Money Markets	20	-	20	21	-
	Commodities	93	93	-	93	-
	Hedge Funds Strategies	37	66	9	43	7
	Credits	10	-	10	11	10
	Corporate Bonds	12	-	12	12	-
	Currencies	50	50	-	50	-
	Multi Assets	70	70	-	70	-
Turnover	All	34.0%	47.4%	5.0%	33.5%	36.3%
	Equities	43.9%	46.5%	18.8%	42.1%	51.3%
	Government Bonds	1.3%	0.5%	1.4%	1.3%	1.6%
	Money Markets	2.2%	-	2.2%	2.2%	-
	Commodities	72.7%	72.7%	-	72.7%	-
	Hedge Funds Strategies	29.7%	60.7%	0.3%	36.0%	0.7%
	Credits	1.4%	-	1.4%	1.6%	1.2%
	Corporate Bonds	2.4%	-	2.4%	2.4%	-
	Currencies	60.8%	60.8%	-	60.8%	-
	Multi Assets	76.7%	76.7%	-	76.7%	-

Notes: This table presents some summary statistics on the size and turnover of the ETFs' collateral portfolios. It displays for all sample ETFs, as well as separately for funded-swap based ETFs vs. unfunded-swap based ETFs, and long ETFs vs. inverse ETFs, the collateral value in USD million, the value-weighted average level of collateralization (collateral value/AUM), the total number of collateral securities, the average number of collateral securities, and the average turnover. Results are also broken down by asset exposure. Turnover is defined as the ratio between the number of different securities, as defined by their ISIN, that enter or exit the collateral portfolio between two dates divided by the total number of collateral securities on both dates. The sample period is July 5, 2012 - November 29, 2012.

Table 3 – Types of Collateral Securities

Panel A: Type of Collateral Securities		Equity	Government Bonds	Corporate Bonds
Number of Collateral Securities		2,591	490	218
ETF Asset Exposure	All	74.9%	19.7%	5.4%
	Equity	**92.5%**	2.7%	4.8%
	Government Bonds	-	**96.5%**	3.5%
	Corporate Bonds	-	100%	**-**
	Others	40.8%	48.8%	10.4%

Panel B: Geographic Origin of the Collateral Securities		Europe	Asia-Pacific	N. America	R. World
ETF Geographic Exposure	All	66.0%	17.5%	16.3%	0.2%
	Europe	**71.8%**	13.9%	13.9%	0.4%
	Asia-Pacific	56.0%	**24.1%**	19.8%	0.1%
	North America	58.3%	22.1%	**19.5%**	0.1%
	Rest of the World	58.1%	25.5%	16.3%	**0.1%**
	World	58.8%	21.7%	19.4%	0.1%

Notes: This table presents some summary statistics on the securities used as collateral. Panel A displays the number of collateral securities per type of collateral and the value-weighted average percentage of collateral that is held in equity, Governments bonds, and corporate bonds, respectively. Panel B presents, for each type of ETF geographic exposure, the value-weighted percentage of collateral that comes from Europe, Asia-Pacific, North America, and Rest of the World, respectively. The Government Bond category also includes Supranational Bonds (5), Government Guaranteed Bonds (3), Government Agency Bonds (2), and German Regional Government Bonds (1). The Corporate Bond category also includes Covered Bonds (16). The number in parentheses indicates the number of different securities in each category. The sample period is July 5, 2012 - November 29, 2012.

Table 4 – Collateral Risk and Fund Characteristics

		Correlation	p	S	ACR
	log(AUM)	-0.0201	1.025**	-0.000123	0.00587
		(-0.75)	(2.30)	(-0.12)	(1.17)
	Funded Swap	-0.267	11.77***	-0.0127	0.126***
		(-1.09)	(2.62)	(-1.53)	(2.64)
	Inverse	-3.084***	4.096*	0.0141***	-0.0170
		(-16.83)	(1.88)	(4.03)	(-0.93)
	Collateralization	0.356**	-51.76***	-0.119**	-0.642***
		(2.42)	(-4.00)	(-2.07)	(-3.47)
	log(# Securities)	0.0610**	-0.545	0.00103	-0.0216
		(2.44)	(-0.59)	(0.41)	(-1.46)
	Equity Fraction	0.00229*	-0.0343	0.000139**	0.00000478
		(1.86)	(-0.69)	(2.35)	(0.01)
	Europe Fraction	0.00352***	-0.00405	-0.0000453	-0.000175
		(4.48)	(-0.19)	(-1.57)	(-0.59)
Asset Exposure	Governement Bonds	-0.116	-16.45***	-0.00934	-0.0259
		(-0.37)	(-2.64)	(-0.97)	(-0.39)
	Money Markets	-1.480***	-21.68***	-0.0140	-0.0501
		(-4.83)	(-3.57)	(-1.37)	(-0.84)
	Commodities	-0.913***	15.90***	-0.00702**	0.000910
		(-3.93)	(6.12)	(-2.02)	(0.03)
	Hedge Funds Strategies	-0.361	-7.926	-0.00175	-0.0229
		(-0.83)	(-1.28)	(-0.18)	(-0.71)
	Credit	-0.0318	-18.23***	-0.0122	-0.0192
		(-0.06)	(-3.12)	(-1.19)	(-0.32)
	Corporate Bonds	-1.217***	-16.80***	-0.00362	-0.0241
		(-3.88)	(-2.63)	(-0.28)	(-0.37)
	Currencies	-1.261***	18.73***	0.00154	-0.0389*
		(-3.98)	(8.60)	(0.35)	(-1.80)
	Multi Assets	-0.445***	16.16***	-0.00764**	-0.0732***
		(-4.70)	(7.15)	(-2.29)	(-3.27)
Geographic Exposure	World	-0.0468	-10.65***	0.00345	-0.0421
		(-0.39)	(-3.94)	(1.28)	(-1.45)
	Asia-Pacific	-0.820***	-9.389***	0.00177	-0.00667
		(-6.17)	(-2.67)	(0.44)	(-0.16)
	North America	-0.104	1.009	0.00583*	-0.0149
		(-0.86)	(0.34)	(1.93)	(-0.66)
	Rest of the World	-0.439*	-9.486	-0.000717	-0.0582*
		(-1.73)	(-1.06)	(-0.22)	(-1.82)
	Constant	0.867***	36.13**	0.139**	0.783***
		(2.77)	(2.49)	(2.53)	(4.05)
	Fund Random Effects	Yes	Yes	Yes	Yes
	Observations	3,447	3,447	2,864	2,864
	R^2	0.758	0.498	0.314	0.225

Notes: This table reports the parameter estimates obtained by regressing the correlation coefficient between the ETF return and the return of the collateral portfolio (column 1) and different collateral risk measures p, S, and ACR (columns 2 to 4) on a series of ETF-specific variables. The estimation is based on all week-fund observations. The explanatory variables are the level of the ETF asset under management (in log), a dummy variable that takes the value of 1 if the ETF is based on a funded swap and 0 otherwise, a dummy variable that takes the value of 1 if the ETF is an inverse ETF and 0 otherwise, the level of collateralization of the fund, the number of securities in the collateral portfolio (in log), the fraction of equities in the collateral portfolio, the fraction of European securities in the collateral portfolio, as well as a dummy variable for each asset exposure and geographic exposure. In all columns, the model is estimated by GLS with random effects and robust standard errors. We display t-statistics in parentheses. ***, **, * represent statistical significance at the 1%, 5% or 10% levels, respectively.

Table 5 – Equities Used as Collateral

Panel A: Equity Issuer

	Europe	Asia-Pacific	N. America	R. World
Region	58.5% (752)	21.6% (1,365)	19.6% (469)	0.3% (5)

	Industrial	Financial	Utility	Transportation
Industry Classification	76.6% (2,047)	11.4% (346)	10.4% (127)	1.6% (71)

	Micro-Cap	Small-Cap	Mid-Cap	Large-Cap
Market Capitalization	1.4% (30)	9.3% (1,634)	9.9% (404)	79.4% (523)

Panel B: Liquidity	mean	median	st.dev.	min	max
Average Daily Spread	0.21%	0.20%	0.48%	0.01%	12.73%
Average Daily Volume	4.67%	0.44%	21.83%	0.01%	115.15%

Panel C: Dependence			mean	median	st.dev.	min	max
Beta	ETF		0.48	0.45	0.70	-1.83	3.04
	Swap Counterparty		0.30	0.28	0.28	-0.06	1.11
Conditional Beta	ETF		0.50	0.47	0.74	-3.05	4.14
	Swap Counterparty		0.35	0.33	0.30	-0.46	1.06
		< 0	$[0; 0.2[$	$[0.2; 0.4[$	$[0.4; 0.6[$	$[0.6; 0.8[$	≥ 0.8
Beta	ETF	7.5%	11.3%	25.3%	21.4%	13.4%	21.1%
	Swap Counterparty	0.4%	36.6%	38.3%	19.3%	4.2%	1.2%
Conditional Beta	ETF	9.2%	9.1%	22.9%	20.9%	14.4%	23.5%
	Swap Counterparty	0.2%	23.9%	40.2%	25.3%	8.8%	1.6%

Notes: This table presents some summary statistics on the equities included in the collateral portfolios. Panel A displays the value-weighted percentage of collateral equities by region, industry, and size of the issuer, along with the number of different equities in parentheses. We use the following definitions for size groups: Micro-Cap: below \$100 million; Small-Cap: \$100 million-\$4 billion; Mid-Cap: \$4 billion-\$10 billion; Large-Cap: Over \$10 billion. These ranges were selected to match the average market capitalization of the MSCI World Index of the respective categories. The size figures are as of November 29th, 2012. Panel B displays value-weighted statistics about the average daily percentage bid-ask spread and the average daily volume in percentage of the market capitalization. For each security, the percentage spread and volume are winsorized at the top 1%. Panel C displays value-weighted statistics on the beta coefficient ($\beta_{i,j}$) and conditional beta coefficient ($\beta_{i,j}|r_j < 0$) of the collateral equities with respect to the ETF return and to the swap counterparty return (Deutsche Bank stock return). We compute the conditional betas by using only days during which the index return or the swap counterparty return is negative. The lower part of Panel C presents a histogram of the betas and conditional betas, weighted by the equity value. In Panels B and C, the sample period is between January 1, 2007 and December 31, 2012.

Table 6 – Bonds Used as Collateral

Panel A: Bond Issuer		Europe	N. America	Asia-Pacific	R. World
Bond Type	All	88.3% (512)	7.1% (118)	4.5% (77)	0.1% (1)
	Gov. Bonds	86.0% (338)	8.6% (101)	5.4% (51)	-
	Corp. Bonds	96.6% (174)	2.0% (17)	1.3% (26)	0.1% (1)

Panel B: Rating		AAA	AA	A	BBB	BB	B	n/a
Bond Type	All	65.5%	14.8%	17.2%	0.8%	0.3%	0.1%	1.3%
	Gov. Bonds	75.2%	9.3%	15.3%	0.1%	0.1%	-	-
	Corp. Bonds	30.4%	34.5%	23.9%	3.5%	1.5%	0.1%	6.1%
Bond Issuer	Europe	66.0%	12.6%	19.2%	0.9%	0.2%	0.1%	1.0%
	North America	93.9%	0.4%	0.1%	0.1%	1.8%	0.2%	3.5%
	Asia-Pacific	19.8%	69.9%	3.5%	0.4%	1.1%	-	5.3%

Panel C: Maturity		<1Y	1-3Y	3-5Y	5-7Y	7-10Y	>10Y
Bond Type	All	16.4%	24.4%	14.7%	6.7%	14.7%	23.1%
	Gov. Bonds	11.7%	19.2%	13.3%	8.5%	18.1%	29.2%
	Corp. Bonds	33.4%	43.0%	19.6%	0.5%	2.3%	1.2%
Bond Issuer	Europe	17.4%	25.3%	14.7%	6.4%	13.2%	23.0%
	North America	4.5%	21.7%	18.8%	8.0%	16.7%	30.3%
	Asia-Pacific	17.9%	14.4%	12.4%	17.4%	28.1%	9.8%
Index Maturity	Short	21.6%	49.5%	23.7%	3.3%	1.0%	0.9%
	Medium	10.7%	8.9%	22.7%	34.0%	23.7%	-
	Long	-	-	-	7.5%	7.6%	84.9%

Notes: This table presents some summary statistics on the bonds included in the collateral portfolios. Panel A displays the value-weighted percentages of collateral bonds by region, along with the number of different bonds in parentheses. Panel B presents the value-weighted percentages of collateral bonds by bond rating, for different bond types and issuers. The n/a category corresponds to unrated bonds. Panel C displays the value-weighted percentages of collateral bonds by bucket of maturity, for different bond types and issuers. Short, Medium, and Long refer to funds that track a bond index with, respectively, a short maturity (less than 3 years), a medium maturity (between 3 and 10 years), and a long maturity (more than 10 years). The sample period is July 5, 2012 - November 29, 2012.