Digital Currencies and Bank Competition

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March 23, 2021

Abstract

This article examines how the issuance of a digital currency by a non-bank operator impacts competition between banks in a cashless society. Unlike banks, the digital currency provider is not allowed to engage in maturity transformation. I analyze how the fee charged for the digital currency impacts the interest rates on loans and the fees charged by banks to depositors for paying from their bank account and opening an account in a bank. I derive the conditions under which consumers use the digital currency to pay. I also discuss how the distribution mode of the digital currency may impact its use for payments.

JEL Codes: G21, L31, L42.

Keywords: Payment systems, cash, digital currencies, CBDC, money demand, banking regulation.

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†I thank Christian Pfister and participants at the Sciences Po/Banque de France seminar for their comments.

1 Introduction

Will banks remain the main providers of payment services? Banks are defined both in the economic literature and by the legislation as institutions that engage both in credit and deposit-taking activities. Along with bookeeping and safety services, banks offer to depositors the option to make electronic payments from their account. Over the recent years, the use of cash has declined in several developed countries. In developing countries, mobile payments offer the promise to foster the financial inclusion of the population that is not currently served by the banking sector.[1] Given the digitalization of payments, private non-bank operators such as Facebook are considering competing with banks either for payments, sometimes deposits if possible, or both.[2] Moreover, several Central Banks (e.g., Bank of England, Norges Bank, the European Central Bank) have started to explore the role of a Central Bank Digital Currency (hereafter, CBDC).

In this paper, I analyze how the presence of a non-bank competitor on the deposit side of banks' balance sheet may disrupt banks' business model of financial intermediation, given that banks incur costs of managing their liquidity risk. I focus on a specific category on non-banks, that I will call digital currency providers. Digital currency providers offer to consumers the option to store deposits in an alternative account and to pay from this account by digital currency. Unlike banks, digital currency providers do not bundle credit and sight deposits. Therefore, they do not engage in maturity transformation. Moreover, they do not incur additional costs of liquidity to settle payment transactions. The issuance of a non-bank digital currency raises several unexplored issues, both from a macroeconomic and a microeconomic perspective. In this paper, I study how the design of the digital currency (i.e., price, interest bearing or not, distribution mode) and the market conditions (number of banks, cost of liquidity) impacts its adoption for payments. I also derive the equilibrium interest rate on bank loans, the fee for deposits and the price of bank transfers that result from competition between banks and a digital currency provider.

A digital currency can either be issued by a Central Bank or by a non-bank private e-money operator that is not allowed by regulation to lend directly to depositors.[3] Central Bank e-money

[1] Examples of developed countries with a decline in the use of cash include Sweden and the United-Kingdom. Examples of developing countries seeing an increase in the adoption of mobile payments include Kenya, Tanzania or Bangladesh.

[2] See for instance the Libra project of a stablecoin issued by Facebook (renamed Diem). Competition for deposits depends on whether non-banks need a licence to engage in deposit-taking activities. In some countries, such as China, competition is already a reality, see the role of Alipay and Wechat in China (explained in Yan, 2020).

[3] A paper by the ECB (2020) explains that the introduction of a digital currency would have a similar impact on financial accounts if it is issued by a financial vehicule that manages a stablecoin (such as the Libra project by

or CBDC refers to any form electronic fiat claim on a Central Bank that can be used to settle payments or as a store of value (Meaning, Barker, Clayton and Dyson, 2017).[4] Central Bank e-money might be held either on a Central Bank account or token-based (Kahn, Rivadeneyra and Wong, 2018) and even pay a rate of interest.[5] If e-money is issued by private providers who are able to keep their clients' funds as Central Bank reserves, e-money users can indirectly transact in a Central Bank liability. Therefore, e-money can be qualified as a synthetic CBDC (Adrian and Mancini-Griffoli, 2019). A digital currency can also be offered by private operators who cannot access Central Bank reserves, such as Internet giants.

Several authors have studied whether the Central Bank should issue CBDCs. One of the motivations mentioned by several Central Banks is the risks associated to the dominance of private companies for the issuance and the distribution of digital currencies (e.g., ECB). According to Kahn, Rivadeneyra and Wong (2018), the most important issues relate to the the effects on the industrial organization of banks. How does the issuance of Central Bank e-money impacts competition for deposits, payment instrument pricing and the amount of credit available in retail lending markets? I are not aware of any research work answering these research questions from an industrial organization perspective, which is the purpose of this paper. Unlike in other fields of the literature (e.g., the search theoretic models of money and payments), the industrial organization approach enables me to focus on the pricing of services bundled to sight deposits, that is, money storage and payment services.

I model the digital currency as units of value that can be stored on separate accounts, which may either be managed by the Central Bank or a private operator, which may be regulated. The digital currency competes with commercial bank deposits both as means of payment and store of value. It can be transferred on demand to pay. Unlike banks, the operator that manages the digital currency does not offer loans to individuals.

At date zero, banks decide how much to hold in reserves, how to price deposits, payments and loans. At date one, they offer loans to consumers, which mature at date two. Loans are funded with demandable deposits that may be transferred by consumers to pay for their expenses between date one and date two. If depositors' external payments exceed banks' reserves, banks may need to incur additional costs of liquidity. The presence of the digital currency impacts

Facebook) or by a narrow bank. A narrow bank is defined as an institution that issues deposits which are fully backed on the asset side with central bank deposits.

[4]CBDCs are distinct from reserves, which are only available to a limited number of financial institutions.

[5]Distinction between account-based and token-based depends on who is liable in case of a fraudulent transaction In token-based payment systems (such as cash), the receiver of the payment is liable for fraud. In account-based, the provider of the account should check the identity of the account-holder and bear the cost of record-keeping by verifying the authenticity of transactions. A discussion on the legal definitions of token-based versus account-based CBDC is provided in Bossu et al., 2020.

banks' management of liquidity. First, the crowding-out effect on bank deposits reduces the amount of reserves held by banks in equilibrium, which raises banks' cost of liquidity. Indeed, if depositors pay more often from their bank account, banks are more likely to incur higher costs of liquidity. Second, the price of transactions in digital currency and the interest rate paid on balances held in the digital currency account impact the depositors' payment decision. When the fee for digital currency transactions becomes lower, consumers are more likely to pay from their digital currency account, which reduces banks' cost of liquidity. By contrast, when the interest rate on digital currency accounts increases, consumers pay more often by bank transfer, which increases banks' cost of liquidity. In my paper, I assume that consumers prefer to pay smaller transactions by digital currency and larger transactions by bank transfer, to capture the fact that a digital currency may partly substitute for cash transactions. Liquidity management is costly for banks, which pass through the cost of liquidity to consumers in the form of higher transaction fees.

In the first part of the paper, I consider that the amount kept by depositors in their bank account is exogenous. I analyze the conditions on the lending and the deposit market such that consumers use the digital currency to pay. Banks choose a bank transfer fee that reflects a trade-off between extracting more surplus from consumers when they open their digital currency account and reducing their expected cost of liquidity. Since consumers cannot completely bypass banks to make payments, banks are able to extract rents from depositors when they open a digital currency account. On the one hand, banks have incentives to decrease the bank transfer fee to extract more surplus from their depositors, who receive interest rates from their digital currency accounts. This effect increases with the interest rate on digital currency accounts. On the other hand, the bank transfer fee has two opposite effects on banks' marginal benefit of liquidity. If consumers pay less frequently from their bank account, the marginal benefit of liquidity is reduced, which implies that banks have incentives to decrease the bank transfer fee. The intensity of this effect increases with the amount of reserves held by banks in equilibrium. However, if consumers pay a higher amount in average from their bank account, the marginal benefit of liquidity increases, which implies that banks have also incentives to increase the bank transfer fee. The magnitude of this last effect is all the more important since the cost of liquidity is high and the number of banks is small, because banks collect a higher share of deposits. The market share of the digital currency reflects these effects and depends on the design of the digital currency, that is, the interest rate on digital currency accounts, and the fee for digital currency payments. It also depends on the market conditions for liquidity, that is, the interest rate on Central Bank reserves and cost of liquidity for banks.

I also study how the quantity of deposits kept in a digital currency account and the fee for digital currency transactions impacts the lending rate. I show that the lending rate increases when the digital currency crowds out a higher amount of bank deposits, because banks need to borrow additional amounts from the Central Bank to meet their liquidity needs. I also show that if banks hold a low amount of reserves, the interest rate on loans decreases with the fee for the digital currency. Indeed, the threshold value of the transaction such that consumers prefer to pay from their bank account impacts banks' marginal cost of liquidity.

Then, I discuss the equilibrium of the game if the fee for payments by digital currency is set at the marginal cost of transactions. I show that even if digital currency accounts do not pay interests, the digital currency may be adopted for payments. There may be equilibria in which consumers use the three payment instruments to pay. The fee for the digital currency should however reflect banks' marginal cost of liquidity and depend on whether banks hold enough reserves to meet demand for large value payments by bank transfers or only hold a small amount of reserves. However, when interest rates on digital currency accounts are too high, consumers do not use the digital currency to pay. In that case, the digital currency provider has no incentives to enter the market, unless it obtains revenues from other activities.

In the last section, I discuss several assumptions of the paper. First, I consider that consumers leave an endogenous amount of wealth in their bank account. Second, I discuss how the results would be modified with an hybrid model of a Central Bank Digital currency, in which banks would be the distributors of a Central Bank Digital Currency. I assume that banks would be the managers of digital currency accounts, backed by a one to one ratio of reserves in a dedicated Central Bank account. I derive the conditions such that the adoption of the digital currency is identical under the hybrid model of a Central Bank Digital currency and in the baseline model of the paper. Third, I discuss the impact of restricting the access to Central Bank accounts to banks. Fourth, I analyze how the threshold value for bank transfer transactions would be modified with partial acceptance of payment media. Fifth, I discuss how the results of the model could be impacted by other features of the digital currency, such as safety. Then, I analyze what happens if banks can be completely bypassed by some consumers who may decide to renounce to opening a bank account. In the end, I comment on the choice of the fee for digital currency transactions. I show that marginal cost pricing of digital currency transactions may not maximize the surplus of depositors.

The rest of the article is organized as follows. In section 2, I introduce the literature that is related to my study. In section 3, I present the model and the assumptions. In section 4, I solve for the equilibrium of the game. In section 5, I discuss the robustness of the results obtained in section 4. Finally, I conclude.

2 The literature

My paper is connected to three main strands of the literature: the literature on competition between currencies, the literature on financial intermediation and FinTech, and the literature on CBDCs.

An extensive literature initiated by Hayek (1976) studies competition between currencies. In my model, I only take into account competition between currencies as store of value and means of exchange. I identify cases in which the digital currency is used as a store of value but not as a means of payment and vice versa. My results also confirm the view of Brunnermeier, James and Landau (2019), who argue that the digital revolution may lead to an unbundling of the separate roles of money as means of exchange and store of value. When switching costs are low, there is no longer a strong incentive to use a single currency both as a store of value, medium of exchange and unit of account. This is indeed the case in the baseline model of my paper because I assume that value can be transferred costlessly from the bank account to the digital currency account.[6]

A wide strand of the literature analyzes how consumer demand for money depends on the behavior of banks as financial intermediaries. In particular, several papers connect consumer demand for money with the fees charged by banks for transactions initiated from deposit accounts (see Towey (1974), Saving (1979), Santomero (1979), Whitesell (1989)). These works rely on an inventory-theoretic approach to model consumer demand for money. My model of consumer demand for payment media is closer to the framework used by Whitesell (1992), in which consumers choose their payment method according to the size of the transaction and their cost of holding a specific asset. In my paper, consumers' transaction costs are endogenous and depend on competition between banks and the digital currency provider. I also enrich the model of the banking firm considered by Whitesell (1992) by taking into account banks' decision to hold reserves. The cost of managing liquidity is passed through to consumers who borrow from the bank, but also to depositors in the form of higher deposit and transaction fees for payments. This is due to the fact that banks' cost function is not separable in the volume of deposits and loans.

The literature analyzing whether banks should be allowed by regulation to distribute both

[6] A similar result has been obtained before in Santomero and Seater (1996) who study the choice of a representative consumer between competing currencies. However, their framework does not model competition between banks.

credit and payment services has also been revived recently both by the COVID-19 pandemic and the expansion of bank-FinTech competition.[7] During the Great Depression, several economists proposed to dissociate the distribution of credit from the provision of means of payment and to back deposits with reserves held at the Central Bank (see Knight et al. (1933), Fisher (1936) or later Friedman (1965)). According to this plan, individual consumers could hold sight deposits at the Central Bank, which would reduce liquidity risks in the financial system. This proposal has also been debatted by Tobin in the context of the Savings and Loans crisis in the mid 1980s in the United-States (see the proposition of Tobin (1987) of a deposited currency). Other economists have studied the role of narrow banking as a means to limit banks' incentives to take risks in the lending market. Unlike in the fractional reserve banking system, narrow banks are not allowed to engage in maturity transformation and need to back sight deposits by a 100% reserve ratio. Shy and Stenbacka (2000) argue that narrow banking improves social welfare by enlarging consumers' investment opportunities, given that they have access to other risky investment opportunities outside the banking system. By contrast, Kashyap and Stein (2002) argue that the bundling of credit and deposits is optimal if banks commit to offer credit lines to their depositors. In my paper, the digital currency provider corresponds to the definition of a narrow bank. However, the focus of my model is not to analyze whether bundling of deposits and credit activities improves social welfare. I am interested in understanding instead which payment instruments may be used by consumers given competition between banks and a narrow bank.

An emerging theoretical literature studies how non-banks and FinTech disrupt banks' traditional intermediation activities. Parlour, Rajan and Zhu (2020) analyze the impact of competition between FinTechs and banks on the disruption of information flows stemming from payments. They study how the entry of payment service providers may impact banks' lending activities. Biancini and Verdier (2020) show that competition between a bank and a lending platform impacts the lending rates offered to borrowers and returns offered to investors. Their work is therefore centered on competition between a bank and a non-bank platform on the asset side of banks' balance sheet. Unlike these two papers, my paper focuses on competition on the liability side of banks' balance sheet. I analyze how the presence of a digital currency provider disrupts banks' management of liquidity and payment instrument pricing.

Several papers address indirectly the issue of bank-FinTech competition by trying to determine the economic value added by banks' competitors. In particular, Prat et al. (2019) or You and Rogoff (2020) study a platform's incentives to issue tokens as substitute for fiat currency. Other papers analyze the mechanism of stablecoins (e.g., Lyons and Viswarath-Natraj, 2020 or

[7]In the United-States, several Congress initiatives have proposed 100% reserve accounts with banks.

Melachrinos and Pfister, 2020). In my paper, both currencies rely on the same unit of account. The digital currency provider's decision to issue a new currency depends on its expected profit of competing with banks by issuing a differentiated currency. The source of differentiation between currencies is related both to their value as a storage instrument and their value as a payment method. Unlike in this literature, I do not discuss whether the digital currency could be used to buy specific services on a platform. In my paper, the digital currency is accepted without restrictions to buy consumption goods. The service provider that I consider is not necessarily organized as a platform. Moreover, I do not discuss whether the digital currency could add more value to consumers if it is distributed on a blockchain, enabling innovative contracts between digital currency users.[8] On the other hand, blockchain distribution of a digital currency may also entail several costs, such as the cost of reaching a consensus and updating the ledger (see Auer, Monnet and Shin, 2021).

My paper is also related to a literature that studies the role of Central Banks in payment systems. An important research question is whether the Central Bank should act as a direct operator in the provision of payment instruments. In the recent years, Central Banks have avoided dealing directly with consumers for the provision of electronic retail payment instruments, relying on tiered arrangements in which commercial banks provide retail payment services.[9] The only direct connection between Central Banks and consumers arises when the latter hold cash in their wallets, because it is a form of Central Bank debt. New technologies have changed the situation, allowing Central Banks to offer retail payment services directly to consumers either through dedicated accounts or in the form of token-based systems (see Kahn et al. 2018).[10] Whether Central Banks should become direct providers of retail payment services remains an open research question, and if so, what would be the design of the digital currency that could be offered by the Central Bank. In particular, Central Bank e-money could compete with commercial bank deposits as means of payment, store of value or both. The Central Bank could offer a digital currency through dedicated accounts or thanks to a token-based system that could be either centralized or delegated to intermediaries. The issuance of a non-bank digital currency would have a crowding-out effect on the commercial provision of deposit by banks. Since banks rely on sight deposits to fund loans, this would also impact bank lending to firms and individ-

[8] In particular, the blockchain could add value to specific types of payments characterized by credit or liquidity risk by implementing smart contracts. I do not discuss in this paper the risks associated to the payment transaction itself. This issue could become relevant in the future for retail payments.It is already very relevant for large value payment systems and wholesale CBDCs, that I do not discuss in this paper.

[9] Historically, many Central Banks have allowed deposits by firms and large citizens (See Fernandez-Villaverde, Sanches, Schilling and Uhlig, 2020 for a survey).

[10] Kahn et al. (2018) define a token-based system as relying on identification of the object being transferred as a means of payment rather than relying on identification of the individual whose account is being debited.

uals. In my paper, I focus on the account-based design of digital currencies. I model the case in which the digital currency account is managed by an alternative provider that may be the Central Bank and discuss the case in which banks act as distributors of the digital currency in the extension section.

The introduction of a CBDC may impact bank lending and the volume of deposits collected by banks. My paper is the first to endogenize the choice of the price of payment instruments in an industrial organization model, in which consumers can choose between paying by bank transfer and by digital currency. Moreover, I am able to analyze how the crowding-out of deposits that follows the introduction of the digital currency impacts banks' margins per depositor. As regards the impact of the introduction of the digital currency on lending, I identify the same negative effect on banks' funding costs as in other papers of the literature. My contribution is to analyze how the price of the digital currency impacts the interest rate on loans and the price of payment transactions when the digital currency provider competes with banks, taking into account banks' cost of liquidity.

The introduction of a CBDC can have both positive and negative effects on lending. On the one hand, if a CBDC leads to a crowding-out of commercial deposits from the banking sector, banks' funding costs become higher, which reduces bank lending. This concern has been expressed in several reports on CBDCs (See the report by the Bank of International Settlements, or the staff note by Mancini-Griffoni et al., 2018 of the International Monetary Fund). The impact of higher funding costs on lending depends on competition between banks. The more competitive the banking sector is, the higher is the pass-through of banks' costs to consumers and the reduction of bank lending (Keister and Sanches, 2018 or Chiu et al., 2019). This effect is also present in my paper. I am able to relate it to the price of the digital currency and the interest rate offered by the digital currency provider. I also identify the conditions on the lending and the deposit market such that the digital currency is used to pay at the equilibrium.

On the other hand, a CBDC can provide consumers with an outside option for payments and deposits, which reduces banks' market power. Banks are then forced to increase the deposit rate to retain their depositors, which leads to more funding and a higher supply of loans (see Chiu et al., 2019). I identify another channel through which banks' market power may be impacted by the introduction of the digital currency. Unlike Chiu et al. (2019) who model Cournot competition in the deposit market, I use a setting in which banks compete in two-part tariffs for depositors on the Hotelling line, through the choice of the fixed deposit fee and the price of bank transfers. Two-part tariff competition implies that banks care about the margin that they obtain per depositor. In my paper, I use a demand function such that the aggregate supply of loans is constant at the equilibrium, with and without a digital currency. Banks adjust their price of loans so that they lend a constant amount. With a Cournot demand function, the aggregate supply of loans could either increase or decrease because of the digital currency, depending on the impact of the digital currency on the use of bank accounts for payments.

A strand of the literature explores the impact of a CBDC on banks' funding costs, lending, investment and output. Several authors use new monetarist models to adress these issues by adding a banking sector to the model of Lagos and Wright (2005). In particular, Keister and Sanches (2018) show that a CBDC reduces bank lending if the banking sector is perfectly competitive. Chiu et al. (2019) extend their work by considering Cournot competition between banks in the deposit market. They show that a CBDC can increase lending and aggregate output. The presence of the CBDC limits banks' market power by offering an outside option to depositors. Banks are forced to increase the deposit rate offered to depositors, which leads to more bank funding, more supply of loans and lower lending rates. Andolfatto (2018) considers an economy with a monopolistic commercial bank in an overlapping generation model. He shows that a CBDC may have a positive effect on banks' deposits but no impact on bank lending if the Central Bank lends to the commercial bank.

A Central Bank digital currency could also impact the monetary transmission mechanism, by alleviating the lower bound on interest rates (see Meaning et al., (2017), Goodfriend (2016), Agarwal and Kimball (2015) and Rogoff (2016)). In my paper, I identify a link between the interest rate on loans and the design of the digital currency as a payment instrument. Therefore, the impact of the digital currency on the monetary transmission mechanism would also depend on competition between payment instruments.

In my paper, I do not analyze the impact of digital currencies on financial stability. Fernandez-Villaverde et al. (2020) adress this issue by modifying the Diamond and Dybvig (1983) model. As in my framework, they also assume that investment opportunities of the Central Bank and the commercial bank are different. The Central Bank is not able to invest itself in long-term technologies. They show that the Central Bank is able to offer the socially optimal deposit contract, provided competition with commercial banks is allowed. However, they demonstrate that the Central Bank's deposit contract may generate a complete crowding-out of deposits in periods of panic, leading to a suboptimal outcome for the economy. In another work, Fernandez-Villaverde et al. (2020) establish an impossibility result that they call the CBDC trilemma. They show that the Central Bank can achieve at most two of the three objectives of efficiency (in terms of investment choices), financial stability and price stability. In my paper, I only adress the withdrawal risk on the liability side of banks' balance sheet. Therefore, I disregard how default risk on the asset side and unsolvency costs may impact bank competition with a digital currency provider. As argued by Baltensperger (1980), banks' incentives to attract deposits also depend on the marginal cost of equity, on the expected return of deposits and banks' marginal cost of unsolvency.

The issuance of a CBDC that substitutes for cash may also impact social welfare. Huynh et al. (2020) estimate a maximum welfare gain of $2 per month for canadian people if the CBDC is fully adopted and accepted by merchants. They also measure how different designs of a CBDC (cash-like or debit-card like or with the best characteristics of the two payment methods) will benefit different people according to their socio-demographic characteristics. Kwon, Lee and Park (2020) argue that a CBDC can improve social welfare if it is issued by an independent Central Bank when there is tax evasion. In my framework, it is also possible to analyze how the design of a CBDC impacts social welfare. Since I do not model explicitly the surplus of lenders, I discuss in the extension section how the choice of the fee for digital currency transactions impacts the surplus of depositors. I show that marginal cost pricing of digital currency transactions may not maximize the surplus of depositors if banks are imperfectly competitive. This result is due to the fact that banks charge a mark-up on deposits that depends on their marginal benefit of liquidity both in the deposit and the lending market.

3 The model

In this section, I construct a theoretical model of monopolistic banking competition to analyze how the issuance of a non-bank digital currency impacts the prices charged by banks for deposits, payment transactions and loans. The digital currency competes with banks' deposits both as means of payment and store of value.

There are three dates in the economy ($t = 0, 1, 2$) and four types of risk neutral agents: $n \geq 2$ banks, one digital currency provider, a continuum of depositors and numerous entrepreneurs. At date 0, banks choose the price of deposits, payment transactions and loans. At date 1, they collect deposits from the public, keep a share of deposits in reserves and invest the rest in loans that mature at date 2.

Depositors choose from which bank to open an account and may decide to leave a share of their funds in another account managed by an alternative provider, who may be either a private non-bank operator (e.g., Facebook, Alipay, a Payment Service Provider) or a private entity operating on behalf of the Central Bank. This entity differs from banks in two dimensions. First, it operates online, without any bank branch. Second, it does not offer credit to consumers. Depositors may use their alternative account to pay with another payment method that I call the digital currency. Payment instruments are denominated in the same unit of account.[11]

Between date 1 and date 2, depositors may transfer a share of their deposits (either bank or digital currency deposits) using a payment method. The depositors' choice of a payment instrument depends on the prices of payment instruments and the value of the transaction that needs to be settled. If the amount of bank deposit transfers exceeds the bank's reserves, the bank incurs an additional cost of liquidity. To simplify the model, all interest and fees are paid out at date 2.

The lending market: At date 1, banks make risk-free term loans, which mature and are paid off at date 2. Banks offer differentiated lending contracts and compete in prices. Following Shubik and Levitan (1980) and Carletti, Hartmann and Spagnolo (2007), I assume that each bank $i \in \{1..n\}$ faces a linear demand for loans such that

$$L_i = L - \gamma(r_i^L - \frac{1}{n}\sum_{k=1}^{n} r_k^L), \tag{1}$$

where r_k^L represents the interest rate charged by bank k, and $\gamma > 0$ measures the degree of substituability between loans. Note that the aggregate demand for loans is constant and equal to nL.[12] Borrowers are distinct from depositors.[13]

The deposit market: There is a continuum of consumers who are uniformely distributed around a circle of length one as in Salop (1979) and n banks which are all located around the circle at a distance $1/n$ from each other. The digital currency provider is not located on the circle. All consumers deposit the same amount of money d either in a bank account or in a digital currency account and keep m in cash.[14] There is no cost nor any benefit of holding cash.[15] If they deposit their money in bank $i \in \{1..n\}$, consumers leave a share $\alpha_i \in (1/2, 1)$ of

[11] In the terminology of Brunnermeier, James and Landau (2019), I model reduced competition of monetary instruments, that is, monetary instruments which as denominated in the same unit of account (as opposed to full currency competition). For example, one of the core principles of the ECB for the issuance of a digital euro is that it is convertible at par with other forms of the euro.

[12] Therefore, I abstract from studying the effect of a digital currency on the aggregate demand for loans, unlike Chiu et al. (2019). I discuss later on how the results would be modified with Cournot competition in the lending market.

[13] This assumption differs from Andolfatto (2020) who assumes that banks create money in the act of lending.

[14] The quantity of deposits d is treated as exogenous in the model and independent of the deposit rate. In practice, the quantity of deposits could increase with the deposit rates offered by banks and the digital currency provider, respectively, and decrease with the opportunity cost of holding deposits instead of alternative saving instruments.

[15] In my model, the allocation of wealth is made at the beginning of the game, while payments to consume are made at the end of the game, as in cash-in-advance models (see Lucas and Stokey, 1987). In the baseline model, the quantity of cash is considered as independent of the quantity of deposits. However, I discuss this assumption in the extension section, because in pratice, liquidity services of deposit accounts and cash are substitutes.

their deposits d in their bank account.[16] The rest $(1-\alpha_i)d$ is stored in their digital currency account.

At date 1, banks compete for depositors.[17] All banks and the digital currency provider offer to depositors both money storage and payment services, which are imperfect substitutes. Consumers obtain a fixed utility $u_b > 0$ (resp., $u_d > 0$) of opening a bank account (resp., a digital currency account). They choose their bank according to i) the linear transportation cost $t_b x > 0$ of opening an account in a bank branch located at a distance x from their location on the circle; ii) the deposit fee $F_i > 0$ charged by each bank $i \in \{1..n\}$; iii) the interest rates that they expect to earn from deposits; iv) the expected transaction cost of making payments from their accounts. There is no fixed cost and no fee for opening a digital currency account.[18] In the baseline model, I assume that all consumers open a bank account and a digital currency account at the equilibrium of the game. Opening a bank account is essential to open a digital currency account.[19]

Money storage: All banks and the digital currency provider offer money storage services. Banks pay the same exogenous interest rate r_b on bank deposits and the digital currency provider pays the interest rate r_d on digital currency deposits.[20] The difference between the interest rate on deposits in the bank account and the digital currency account is given by

$$\Delta r = r_b - r_d.$$

I do not make any assumption on the sign of the interest rate on digital currency r_d as several papers discuss the possibility for a CBDC to pay negative interest rates.[21] Though most

[16] I focus on an equilibrium in which consumers leave more than half of their wealth in their bank account, which seems a realistic assumption, given that several Central Banks intend to set up limits on the amount that consumers can hold in their digital currency account.

[17] There is no equity in the model. The bank could use equity capital or alternative funding sources if their marginal cost is lower than the marginal cost of deposits. My model could therefore be extended by adding default risk and unsolvency costs, as discussed in Baltensperger (1980).

[18] This assumption is motivated by the fact that if the digital currency is offered by a private provider, consumers already have a relationship with most Internet giants that would be able to offer a digital currency. If the digital currency is supplied by the Central Bank, it seems unrealistic that the Central Bank starts to compete with banks for deposits by charging fixed fees for deposits.

[19] This assumption corresponds to the situation of developed countries, where the proportion of unbanked consumers is low. I discuss the case of developing countries in the extension section. In several countries where non-banks offer payment services, a bank account is required to register for non-bank account services (e.g., for Alipay and Wechat in China for Chineese citizens. For foreigners, an international payment card is required). As regards CBDC accounts, it is often argued that the Central Bank could not bear the costs of KYC investigations.

[20] The quantity of deposits d is treated as exogenous in the model and independent of the deposit rate. In practice, the quantity of deposits could increase with the deposit rates offered by banks and the digital currency provider, respectively, and decrease with the opportunity cost of holding deposits instead of alternative saving instruments.

[21] In a speech at the Bruegel online seminar, F.Panetta from the ECB argued that a negative rate on the CBDC would foster its use as a payment instrument (10th February, 2021).

Central Banks mention that an interest-bearing CBDC would raise several technical issues, some countries (e.g., Sweden) contemplate designing a CBDC with a built-in ability to pay interests. However, several authors argue that the interest rate on a CBDC could not exceed the interest rate on deposits. The ECB report (2020) states that there may be reasons to remunerate the digital euro at a variable rate, namely to prevent the Central Bank from becoming a large-scale financial intermediary. As regards non-bank private digital currency providers, they also may offer to consumers interest-bearing accounts (e.g., Alipay in China). Moreover, one could argue that non-bank digital currency private providers are able to offer additional value to consumers who leave funds in their digital currency accounts (such as rewards or discounts on other services).[22]

Transactions: Between date 1 and date 2, all consumers of bank i face a consumption shock and need to make a transaction of size $T_i \in (0, d)$. The size of the transaction is expressed as a share s_i of the quantity of deposits d held by each depositor, such that $s_i d = T_i$. As the quantity of deposits held by each consumer is constant, I refer to s_i as the size of the transaction in the rest of the analysis. The ex ante distribution of s_i is common knowledge and given by the probability density h with cumulative H on $[0, 1]$. In an example, I assume that s_i follows a uniform distribution on $[0, 1]$.[23] The consumption shocks of all banks are mutually independent.

The choice of a payment instrument: Payment instruments are means to transfer money, either in cash or by transfers of deposits. Consumers may use three payment instruments denoted by $k \in \{c, d, b\}$: i) cash ($k = c$), ii) the bank payment instrument ($k = b$, that is, a payment card or a bank transfer), iii) the digital currency ($k = d$). Consumers decide how to pay for a transaction of size s by comparing the net benefit $b_k(s)$ of paying with each payment instrument k. The variable convience benefit of paying with payment instrument k a transaction of size s is $v_k(s) = v_k s$ for $k \in \{c, d, b\}$. I normalize v_c to $v_c = 0$ and denote by $\Delta v = v_b - v_d > 0$. The variable convenience benefits of payment instruments are such that if the value of the transaction is small, consumers prefer to pay in cash, if it is higher, they pay from their digital currency account, and if it is very high, they pay from their bank account.[24]

[22] In the paper, interest rates on deposits are exogenous. I discuss this assumption in the extension section.

[23] The framework of analysis is a discrete-time model, that simplifies consumers' choices of payment media to one withdrawal decision. With a higher number of withdrawals, consumers would need to make inventory-management decisions as in Baumol (1953), Tobin (1956), Santomero (1979), Whitesell (1989) or Alvarez and Lippi (2009).

[24] In such a setting, the choice of a payment method depends only on the value of the transaction and not on the particular commodities included in the transaction. Without any knowledge of what would be the demand for DC, I model the demand for DC as a substitute for cash transactions, that impacts the volume of transactions paid by bank transfer. I motivate this assumption by the argument mentioned in several papers (e.g., IMF, 2019)

To obtain equilibria in which cash, the digital currency and the bank payment instrument may be used by consumers, I assume that $\Delta v > \max(\Delta r, 0)$ and $v_k > r_k$ for $k = d, b$.[25] All payment instruments are accepted everywhere.[26]

If he pays in cash a transaction of size s_i, the consumer obtains the benefits of keeping his funds in his deposit accounts (i.e., the interest rates r_b and r_d, respectively) and does not pay any transaction fee. Therefore, the benefit of paying in cash a transaction of size s_i is given by

$$b_c(s_i) = \alpha_i d r_b + (1 - \alpha_i) d r_d. \tag{2}$$

If he pays by digital currency a transaction of size s_i, he obtains a variable benefit v_d, but incurs a transaction fee f_d and renounces to the interest rate on digital currency deposits.[27] Hence, the net benefit $b_d(s_i)$ of paying by digital currency a transaction of size s_i is given by

$$b_d(s) = b_c + (v_d - r_d) s d - f_d. \tag{3}$$

If he pays by bank transfer a transaction of size s_i, he obtains a benefit v_b, but incurs a transaction fee f_b^i and renounces to the interest rate on bank deposits. Therefore, the net benefit $b_b(s_i)$ of paying by bank transfer a transaction of size s_i is given by

$$b_b(s_i) = b_c + (v_b - r_b) s d - f_b^i. \tag{4}$$

The threshold value of the consumption shock such that consumers of bank i prefer to pay by bank transfer rather than by digital currency is denoted by s_{dc}^i. The threshold value of the consumption shock such that consumers of bank i prefer to pay by a transfer of deposits with payment instrument $k = d, b$ rather than cash is s_k^i. The expected share of payment instrument k for consumers of bank i is β_k^i for $k \in \{c, d, b\}$.[28] If consumers do not have enough funds either in their digital currency account or in their bank account to pay, they are able to make a

that access to a digital currency will be more convenient than travelling to an ATM. In the ECB report (2020), it is also argued that ideally, a digital euro should allow citizens to make payments such as they do today with cash. Moreover, the People's Bank of China has started to run an experiment on CBDCs for small retail transactions. As regards the role of the Central Bank, it is for the moment unrealistic to assume that it would wish to supply all payment instruments itself.

[25] This specification is similar to Whitesell (1992).

[26] This assumption is relaxed in the extension section.

[27] There are divergent views on the amount of fees that could be charged for CBDC payments. For example, the Central Bank of Brazil has decided to offer CBDC transactions for free. The National Bank of Ukraine (2019) in its pilot project has decided that there would be no fees for P2P transactions and that PSP would be able to charge 1% of the transaction for P2B and B2B payments.

[28] Huynh et al. (2020) analyze what would be the adoption and the use of a CBDC if it is designed with cash-like features, debit-card like features or the best of both. They conclude that even with the best features of cash and debit cards, the CBDC would leave a positive market share for the existing payment instruments.

transfer of value from their digital currency account to their bank account at no cost (and vice versa).

Transaction costs and benefits: The expected transaction costs TC_i incurred by consumers of bank i for payments are given by the sum of the transaction fees. If consumers trade off between paying by cash, by digital currency or by bank transfer, we have

$$TC_i = f_b^i \beta_b^i (s_{dc}^i, s_b^i) + f_d \beta_d^i (s_{dc}^i, s_d^i). \tag{5}$$

The consumer surplus of opening an account: The total expected surplus S_i of opening an account in bank i and a digital currency account is the sum of the fixed utility of opening a bank account u_b, the fixed utility of opening a digital currency account u_d, the expected revenues from interest rates IR_b^i on deposits in bank i, the expected revenues from interest rates IR_d^i on deposits in the digital currency account, less the transaction costs TC_i incurred for payments and the fixed deposit fee F_i. Therefore, the total expected surplus of opening an account in bank i is given by

$$S_i = u_b + u_d + IR_b^i(\alpha_i, s_{dc}^i, s_b^i) + IR_d^i(\alpha_i, s_{dc}^i, s_d^i) - TC_i(f_b^i, f_d, \alpha_i, s_{dc}^i, s_d^i, s_b^i) - F_i, \tag{6}$$

where TC_i is given by (5). The expected revenues from interest rates on bank accounts and digital currency accounts, IR_b^i and IR_d^i, depend on the consumer's choice of a payment method and are given in Appendix B-0, respectively.

Bank profits:

The profits from deposits: Each bank i obtains a margin per depositor that is the sum of the fixed deposit fee, the revenues from bank transfers, from which are substracted the interest rates paid to depositors. Banks incur a marginal cost c_b for bank transfers. Therefore, bank i's margin per depositor is given by

$$\mu_i \equiv F_i + \beta_b^i(s_{dc}^i, s_b^i)(f_b^i - c_b) - IR_b^i(s_{dc}^i, \alpha_i). \tag{7}$$

Since each depositor possesses an amount of wealth d and deposits $\alpha_i d$ in a bank account, given that each bank i has a share d_i of deposits, the total volume of deposits in bank i is given by

$$D_i \equiv \alpha_i d d_i, \tag{8}$$

and each bank i makes a profit $\mu_i d_i$ from deposits.

The profits from lending: Banks incur a marginal cost c_L of lending to consumers. Therefore, each bank i makes a profit $(r_i^L - c_L)L_i$ from lending.

The revenues and costs of liquidity management:

- *Transfers of deposits:*

Between date 1 and date 2, each bank i may lose some deposits when its consumers make payments to another bank. Given that loans mature at date 2, bank i needs to borrow additional liquidity at date 1 if it does not hold enough reserves to meet its consumer demand for payments.

If consumers pay in cash or by digital currency, the quantity of deposits in bank i does not vary. I assume that the receivers of cash payments keep the funds in cash until date 2 and do not deposit them in a bank account. The receivers of a payment in digital currency do not transfer them to their bank account. If consumers pay with the bank payment instrument, there may be a variation of the quantity of deposits in bank i. When a consumer initiates a payment from his bank account, his deposit account is instantly debited. The receiver of the payment only obtains the funds at date 2.[29] With probability φ, the receiver holds an account in another bank. In that case, the quantity of deposits in bank i is reduced between date 1 and date 2. With probability $1 - \varphi$ (and independently from the size of the transaction), the receiver holds an account in the same bank i. In that case, the quantity of deposits in bank i does not vary between date 1 and date 2 and bank i keeps its reserves in the Central Bank until date 2.

- *Liquidity management:*

Each bank i determines the quantity $R_i \geq 0$ of reserves to hold out of a total amount D_i of deposits collected. As in Klein (1971), the reserves are held at the Central Bank to satisfy the bank's liquidity needs when deposits are transferred from bank i for payments.[30] If the bank's liquidity needs are lower than R_i, the bank uses its reserves to meet the demand of depositors for payments. If the bank's liquidity needs are higher than R_i, the bank has a liquidity shortage and needs to borrow additional funding sources at a rate ρ. The cost ρ reflects all the additional

[29] This assumption simplifies the model. If payments occur instantly between the issuer of the bank transfer and the receiver, each bank takes into account the probability that it receives funds from the other banks when it manages its liquidity risk. If banks are sufficiently differentiated in the market for deposits (i.e., if t_b is high enough), this does not affect the results of the model. The results change if competition for deposits becomes more intense.

[30] I consider that banks already hold the amount of reserves required by the regulator and need to decide how much reserves to hold in excess of that amount.

costs of liquidity adjustements including transaction costs.[31] If the realization of the liquidity shock is lower than R_i, the bank has an excess of reserves and may lend to the Central Bank at a rate $\tau < \rho$. The rate τ is referred to as the interest-on-reserves (IOR).

Each bank i raises an amount D_i from depositors, keeps a quantity R_i of reserves and invests the rest of its funds L_i in the lending market, such that

$$L_i + R_i = D_i. \tag{9}$$

Replacing for bank i's deposits given by (8), each bank i keeps an amount of reserves given by

$$R_i = \alpha_i d d_i - L_i. \tag{10}$$

The expected net benefit of liquidity management for bank i is denoted by EL_i and it is detailed in Appendix B-1. It depends on bank i's share of deposits $\alpha_i d_i$, on the amount of loans L_i offered by bank i, and on whether consumers of bank i need to transfer deposits from bank i between date 1 and date 2 to make payments.

Bank profits: Bank i's expected profit π_i is the sum of the profit on loans and deposits, and the expected net benefit of liquidity management. It is given by

$$\pi_i = (r_L^i(L_i) - c_L)L_i + \mu_i(f_b^i, F_i, \alpha_i, s_{dc}^i, s_b^i)d_i + EL_i(d_i, L_i, \alpha_i, s_{dc}^i, s_d^i, s_b^i), \tag{11}$$

where the interest rate on loans r_L^i is given by (1), the amount of loans L_i is given by (10), the margin per depositor is given by (7) and the share of deposits d_i is given by competition between banks. The amount of reserves R_i, the share of deposits d_i and the amount of lending L_i are linked by the balance sheet identity given in Eq. (10).

Digital currency provider profit: The digital currency is distributed by a private provider, which is distinct from banks.[32] It offers to consumers the possibility to store money in a digital currency account and pay by digital currency. The DC provider holds a Central Bank account, in which it invests the funds of its consumers, remunerated at the interest rate on reserves

[31] As argued by Baltensperger (1980), the rate ρ cannot strictly be identified as the discount rate, because some banks "cannot borrow freely from the Central Bank or (have) an aversion against borrowing from it".

[32] The case in which banks act as distributors of a Central Bank Digital Currency is discussed in the extension section. The Central Bank may use a richer set of instruments than a private provider to design the digital currency, because it may choose the return offered to banks on Central Bank accounts (i.e., τ) and the cost of borrowing from the Central Bank (i.e., ρ). Whether the Central Bank will be allowed by its mandate to issue a digital currency is still an opened question in several countries that is analyzed in Bossu et al. (2020).

in digital currency τ_d (IOR-DC).[33] As it is not allowed by regulation to engage in maturity transformation, it must keep a ratio of 100% of reserves. The case in which the digital currency provider does not have access to Central Bank accounts is discussed in Section 5.3.[34]

The DC provider makes a profit from digital currency payments thanks to the fee f_d for each digital currency transaction and incurs a marginal cost $c_d > 0$ per transaction.[35] It also obtains a revenue from its money storage activity and incurs the costs of paying the interest rate r_d on deposits in digital currency.

Therefore, the profit of the digital currency provider π_d is the sum of the revenues from payment transactions and digital currency accounts from all consumers of each bank $i = 1..n$. Since each bank i has a share d_i of consumers, it is given by

$$\pi_d = \sum_{i \in (1,n)} d_i((f_d - c_d)\beta_d^i(s_i) + (\tau_d/r_d - 1)IR_d^i(s_{dc}^i, \alpha_i)). \tag{12}$$

As the provider always keeps 100% of the deposits in reserves, it does not need to borrow additional liquidity to settle payment transactions.[36]

I assume that the fee for digital currency transactions f_d is regulated at marginal cost (as analyzed in the report by the Bank of England, 2020), that is, $f_d = c_d$. As discussed in the ECB report (2020), the Central Bank may be guided by a cost recovery principle for pricing digital currency transactions. Other Central Banks may decide to set zero transaction fees for a CBDC.

Assumptions: I make the following assumptions:

(A0) The fixed utility u_b that each depositor obtains from opening an account in a bank is sufficiently large such that all consumers prefer to open an account in a bank in equilibrium, even if they do not leave any money on it.

[33] As discussed by Cukierman (2020), allowing a Central Bank to compete with banks both in the credit and in the deposit market would be politically challenging, though this proposal has been discussed before (see Benes and Kumhof, 2012). In China, Alipay and Wechat are obliged by the People's Bank of China to hold a 100% reserve ratio on their assets.The designers of the Diem project of a stablecoin distributed by Facebook argue that 80% of Diem reserves are invested in short-term treasury bonds.

[34] Adrian and Mancini-Griffoli (2019) argue that e-money providers could have access to Central Bank reserves under strict conditions. In that case, consumers transact and hold a synthetic CBDC.

[35] According to a survey of research on retail Central Bank Digital Currencies conducted by several authors from the IMF (2020), the costs of supplying a CBDC include labor and infrastructure costs, operation of software, costs of cyber security and online support.

[36] The underlying assumption is that consumers paying by digital currency need to send their funds to another consumer who has a digital currency account. However, it could be that the consumer transfers funds from his digital currency account to another receiver who has only a bank account.

This assumption is motivated by the fact that in most countries, each depositor needs to have a bank account for several kinds of transfers and payments (tax payments, transfers from the government) or to obtain loans via commitments or credit lines (see Kashyap and Stein, 2002). A sufficient condition for Assumption (A0) to hold is that

$$u_b \geq (3t_b/2n) + \rho d \int_0^1 s h(s) ds.$$

I relax this assumption in the extension section by assuming that depositors may renounce to open a bank account and open only a DC account.

(A1) The fixed utility u_d that each depositor obtains from opening a digital currency account is sufficiently large, such that all consumers open a digital currency account if they open a bank account.

A sufficient condition for Assumption (A1) to hold in equilibrium is that the fee for the digital currency f_d in equilibrium is low enough with respect to u_d.

Timing: The timing of the game is as follows:

0. At date 0, each bank $i \in \{1..n\}$ decides on the deposit fee F_i, the price of bank transfers f_b^i and the price of loans r_L^i. The digital currency provider decides on the price of the digital currency f_d.

1. At date 1, consumers choose in which bank to deposit money, then they decide how much to deposit in their bank account and in their digital currency account, respectively.

1/2. Between date 1 and date 2, consumption shocks are revealed and depositors decide how to pay for their expenses. With probability φ, each bank i initiates transfers of deposits if its consumers pay from their bank accounts.

2. At date 2, Consumers receive interest rate payments from their bank and from the digital currency provider. Banks incur the cost of liquidity or receive the interest rate on overnight deposits. All banks receive the transfers of deposits.

In the following section, I look for the subgame perfect equilibrium and solve the game by backward induction. All the variables of the model are summarized in Appendix A.

4 Bank competition with a digital currency

In this section, I study whether consumers have incentives to pay with a digital currency given the interest rates on deposits and the number of banks that compete for deposits.

4.1 The choice of a payment instrument

Between date 1 and date 2, each consumer of bank i chooses whether to pay by cash, by digital currency or by bank transfer. In Lemma 1, I give the threshold value of the consumption shock that maximizes the consumer surplus at the payment stage. For this purpose, I define

$$\underline{f_b}(f_d) = (v_b - r_b) f_d / (v_d - r_d),$$

and

$$\overline{f_b}(f_d) = f_d + d(\Delta v - \Delta r),$$

the minimum and the maximum values of the bank transfer fee, respectively, such that consumers trade off between paying by cash, by digital currency and by bank transfer.

Lemma 1 *Let $\widehat{s_d} = f_d/(d(v_d - r_d))$, $\widehat{s_b} = f_b^i/(d(v_b - r_b))$ and $\widehat{s_{dc}} = (f_b^i - f_d)/(d(\Delta v - \Delta r))$. If $f_b^i \in (\underline{f_b}(f_d), \overline{f_b}(f_d))$, the consumer trades off between paying by cash, by digital currency or by bank transfer. He pays by cash if $s_i \leq \widehat{s_d}$, by digital currency if $s_i \in (\widehat{s_d}, \widehat{s_{dc}})$ and by bank transfer if $s_i \geq \widehat{s_{dc}}$. If $f_b^i < \underline{f_b}(f_d)$, the consumer does not pay by digital currency. He pays by cash if $s < \widehat{s_b}$ and by bank transfer if $s_i \geq \widehat{s_b}$. If $f_b^i > \overline{f_b}(f_d)$, the consumer does not pay by bank transfer. He pays by cash if $s_i < \widehat{s_d}$ and by digital currency if $s_i > \widehat{s_d}$.*

Proof. See Appendix B-0. ■

In Corollary 1, I study how the market share of the digital currency varies with the transaction fees incurred by consumers when they make payments and with the interest rates on bank and digital currency deposits.

Corollary 1 *Suppose that $f_b^i \in (\underline{f_b}(f_d), \overline{f_b}(f_d))$. The market share of the digital currency for consumers of bank i is increasing with the fee f_b^i chosen by bank i for bank transfers. It is also increasing with Δr, the difference between the interest rates on bank deposits and digital currency deposits. It is decreasing with the fee for the digital currency if $\Delta v - \Delta r > v_d - r_d$.*

If bank i increases the fee for bank transfers (compared to the fee for the digital currency), the market share of the digital currency for consumers of bank i increases. If the interest rate revenues from the bank account increase with respect to the digital currency account, the market share of the digital currency for consumers of bank i increases. This implies that consumers prefer to use their bank account as a means to store of value and their digital currency account as a means to initiate payments.

4.2 Competition for deposits

At date 1, prior to making transactions, consumers have to decide in which bank to open an account. For this purpose, consumers take into account the expected transaction costs at date 2, the interest rates on deposits, the fixed deposit fee F_i, and the transportation cost t_b, which depends on their location. A consumer located at point $x \in [0; 1/n]$ trades off between opening an account in bank k located at point 0 and bank i located at $1/n$. If he opens an account in bank i, he incurs a travelling cost $t_b(1/n - x)$, and obtains a net surplus S_i. If he opens an account in bank k, he incurs a travelling cost $t_b x$ and obtains a net surplus S_k. The indifferent consumer between bank i and bank k is given by

$$x_{ik} = \frac{1}{2n} + \frac{1}{2t_b}(S_k - S_i). \tag{13}$$

A consumer located at point $y \in [1/n; 2/n]$ trades off between opening an account in bank i located at $1/n$ and bank l located at $2/n$. The indifferent consumer between bank i and bank l is given by

$$y_{il} = \frac{3}{2n} + \frac{1}{2t_b}(S_i - S_l). \tag{14}$$

The total market share of bank i is $d_i = y_{il} - x_{ik}$. Replacing for y_{il} and x_{ik} given by (13) and (14), the market share of bank i is given by

$$d_i = \frac{1}{n} + \frac{S_i}{t_b} - \frac{(S_k + S_l)}{2t_b}. \tag{15}$$

4.3 The profit-maximizing prices

At date 0, banks compete for loans and deposits. In this section, I analyze how the presence of a digital currency impacts banks' prices for loans, deposits and payment transactions.

I study a benchmark in which I assume that the share of wealth $\alpha > Ln/d$ stored by a consumer in his bank account is exogenous and identical in all banks. This benchmark enables me to focus on competition between payment instruments, leaving the analysis of competition for money as a store of value for the next section. This assumption could also be relevant for markets in which households' deposits tend to be sticky, that is, depositors tend to stay with their bank and do not move their savings from one acount to the other very frequently. Alternatively, this assumption is also relevant if the consumer's decision to leave a positive balance in his digital currency account is not mainly driven by his choice of a payment instrument, and rather by other exogenous considerations such as safety.[37]

I focus on an equilibrium in which consumers always have enough funds in their digital currency account to pay by digital currency, that is, I assume that $\widehat{s_{dc}} \leq 1-\alpha_i$. This corresponds to a scenario in which the digital currency is used as a store of value and for small retail payment transactions.[38]

4.3.1 The banks' best-responses

If there exists an equilibrium in which consumers trade off between paying by cash, by digital currency and by bank transfer, each bank i chooses the fixed fee for deposits F_i, the fee for bank transfers $f_b^i \in (\underline{f_b}(f_d), \overline{f_b}(f_d))$ and the interest rate on loans r_L^i that maximize its expected profit given by Eq. (11) for $i = 1..n$, that is,

$$\pi_i = (r_L^i(L_i) - c_L)L_i + \mu_i(f_b^i, F_i, \alpha_i, s_i, s_d)d_i + EL_i(d_i, L_i, \alpha_i, s_i),$$

subject to the balance sheet identity given by Eq. (9) and to the constraint that the bank is able to fund term loans after depositors' withdrawals between date 1 and date 2. I focus on symmetric best-responses to the fee f_d chosen by the digital currency provider and denote the vector of profit-maximizing prices by $P^* = (F^*, f_b^*, r_L^*)$.

In a symmetric equilibrium, if $\alpha > Ln/d$, each bank i obtains an amount of deposits given by $D^* = \alpha d/n$, lends $L^* = L$ and keeps a quantity $R^* = \alpha d/n - L$ of reserves in its Central Bank account for $i = 1..n$. From the first-order conditions given in Appendix B-2, if there are neither costs nor benefits of holding liquidity (i.e., if $\rho = \tau = 0$), in equilibrium, each bank i chooses an interest rate on loans given by the sum of its marginal cost and a mark-up that depends on the number of banks and the degree of substituability between bank loans, that is, we have

$$r_L^* = \underline{r_L} = c_L + \frac{nL}{\gamma(n-1)}.$$

If liquidity is costly, banks' expected net benefit of liquidity management depends on depositors' trade-off between paying by bank transfer or by digital currency. As this decision depends on the variable benefit of paying by bank transfer and by digital currency, respectively, it depends on the optimal threshold of the consumption shock such that consumers pay by bank

[37] I discuss this motivation in the extension Section.

[38] The analysis of the other scenarii is left for the extension section and the supplementary material.

transfer given by

$$s_{dc}^* = \widehat{s_{dc}}(f_b^*, f_d).$$

Thus, in equilibrium, the vector of profit-maximizing prices $P^* = (F^*, f_b^*, r_L^*)$ depends on the design of the digital currency (i.e., the fee f_d and the interest rate r_d). From the first-order conditions given in Appendix B-2, each bank i chooses an interest rate on loan that is the sum of $\underline{r_L}$ and the marginal cost of liquidity implied by an increase in bank lending, that is, we have

$$r_L^* = \underline{r_L} - \left.\frac{\partial EL_i}{\partial L_i}\right|_{P^*}.$$

Each bank i makes a margin μ^* per depositor that is the sum of a mark-up equal to the average transportation cost t_b/n and the marginal cost of liquidity implied by an increase in bank i's market share, that is, we have

$$\mu^* = \frac{t_b}{n} - \left.\frac{\partial EL_i}{\partial d_i}\right|_{P^*}.$$

If there is an interior solution, the optimal threshold of the consumption shock such that consumers pay by bank transfer, s_{dc}^*, is implicitly defined by

$$\frac{1}{n}(c_b - f_d - dr_d s_{dc}^*)h(s_{dc}^*) + \left.\frac{\partial EL_i}{\partial \widehat{s_{dc}}}\right|_{P^*} = 0. \tag{16}$$

The interpretation of Eq. (16) is as follows. Any increase in the bank transfer fee enables a bank to save the marginal cost of bank payments c_b for the transactions of its depositors (in share $1/n$). However, as consumers substitute a payment by digital currency for a payment by bank transfer, depositors receive lower interest revenues from their digital currency account and have to pay the fee f_d, which reduces the marginal surplus that the bank is able to extract from depositors by $f_d + dr_d s_{dc}^*$. Moreover, an increase in the bank transfer fee impacts banks' marginal benefit of liquidity (see the last term of Eq. (16)).

At the equilibrium, each bank makes a profit given by

$$\pi^* = \frac{t_b}{n^2} + \frac{nL^2}{\gamma(n-1)}. \tag{17}$$

Since banks trade off between the profits from deposits and loans, as the deposit market is covered, there exists several combinations of bank transfer fees, fixed deposit fees and interest rates on loans that yield the same profit for banks given in Eq. (17).[39]

[39] In this baseline model, competition with the digital currency provider has no impact on banks' profits, provided that banks remain active in the market. This is due to the tariff structure with fixed deposit fees and the assumption that the market for deposits remains covered even if banks compete with a DC provider. This

As can be seen from the expression of EL_i given in Appendix B-1, each bank i's profit is defined by parts, depending on the amount of reserves held in a Central Bank account. Therefore, the nature of the solution to the model depends on the magnitude of the optimal value of reserves R^* relative to the minimum share of deposits $s^*_{dc}d/n$ that is transferred by depositors from their bank account when they pay. Banks may choose between holding a low amount of reserves that never meets consumer demand for payments from their bank account and hold enough reserves to meet some but not all consumer demand for payments. The distinction between these two cases is relevant to assess the impact of the digital currency on banks' marginal cost of liquidity. In the next subsection, I choose to focus on the first case (i.e., a low amount of reserves), and explain in subsection 4.3.5 how the mechanism that I highlight changes if banks hold a high amount of reserves.[40]

4.3.2 The banks' best-responses with a low amount of reserves

Suppose that each bank i holds a low amount of reserves that never meets consumer demand for bank transfers (i.e., $R_i \in (0, \widehat{s_{dc}}dd_i)$). In Proposition 1, I characterize banks' profit-maximizing strategy in a symmetric equilibrium according to the fee for the digital currency.

For this purpose, I define the bank transfer fee $\widetilde{f_b}(f_d)$ such that the size of the transaction above which consumers pay from their bank account equals the average amount of reserves per unit of deposits, that is, $\widehat{s_{dc}}(\widetilde{f_b}(f_d), f_d) = nR^*/d$. I also define the bank transfer fee $f_b^m(f_d)$ such that consumers always pay by digital currency when they have enough funds in their digital currency account, that is, $\widehat{s_{dc}}(f_b^m(f_d), f_d) = 1 - \alpha$. Since I chose to focus on determining an equilibrium in which $\widehat{s_{dc}} \leq 1 - \alpha$ (i.e., a digital currency for small retail payments), if $f_b^i \geq f_d$, Lemma 1 implies that

$$\widetilde{f_b}(f_d) = f_d + (\Delta v - \Delta r)nR^*,$$

and

$$f_b^m(f_d) = f_d + (1 - \alpha)d(\Delta v - \Delta r).$$

Moreover, to focus on a case in which consumers trade-off between paying by cash, digital currency or by bank transfer (i.e., such that $\widetilde{f_b}(f_d) \geq \underline{f_b}(f_d)$), I assume that $f_d \leq (v_d - r_d)nR^*$ and discuss the case in which $\widetilde{f_b}(f_d) < \underline{f_b}(f_d)$ in the Appendix.[41]

assumption could be relaxed if some depositors would receive a very low fixed utility of opening an account in a bank, such that they would always prefer to open a DC account instead. Moreover, if banks do not hold any reserves and only lend the amount of deposits that they have collected, banks' profit would be reduced at the equilibrium by competition with the DC provider.

[40] The proof can be found in Appendix B-2.

[41] If $\widetilde{f_b}(f_d) < \underline{f_b}(f_d)$, if the fee for digital currency payments is sufficiently high, each bank i sets a bank transfer

Proposition 1 *Assume that banks never hold enough excess reserves to meet consumer demand for payments. Each bank i chooses an interest rate on loans given by*

$$r_L^* = \underline{r_L} + \varphi(\rho - (\rho - \tau)H(s^*)) + (1 - \varphi)\tau,$$

makes a margin per depositor given by

$$\mu^* = \frac{t_b}{n} - d\varphi \left(\tau\alpha H(s^*)) - \rho \int_{s^*}^{\alpha} (s_i - \alpha)h(s_i)ds_i \right) - (1 - \varphi)\alpha d,$$

and charges a fee for deposits given by $F^* = \mu^* - f_c^* \beta_b^i(s^*) + IR_b^i(s^*, \alpha)$.
There exists $f_d^{\min}$ *and* $f_d^{\max}$ *such that if* $f_d \in (f_d^{\min}, f_d^{\max})$ *and* $\rho\varphi \neq r_d$*, each bank i sets the bank transfer fee* f_b^* *so that consumers pay from their bank account if the size of the transaction exceeds*

$$s_{dc}^* = \frac{f_d - c_b + (\rho - \tau)n\varphi R^*}{d(\rho\varphi - r_d)}.$$

If $f_d \geq f_d^{\max}$*, each bank i sets* $f_b^* = \widetilde{f}_b(f_d)$ *and* $s_{dc}^* = nR^*/d$.
If $f_d \leq f_d^{\min}$*, each bank i chooses* $f_b^* = f_b^m(f_d)$*. Consumers always pay by digital currency when they have money in their digital currency account, that is,* $s^* = 1 - \alpha$.
If $r_d < \rho\varphi$, $f_d^{\min} = c_b - (r_d - \tau\varphi)nR^*$ *and* $f_d^{\max} = c_b - (\rho - \tau)n\varphi R^* + d(\rho\varphi - r_d)(1 - \alpha)$.
If $r_d > \rho\varphi$, $f_d^{\max} = c_b - (r_d - \tau\varphi)nR^*$ *and* $f_d^{\min} = c_b - (\rho - \tau)n\varphi R^* + d(\rho\varphi - r_d)(1 - \alpha)$.
If $r_d = \rho\varphi$, $f_d^{\max} = f_d^{\min} = c_b - \varphi(\rho - \tau)nR^*$*. Consumers pay by digital currency if* $f_d < f_d^{\min}$*, by bank transfer if* $f_d > f_d^{\min}$*, and are indifferent between both payment instruments if* $f_d = f_d^{\min}$.

Proof. See Appendix B-2. ■

In a symmetric equilibrium, bank set an interest rate on loans that is the sum of the marginal cost of lending, the marginal cost of liquidity and a mark-up that depends on the number of banks and the degree of substituability between loans (i.e., the parameter γ). Since the benefit of liquidity EL_i for banks is not separable in the bank transfer fee f_b^i and the amount of loans L_i, the depositors' trade-off between paying by bank transfer and by digital currency impacts banks' marginal cost of liquidity.[42] When consumers transfer funds more frequently from their bank account to another bank (i.e., the threshold s_{dc}^* decreases or if φ increases), banks' marginal cost of liquidity increases if $\rho > \tau$, and, therefore, the interest rate on loans becomes higher.[43]

fee equal to $\underline{f_b}(f_d)$ in equilibrium, such that consumers never use the digital currency to pay, and trade off between paying by cash and by bank transfer instead. See the section in Appendix B-2 on the trade-off between cash and bank transfer.

[42] A standard result in microeconomics of banking is that if the bank's cost function is separable in the amount of deposits and the amount of loans, the prices on the lending market and on the deposit market are independent (see Monti and Klein,1972).

[43] Remark that, in equilibrium, the interest rate on loans is higher than the IOR. The lending market is more

As the aggregate demand for loans is constant in my model, the digital currency has no impact on the aggregate volume of lending offered by banks, which adjust their prices to maintain a constant amount of lending at the equilibrium. With a Cournot demand for loans, I would obtain results that would be comparable to Chiu et al. (2019), who show that the aggregate demand of lending can increase following the introduction of the digital currency.[44] However, in my paper, the aggregate demand of lending would increase only if consumers pay less from their bank account at the equilibrium of the game, which, as I shall demonstrate, is not obvious.

The choice of the bank transfer fee reflects banks' trade-off between extracting the consumer surplus from deposits and reducing their marginal cost of liquidity. The mechanism is the following. A higher bank transfer fee has three effects on a bank's profit through: i) the bank's share of deposits, ii) the bank's revenues from depositors, iii) the expected benefit of liquidity.

First, when a bank increases the bank transfer fee, it changes the depositors' surplus of opening a bank account and a digital currency account. As depositors prefer to transfer funds from their digital account rather than their bank account, they expect to earn higher interest rates from their bank account and lower interest rates from their digital currency account, respectively. Moreover, they expect to make a lower volume of bank transfers and a pay a higher bank transfer fee.

Second, a higher bank transfer fee changes the margin that each bank earns from depositors. Two-part tariff competition implies that banks are able to internalize perfectly the impact of the bank transfer fee on the consumer surplus of opening a bank account. However, each bank is not able to internalize the impact of a higher bank transfer fee on the consumer surplus from payments in digital currency (which depend on the fee for the digital currency) and from the digital currency account (which depend on the interest rate r_d on deposits in digital currency). A higher bank transfer fee reduces the consumer surplus from payments in digital currency and from the digital currency account. Therefore, banks' profits from depositors decrease. Because of the first two effects ((i) and (ii)), banks have incentives to decrease the bank transfer fee, which reduces the market share of the digital currency. The magnitude of the first two effects increases with the interest rate on the digital currency account r_d and the fee for the digital currency f_d. Hence, if the value of the digital currency as a storage instrument increases for

profitable for banks than investment in Central Bank reserves.

[44] In my paper, the amount of lending can either increase or decrease following the introduction of the digital currency. In Chiu et al. (2019), the reason for the increase in the aggregate amount of lending is that consumer demand for deposits becomes more elastic to the deposit rate with a digital currency. Banks increase the deposit rate, which attracts a higher demand of depositors, and therefore, increase their supply of lending at the equilibrium. In my paper, the results are comparable. However, I make a distinction between consumer demand for bank deposits before and afeter the consumption shock. The demand for bank deposits after the consumption shock depends on the fees for payment transactions and the interest rates on bank and digital currency accounts, respectively.

consumers (i.e., increases r_d), banks compete more intensively with the digital currency provider for payments.

A third effect is caused by the impact of the consumer's decision to pay by bank transfer on the expected net benefit of liquidity management (which impacts bank's liquidity with probability φ). A higher bank transfer fee has two opposite effects on banks' marginal benefit of liquidity. If $\rho > \tau$, as consumers transfer funds less frequently from their bank account to pay, the marginal benefit of liquidity is reduced. The intensity of this effect increases with the amount of reserves R^* held by banks in equilibrium. As the amount of reserves becomes lower when the digital currency crowds out a higher share of deposits, this effect tends to be smaller when the use of the digital currency as a means to store value increases. At the same time, when the bank transfer fee increases, consumers transfer a larger amount from their bank account in average when they pay, which raises the marginal benefit of liquidity. The magnitude of this effect is all the more important since the expected cost of liquidity $\varphi\rho$ is high and the number of banks is small. Indeed, the marginal benefit of liquidity is higher when banks attract a higher share of deposits. Banks have incentives to lower the bank transfer fee if the first effect is dominant (resp., to raise the bank transfer fee if the second effect is dominant). Therefore, banks have stronger incentives to compete with the digital currency as a means of payment when the use of the digital currency as a store of value increases.

The design of the digital currency impacts banks' trade-off between extracting surplus from depositors and reducing their cost of liquidity. If the fee for the digital currency is low enough, the third effect may become dominant and positive. Banks have incentives to set a high bank transfer fee such that consumers pay for all their transactions by digital currency when they have enough money in their digital currency account. If the fee for the digital currency is high enough, the first two effects are dominant. Banks set a low bank transfer fee such that consumers pay as much as possible from their bank account, given the constraint on reserves. For intermediary values of the fee for the digital currency, banks set a bank transfer fee such that consumers pay some transactions by digital currency and other by bank transfer.[45]

Remark that the digital currency impacts banks' margin per depositor through the marginal cost of liquidity. If depositors leave a lower share of their wealth in their bank account, the marginal cost of liquidity increases, and so does the bank's margin per depositor. This implies that the presence of the digital currency may also soften banks' competition for deposits.[46]

[45] The model does not say whether banks prefer to hold a high or a low amount of reserves in a symmetric equilibrium. The choice between case i) and case ii) in Proposition 2 could be driven by regulatory requirements.

[46] A higher fee for the digital currency has two effects on banks' margin per depositor through the marginal cost of liquidity. On the one hand, if consumers pay more often by card, banks' marginal cost of liquidity increases, which raises banks' margin per depositor. On the other hand, the marginal transfer to pay by card becomes lower

4.3.3 The adoption of the digital currency as a means to store of value

If consumers do not adopt the digital currency for payments, they trade off between paying by bank transfer and paying by cash. In that case, the first two effects that we identified in subsection 4.3.3 for the choice of the bank transfer fee are cancelled. Indeed, the fee for bank transfers has no impact on the interest rate that the consumer receives from his digital currency account. The choice of the bank transfer fee reflects only the third effect, that is, its impact on banks' marginal cost of liquidity. In Proposition 2, I give the threshold value of the transaction such that consumers pay by bank transfer if depositors trade off between paying by cash and bank transfer.

Proposition 2 *Suppose that banks hold a low amount of reserves, that never meets consumer demand for payments from their bank account. If depositors trade off between paying by bank transfer and paying by cash, the size of the transaction such that consumers pay from their bank account is given by*

$$s_b^* = \frac{(\rho - \tau)n\varphi R^* - c_b}{d\rho\varphi}.$$

Proof. See Appendix B-2. Since $nR^* = \alpha d - Ln$, the threshold s_b^* increases with α, and so does the market share of the digital currency. This implies that the market share of bank transfers increases when the share of deposits left in a bank account decreases (that is, if α is reduced).
■

The trade-off between paying by cash and by bank transfer is equivalent to the trade-off between paying by the digital currency and by bank transfer if there are no interests on digital currency accounts and if the fee for digital currency transactions is equal to zero (i.e., that is, if $f_d = 0$ and $r_d = 0$, we have $s_{dc}^* = s_b^*$).

Even if the digital currency is not used to pay, its use as a means to store of value impacts the size of the transaction such that consumers pay by bank transfer. When the digital currency crowds out a higher amount of deposits (i.e., when α is reduced), banks hold a lower amount of reserves in equilibrium, which reduces their marginal benefit of liquidity. Hence, banks choose a bank transfer fee such that consumers pay more often from their bank account. Note that if $\alpha = 1$, we obtain for s_b^* the benchmark case in which the digital currency is not available (either as a store of value or as a means of payment).

when consumers pay more often by bank transfer, which reduces banks' marginal cost of liquidity.

4.3.4 The adoption of the digital currency with a high amount of reserves

If banks hold enough reserves to cover some but not all transfers of deposits, the size of the transaction such that consumers pay from their bank account does not impact banks' management of reserves.[47] Therefore, unlike in Proposition 1, the fee for digital currency transactions has no impact on the interest rate on loans.

As in Proposition 1, when they choose the bank transfer fee, banks trade off the benefits from lower costs of liquidity and the losses due to the reduction in consumer surplus of opening a digital currency account. However, the third effect that we identified is always positive for the use of the digital currency. Indeed, banks always have incentives to increase the bank transfer fee. In this case, they have no incentives to decrease the bank transfer fee to reduce their marginal cost of liquidity, because the substitution between payment instruments has no impact on banks' management of reserves.

If the fee for the digital currency is high or if the interest rate on the digital currency account is high enough, the first two effects are dominant. Banks set a low bank transfer fee such that consumers do not use digital currency to pay. This situation is all the more likely to happen since the interest rate on digital currency accounts is high with respect to the expected return offered by the Central Bank on bank deposits (i.e., if r_d is high with respect to τ). If the fee for digital currency transactions is low enough, the third effect may become dominant and banks may choose a bank transfer fee such that consumers always pay by digital currency when they have enough money in their digital currency account. For intermediary values of the fee for the digital currency, consumers trade off between paying by bank transfer, by cash and by digital currency.

4.3.5 The equilibrium

I characterize the equilibrium of the game if the fee for digital currency transactions is regulated at marginal cost, and if the interest rate on DC accounts is set to zero.

Proposition 3 *Suppose that there are no interest rates on DC account and that the marginal cost of bank payments is equal to zero. If banks hold a low amount of reserves, there is an equilibrium in which consumers trade off between paying by cash, by DC and by bank transfer if c_d belongs to $(\tau\varphi nR^*, (\tau-\rho)n\varphi R^* + d\rho(1-\alpha))$ and $c_d \leq v_d nR^*$. If c_d belongs to $(0, \tau\varphi nR^*)$, the digital currency crowds out the use of bank deposits for payments.*

[47] Technically, the expected benefit of liquidity management is separable in $\widehat{s_{dci}}$ and L_i. See the Appendix B-2 for details.

If banks hold a high amount of reserves, there is an equilibrium in which consumers trade off between paying by cash, by DC and by bank transfer if c_d *belongs to* $(0, \tau\varphi n R^*)$.

Proof. From Proposition 1 and Appendix B-2. ■

One policy implication of this result is that, even if there are no interest rates on digital currency accounts, digital currencies may be adopted by consumers to pay, given the assumption that consumers obtain variable benefits of paying with the digital currency instead of cash.

Depending on the value of the parameters, there are also equilibria in which the digital currency is not adopted by consumers for payments. For example, if the interest rate on digital accounts is higher than the IOR, if the marginal cost of bank transfers is equal to zero and if banks hold a high amount of reserves, the digital currency is not used to pay. In that case, the digital currency provider has no incentive to enter the market if it does not make any revenue from other activities because, otherwise, it makes a negative profit. This result is consistent with a statement of the ECB report (2020) which acknowledges that it might have to subsidise the services offered by the digital currency provider in order to ensure that consumers do not have to bear any costs.[48]

If the fee for digital currency transactions is not regulated, the DC provider chooses the fee for digital currency transactions that maximizes its profit. There may be various situations according to its business model. The DC provider could decide to subsidize digital currency payments, which would serve as loss-leaders to attract consumers for other services.[49] The DC provider could also decide to charge merchants for the acceptance of the CBDC, while subsidizing consumer adoption, as in other two-sided markets (see Verdier, 2011, for a survey).

4.3.6 The digital currency and the use of payment instruments

I analyze the determinants of the use of the digital currency and how the presence of the digital currency impacts the use of transfers of deposits for payments.

The market share of the digital currency Given banks' optimal choice of the bank transfer fee (in Proposition 1), the market share of the digital currency as a payment instrument is given by

$$\beta_d^i = H(s_{dc}^*) - H(\widehat{s_d}), \tag{18}$$

where s_{dc}^* is given in Proposition 1 and $\widehat{s_d}$ is given in Lemma 1.

[48] The ECB makes an analogy with the distribution of banknotes. In pratice, some consumers pay fees to their banks to withdraw banknotes. Therefore, the use of cash is not completely free for consumers.

[49] The Bank of England (2020) mentions the role of Payment Interface Providers, that is, private sector firms that would manage all the interactions with users of CBDC.

The market share of the digital currency depends on its design, namely, its interest rate r_d and its transaction fee f_d, on the structure of the deposit market (through the number of banks) and the costs of liquidity for banks. Even if there are positive transaction fees for the digital currency, consumers may decide to adopt it for payments because it is not a perfect substitute for cash transactions. The adoption of the digital currency for payments depends on the magnitude of the interest rate on digital currency accounts r_d with respect to expected marginal cost of liquidity for banks $\rho\varphi$ and the IOR τ. This results echoes the analysis of the report written by the European Central Bank (2020). Indeed, to issue a digital euro, the Central Bank may need to offer long-term lending to banks that lose deposits (e.g., via Long Term Refinancing Operations, LTROs). As a consequence, the differential between the remuneration of the digital euro and the interest rate applied to LTROs would be critical to determining the profitability and the use of the digital euro (i.e., $\rho\varphi - r_d$). My framework complements this view by showing that it is crucial to apply coherent transaction fees given the difference between $\rho\varphi$ and r_d, but also given the difference between r_d and τ.

In Corollary 2, I discuss how the market conditions (IOR, number of banks, cost of liquidity for banks) impact the share of payments by digital currency.

Corollary 2 *If banks hold a low amount of reserves, if $r_d < \rho\varphi$, the market share of the digital currency decreases with the number of banks n, the IOR τ, with the interest rate on digital currency account r_d, the probability that deposits are transferred to another bank φ, and the share of deposits left in the bank account α. It varies non-monotonically with the cost of liquidity ρ.*

Proof. From Proposition 1, we have $nR^* = \alpha d - nL$. Therefore, the threshold s^* decreases with n, and so does the market share of the digital currency. ■

When the number of banks increases, each bank captures a lower share of deposits. Since each bank lends a constant amount (i.e., L), the probability that a bank needs to borrow from the Central Bank when a consumer makes a transaction of high value decreases with the number of banks. This reduces the impact of a decrease in the bank transfer fee on banks' marginal cost of liquidity (if $\rho > \tau$). Hence, all else being equal, banks have more incentives to reduce the bank transfer fee when the number of banks increases, and therefore, the market share of the digital currency declines.

A higher cost of liquidity for banks has an ambiguous impact on the market share of the digital currency. On the one hand, a higher cost of liquidity raises the marginal benefit of holding reserves, which provides banks with incentives to increase the bank transfer fee, such that consumers use their digital currency account more often for payments. On the other hand, banks have also incentives to reduce the bank transfer fee, in order to reduce their average liquidity needs, which lowers the market share of the digital currency.

The impact of the digital currency on the use of bank deposits for payments The digital currency impacts the use of bank deposits for payments. In Corollary 3, I analyze whether the presence of the digital currency reduces the use of bank deposits for payments.

Corollary 3 *If* $0 < r_d \leq \varphi\rho$, *the digital currency reduces the use of bank deposits for payments if* $c_b r_d \leq \varphi(c_d\rho + (\rho - \tau)(r_d(d - Ln) - (1 - \alpha)\rho\varphi d)$.
If $r_d < 0$, *the digital currency reduces the use of bank deposits for payments if* $c_b r_d > \varphi(c_d\rho + (\rho - \tau)(r_d(d - Ln) - (1 - \alpha)\rho\varphi d)$.

Proof. We have

$$s_{dc}^* - s_b^*|_{\alpha=1} = \frac{-c_b r_d + \varphi(c_d\rho + (\rho - \tau)(r_d(d - Ln) - (1 - \alpha)\rho\varphi d)}{d\rho\varphi(-r_d + \varphi\rho)}.$$

■

The digital currency may either increase or decrease the use of bank deposits for payments. Without a digital currency, the use of bank deposits for payments depends on competition between bank transfers and cash. Since the use of cash is costless, the size of the transaction such that consumers pay from their bank account depends on the bank transfer fee, the cost of liquidity and the benefits from reserves. With a digital currency, bank transfers (as a payment instrument) compete with another payment instrument (the digital currency) that may also pay an interest as a store of value. This implies that competition between payment instruments may not reduce the fee for bank transfers.

As explained before, the choice of the bank transfer fee responds to different effects. With a higher fee fee for digital currency transactions, consumers incur higher transaction costs of paying from their digital currency account. Thus, their surplus from deposits in digital currency decreases. Because banks extract lower rents from depositors, they have incentives to react by increasing the bank transfer fee. I label this first effect as a transaction cost effect.

However, as the fee for digital currency payments increases, the size of the transaction such that consumers pay by from their bank account falls. This implies that consumers pay more often by bank transfer, and less by digital currency. I label this second effect as a payment substitution effect. Because of the payment substitution effect, the depositors' surplus of storing money in a digital currency account increases. Hence, banks are able to extract higher rents from depositors, which gives them incentives to reduce the bank transfer fee. Moreover, banks borrow lower amounts in average from the Central Bank, which reduces their marginal benefit of liquidity. This last effect provides banks with incentives to increase the bank transfer fee. The resultant of these two effects depends on the sign of $r_d - \rho\varphi$. If $r_d - \rho\varphi < 0$, banks have incentives to increase the bank transfer fee when s^* increases, whereas the reverse is true otherwise.

Therefore, if $r_d - \rho\varphi < 0$, banks prefer unambiguously to increase the bank transfer fee when the fee for digital currency payments increases - and to reduce it when it falls. If $r_d - \rho\varphi > 0$, banks' reaction to an increase in the fee for digital currency payments depends on the elasticity of substitution between bank transfers and digital currency payments.[50] If $(r_d - \rho\varphi)/(\Delta v - \Delta r) > 1$, the transaction cost effect outweights the payment substitution effect. Therefore, banks increase the bank transfer fee when the fee for digital currency payments increases, whereas the reverse is true otherwise.

5 Extensions and discussion

In this section, I discuss several assumptions of the model and their implications for the design of the digital currency.

5.1 Competition between currencies as storage instruments

So far, I have assumed that depositors keep an exogenous share of their wealth in their bank account and in their digital currency account, respectively. If α_i is endogenous, depositors choose how much to keep in their bank and in their digital currency account so as to maximize their expected surplus. I assume that depositors are not allowed to deposit more that $\overline{\alpha_d} < 1/2$ in their digital currency account. Indeed, several Central Banks are considering setting up limits on the amount that consumers can hold in their digital currency account.[51] I denote by α_i^* the share of wealth that is left by depositors in their bank account if this decision is endogenous.

If α_i^* depends on f_b^i, the results of Proposition 1 is modified by an additional effect. Banks take into account how the consumers' decisions to leave some funds in their bank account rather than in their digital currency account impact their profits. A higher bank transfer fee reduces the share of wealth that consumers leave in their bank account (see Appendix C). This has two effects on banks' profits. On the one hand, banks are able to extract more surplus from consumers when the latter leave a higher share of wealth in their digital currency account. This provides banks with incentives to increase the bank transfer fee, which raises the market share of the digital currency. On the other hand, the cost of liquidity for banks increases. This

[50] From Lemma 1, the sensitivity of s^* to f_d is invertly proportional to $\Delta v - \Delta r$.
[51] See the speech by F.Panetta (ECB), Bruegel online seminar 10th February, 2021.

provides banks with incentives to reduce the bank transfer fee, which lowers the market share of the digital currency. Whether banks choose a bank transfer fee such that consumers use the digital currency to pay depends on how both effects compensate for each other. The first effect increases with the interest rate on the digital currency. Therefore, a higher interest rate on the digital currency tends to increase the market share of the digital currency, and there may be an equilibrium in which consumers pay both by bank transfer and by digital currency. In that case, both functions of the digital currency (i.e., storage of value and payment instrument) complement each other.

5.2 The distribution mode of the digital currency

In my model, I have assumed that the digital currency is not distributed by banks. An alternative would be that banks distribute themselves the digital currency in dedicated accounts. One possible organization is to allow banks to hold reserves in two separate accounts, one for the digital currency (reserves of type d) and one for standard bank accounts (reserves of type b). I denote the amount of reserves held for standard bank accounts by R_i^b and the amount of reserves held for digital currency accounts by R_i^d. Digital currency accounts are still backed by a ratio of 100% of reserves and the fee for digital currency transactions is still regulated at marginal cost.

I analyze how the distribution mode for the digital currency impacts the market share of the digital currency. I denote by R_b^* the amount of reserves of type b held by banks in equilibrium. In Proposition 3, I give the size of the transaction s_{bd}^* such that consumers pay from their bank account if banks distribute the digital currency themselves and if they hold a low amount of reserves.

Proposition 4 *If $\varphi\rho > \tau_d$, if banks distribute the digital currency themselves, the size of the transaction such that consumer prefer to pay from their bank account is given by*

$$s_{bd}^* = \frac{n\varphi(\rho - \tau_b)R_b^* + (c_d - c_b)}{d(\varphi\rho - \tau_d)}.$$

Proof. See Appendix D. ■

Compared to Proposition 1, banks are able to internalize the impact of the fee for the digital currency and the interest rates on digital currency accounts on their profit. Therefore, the effects of r_d and f_d on the choice of the bank transfer fee are cancelled. However, banks take into account the impact of the consumer's choice of a payment method on their marginal benefit of liquidity, which differs for reserves of type b and reserves of type d.

We analyze whether the adoption of the digital currency is higher if it is distributed by banks rather than a digital currency provider. The answer to this question depends on the pass-through of IOR-DC to digital currency depositors. If the pass-through is perfect, that is, if $\tau_d = r_d$, the adoption of the digital currency is exactly identical if banks distribute the digital currency themselves (see Proposition 1). However, if $\tau_d > r_d$, banks make profits on reserves in digital currency. Hence, when banks distribute themselves the digital currency, this reduces its adoption for payments, compared to a situation in which the digital currency is priced at marginal cost and distributed by a non-bank provider.

So far, also, I have not discussed how the revenues from seignorage obtained by the Central Bank could be impacted by the distribution mode of the CBDC, which is an issue that would deserve more investigation in the future.

5.3 The access to Central Bank Reserves

I have so far assumed that the digital currency provider is allowed to deposit reserves in a Central Bank account. However, the access to Central Bank accounts may be restricted to banks. In that case, the digital currency provider may prefer to open an account in a bank to store the money of its depositors, if banks are allowed to be the custodians of funds in digital currency. Banks would compete à la Bertrand to attract the deposits of the digital currency provider, and store them in a dedicated Central Bank account. Each bank should be able to offer to the digital currency provider a deposit rate equal to its marginal benefit of storing funds in digital currency, that is, the IOR-DC, and would not make any profit on deposits in digital currency. At the equilibrium, the digital currency provider splits his deposits between all banks, such that each bank holds a quantity $(1-\alpha)d/n$ of deposits in digital currency. Therefore, the situation would be equivalent to the discussion of Section 5.2, with $\tau_d = 0$.

5.4 The acceptance of the digital currency

In the model, I have also assumed that both the digital currency and the bank payment instrument are universally accepted for payments. However, it may be the case that some merchants refuse the digital currency. I denote by γ_d the probability that the digital currency is accepted at the transaction stage. If banks hold a low amount of reserves, if there is an interior solution, the threshold value of the transaction such that consumers pay from their bank account is given by

$$s^*_{dc} = \frac{\gamma_d f_d + (\rho - \tau) n\varphi R^* - c_b}{d(\rho\varphi - \gamma_d r_d)}.$$

If γ_d is close to zero, consumers trade off between cash and bank transfers and if γ_d is close to one, the threshold value of the transaction such that consumers pay from their bank account is identical to Proposition 1. The market share of the digital currency increases with its acceptance by merchants. Moreover, a higher acceptance of the digital currency by merchants (i.e., a higher γ_d) increases the marginal impact of the interest rate on digital currency accounts on consumers' trade-off between paying by bank transfer and by digital currency.

5.5 Cash holding costs and safety considerations

To simplify the model, I have considered that there are neither costs nor benefits of holding and using cash. In reality, the amount d that consumers deposit in their bank and digital currency accounts is increasing with the opportunity cost of holding cash (with respect to the cost of holding deposits either in a bank account or in a digital currency account). In my paper, higher costs of holding cash would reduce the size of the transaction such that consumers pay by digital currency instead of paying by cash. Costs and benefits of holding cash for consumers include safety and privacy considerations. Cash detention is often considered as risky because of possible theft or losses, while respecting consumer privacy.

In my model, I have supposed that bank accounts and digital currency accounts offer different benefits with respect to cash. The source of differentiation between the digital currency account and the bank account is linked to: i) the different interest rates borne by each account, ii) the surplus that consumers expect from making transactions. However, there is no intrinsic value for consumers of depositing money in the digital currency account. Consumers could value the safety of digital currency accounts in crisis times, as in Fernandez-Villaverde et al. (2020), or value differently the respect of their privacy in digital currency accounts and in bank accounts as in Agur et al. (2020). This would add another dimension of differentiation in competition between banks and digital currency providers.

For example, in my model, it would be possible to add a cost $C(\alpha_i)$ of depositing an amount α_i in a bank account (instead of a CBDC account), motivated by the agency cost of monitoring the bank's behavior. Such a cost would not be incurred for deposits in a Central Bank account, deemed as safer and guaranteed in crisis times. In that situation, consumers would choose to deposit an amount α_i^* in their bank account reflecting a trade-off between the marginal benefits of higher interest rates paid by the bank account if $\Delta r \geq 0$ and the marginal costs of maintaining a positive balance in a bank account. Thus, the consumers' decision to maintain a positive balance in a digital currency account would not be related to their choice of a payment instrument and therefore, the results of Proposition 1 would remain identical.

5.6 Complete bypass of banks

So far, I have assumed that all consumers prefer to open both a bank account and a digital currency account. Since all consumers open a digital currency account, the presence of the digital currency account impacts competition between banks who are located near each other. Banks take into account the expected surplus that consumers obtain from their digital currency account in their pricing decisions.

The results of the model would change if some consumers obtain a low utility u_b of opening a bank account, such that they prefer to open a digital currency account only. In that case, banks compete against the digital currency provider for deposits, which softens competition between banks. Consumers who are located close to bank k prefer to open both a bank account and a digital currency account. Consumers who are located further away from bank k - but not close enough to bank i - open only a digital currency account. The indifferent consumer between opening both deposit accounts and only a digital currency account obtains a zero surplus of opening a bank account given his location. I denote this consumer by x_{kd}. We have

$$x_{kd} = \frac{u_b + IR_k^b - TC_k^b - F_k}{t_b},$$

where $TC_k^b = f_b^k \beta_b^k$ denotes the costs of making transactions from a bank account.

Bank k's market share is equal to $2x_{kd}$ and it neither depends on the interest rates from digital currency accounts, nor on the transaction costs in digital currency. In that case, the size of the transaction such that consumers pay from their bank account is given by

$$s_{dc}^* = \frac{n\varphi(\rho - \tau)R^* - nx_{kd}^* c_b}{\varphi\rho},$$

where x_{kd}^* denotes the indifferent consumer at the equilibrium. Then, the fee for digital currency payments does not impact consumers' decision to pay from their bank account. However, the presence of the digital currency still impacts bank reserves, and therefore, banks' marginal cost of liquidity, which is passed through to consumers into the bank transfer fee.[52]

5.7 A discussion on marginal cost pricing of digital currency transactions

Along with price stability and financial stability, depending of its mandate, the Central Bank may pursue other objectives, such as fostering an efficient use of payment instruments. Several

[52] Another possibility is that consumers trade off between opening both accounts and only a bank account. In that case, the digital currency is not fully adopted by consumers as a storage instrument. Such an assumption can be discussed by extending our baseline model with heterogeneous consumers, who differ across their fixed utility of opening a digital currency account u_d.

papers argue that the Central Bank could regulate the fee for digital currency transactions at marginal cost or set a zero transaction fee for payments in digital currency. However, this may not maximize the surplus of depositors.

Indeed, the fee that maximizes the depositors' surplus (ex ante) is such that the marginal benefit of opening a digital currency account is equal to the marginal cost of opening a bank account, that is, we have

$$nL(\tau-\rho)h(\widehat{s_{dc}})\frac{d\widehat{s_{dc}}}{df_d}+f_d\widehat{s}_dh(\widehat{s}_d)\frac{d\widehat{s_{dc}}}{df_d}-\beta_d^i=0. \tag{19}$$

When the fee for digital currency payments increases, this has two effects on the depositor surplus: a payments substitution effect and a transaction cost effect. The depositor substitutes payments by bank transfer *and* by cash for payments by digital currency and incurs higher transaction costs of paying by digital currency.

Banks internalize partly the substitution effect for payments by bank transfers when they choose the bank transfer fee.[53] However, their choice of a threshold value for payments by bank transfer includes the marginal cost of liquidity both on the deposit and the lending market. Therefore, the first term of Eq. (19) corresponds to the marginal impact of a higher f_d on the lending market. The second term of Eq. (19) corresponds to the marginal impact of the substitution between payments by cash and by digital currency on the depositor's surplus (that is not internalized by banks at the next stage). The last term of Eq. (19) corresponds to the transaction cost effect.

Either zero pricing or marginal cost pricing of digital currency transactions may be too high or too low to maximize the surplus of depositors who open both a bank account and a digital currency account.[54] However, the objective of the Central Bank should be carefully discussed in a more complete framework that would also include the surplus of lenders, given that the fee for digital currency transactions might impact the interest rate on loans.

6 Conclusion

Whether consumers will use a digital currency to pay depends crucially on banking regulation (i.e., liquidity requirements), on the possibility for non-banks to obtain revenues from Central Bank accounts and on the degree of competition in the deposit market. My paper identifies

[53] Indeed, the payment card fee such that the marginal cost of card payments (including the marginal cost of liquidity) is equal to the marginal surplus that the bank extracts from depositors thanks to their digital currency account. The marginal cost of liquidity includes the impact of the substitution between payment methods both on the deposit and on the lending market.

[54] Depending on the sign of the left-hand side of Eq. (19) evaluated at $f_d = c_d$.

the conditions such that consumers will use digital currencies to pay in a market where banks compete for loans and deposits, while incurring costs of managing liquidity. I also discuss how the distribution mode of the digital currency may impact its market share.

More research will be needed to understand the welfare effects of digital currencies. From a theoretical perspective, it would be valuable to construct a framework that would take into account not only their impact on price stability, financial stability and efficient risk-sharing in crisis times (as in Fernandez-Villaverde, Uhlig and Schilling, 2020), but also efficient use of payment instruments when there is no specific stress on liquidity. From an empirical perspective, it is necessary to measure the elasticity of substitution between payment instruments, which can be achieved by understanding the determinants of consumer and merchant adoption of digital currencies.

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Appendix A: summary of the variables used in the model

The table below summarizes the various parameters used in the model:

	Exogenous variables:	Endogenous variables:
Consumers	v_b variable benefit of bank transfer	$(1-\alpha_i)d$ wealth in CBDC account
	v_d variable benefit for DC	$\alpha_i d$ wealth in bank account
	r_b interest rate on deposits	S_i expected surplus of deposits
	r_d interest rate on DC	
	s share of deposit used of payments	
	f_d payment fee for DC	
	d amount of wealth	
Banks/DC	ρ cost of liq. φ probability of transfer	f_b^i bank transfer fee
	τ IOR and τ_d IOR-DC	f_d payment fee for DC
	c_b marginal cost of bank transfer	F_i deposit fee
	γ degree of substituability for loans	r_L^i interest rate on loans
	c_d marginal cost of DC payments	d_i market share on the circle

Appendix B:

B-0: Proof of Lemma 1

- **Trade-off between cash, digital currency and bank transfer:**

Between date 1 and date 2, each consumer of bank i choose the threshold value of the consumption shock s_{dc} such that he prefers to pay by bank transfer rather than by digital currency, the threshold value of the consumption shock s_d such that he prefers to pay by digital currency rather than by cash and the threshold value of the consumption shock s_b such that he prefers to pay by bank transfer rather than by cash. I focus on a scenario in which there the digital currency is used for low value retail payments (i.e., $\widehat{s_{dc}} \leq 1-\alpha_i \leq \alpha_i$). If the consumer trades off between paying from his bank account and his digital currency account, he obtains the same surplus by paying by bank transfer and by digital currency if and only if

$$dv_b\widehat{s_{dc}} - f_b^i + (\alpha_i - \widehat{s_{dc}})dr_b + (1-\alpha_i)dr_d = dv_d\widehat{s_{dc}} - f_d + \alpha_i dr_b + (1-\alpha_i-\widehat{s_{dc}})dr_d.$$

Therefore, we have

$$\widehat{s_{dc}} = \frac{f_b^i - f_d}{d(\Delta v - \Delta r)}.$$

If the consumer trades off between the digital currency and cash, he obtains the same surplus of paying by cash and by digital currency if and only if

$$dv_d\widehat{s}_d - f_d + \alpha_i dr_b + (1-\alpha_i-\widehat{s}_d)dr_d = \alpha_i dr_b + (1-\alpha_i)dr_d.$$

Therefore, we have

$$\widehat{s_d} = \frac{f_d}{d(v_d - r_d)}.$$

If the consumer trades off between paying by bank transfer and cash, he obtains the same surplus of paying by bank transfer and by cash if and only if

$$dv_b\widehat{s_b} - f_b^i + (\alpha_i - \widehat{s_b})dr_b + (1-\alpha_i)dr_d = dv_c\widehat{s_b} + \alpha_i dr_b + (1-\alpha_i)dr_d.$$

Therefore, we have

$$\widehat{s_b} = f_b^i/(d(v_b - r_b)).$$

- **The choice of a payment instrument and the expected interest rate revenues from the bank account and the digital currency account:**

a) If $f_b^i \geq f_d(v_b - r_b)/(v_d - r_d)$, we have $\widehat{s_d} \leq \widehat{s_{dc}}$. The consumer pays by bank transfer if $s \geq \widehat{s_{dc}}$, by digital currency if s belongs to $(\widehat{s_d}, \widehat{s_{dc}})$ and by cash if $s \leq \widehat{s_d}$. As $\widehat{s_{dc}} \leq 1 - \alpha_i \leq \alpha_i$, since the consumer trades off between paying by cash, with the digital currency and by bank transfer, the interest rate revenues from the bank account are given by

$$IR_b^i(\widehat{s_{dc}}, \alpha_i) = dr_b(\alpha_i H(\alpha_i) - \int_{\widehat{s_{dc}}}^{\alpha_i} sh(s)ds), \tag{20}$$

and the interest rate revenues from the digital currency account are given by

$$IR_d^i(\widehat{s_{dc}}, \alpha_i) = dr_d(1 - \alpha_i H(\alpha_i) - \int_{s_d}^{\widehat{s_{dc}}} sh(s)ds - \int_{\alpha_i}^{1} sh(s)ds). \tag{21}$$

b) If $f_b^i < f_d(v_b - r_b)/(v_d - r_d)$, the consumer pays by bank transfer if $s \geq \widehat{s_b}$ and by cash if $s < \widehat{s_b}$. The consumer obtains the interest rate revenues from his bank account given by

$$IR_b^i(\widehat{s_b}, \alpha_i) = dr_b(\alpha_i H(\alpha_i) - \int_{\widehat{s_b}}^{\alpha_i} sh(s)ds),$$

and the interest rate revenues from his digital currency account given by

$$IR_d^i(\widehat{s_b}, \alpha_i) = dr_d(1 - \alpha_i H(\alpha_i) - \int_{\alpha_i}^{1} sh(s)ds).$$

B-1: Expected net benefit of liquidity management The expected net benefit of liquidity management depends on the amount of reserves held by bank i to meet the demand of depositors for payment transactions. If bank i does not have enough reserves to meet the demand for payments, it needs to borrow additional liquidity. If bank i has an excess of reserves, it receives the IOR. I denote by $\underline{a_i}$ the amount of funds held by bank i in a Central Bank account after the shock if the amount of payments made by depositors is lower than the amount of reserves of bank i before the shock. I denote by $\overline{a_i}$ the liquidity needs of bank i if the amount of payments made by its depositors are higher than the amount of reserves of bank i before the shock.

Case 1 - no reserves for payments by bank transfer: $R_i \in (0, \widehat{s_{dc}}dd_i)$. Bank i has never enough reserves to meet the demand of depositors when they need to pay by bank transfer (that is, if $s_i \geq \widehat{s_{dc}}$). With probability φ, bank i initiates a transfer to the other banks. If $s_i \in (0, \widehat{s_{dc}})$, there is no transfer of deposits. Bank i lends R_i to the Central Bank at a rate τ, because consumers pay by digital currency. If $s_i \in (\widehat{s_{dc}}, \alpha_i)$, the consumer transfers funds from his bank account. Bank i needs to borrow $s_i dd_i - R_i$ at a rate ρ. If $s_i \in (\alpha_i, 1)$, consumers transfer all their funds from their bank account and need to borrow $\alpha_i dd_i - R_i$. With probability $1-\varphi$, bank i does not initiate any transfer to the other banks and lends R_i to the Central Bank until date 2.

Therefore, the expected net benefit of liquidity management is given by

$$EL_i = \tau \underline{a_i} + \rho \overline{a_i}, \tag{22}$$

where

$$\underline{a_i} = (\varphi H(\widehat{s_{dc}}) + (1-\varphi))R_i,$$

and

$$\overline{a_i} = \int_{\widehat{s_{dc}}}^{\alpha_i} (s_i dd_i - R_i)h(s_i)ds_i + (1 - H(\alpha_i))(\alpha_i dd_i - R_i).$$

Case 2 - reserves to cover some but not all payments by bank transfer: $R_i \in (\widehat{s_{dc}}dd_i, \alpha_i dd_i)$ With probability φ, bank i initiates a transfer to the other banks.

i) If s belongs to $(0, \widehat{s_{dc}})$, bank i does not use any reserves from its Central Bank account because consumers pay by digital currency.

ii) If s_i belongs to $(\widehat{s_{dc}}, R_i/(dd_i))$, bank i has enough reserves to cover the demand of depositors given by $s_i dd_i$.

iii) If s_i belongs to $(R_i/(dd_i), \alpha_i)$, bank i does not have enough reserves to cover the demand of depositors and needs to borrow $s_i dd_i - R_i$.

iv) If s_i belongs to $(\alpha_i, 1)$, a consumer of bank i transfers all his wealth $\alpha_i d$ from his bank account and transfers funds from his digital currency account to his bank account to pay by bank transfer. Bank i needs to borrow all the funds lent to borrowers. Since bank i has a share

d_i of deposits, it borrows $L_i = \alpha_i d d_i - R_i$.

With probability $1-\varphi$, bank i does not initiate any transfer to the other banks and lends R_i to the Central Bank until date 2.

From i), ii), iii) and iv), the expected net benefit of liquidity management is given by

$$EL_i = \tau \underline{a_i} + \rho \overline{a_i}, \tag{23}$$

where

$$\underline{a_i} = \varphi H(\widehat{s_{dc}}) + \varphi \int_{\widehat{s_{dc}}}^{R_i/(dd_i)} (R_i - s_i dd_i) h(s_i) ds_i + (1-\varphi)) R_i,$$

and

$$\overline{a_i} = \int_{R_i/(dd_i)}^{\alpha_i} (s_i dd_i - R_i) h(s_i) ds_i + (1 - H(\alpha_i))(\alpha_i dd_i - R_i).$$

B-2: Proof of Propositions 1 and 2 The expected net benefit of liquidity management EL_i is defined by parts according to the amount of reserves R_i held by bank i. Therefore, we maximize EL_i on each segment given in Appendix B-1.

- **Trade-off between cash, the digital currency and bank transfer**

Suppose that consumers trade off between paying by cash, by bank transfer and by digital currency, that is, from Lemma 1, that $f_b^i \geq \underline{f_b}(f_d)$. Using Lemma 1, the choice of the fee for bank transfers f_b^i amounts to choosing a threshold $\widehat{s_{dc}} \in (\widehat{s_d}, 1-\alpha_i)$ above which consumers prefer paying by bank transfer rather than digital currency. If there is an interior solution, using Leibniz's rule and Eq. (11), the first-order conditions with respect to the choice variables F_i, $\widehat{s_{dc}}$ and r_L^i are given by

$$\frac{\partial \pi_i}{\partial F_i} = \frac{\partial \pi_i}{\partial d_i}\frac{\partial d_i}{\partial F_i} + \frac{\partial \mu_i}{\partial F_i} d_i = 0, \tag{FOC-1}$$

$$\frac{\partial \pi_i}{\partial \widehat{s_{dc}}} = \frac{\partial \pi_i}{\partial d_i}\frac{\partial d_i}{\partial \widehat{s_{dc}}} + d_i \frac{\partial \mu_i}{\partial \widehat{s_{dc}}} + \frac{\partial EL_i}{\partial \widehat{s_{dc}}} = 0, \tag{FOC-2}$$

and

$$\frac{\partial \pi_i}{\partial r_L^i} = L_i + \frac{\partial \pi_i}{\partial L_i}\frac{\partial L_i}{\partial r_L^i} = 0. \tag{FOC-3}$$

for all $i = 1..n$. In a symmetric equilibrium, banks' best-responses to the fee f_d chosen by the digital currency provider are identical. The vector of profit-maximizing prices is denoted by $P^* = (F^*, f_b^*, r_L^*)$. At P^*, we have $d_i^* = 1/n$ and $L_i^* = L$ for all $i = 1..n$. If $\alpha d/n - L \geq 0$, we denote by $R^* = \alpha d/n - L$ the amount of reserves held by each bank i in a symmetric equilibrium for all $i = 1..n$.

The second-order conditions hold if the Hessian matrix is semi-definite negative at $P^* = (F^*, f_b^*, r_L^*)$. We show that this is the case if the transportation cost is sufficiently high in a supplementary material that is available upon author's request.

Replacing for $\partial d_i/\partial F_i = -1/t_b$ given by Eq. (15) and $\partial \mu_i/\partial F_i = 1$ given by Eq. (7) into (FOC-1) gives

$$\partial \pi_i/\partial d_i|_{P*} = t_b/n. \tag{FOC-1-B}$$

From Eq. (11), since $\partial \pi_i/\partial d_i = \mu_i + \partial EL_i/\partial d_i$, Eq. (FOC 1-B) implies that in a symmetric equilibrium, we have

$$\mu_i = \frac{t_b}{n} - \left.\frac{\partial EL_i}{\partial d_i}\right|_{P^*}. \tag{C1}$$

Replacing at P^* for $d_i^* = 1/n$ and (FOC-1-B) into (FOC-2), we obtain that

$$\frac{\partial d_i}{\partial \widehat{s_{dc}}}\frac{t_b}{n} + \frac{\partial \mu_i}{\partial \widehat{s_{dc}}}\frac{1}{n} + \frac{\partial EL_i}{\partial \widehat{s_{dc}}} = 0. \tag{FOC-2-B}$$

From Eq. (15), we have $\partial d_i/\partial \widehat{s_{dc}} = (\partial S_i/\partial \widehat{s_{dc}})(1/t_b)$. Replacing for

$$\frac{\partial S_i}{\partial \widehat{s_{dc}}} = \frac{\partial IR_b^i}{\partial \widehat{s_{dc}}} + \frac{\partial IR_d^i}{\partial \widehat{s_{dc}}} - (\beta_b^i \frac{\partial f_b^i}{\partial \widehat{s_{dc}}} + \frac{\partial \beta_b^i}{\partial \widehat{s_{dc}}}(f_c^i - f_d))$$

given by Eq. (5) and Eq. (6), for

$$\frac{\partial \mu_i}{\partial f_c^i} = \frac{\partial f_b^i}{\partial \widehat{s_{dc}}}\beta_b^i + (f_b^i - c_b)\frac{\partial \beta_b^i}{\partial \widehat{s_{dc}}} - \frac{\partial IR_b^i}{\partial \widehat{s_{dc}}}$$

given by Eq. (7) into (FOC-2-B) gives

$$\frac{\partial \pi_i}{\partial s_{dc}} = \frac{1}{n}(\frac{\partial IR_d^i}{\partial \widehat{s_{dc}}} + \frac{d\beta_b^i}{d\widehat{s_{dc}}}(f_d - c_b)) + \frac{\partial EL_i}{\partial \widehat{s_{dc}}}.$$

Since $\beta_b^i = 1 - H(\widehat{s_{dc}})$, we have $d\beta_b^i/d\widehat{s_{dc}} = -h(\widehat{s_{dc}})$. After simplification by $1/n > 0$, this implies that at P^*, we have

$$\frac{\partial IR_d^i}{\partial \widehat{s_{dc}}} + n\frac{\partial EL_i}{\partial \widehat{s_{dc}}} - (f_d - c_b)h(\widehat{s_{dc}}) = 0. \tag{C2}$$

Replacing for $\partial L_i/\partial r_L^i = -\gamma(n-1)/n$ given by Eq. (1) and for $\partial \pi_i/\partial L_i = r_L^i - c_L + \partial EL_i/\partial L_i$ given by Eq. (11) into (FOC-3), we find that

$$\left.\frac{\partial \pi_i}{\partial r_L^i}\right|_{P^*} = L - \gamma\frac{(n-1)}{n}(r_L^i - c_L + \left.\frac{\partial EL_i}{\partial L_i}\right|_{P^*}) = 0. \tag{FOC-3-B}$$

Eq. (FOC-3-B) implies that in a symmetric equilibrium, we have

$$r_L^* = \underline{r_L} - \left.\frac{\partial EL_i}{\partial L_i}\right|_{P^*}, \tag{C3}$$

where

$$\underline{r_L} = c_L + \frac{nL}{\gamma(n-1)}. \tag{24}$$

If there is a symmetric equilibrium, if there is an interior solution, banks' best-responses to the fee f_d are given by (C1), (C2) and (C3).

Replacing for (C1), (C2) and (C3) into Eq. (11), in cases 1 and 2 defined in Appendix B-1, banks make identical profits given by

$$\pi^* = \frac{t_b}{n^2} + \frac{n}{\gamma(n-1)}L^2.$$

The choice of the fee f_d for the digital currency chosen by the digital currency provider impacts the interest rate on loans, the fee for bank transfers and the fixed fee paid by depositors to open an account.

- **Trade-off between cash and bank transfer**

From Lemma 1, if $f_b^i < \underline{f_b}(f_d)$, consumers trade off between paying by cash and by bank transfer, a similar method of analysis applies. The market share of the digital currency is equal to zero and the threshold above which consumers transfer funds from their bank account to pay by bank transfer is defined by $\widehat{s_b}$. Equation (C1) and (C3) still hold (except that $\widehat{s_{dc}}$ is replaced by $\widehat{s_b}$). I denote by s_b^* the size of the transaction such that consumers pay by bank transfer in an equilibrium in which consumers trade off between paying by cash and by bank transfer. Since the interest rates on digital currency accounts do not depend on the fee for bank transfer, equation (C2) is modified as follows

$$n \left.\frac{\partial EL_i}{\partial \widehat{s_b}}\right|_{\widehat{s_b}=s_b^*} + c_b h(s_b^*) = 0. \tag{C2-CC}$$

Case 1 - no reserves for payments by bank transfer: $R_i \in (0, \widehat{s_{dc}} dd_i)$.

- **Trade-off between cash, the digital currency, bank transfer**

Suppose that consumers trade off between paying by bank transfer, by digital currency and by cash (i.e., that $f_b^i \geq \underline{f_b}(f_d)$). For an interior solution to exist, it must be that at f_b^* given by condition (C2) the amount of reserves R^* held by bank i belongs to $(0, s_{dc}^* d/n)$ (or equivalently $s_{dc}^* \geq nR^*/d$ and $\alpha - Ln/d \geq 0$). Since $s_{dc}^* \leq 1-\alpha$, in an interior solution, it must be that $s_{dc}^* \in (nR^*/d, 1-\alpha)$

There are two cases. In case 2-1, $nR^*/d \geq \widehat{s_d}$ and there may be an interior solution such that consumers trade off between paying by cash, by digital currency and by bank transfer.

In case 2-2, $nR^*/d < \widehat{s_d}$. There is no interior solution in which consumers trade off between the three payment instruments.

If $R_i \in (0, \widehat{s_{dc}} dd_i)$, from (22), we have

$$\left.\frac{\partial EL_i}{\partial s_{dc}}\right|_{P^*} = \varphi\left(\tau R^* + \rho(\frac{d}{n}s^* - R^*)\right)h(s^*), \tag{25}$$

$$\left.\frac{\partial EL_i}{\partial L_i}\right|_{P^*} = -\varphi\rho(1 - H(s^*)) - \tau(\varphi H(s^*) + 1 - \varphi), \tag{26}$$

and

$$\left.\frac{\partial EL_i}{\partial d_i}\right|_{P^*} = \varphi(\tau\alpha d H(s^*) - \rho d \int_{s^*}^{\alpha} (s_i - \alpha)h(s_i)ds_i) + (1-\varphi)\alpha d. \tag{27}$$

Replacing for Eq. (27) into Eq. (C1) implies that bank i's margin per depositor is given by

$$\mu_i = \frac{t_b}{n} - \varphi\left(\tau\alpha d H(s^*) - \rho d \int_{s^*}^{\alpha} (s_i - \alpha)h(s_i)ds_i\right) - (1-\varphi)\alpha d.$$

Replacing for Eq. (26) into Eq. (C3) implies that in a symmetric equilibrium, we have

$$r_L^* = \underline{r_L} + \varphi(\rho - (\rho - \tau)H(s^*)) + (1-\varphi)\tau, \tag{28}$$

where $\underline{r_L}$ is given by (24). If there is an interior solution, since $IR_d^i/\partial s_{dc} = -dr_d\widehat{s_{dc}}h(\widehat{s_{dc}})$ if $\widehat{s_{dc}} \leq 1-\alpha$, replacing for Eq. (25) into Eq. (C2), s_{dc}^* (or equivalently f_b^*) is implicitly defined by

$$(n\varphi(\tau - \rho)R^* + d(\varphi\rho - r_d)s_{dc}^* - f_d + c_b)\, h(s_{dc}^*) = 0. \tag{29}$$

We denote by

$$\lambda(f_b^i) \equiv c_b - f_d - n\varphi(\rho - \tau)R^* + d(\varphi\rho - r_d)\widehat{s_{dc}}(f_b^i, f_d).$$

After simplification by $h(s_{dc}^*) > 0$, from Eq. (29), f_b^* is implicitly defined by $\lambda(f_b^*) = 0$. We determine whether there exists a bank transfer fee $f_b^* \in (\widetilde{f}_b(f_d), f_b^m(f_d))$ such that $\lambda_i(f_b^*) = 0$. For this purpose, we analyze the sign of $\lambda'(f_b^i) = d(\varphi\rho - r_d)\partial\widehat{s_{dc}}/\partial f_b^i$.

- **Case 2-i): low interest rate on DC account:** $r_d \leq \varphi\rho$

Since $\partial\widehat{s_{dc}}/\partial f_b^i > 0$, if $r_d < \varphi\rho$, for all $f_b^i \in (\widetilde{f_b}(f_d), f_b^m(f_d))$ we have $\lambda'(f_b^i) > 0$. Since $\widehat{s_{dc}}(\widetilde{f_b}(f_d), f_d) = nR^*/d$, we have

$$\lambda(\widetilde{f_b}(f_d)) = c_b - f_d - n(r_d - \tau\varphi)R^*.$$

Since $f_c^m(f_d) = f_d + (1-\alpha)d(\Delta v - \Delta r)$, we have

$$\lambda(f_b^m(f_d)) = c_b - f_d - n\varphi(\rho - \tau)R^* + d(\varphi\rho - r_d)(1-\alpha).$$

If $\lambda(f_b^m(f_d)) \leq 0$ (i.e., if $f_d \geq c_b - n\varphi(\rho-\tau)R^* + d(\varphi\rho - r_d)(1-\alpha)$), since λ is increasing with f_b^i, for all $f_b^i \in (\widetilde{f_b}(f_d), f_b^m(f_d))$, we have $\lambda(f_b^i) \leq 0$. From (29), bank i's profit is decreasing with f_b^i. Hence, bank i chooses $(f_b^i)^* = \widetilde{f_b}(f_d)$ such that $\widehat{s_{dc}}(\widetilde{f_b}(f_d), f_d) = nR^*/d$.

If $\lambda(\widetilde{f_b}(f_d)) \leq 0$ and $\lambda(f_b^m(f_d)) > 0$ (i.e., if $f_d < c_b - n\varphi(\rho-\tau)R^* + d(\varphi\rho - r_d)(1-\alpha)$ and $f_d > c_b - n(r_d - \tau\varphi)R^*$), there is an interior solution such that $\lambda(f_b^*) = 0$ and

$$s_{dc}^* = \frac{f_d - c_b + n\varphi(\rho-\tau)R^*}{d(\varphi\rho - r_d)}.$$

If $\lambda(\widetilde{f_b}(f_d)) > 0$ (i.e., if $f_d < c_b - n(r_d - \tau\varphi)R^*$), since λ_i is increasing, we have $\lambda(f_b^i) > 0$. Therefore, bank i's profit is increasing in f_b^i and there is a corner solution. Each bank i chooses $(f_b^i)^* = f_b^m(f_d)$ such that $s_{dc}^* = 1-\alpha$. In that case, consumers pay for all their transactions by digital currency when they have enough money on their digital currency account.

- **Case 2-ii): High interest rate on DC account:** $r_d > \varphi\rho$

If $r_d > \varphi\rho$, for all $f_b^i \in (\widetilde{f_b}(f_d), f_b^m(f_d))$ and for any $f_d \geq 0$, we have $\lambda_i'(f_b^i) < 0$. If $\lambda(\widetilde{f_b}(f_d)) \leq 0$, bank i's profit is decreasing with f_b^i and there is a corner solution. Each bank i chooses $(f_b^i)^* = \widetilde{f_b}(f_d)$ such that $s_{dc}^* = nR^*/d$. If $\lambda(f_b^m(f_d)) > 0$, bank i's profit is increasing in f_b^i and there is a corner solution. Each bank i chooses $(f_b^i)^* = f_b^m(f_d)$ such that $s^* = 1-\alpha$. If $\lambda(\widetilde{f_b}(f_d)) > 0$ and $\lambda(f_b^m(f_d)) < 0$, there is an interior solution $\lambda(f_b^*) = 0$ and

$$s_{dc}^* = \frac{f_d - c_b + n\varphi(\rho-\tau)R^*}{d(\varphi\rho - r_d)}.$$

- **Trade-off between cash and bank transfer with a low amount of reserves**

Suppose that consumers trade off between paying by cash and by bank transfer. Replacing for Eq. (25) into Eq. (C2-CC), if there is an interior solution, s_b^* is implicitly defined by

$$\varphi n(\tau - \rho)R^* + \rho d\varphi s_b^* + c_b = 0.$$

This implies that

$$s_b^* = \frac{\varphi n(\tau - \rho)R^* - c_b}{\rho\varphi d}.$$

Since $nR^* = \alpha d - Ln$, we have

$$f_b^* = (v_b - r_b)\frac{\varphi(\rho - \tau)(\alpha d - Ln) - c_b}{\rho\varphi}.$$

There is an interior solution if given the design of the digital currency, we have $f_b^* < \underline{f_b}(f_d)$. Otherwise, banks choose $f_b^* = \underline{f_b}(f_d)$.

Case 2 - reserves for some but not all payments by bank transfer $R_i \in (\widehat{s_{dc}}dd_i, \alpha_i dd_i)$.

- **Trade-off between cash, the digital currency, bank transfer**

Suppose that consumers trade off between paying by bank transfer, by digital currency and by cash (i.e., that $f_b^i \geq \underline{f_b}(f_d)$). For an interior solution to exist, it must be that at f_b^* given by condition (C2) the amount of reserves R^* held by bank i belongs to $(s_{dc}^* d/n, \alpha d/n)$ (or equivalently if $s_{dc}^* \leq nR^*/d$ and $\alpha d/n - L \geq 0$). Therefore, it must be that f_b^* belongs to $(\underline{f_b}(f_d), \widetilde{f_b}(f_d))$, where $\widetilde{f_b}(f_d)$ is implicitly defined by $\widehat{s_{dc}}(\widetilde{f_b}(f_d), f_d) = nR^*/d$.

If $R_i \in (\widehat{s_{dc}}dd_i, \alpha_i dd_i)$, from Eq. (23), we have

$$\left.\frac{\partial EL_i}{\partial \widehat{s_{dc}}}\right|_{P^*} = \frac{\varphi d\tau s_{dc}^*}{n} h(s_{dc}^*), \tag{30}$$

$$\left.\frac{\partial EL_i}{\partial L_i}\right|_{P^*} = \varphi((\rho - \tau)H(\frac{nR^*}{d}) - \rho) - (1 - \varphi)\tau, \tag{31}$$

and

$$\left.\frac{\partial EL_i}{\partial d_i}\right|_{P^*} = d\varphi(\tau\alpha H(s^*) + \tau \int_{s_{dc}^*}^{\alpha - nL/d} (\alpha - s_i)h(s_i)ds_i - \rho \int_{\alpha - nL/d}^{\alpha} (s_i - \alpha)h(s_i)ds_i) + (1 - \varphi)d\alpha. \tag{32}$$

Replacing for Eq. (32) into Eq. (C1) implies that bank i's margin per depositor is given by

$$\mu_i = \frac{t_b}{n} - d\left(\tau\alpha(\varphi H(s_{dc}^*) + 1 - \varphi) + \tau \int_{s^*}^{\alpha - nL/d} (\alpha - s_i)h(s_i)ds_i - \rho \int_{\alpha - nL/d}^{\alpha} (s_i - \alpha)h(s_i)ds_i\right).$$

Replacing for Eq. (31) into Eq. (C3) gives

$$r_L^* = \underline{r_L} + \varphi(\rho - (\rho - \tau)H(\frac{nR^*}{d})) + (1 - \varphi)\tau, \tag{33}$$

where $\underline{r_L}$ is given by Eq. (24). If there is an interior solution, replacing for Eq. (30) and for $IR_d^i/\partial\widehat{s_{dc}} = -dr_d\widehat{s_{dc}}h(\widehat{s_{dc}})$ into Eq. (C2), we find that f_b^* is implicitly defined by

$$(d(\varphi\tau - r_d)s_{dc}^* - f_d + c_b)h(s_{dc}^*) = 0. \tag{34}$$

If there is an interior solution, we have

$$s_{dc}^* = \frac{f_d - c_b}{d(\varphi\tau - r_d)}.$$

We determine the conditions under which there is an interior solution. We denote by

$$\phi(f_c^i) \equiv d(\varphi\tau - r_d)\widehat{s}(f_b^i, f_d) - f_d + c_b.$$

After simplification by $h(s_{dc}^*) > 0$, Eq. (34) implies that

$$\phi(f_b^*) = 0. \tag{35}$$

We determine whether there exists a bank transfer fee $f_b^* \in (\underline{f_b}(f_d), \widetilde{f_b}(f_d))$ such that $\phi(f_b^*) = 0$. For this purpose, we analyze the sign of $\phi^{'}(f_b^i) = d(\varphi\tau - r_d)\partial\widehat{s_{dc}}/\partial f_b^i$.

- **Case 3-i): low IR on DC account:** $r_d < \varphi\tau$.

Since $\partial\widehat{s}/\partial f_b^i > 0$, if $r_d < \varphi\tau$, we have $\phi_i^{'}(f_b^i) > 0$. Since $\widehat{s_{dc}}(\underline{f_b}(f_d), f_d) = s_d$, for any $f_d \geq 0$, we have $\phi(\underline{f_b}(f_d)) = d(\varphi\tau - r_d)s_d - f_d + c_b$. Since $s_d = f_d/(d(v_d - r_d))$, we have

$$\phi(\underline{f_c}(f_d)) = (\varphi\tau - v_d)f_d/(v_d - r_d) + c_b.$$

If $\varphi\tau \leq v_d$, we have $\phi(\underline{f_b}(f_d)) \leq 0$ if and only if $f_d \geq c_b(v_d - r_d)/(v_d - \varphi\tau)$.

If $\varphi\tau > v_d$, we have $\phi(\underline{f_c}(f_d)) > 0$. Moreover, since $\widehat{s_{dc}}(\widetilde{f_b}(f_d), f_d) = nR^*/d$, we have $\phi(\widetilde{f_c}(f_d)) = (\varphi\tau - r_d)nR^* - f_d + c_b$.

If $\phi(\widetilde{f_c}(f_d)) \leq 0$ (i.e., if $f_d \geq c_b + (\varphi\tau - r_d)nR^*$), since ϕ is strictly increasing with f_c^i, for all $f_b^i \in (\underline{f_b}(f_d), \widetilde{f_b}(f_d))$, we have $\phi(f_b^i) \leq 0$. From (35), this implies that bank i's profit is decreasing with f_b^i. Therefore, bank i's best-response to f_d is to choose $(f_b^i)^*$ such that consumers pay for all their transactions by bank transfer.

If $\phi(\widetilde{f_b}(f_d)) > 0$ and $\phi(\underline{f_b}(f_d)) \leq 0$ (i.e., if $f_d < c_b + (\varphi\tau - r_d)nR^*$) and if $\varphi\tau \leq v_d$ and $f_d \geq c_b(v_d - r_d)/(v_d - \varphi\tau)$), since ϕ is increasing with f_b^i, there exists a unique $(f_b^i)^* \in (\underline{f_b}(f_d), \widetilde{f_b}(f_d))$ such that $\phi(f_b^*) = 0$. Therefore, there is an interior solution such that consumers pay some transactions by bank transfer and other by digital currency. Since $\phi(f_b^*) = 0$, we have $\widehat{s_{dc}}(f_b^*, f_d) = (f_d - c_b)/(d(\varphi\tau - r_d))$.

If $\phi(\underline{f_b}(f_d)) > 0$ (i.e., if $\varphi\tau \leq v_d$ and $f_d < c_b(v_d - r_d)/(v_d - \varphi\tau)$ or $\varphi\tau > v_d$), since ϕ is strictly increasing with f_b^i, for all $f_b^i \in (\underline{f_b}(f_d), \widetilde{f_c}(f_d))$, we have $\phi(f_b^i) > 0$. This implies that bank i's profit is increasing with f_b^i. Therefore, bank i's best-response to f_d is to choose $(f_b^i)^*$ such that $s^* = nR^*/d$.

- **Case 3-ii): high interest rate on DC account:** $v_d \geq r_d \geq \varphi\tau$

If $r_d \geq \varphi\tau$, we have $\phi^{'}(f_b^i) \leq 0$. If $\varphi\tau \leq v_d$ and $f_d \geq c_b(v_d - r_d)/(v_d - \varphi\tau)$, we have $\phi(\underline{f_b}(f_d)) \leq 0$. This implies that for all $f_b^\iota \in (\underline{f_b}(f_d), \widetilde{f_b}(f_d))$ we have $\phi(f_b^i) \leq 0$. From (35), bank i's profit is decreasing with f_b^i. Therefore, bank i's best-response to f_d is to choose $(f_b^i)^*$ such that consumers pay for all their transactions by bank transfer.

If $f_d < c_b + (\varphi\tau - r_d)nR^*$, we have $\phi(\widetilde{f_b}(f_d)) > 0$. Therefore, for all $f_b^i \in (\underline{f_b}(f_d), \widetilde{f_b}(f_d))$, we have $\phi(f_b^i) > 0$. This implies that bank i's profit is increasing with f_b^i. Therefore, bank i's best-response to f_d is to choose $(f_b^i)^*$ such that $s_{dc}^* = nR^*/d$.

If $\phi(\underline{f_b}(f_d)) > 0$ and $\phi(\widetilde{f_b}(f_d)) < 0$, there exists a unique $(f_b^i)^* \in (\underline{f_b}(f_d), \widetilde{f_b}(f_d))$ such that $\phi(f_b^*) = 0$. Therefore, there is an interior solution such that consumers pay some transactions by bank transfer and other by digital currency. Since $\phi(f_c^*) = 0$, we have $\widehat{s}(f_b^*, f_d) = (f_d - c_b)/(d(\varphi\tau - r_d))$.

Note that the case in which $\varphi\tau > v_d$ is impossible because we have $v_d - r_d \geq 0$.

Summary of the results in Case 2:

Suppose that banks hold enough excess reserves to cover some but not payments by bank transfer. The interest rate on loans does not depend on the fee for digital currency payments. Each bank i chooses an interest rate on loans given by

$$r_L^* = \underline{r_L} + \varphi(\rho - (\rho - \tau)H(nR^*/d)) + (1 - \varphi)\tau,$$

makes a margin per depositor given by

$$\mu^* = \frac{t_b}{n} - d\varphi\left(\tau\alpha H(s^*) + \tau\int_{s_{dc}^*}^{\alpha - nL/d}(\alpha - s_i)h(s_i)ds_i - \rho\int_{\alpha - nL/d}^{\alpha}(s_i - \alpha)h(s_i)ds_i\right) - (1 - \varphi)\alpha d,$$

and charges a fee for deposits given by $F^* = \mu^* - f_b^* \beta_b^i(s_{dc}^*) + IR_b^i(s_{dc}^*, \alpha)$.

There exists $f_d^{\min}$ and $f_d^{\max}$ such that if $f_d \in (f_d^{\min}, f_d^{\max})$ and $\tau/n < v_d$, each bank i sets the bank transfer fee f_b^* so that the threshold value of bank transfers is given by

$$s_{dc}^* = \frac{f_d}{d(\varphi\tau - r_d)}.$$

If $\varphi\tau \geq v_d$ or $\varphi\tau < v_d$ and $f_d \leq f_d^{\min}$, each bank i sets $f_b^* = \widetilde{f}_b(f_d)$ such that $s_{dc}^* = nR^*/d$.

If $f_d \geq f_d^{\max}$, each bank i sets f_b^* such that consumers never use the digital currency to pay.

If $r_d < \varphi\tau$, $f_d^{\min} = c_b(v_d - r_d)/(v_d - \varphi\tau)$ and $f_d^{\max} = c_b + (\varphi\tau - r_d)nR^*$.

If $r_d > \varphi\tau$, $f_d^{\max} = c_b(v_d - r_d)/(v_d - \varphi\tau)$ and $f_d^{\min} = c_b + (\varphi\tau - r_d)nR^*$.

If $r_d = \varphi\tau$, $f_d^{\min} = f_d^{\max}$. If $f_d < f_d^{\min}$, consumers always use the digital currency to pay. If $f_d > f_d^{\min}$, consumers never use the digital currency to pay. If $f_d = f_d^{\min}$, consumers are indifferent between both payment instruments.

- **Trade-off between cash and bank transfer:**

Suppose that consumers trade off between paying by cash and by bank transfer. Replacing for Eq. (25) into Eq. (C2-CC), we have that

$$n \left. \frac{\partial EL_i}{\partial \widehat{s_b}} \right|_{\widehat{s_b} = s_b^*} + c_b > 0.$$

Therefore, banks choose the maximum bank transfer fee that satisfies to the constraint on reserves, that is $s_b^* = nR^*/d$ and $f_b^* = n(v_b - r_b)R^*$, if $n(v_b - r_b)R^* \leq \underline{f_b}(f_d)$. If $n(v_b - r_b)R^* > \underline{f_b}(f_d)$, banks choose $f_b^* = \underline{f_b}(f_d)$.

Appendix C: Endogenous α_i: If $s_i \leq 1 - \alpha_i$, if $\Delta r \geq 0$, the expected surplus of a consumer S_i is increasing with the amount left in his bank account, because we have $dS_i/d\alpha_i = d(\Delta r)H(\alpha_i)$. Therefore, the consumer leaves the maximum share of his wealth in his bank account that satisfies to the constraints $s_i \leq 1 - \alpha_i$ and $1 - \alpha_i \leq \overline{\alpha_d}$, that is, $\alpha_i^* = \max(1 - s_i, 1 - \overline{\alpha_d})$. The consumer keeps in his digital currency account a share $1 - \alpha_i^* = \min(s_i, \overline{\alpha_d})$, that is, the maximum share of his wealth that may be withdrawn by the consumer. If $\Delta r < 0$, the consumer keeps in his digital currency account the maximum share of wealth that is authorized by regulation, that is, $1 - \alpha_i^* = \overline{\alpha_d}$.

If α_i^* is independent of f_b^i (e.g., if $\Delta r < 0$), the fact that consumers leave an endogenous share of their wealth in their digital currency account has no impact on banks' choice of the bank transfer fee. Hence, the results of Proposition 1 remains unchanged.

If α_i^* depends on f_b^i, we have

$$\frac{\partial \pi_i}{\partial f_c^i} = \frac{1}{n}(\frac{\partial IR_d^i}{\partial f_c^i} + \frac{\partial IR_d^i}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial f_c^i} + \frac{d\beta_b^i}{df_c^i} f_d) + \frac{\partial EL_i}{\partial \widehat{s_{dc}}}\frac{\partial \widehat{s_{dc}}}{\partial f_c^i} + \frac{\partial EL_i}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial f_c^i}, \tag{C2-Bis}$$

where $\partial IR_d^i/\partial\alpha_i = -dr_d H(\alpha_i)$, $\partial IR_d^i/\partial f_c^i = -dr_d \widehat{s_{dc}} h(\widehat{s_{dc}})$ and $d\beta_b^i/df_c^i = -h(\widehat{s_{dc}})\partial\widehat{s_{dc}}/\partial f_c^i$. This implies that

$$\frac{\partial \pi_i}{\partial f_c^i} = \frac{1}{n}(-dr_d\widehat{s_{dc}}h(\widehat{s_{dc}}) - dr_d H(\alpha_i)\frac{\partial \alpha_i}{\partial f_c^i} - \frac{\partial \widehat{s_{dc}}}{\partial f_c^i} f_d h(\widehat{s_{dc}})) + \frac{\partial EL_i}{\partial \widehat{s_{dc}}}\frac{\partial \widehat{s_{dc}}}{\partial f_c^i} + \frac{\partial EL_i}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial f_c^i}. \tag{36}$$

Case 1 - no excess reserves for payments by bank transfer: $R_i \in (0, \widehat{s_{dc}} dd_i)$. In case 1, from (22), we have

$$\left.\frac{\partial EL_i}{\partial \alpha_i}\right|_{P^*} = \frac{d\varphi}{n}\left(\rho H(\alpha_i) - (\rho - \tau)H(s^*)\right) + \frac{d(1-\varphi)\tau}{n}.$$

Replacing for $\partial EL_i/\partial\alpha_i$ and $\partial EL_i/\partial\widehat{s_{dc}}$ into Eq. (C2-Bis), after multiplication by $n > 0$, we find that

$$\frac{\partial \widehat{s_{dc}}}{\partial f_b^i}\left(-(\rho-\tau)n\varphi R^* + d(\varphi\rho - r_d)s_{dc}^* - f_d\right)h(s_{dc}^*)$$
$$+(\varphi d\left((\rho - r_d)H(\alpha_i^*) - (\rho-\tau)H(s_{dc}^*)\right) + (1-\varphi)\tau d)\frac{\partial \alpha_i^*}{\partial f_c^i} = 0.$$

If α_i^* depends on f_c^i, since $\alpha_i^* = 1 - \widehat{s_{dc}}$, we have $\partial\alpha_i^*/\partial f_c^i = -\partial\widehat{s_{dc}}/\partial f_c^i$. After simplification by $\partial\widehat{s_{dc}}/\partial f_c^i > 0$, this implies that, if there is an interior solution, the bank transfer fee is chosen such that

$$\left(-(\rho-\tau)n\varphi R^* + d(\varphi\rho - r_d)s_{dc}^* - f_d\right)h(s_{dc}^*) = d\left((\rho - r_d)H(\alpha_i^*) - (\rho-\tau)H(s_{dc}^*)\right).$$

Case 2 - excess reserves for some but not all payments by bank transfer: $R_i \in (\widehat{s_{dc}} dd_i, \alpha_i dd_i)$. In case 2, we have

$$\left.\frac{\partial EL_i}{\partial \alpha_i}\right|_{P^*} = \frac{d\varphi}{n}\left(\rho H(\alpha_i) - (\rho - \tau)H(\frac{nR^*}{d})\right) + \frac{d(1-\varphi)\tau}{n}.$$

Replacing for $\partial EL_i/\partial\alpha_i$ and $\partial EL_i/\partial\widehat{s_{dc}}$ into Eq. (C2-Bis), after multiplication by $n > 0$, we find that

$$\frac{\partial \widehat{s_{dc}}}{\partial f_c^i}(d(\tau\varphi - r_d)\widehat{s_{dc}} - f_d)h(\widehat{s_{dc}}) + d\varphi\left((\rho - r_d)H(\alpha_i) - (\rho-\tau)H(s_{dc}^*) + d(1-\varphi)\tau\right)\frac{\partial \alpha_i^*}{\partial f_c^i} = 0.$$

If α_i^* depends on f_c^i, since $\alpha_i^* = 1 - \widehat{s_{dc}}$, we have $\partial\alpha_i^*/\partial f_c^i = -\partial\widehat{s_{dc}}/\partial f_c^i$. After simplification by $\partial\widehat{s_{dc}}/\partial f_c^i > 0$, this implies that, if there is an interior solution, the bank transfer fee is chosen such that

$$(d(\tau\varphi - r_d)\widehat{s_{dc}} - f_d)h(s_{dc}^*) = d\varphi\left((\rho - r_d)H(\alpha_i^*) - (\rho - \tau)H(s_{dc}^*)\right) + d(1-\varphi)\tau.$$

In particular, if the distribution H is uniform on $(0,1)$, we have

$$(d(\tau\varphi - r_d)s_{dc}^* - f_d) = d\varphi\left((\rho - r_d)(1 - s_{dc}^*) - (\rho - \tau)(s_{dc}^*)\right) + d(1-\varphi)\tau.$$

This implies that, if there is an interior solution, the share of digital currency payments is defined by

$$s_{dc}^* = (d\varphi(\rho - r_d) + f_d + d(1-\varphi)\tau)/(2d(\varphi\rho - r_d)).$$

For an interior solution to exist, it must be that $s_{dc}^* \leq 1/2 - (Ln)/(2d)$.

Appendix D: banks as distributors of the Central Bank Digital Currency We have $R_i^d = (1-\alpha_i)dd_i$ and $L_i = dd_i - (R_i^b + R_i^d)$. This implies that $R_i^b = \alpha_i dd_i - L_i$. Since banks may pay interests to consumers on the digital currency holdings, Eq. (C2) is modified as follows

$$n\left.\frac{\partial EL_i}{\partial\widehat{s_{dc}}}\right|_{s_{dc}^*} - (c_d - c_b)h(s_{dc}^*) = 0. \qquad \text{(D-1)}$$

I analyze now how the consumer's decision to pay by digital currency impacts bank i's marginal cost of liquidity. If the consumer pays from his standard bank account, if the bank has enough reserves of type b, the bank reduces its amount of reserves of type b from its Central Bank account. If the bank does not have enough reserves of type b, it borrows additional liquidity. If the consumer pays both from his standard bank account and from his digital currency account, the bank borrows additional liquidity for payments from the standard bank account and uses its reserves of type d for the share of payments that comes from the digital currency account.

If $\widehat{s_{dc}} \leq \alpha_i - L/(dd_i)$, bank i's expected benefit of liquidity management is given by

$$\begin{aligned} EL_i &= \varphi(\tau_d \int_0^{\widehat{s_{dc}}} ((1-\alpha_i)dd_i - s_i dd_i)h(s_i)ds_i + \tau_d \int_{\widehat{s_{dc}}}^{\alpha_i} ((1-\alpha_i)dd_i)h(s_i)ds_i \\ &\quad + \tau_d \int_{\alpha_i}^{1} (s_i dd_i - (\alpha_i dd_i) - (1-\alpha_i)dd_i)h(s_i)ds_i + \tau_b \int_0^{\widehat{s_{dc}}} R_i^b h(s_i)ds_i \\ &\quad \tau_b \int_{\widehat{s_{dc}}}^{\alpha_i - L/(dd_i)} (R_i^b - s_i dd_i)h(s_i)ds_i - \rho \int_{\alpha_i - L/(dd_i)}^{\alpha_i} (s_i dd_i - R_i^b)h(s_i)ds_i \\ &\quad -\rho \int_{\alpha_i}^{1} (\alpha_i dd_i - R_i^b)h(s_i)ds_i) + (1-\varphi)(\tau_d R_i^d + \tau_b R_i^b). \end{aligned}$$

We have

$$\frac{\partial EL_i}{\partial \widehat{s_{dc}}} = \varphi \widehat{s_{dc}} h(\widehat{s_{dc}}) dd_i (\tau_b - \tau_d),$$

$$\frac{\partial EL_i}{\partial L_i} = \varphi(\rho + (\tau_b - \rho)H(\alpha_i - L/(dd_i))) - (1-\varphi)\tau_b.$$

Therefore, from Eq. (D-1), if there is an interior solution, the threshold value such that consumers pay by digital currency is given by

$$s_{bd}^* d(\tau_b - \tau_d) - (c_d - c_b) = 0,$$

that is, $s_{bd}^* = (c_d - c_b)/(d(\tau_b - \tau_d))$, provided that $(c_d - c_b)/(d(\tau_b - \tau_d))$ belongs to $(0, \min(\alpha_i - Ln/d, 1 - \alpha_i))$. If $\tau_b - \tau_d < 0$ and $c_d - c_b > 0$, each bank i chooses a bank transfer fee such that consumers never use the digital currency to pay. If $\tau_b - \tau_d > 0$ and $c_d - c_b < 0$, each bank i chooses a bank transfer fee such that consumers use the digital currency to pay, as long as $s_{bd}^* \leq \min(\alpha_i - Ln/d, 1 - \alpha_i))$.

If $\widehat{s_{dc}} \leq \alpha_i - L/(dd_i)$, bank i's expected benefit of liquidity management is given by

$$\begin{aligned} EL_i &= \varphi(\tau_d \int_0^{\widehat{s_{dc}}} ((1-\alpha_i)dd_i - s_i dd_i)h(s_i)ds_i + \tau_d \int_{\widehat{s_{dc}}}^{\alpha_i} ((1-\alpha_i)dd_i)h(s_i)ds_i \\ &\quad + \tau_d \int_{\alpha_i}^{1} (s_i dd_i - (\alpha_i dd_i) - (1-\alpha_i)dd_i)h(s_i)ds_i + \tau_b \int_0^{\widehat{s_{dc}}} R_i^b h(s_i)ds_i \\ &\quad -\rho \int_{\widehat{s_{dc}}}^{\alpha_i} (s_i dd_i - R_i^b)h(s_i)ds_i - \rho \int_{\alpha_i}^{1} (\alpha_i dd_i - R_i^b)h(s_i)ds_i) + (1-\varphi)(\tau_d R_i^d + \tau_b R_i^b. \end{aligned}$$

We have

$$\frac{\partial EL_i}{\partial \widehat{s_{dc}}} = h(\widehat{s_{dc}})(\widehat{s_{dc}}(\rho - \tau_d)dd_i - (\tau_b - \rho)(\alpha_i dd_i - L_i)),$$

and

$$\frac{\partial EL_i}{\partial L_i} = \varphi(-\tau_b H(\widehat{s_{dc}}) - \rho(1 - H(\widehat{s_{dc}}))) - (1-\varphi)\tau_b.$$

Eq. (C3) implies that $r_L^* = \underline{r_L} + \rho - (\rho - \tau_b)H(s^*) + (1-\varphi)\tau_b$. Eq. (D-1) of the beginning of the Appendix implies that, if there is an interior solution, the threshold value such that consumers pay by bank transfer is given by

$$s_{bd}^*(\varphi\rho - \tau_d)d + \varphi n(\tau_b - \rho)(R_b^*) - (c_d - c_b) = 0,$$

that is, that

$$s_{bd}^* = \frac{\varphi n(\rho - \tau_b)(R_b^*) + (c_d - c_b)}{d(\varphi\rho - \tau_d)}.$$

Hence, it is interesting to note that this solution is equivalent to the solution obtained in Proposition 2 if the fee for the digital currency is priced (or regulated) at its marginal cost c_d and the interest rate on the digital currency account r_d is identical to τ_d.

Appendix E: partial acceptance of payment media I denote by γ_d the probability that the digital currency is accepted at the transaction stage. Suppose that banks never hold enough reserves to meet consumer demand for payments by bank transfer. With probability γ_d, both payment instruments are accepted by merchants and Eq. (C2) is given by

$$\left(-(\rho - \tau)n\varphi R^* + d(\varphi\rho - r_d)s^* - f_d + c_b\right)h(s^*) = 0.$$

With probability $(1 - \gamma_d)$, bank transfers are accepted and the digital currency is refused. Therefore, Eq. (C2) is given by

$$(\tau n R^* + \varphi\rho(ds^* - nR^*) + c_b)h(s^*) = 0.$$

Therefore, if there is an interior solution, after simplification by $h(s^*) > 0$, the threshold value such that consumers pay by bank transfer is given by

$$-(\rho - \tau)n\varphi R^* + c_b + (d(\varphi\rho - \gamma_d r_d)s^* - \gamma_d f_d) = 0.$$

Therefore, we have

$$s_{dc}^* = (\gamma_d f_d + (\rho - \tau)n\varphi R^* - c_b)/d(\rho\varphi - \gamma_d r_d).$$