Capital Regulation and Credit Fluctuations*

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Abstract

We provide a rationale for imposing counter-cyclical capital ratios on banks. In our simple model, bankers cannot pledge the entire future revenues to investors, which limits borrowing in good and bad times. Complete markets do not sufficiently stabilize credit fluctuations, as banks allocate too much borrowing capacity to good states and too little to bad states. As a consequence, bank credit, output, capital prices or wages are excessively volatile. Imposing a (stricter) capital ratio in good states corrects the misallocation of the borrowing capacity, increases expected output and can be beneficial to all agents in the economy. Although in our economy, all agents are risk-neutral, counter-cyclical capital ratios are an effective stabilization tool. To ensure this effectiveness, capital ratios have to be based on ex ante equity capital, as classical capital ratios can be bypassed.

Keywords: Credit Fluctuations, Complete Markets, Macroprudential Regulation, Misallocation of Borrowing Capacity.

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1 Introduction

The implementation of counter-cyclical capital ratios, i.e. imposing (stricter) capital ratios in booms, is a central theme of macroprudential regulation.[1] Yet, the conceptional foundations for such a regulation are not entirely clear.

In this paper, we provide a conceptional foundation for counter-cyclical capital ratios by examining how banks allocate their borrowing capacity across good and bad states in a complete markets setting. We find that banks allocate too much borrowing capacity to boom states and too little to bad states, creating excessive fluctuations of credit, output, asset prices and wages. This can be corrected by capital regulation in boom states. Imposing tighter capital ratios in booms increases expected output.

More specifically, we consider a simple two-sector economy in which a capital good can be used to produce a consumption good. A fraction of the capital good is owned by investors and the remaining part by bankers. All individuals are risk-neutral. In one sector, capital can be lent and borrowed frictionlessly. There are diminishing returns of capital in this sector. In the other sector, banks lend to entrepreneurs, who have access to a constant returns to scale technology. Banks alleviate the moral hazard problems of the entrepreneurs. In this sector, output and lending rates are affected by macroeconomic shocks. Good states and bad states refer to high or low capital productivity. Banks can only pledge a fraction of their future revenues to investors. The specific form of this financial friction - moral hazard of bankers, asset diversion or non-alienability of human capital - does not matter. The financial friction, however, limits borrowing of banks in good states and bad states. Before macroeconomic shocks occur, agents can trade in complete financial markets. This simple model yields the following insights.

First, complete financial markets allow bankers to reallocate borrowing capacity between good and bad states. This reallocation decision is governed by the bankers' objective to maximize their rents. As a rule, the access to complete markets reduces the volatility of bank-lending and capital prices.

Second, with complete markets, however, banks allocate too much borrowing capacity to good states and too little to bad states. The reason is as follows. Bankers aim at maximizing their rents from lending, taking capital prices and prices of financial assets as given. At the competitive equilibrium, capital prices are such that bankers are indifferent between shifting one

[1] The reasoning behind different notions and possible foundations of macroprudential regulation is outlined in Borio (2003, 2010).

additional unit of borrowing capacity across states. A social planner facing the same borrowing constraints would recognize that reallocating borrowing capacity from good states to bad states would reduce capital prices in good states and increase them in bad states. This would increase the expected borrowing capacity of banks and would allow to increase the expected lending by banks to entrepreneurs that have higher expected capital productivity than in the other sector. As a consequence, the expected output in the economy would increase.

Third, a regulatory capital ratio that is activated only in the boom can implement the expected output gain a social planner could achieve. Such capital regulation limits lending in the boom, and thus corrects the misallocation of borrowing capacity across good and bad states, thereby reducing fluctuations of lending and capital prices.

Fourth, financial markets allow banks to reallocate their initial equity across states. Now it turns out that regulatory capital ratios are based on interim equity. We show that this is not effective in implementing the social planner solution. This is because bankers can use contingent markets and partially bypass this regulation. Effective capital regulation requires to relate lending to initial equity. Such ratios continue to be binding if bankers trade in contingent markets.

Fifth, we examine some simple extensions of the model. In a first extension we introduce financial frictions between banks and entrepreneurs. In such circumstances, the misallocation of borrowing capacity of banks becomes more pronounced and the welfare gains that can be achieved by macroprudential regulation increase. In a second extension, we introduce labor as a further factor of production supplied by workers. We obtain a broader conclusion that tighter capital requirements for booms stabilize other factor prices such as wages. This may be particularly valuable socially when these workers are risk-averse and do not have access to financial markets to insure themselves against wage fluctuations. In a further extension, we explore various ways how output gains obtained via macroprudential regulation can be distributed in the economy, so that all agents benefit and that such regulations engineer a Pareto improvement. Finally, we sketch in the concluding section how defaults of banks could be introduced into the model.

Sixth, we present a dynamic version of our model with a continuous distribution of shocks. We show that the competitive equilibrium is generically constrained inefficient. We establish that simple countercyclical capital ratios continue to be welfare-improving in the dynamic version of the model.

The paper is organized as follows. In section 2, we discuss the related literature. In section 3, we introduce the model. In section 4, we characterize the competitive equilibrium with and without contingent markets. In section 5, we provide the rationale for capital regulation. Section 6 contains several extensions of the model. In section 7, we present a fully dynamic version of the model. Section 8 concludes.

2 Relation to the literature

2.1 Empirical Evidence

Empirical work has identified several phenomena that motivate our analysis and for which it can provide an explanation. First, volatility of bank lending is typically a multiple of the volatility of GDP. In Figure 1 we provide evidence for a variety of countries since World War II. In almost all countries, bank lending is more volatile than GDP.[2]

Second, credit cycles characterized by successions of credit booms and credit busts have been thoroughly investigated in a series of papers (Igan et al. (2009), Elekdag and Wu (2011) and Claessens et al. (2011)). Schularick and Taylor (2012) present new evidence on the rapid rise of leverage in the financial sector in the second half of the twentieth century. They also demonstrate that credit booms predict busts.

Third, Jiménez et al. (2011) provide evidence on how macroprudential policies in the form of counter-cyclical bank capital buffers have reduced fluctuations in total credit supply of banks in Spain. Our model provides a theoretical explanation for this finding.

2.2 Financial Intermediaries and Aggregate Economic Activity

There is a vast literature on financial contracting and aggregate economic activity. We highlight one important line of research that emphasizes how borrowers facing shocks to their net worth may become more or less credit constrained, causing shocks to amplify and persist. Two classical contributions have derived such mechanisms from first principles.

Bernanke and Gertler (1989) examine an overlapping generation model in which risk-neutral entrepreneurs, with private information about their project outcomes, borrow from lenders who

[2] We have used CPI data to deflate the series. Using the GDP deflator for the period 1961-2009 yields the same pattern with small variations of the multiples. We note that the multiple reported in Meh and Moran (2010) for the US in the last decades is over four.

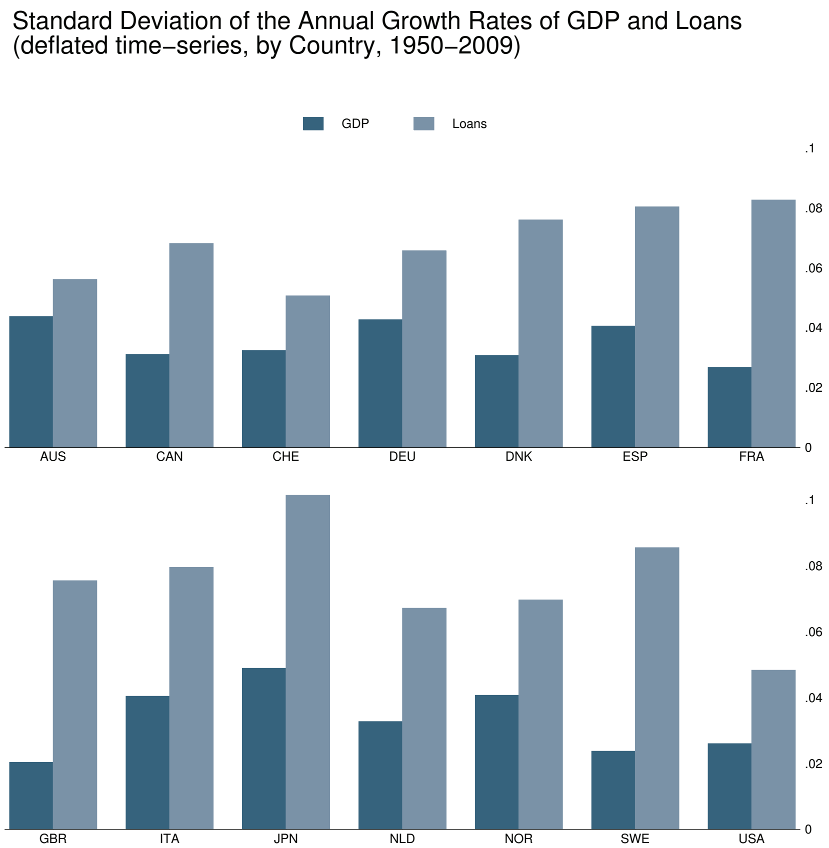

Figure 1: Evidence for Credit Cycles, Source: Calculations based on Schularick and Taylor (2012).

have access to a costly auditing technology. Higher net worth of borrowers lowers financing costs. As a consequence, a positive technology shock not only increases current real capital investments, but it also propagates over time as it raises future net worth of borrowers. The opposite occurs in downturns.

Kiyotaki and Moore (1997) significantly extend these insights by considering shocks to net worth arising from changes in the value of a firm's asset. A small, temporary technology shock causes credit constrained firms to cut back on their investment expenditures in the current and in subsequent periods. As prices of assets reflect future revenue conditions, the shock may cause a significant decline in asset prices and thus in the net worth of credit constrained firms. As a consequence, those firms need to reduce further their investments. This intertemporal multiplier process can generate large, persistent changes of output and asset prices.

Recent work in the tradition of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) has focused on overborrowing coupled with insufficient insurance. In Krishnamurthy (2003) and Lorenzoni (2008) entrepreneurs cannot protect themselves against the risk that they become financially constrained (see also Gromb and Vayanos (2002)). In Korinek (2011) such insurance is available but costly. As a result, in these papers, entrepreneurs invest too much. Bianchi (2011) assesses the aggregate and welfare consequences of such overborrowing phenomena quantitatively.

We focus on an environment with complete financial markets allowing banks to allocate their borrowing capacity across good and bad states flexibly. Atomistic banks allocate too much borrowing capacity to good times at the expense of borrowing capacity in bad times.[3] An alternative mechanism that produces inefficient investment cycles has been set out by He and Kondor (2012). Constrained inefficiency of investment cycles results from the interaction of two frictions: agents cannot raise outside capital and investment opportunities are not contractible. In contrast, in our model, moral hazard of bankers induces inefficient booms and busts.

Our model replicates well-known stylized facts on fluctuations of credit and bank balance sheets as succinctly described in Adrian, Colla and Shin (2012). For instance, bank leverage is procyclical and in downturns increasing bond financing to non-financial firms (in our model credit to the other sectors) substitutes for declining bank lending.[4]

Cyclical adjustments of capital requirements developed by Repullo and Suarez (2013) and Repullo (2012) provide a rationale for cyclical adjustments of (risk-sensitive) capital requirements. Without such adjustments, aggregate investments may decline excessively in the presence of negative shocks to bank capital. We focus on the allocation of borrowing capacity by banks, across good and bad states.

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[3] As banks take capital prices as given and as the allocation decision of a subset of banks impacts on the borrowing capacity allocation of other banks, our paper is broadly related to the literature on pecuniary externalities. At least since Scitovsky (1954) it is well-known that pecuniary externalities may matter for welfare when we depart from the assumptions that guarantee the validity of the first welfare theorem in general equilibrium. Such pecuniary externalities have been the focus of classical contributions (see e.g. Geanakoplos and Polemarchakis (1986), Stiglitz (1982), Greenwald and Stiglitz (1986), and in more recent work, e.g., Caballero and Krishnamurthy (2001), Allen and Gale (2004), and Farhi et al. (2009) and in the literature discussed in subsection 2.1). In our model, pecuniary externalities operate through two relative prices: the relative price of capital in the good and bad state. To achieve welfare gains both prices need to adjust. Therefore, the inefficiency in our model can be called a two-dimensional welfare-reducing pecuniary externality.

[4] Moreover, there is an advanced literature on dynamic stochastic general equilibrium models with financial frictions to which our paper is complementary. Early contribution by Aikman and Paustian (2006) and more recent work by Meh and Moran (2010), Gertler and Karadi (2009), Gertler and Kiyotaki (2011), Brunnermeier and Sannikov (2012) and Angeloni and Faia (2012) provide a rich menu of models.

3 The Model

Our model is closely related to the ones used in Lorenzoni (2008) and Gersbach and Rochet (2012). Like in these two models, we have three dates $t = 0, 1, 2$ and two goods, consumption and capital. We simplify Gersbach and Rochet (2012) by assuming that adjustment costs are zero. We simplify Lorenzoni (2008) by replacing collateral constraints by a simpler form of financial frictions. This allows us to have only one production period (namely between $t = 1$ and $t = 2$) instead of two. Date $t = 0$ is only there to allow ex-ante trade on contingent financial markets. On the other hand, we introduce new features in each of these models: banks into Lorenzoni's model and capital regulation into Gersbach and Rochet's. The other new crucial feature in our model is bankers' possibility to trade ex ante in complete contingent markets and to allocate their borrowing capacity across good and bad times. This last feature justifies macroprudential policies.

As in Lorenzoni (2008), there is a traditional sector[5] characterized by a concave, twice continuously differentiable production function $F(\cdot)$, and another sector, which is interpreted as the banking sector in a broad sense that includes all financial intermediaries that contribute to providing credit to the economy.

There is a continuum of bankers with mass one, each endowed with some physical capital e at date 0. They have access to a lending technology with constant returns to scale.

The expected return on banks' assets is denoted by R.[6] Initially, capital e is distributed according to some distribution with bounded support and aggregate amount $E < 1$. A crucial ingredient is the presence of a financial friction: Bankers can borrow from outside investors, but they cannot fully pledge their future income to these outside investors.

Like Holmström and Tirole (1997) we assume that the non-pledgeable income is a multiple bk of

[5]Our model is isomorphic to a one sector model where the capital good can also be consumed at date 0. The simplicity of our results go through to that model if one assumes that consumers' utility is strictly concave at date 1 and linear at date 2.

[6]This is a summary of the lending activities of banks. At a more detailed level, we envision that there is a continuum of entrepreneurs operating a technology, with expected return R. Those entrepreneurs cannot raise funds directly from investors because of moral hazard. Each banker can alleviate the moral hazard of these entrepreneurs by monitoring them and enforcing contractual obligations. For simplicity, we assume that the costs of these activities are sufficiently small and can be neglected, or that R is the net return after these costs have been taken into account. We assume that banks are efficient in eliminating moral hazard, and can secure R.

the size k of the investment.[7] The parameter b measures the intensity of financial frictions. In Appendix 1, we show that this non-pledgeability can be generated by several forms of financial frictions such as moral hazard, asset diversion, or non-alienability of human capital.

Like in Lorenzoni (2008) and Gersbach and Rochet (2012), there is an aggregate shock at $t = 1$. Conditionally on this shock, the expected return R on the banking technology can be either R_h (high return) or R_l (low return, $R_l < R_h$) with probabilities π_h and π_l (with $\pi_h + \pi_l = 1$). The aggregate state $s = h, l$ that determines R_s is called a "boom" (when $s = h$) or a "recession" (when $s = l$). By contrast, the traditional sector is not subject to aggregate shocks[8] nor to financial frictions.

The total physical capital stock of the economy is normalized to 1. Bankers own (on aggregate) a fraction E of this capital. E can be interpreted as the aggregate capitalization of the banking sector. Investors own the remaining capital $1 - E$. They also own the firms in the traditional sector. All agents are risk-neutral and endeavor to maximize their expected consumption at date 2.

The timing of the model is as follows:

- at date 0, all agents can trade on contingent markets for capital. There is a market for each state, i.e. financial markets are complete.[9] Thanks to these markets, banks can obtain a state dependent capital endowment, denoted by e_s in states $s = h, l$.
- at date 1, all agents observe the aggregate state s. A typical banker borrows $k_s - e_s > 0$ units of capital from investors/depositors, and invests k_s units of capital. The aggregate size of the banking sector (the integral of k_s over all banks) is denoted by K_s. We interpret K_s as the aggregate volume of credit to the economy in state s. The remaining amount of capital $1 - K_s$ is invested in the traditional sector.
- at date 2, output in the banking sector, $R_s K_s$, is shared between investors and bankers. More specifically, the investors who, at date 1, have lent $k_s - e_s$ units of capital to a typical bank receive $p_s(k_s - e_s)$ units of consumption at date 2. Since consumption is taken as a numeraire, the rate of return on banks' deposits, namely p_s, can also be interpreted as

[7] Lorenzoni (2008) assumes instead that the non-pledgeable income is a constant fraction $(1-\theta)Rk$ of the asset's return.

[8] This assumption is not crucial and is only made for the sake of simplicity. It is relaxed in Section 7.

[9] This is an important difference with Lorenzoni (2008) who, like in most of the literature on credit cycles, has to assume some form of market incompleteness.

the price of capital in state s, as it corresponds to the number of units of consumption that are delivered at date 2 in exchange for one unit of capital at date 1. The output in the traditional sector $F(1-K_s)$ accrues to investors.[10] The total output in the economy is $F(1-K_s)+R_sK_s$.

An important remark regarding the concepts and the language is in order. At the beginning, a banker owns capital e: we call this "initial" equity. Through trading in complete financial markets, bankers end up with a state-dependent capital endowment e_s. We call e_s the "interim" equity of the bank.[11] The distinction is important because current regulations are typically based on "interim" equity, whereas we show that they should be based on "initial" equity of banks.

4 The Competitive Equilibrium

Our simple economy has a unique competitive equilibrium, which is easy to characterize. As a first benchmark, we start by the case where there are no contingent markets at date 0.

4.1 The case without contingent markets

Consider a banker who has equity e (here it is the same in both states since we do not have any contingent markets) and assume that the macro state is s. Investors/depositors agree to provide additional capital $k_s - e$ if and only if they are promised an expected repayment of (at least) $p_s(k_s - e)$ units of consumption at date 2. This is only possible if this promised repayment does not exceed the maximum pledgeable income of the bank, namely $(R_s - b)k_s$: When $R_s < p_s + b$, which will always be satisfied at equilibrium, the participation constraint of investors implies that the bank is constrained in its investment choice by what we call a **market imposed solvency ratio**:

$$k_s \leq \frac{e}{1-\frac{R_s-b}{p_s}}. \tag{1}$$

[10] It accrues in two forms: return on invested capital $F'(1-K_s)K_s$ and profits of firms operating the technology $F(1-K_s)-F'(1-K_s)K_s$. For details see Gersbach and Rochet (2012).

[11] In practice, banks use derivatives and other contingent securities to hedge against macro shocks. In the dynamic version of the model (Section 7), banks can costlessly issue new equity when they need it, and also use contingent capital instruments to finance themselves.

Note that the banks in our model can be interpreted as investment banks or hedge funds, that are financed by sophisticated investors. Thus there is no need, at this stage, for a regulation of capital ratios, imposed by a (micro-prudential) regulator. Investors themselves impose a limit to the banks' volume of lending. An alternative interpretation would be that these banks are traditional deposit-taking institutions and that (1) is a micro-prudential capital ratio, imposed by a traditional bank regulator. Both interpretations are possible, and our focus is different: Our objective is to find a conceptual foundation for a macro-prudential regulation of banks' capital. Thus we do not discuss possible motivations for micro-prudential regulation.

Since bankers take on as much leverage as they can, constraint (1) is binding for each bank. The aggregate size K_s of the banking sector in state s is obtained by integrating constraint (1) over all banks:

$$K_s = \frac{E}{1 - \frac{R_s - b}{p_s}} \equiv D_s(p_s). \tag{2}$$

The right hand-side of this equation is the demand for capital by the banking sector in state s. By absence of arbitrage opportunity, the price of capital in state s, namely p_s, must be equal to the marginal productivity of capital in the traditional sector:

$$p_s = F'(1 - K_s), \tag{3}$$

which can be rewritten as

$$K_s = S(p_s), \tag{4}$$

where $S(\cdot)$ is the supply of capital to the banking sector. The equilibrium price p_s is determined by equaling (2) and (4). The demand for capital decreases from $+\infty$ (when $p_s = R_s - b$) to $\frac{R_s E}{b}$ (when $p_s = R_s$). On the same interval $(R_s - b, R_s)$ the supply of capital increases from $S(R_s - b)$ to $S(R_s)$. Thus when $E < b\frac{S(R_s)}{R_s}$, that is if bank capital is relatively scarce,[12] the intermediate value theorem implies that there is a unique value of p_s in $(R_s - b, R_s)$ that equalizes supply and demand. We summarize these findings in the following proposition.

[12] When bank capital is abundant (a case we rule out as empirically irrelevant), $E > b\frac{S(R_s)}{R_s}$ and the constraint (1) does not matter anymore. The competitive equilibrium $(p_s = R_s, K_s = S(R_s))$ coincides with the first-best allocation.

Proposition 1

When there are no contingent markets, there is a unique competitive equilibrium, with credit volumes $K^0 = (K_h^0, K_l^0)$ *and capital prices* (p_h^0, p_l^0)*. When* $E < b\frac{S(R_s)}{R_s}$*, the price of capital in state* s *is the unique value* p_s^0 *that equalizes supply and demand for bank capital in each state:*

$$S(p_s^0) = \frac{E}{1 - \frac{R_s - b}{p_s^0}}. \tag{5}$$

Figure 2 illustrates some properties of the competitive equilibrium in the absence of contingent markets.

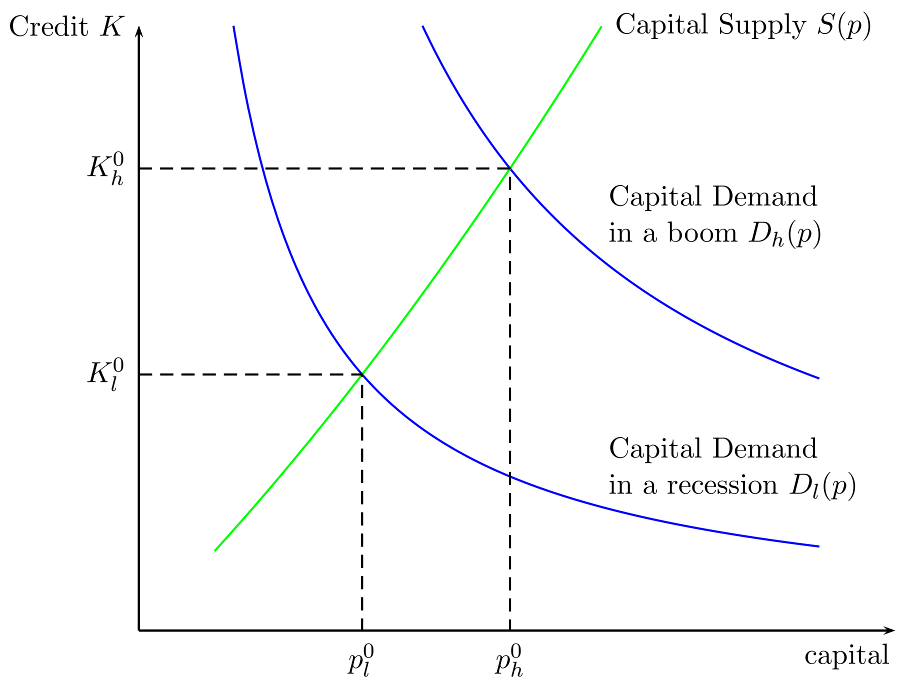

Figure 2: Equilibrium in the absence of contingent markets. Capital price and credit volume are pro-cyclical.

Note that capital price p_s^0 and aggregate credit K_s^0 are pro-cyclical: $p_l^0 < p_h^0$ and $K_l^0 < K_h^0$. This is because the supply curve $K = S(p)$ is state independent, while the demand curve in state h is above the demand curve in state l.

In fact, some procyclicality is not necessarily bad: the first-best allocation (that would prevail in our model if bank capital was abundant) is also pro-cyclical since $p_s^{FB} = R_s$ and $K_s^{FB} = S(R_s)$. However, the volatility of capital prices and credit volumes is typically higher in the competitive equilibrium than in the first-best allocation.

To illustrate this feature, consider the simple specification $S(p) = p$ (elasticity of credit supply identically equal to 1). In this case the competitive price can be computed explicitly:

$$p_s^0 = R_s - \max(b - E, 0). \tag{6}$$

To assess the variability of capital prices, we can use the coefficient of variation $\sigma_p = \frac{p_h - p_l}{\mathbb{E}[p_s]}$, which, by a slight abuse of language, we call the volatility of these prices. Formula (6) immediately shows that financial frictions exacerbate the volatility of capital prices (and credit volumes):

$$\sigma_p^0 = \frac{R_h - R_l}{R - \max(b - E, 0)} \geq \sigma_p^{FB} = \frac{R_h - R_l}{R},$$

where $R \equiv \mathbb{E}[R_s]$.

Moreover, we see that the equilibrium volatility of capital prices σ_p^0 increases with the severity of financial frictions (measured by b) and decreases with the capitalization of the banking sector (measured by E). Thus imposing more transparency in the financial sector (reducing b) or increasing bank capital has a stabilizing effect on capital prices and credit volumes.

However, the absence of contingent markets may not be a reasonable assumption, since modern banks have access to complex financial instruments that allow them to hedge against macro-shocks. This is why Section 4.2 below examines another benchmark, namely the case where financial markets are complete.

4.2 The Case of Complete Financial Markets

Suppose now that at date 0, banks and investors can trade on contingent capital markets. Denote by e_h and e_l the after-trade levels of capital of a typical bank with initial capital e. The simplest way to generate these trades is to consider that the bank will swap $e_h - e$ units of capital in state h against $e - e_l$ units of capital in state l.[13] Since all agents are risk-neutral, the swap rate must be equal to $\frac{\pi_h p_h}{\pi_l p_l}$. Thus the budget constraint of the bank writes

$$\pi_h p_h (e_h - e) = \pi_l p_l (e - e_l),$$

or more simply

$$\mathbb{E}[p_s(e_s - e)] = 0. \tag{7}$$

[13] In practice, such trades are often achieved through derivatives or contingent capital contracts.

Rationally anticipating the values of equilibrium capital ratios that will prevail in each state at date 1, namely

$$e_s = k_s \left(1 - \frac{R_s - b}{p_s}\right), \tag{8}$$

the bank will select the contingent credit volumes (k_h, k_l) that maximize expected profit $b\mathbb{E}[k_s]$ (which is equal to the expectation of non-pledgeable income) under the single constraint obtained by combining (7) and (8):

$$\mathbb{E}[k_s(p_s + b - R_s) - p_s e] = 0. \tag{9}$$

Note that both the objective function of the bank and constraint (9) are linear in (k_h, k_l). Thus if we exclude corner solutions,[14] the only possible equilibrium is such that the coefficient of k_s in constraint (9) is the same in both states:

$$p_h + b - R_h = p_l + b - R_l.$$

Denoting by $p \equiv \mathbb{E}[p_s]$ the expected price of capital (recall that $R \equiv \mathbb{E}[R_s]$ denotes the expected return on assets), this condition can be rewritten as

$$p_s = R_s - R + p. \tag{10}$$

Using (10), constraint (9) can be simplified:

$$\mathbb{E}[k_s](p + b - R) = pe.$$

By aggregating this condition over all banks, we obtain

$$\mathbb{E}[K_s] = \frac{E}{1 - \frac{R-b}{p}}. \tag{11}$$

Since $K_s = S(R_s - R + p)$, the expected price of capital p can be determined by equaling expected supply and demand:

$$\mathbb{E}[S(R_s - R + p)] = \frac{E}{1 - \frac{R-b}{p}}. \tag{12}$$

These results are summarized in the following proposition.

[14] Such corner solutions are empirically irrelevant, since they imply that the banking sector is completely closed down in one of the states. In the theoretical model, they can be excluded if we assume that the production function F satisfies Inada's conditions.

Proposition 2

When financial markets are complete, there is a unique competitive equilibrium denoted by $K^c = (K_h^c, K_l^c)$ *with prices* (p_h^c, p_l^c)*. When bank capital is scarce (specifically* $E < b\frac{\mathbb{E}[S(R_s)]}{R}$*), this equilibrium is characterized by*

$$p_s^c = R_s - R + p^c, \tag{13}$$

where p^c *solves (12).*

Note that banks use contingent markets at $t = 0$ to reallocate their equity across states in such a way that their returns on equity (ROE_s) in each state are proportional to their return on assets (ROA_s):

$$ROE_s = \frac{bk_s}{e_s} = \frac{bp_s^c}{p_s^c - R_s + b} = \frac{b}{p^c - R + b}p_s^c > p_s^c = ROA_s.$$

Thus our model predicts that, when financial markets are complete, banks will select their contingent plans so as to equalize the ratios $\frac{ROE_s}{ROA_s}$ in both states. Note that this ratio is larger than 1 when $E < b\frac{\mathbb{E}[S(R_s)]}{R}$: There is a wedge between the return on informed capital (equity) and the return on uninformed capital (deposits), due to the scarcity of informed capital. This wedge would disappear if E was larger than $b\frac{\mathbb{E}[S(R_s)]}{R}$.

In Appendix 2, we provide a simple numerical calibration of our model with complete markets. In this example, capital prices fluctuate 40% more than returns and bank credit fluctuates 40% more than GDP.

We conclude this subsection by the observation that our model can explain several well-known stylized facts on fluctuations of credit and bank balance sheets over the business cycle. For instance, when negative shocks occur, aggregate financing provided to the traditional sector $(1 - K_l)$ increases and substitutes for declining bank lending. Moreover, banks' leverage is procyclical. Specifically, using (8) and (13), leverage in the banking system is given by

$$\frac{K_s}{E_s} = \frac{p_s}{p_s - R_s + b} = \frac{p_s}{p^c + b - R}.$$

As $p_h > p_l$, leverage is higher in upturns.

4.3 Comparison

A natural question is whether the existence of contingent markets stabilizes the economy. In fact this is not necessarily the case, as illustrated by the simple, linear specification $S(p) = p$ that we already used. Equation (12) gives in this case

$$\mathbb{E}[S(R_s - R + p^c)] = p^c = \frac{E}{1 - \frac{R-b}{p^c}},$$

which gives an explicit solution:

$$p^c = R - \max(b - E, 0).$$

If we assume $E < b\frac{\mathbb{E}[S(R_s)]}{R} = b$, $p^c < R$. Moreover,

$$p_s^c = R_s - R + p^c,$$

thus

$$p_s^c = R_s - \max(b - E, 0).$$

This is the same expression as in equation (6) without contingent markets. With this particular specification of $S(\cdot)$, contingent capital markets do not make any difference: the two competitive equilibria $((p_h^c, p_l^c), K^c)$ and $((p_h^0, p_l^0), K^0)$, i.e. with and without contingent markets, are identical. However, this feature is not robust:

Proposition 3

When the elasticity of capital supply is less than one and the volatility of aggregate shocks $\sigma_R = \frac{R_h - R_l}{R}$ is small, capital prices are less volatile with contingent markets than without:

$$\sigma_p^c = \frac{p_h^c - p_l^c}{p^c} < \sigma_p^0 = \frac{p_h^0 - p_l^0}{p^0},$$

where $p^c = \mathbb{E}[p_s^c]$ and $p^0 = \mathbb{E}[p_s^0]$.

The proof of Proposition 3 is given in Appendix 3.

5 A Role for Macroprudential Regulation

5.1 Characterizing constrained efficiency

Because of financial frictions and scarcity of bank capital, the equilibrium volume of credit K_s and capital price p_s fluctuate more than the (first-best) optimal volume of credit $K_s^{FB} = S(R_s)$ and capital price $p_s^{FB} = R_s$. Of course the relevant comparison is with the (second-best) optimum[15], which is subject to the same constraints as the competitive equilibrium:

$$p_s(K_s - E_s) = (R_s - b)K_s, \tag{14}$$

$$\text{where} \quad p_s = F'(1 - K_s),$$

$$\text{and} \quad \mathbb{E}[p_s(E_s - E)] = 0. \tag{15}$$

Constraint (14) means that investors obtain, in each state s the same return on bank deposits and on their direct investments in the traditional sector. Constraint (15) expresses the equilibrium in contingent capital markets at date 1. By eliminating E_s, these two constraints can be combined into a single constraint that applies both to the competitive equilibrium and to the regulator. Indeed, the regulator can effectively control (through capital regulation) the volumes of lending K_h and K_l of the banks in each state, but he is subject to the same (ex-ante) financing constraint as the banks, namely the aggregate form of condition (9):

$$\mathbb{E}[K_s(p_s - R_s + b)] \leq \mathbb{E}[p_s]E, \tag{16}$$

$$\text{where} \quad p_s = F'(1 - K_s),\ s = h, l. \tag{17}$$

Definition 1
A constrained efficient capital allocation is a vector (K_l, K_h) that maximizes the aggregate expected output of the economy under constraints (16) and (17).

5.2 The Competitive Allocation is not constrained efficient

We now establish the main result of the paper.

[15] If a central planner could impose lump-sum taxes on investors and distribute the proceeds to bankers (in such way that $E \geq b\frac{\mathbb{E}[S(R_s)]}{R}$) the competitive allocation would coincide with the first-best allocation. However such a forced redistribution seems hardly politically feasible.

Proposition 4

The competitive allocation (K_l^c, K_h^c) is generically constrained inefficient. In particular if capital supply is log-concave, aggregate expected output can be increased by reducing K_h and increasing K_l.

Proof of Proposition 4:

Suppose by contradiction that $K^c = (K_l^c, K_h^c)$ maximizes aggregate expected output,

$$Y(K_l, K_h) = \mathbb{E}[F(1-K_s) + R_s K_s]$$

under the participation constraint of investors

$$G(K_l, K_h) = \mathbb{E}[F'(1-K_s)(K_s - E) + (b - R_s)K_s] \leq 0.$$

By the Kuhn-Tucker Theorem, the gradients of Y and G at K^c must be colinear. Now

$$\begin{aligned}
\frac{\partial Y}{\partial K_s} &= \pi_s[-F'(1-K_s^c) + R_s] = \pi_s[-p^c + R], \\
\frac{\partial G}{\partial K_s} &= \pi_s[F'(1-K_s^c) + b - R_s - F''(1-K_s^c)(K_s^c - E)] \\
&= \pi_s[p^c + b - R - F''(1-K_s^c)(K_s^c - E)].
\end{aligned}$$

These gradients are colinear if and only if $F''(1-K_s^c)(K_s^c - E)$ is the same in both states. This is generically not true, since $K_h^c > K_l^c$. Moreover, when $\log S$ is concave, $\frac{S'}{S}$ is a decreasing function and the direction of improvement can be determined. Indeed, $-F''(1-K_s^c) = \frac{1}{S'(p_s^c)}$ and

$$-K_h^c F''(1-K_h^c) = \frac{S(p_h^c)}{S'(p_h^c)} > \frac{S(p_l^c)}{S'(p_l^c)} = -K_l^c F''(1-K_l^c).$$

This is because $p_h^c - p_l^c = R_h - R_l > 0$. Now we also have that $K_h^c > K_l^c$ and thus

$$\frac{K_h^c - E}{K_h^c} > \frac{K_l^c - E}{K_l^c} > 0.$$

By multiplying these two inequalities and using that $F''(1-K_s^c) < 0$, we obtain

$$-F''(1-K_h^c)(K_h^c - E) > -F''(1-K_l^c)(K_l^c - E),$$

and thus

$$\left(\frac{\partial G}{\partial K_h} / \frac{\partial G}{\partial K_l}\right)(K^c) > \left(\frac{\partial Y}{\partial K_h} / \frac{\partial Y}{\partial K_l}\right)(K^c).$$

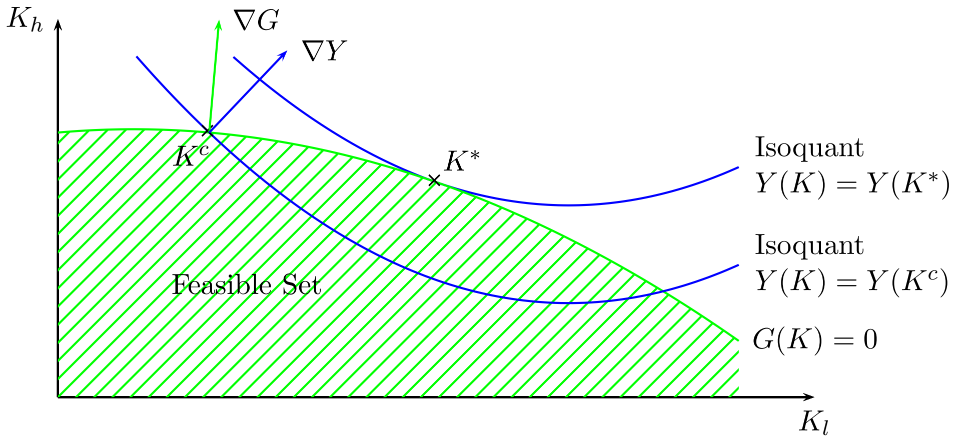

Figure 3: The competitive equilibrium K^c is not constrained efficient. The constrained optimum is K^*.

As shown in Figure 3, aggregate expected output can be increased marginally around K^c by reducing K_h while increasing K_l so that $G(K_l, K_h)$ remains constant.

■

We note that the log-concavity of capital supply can be related to the hazard rate of entrepreneurs'productivity distribution. Suppose that there is a continuum of entrepreneurs, and each entrepreneur has access to an indivisible project of size one. The productivity of these projects is distributed according to some differentiable distribution function that generates the production function $F(\cdot)$.[16] In such a set-up, log-concavity of capital supply is identical to the condition that the hazard rate of entrepreneurs' productivity is declining.[17]

5.3 A counter-cyclical capital ratio can improve social welfare

Suppose that the regulator imposes to all banks, but only during booms, a capital ratio based on their **initial** capital e:

$$e \geq \rho^* k_h. \tag{18}$$

[16]Formally, if $G(x)$ is the distribution function, then $F(1 - K_s) = \int_{G^{-1}(K_s)}^{\infty} x dG(x)$.

[17]See Garicano, Lelarge and Van Reenen 2013 for evidence on the distribution of firm productivities. They indicate that the distribution is between log normal and power (and there is a good fit for a power distribution). Such distributions imply log-concavity.

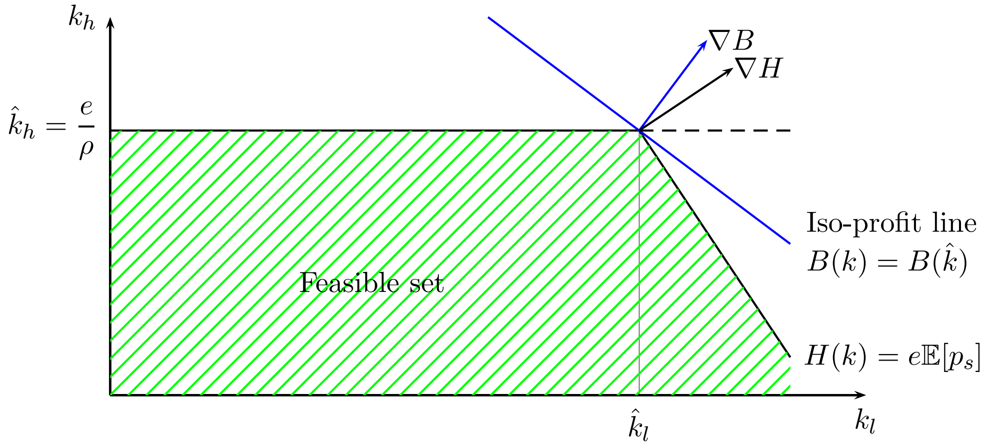

Figure 4: Feasible plans for a regulated bank

By appropriately selecting ρ^*, and letting banks compete to attract investors, the regulator can implement the constrained efficient allocation of capital K^* as we show next:

Proposition 5
We assume that S is log-concave. If the regulator sets $\rho^* = \dfrac{E}{K_h^*}$, the regulated competitive equilibrium allocation coincides with the constrained efficient allocation K^*.

Proof of Proposition 5:
Consider first the behavior of banks. A typical bank, with capital e, will select contingent plans for lending k_s and contingent capital e_s so as to maximize $b\mathbb{E}[k_s]$ under two constraints: the regulatory constraint (18) and the participation constraint of investors:

$$H(k_l, k_h) \equiv \mathbb{E}[k_s(p_s - R_s + b)] \leq e\mathbb{E}[p_s]. \tag{19}$$

The feasible set of the bank now has a kink $\hat{k}$. It is represented in Figure 4, together with the iso-profit line that passes through the kink $\hat{k}$.

Let the minimal capital ratio $\rho^* = \frac{E}{K_h^*}$ be imposed only in state h. Since $\hat{p}_s = F'(1 - K_s^*)$, where (K_l^*, K_h^*) are the aggregate amounts invested in the banking sector, the kink $\hat{k}$ coincides with the contingent plans of the bank in the constrained efficient allocation:

$$\hat{k}_s = \frac{e}{E} K_s^*.$$

This property is tautological for $s = h$, and results from equation (19) for $s = l$. If we prove that $\hat{k}$ indeed maximizes the bank's profit on the feasible set, Proposition 5 will be established. Now recall that, by definition, K^* maximizes expected output $Y(K)$ under the aggregate feasibility constraint $G(K) \leq 0$. Thus ∇Y and ∇G are colinear at K^*, implying the existence of a multiplier λ such that:

$$\frac{\partial Y}{\partial K_s} = \pi_s(-p_s + R_s) = \lambda \frac{\partial G}{\partial K_s} = \lambda \pi_s \left[p_s + b - R_s + \frac{K_s^* - E}{S'(p_s)} \right]$$

where we have used the fact that $-F''(1-K_s) = \dfrac{1}{S'(p_s)}$. Solving for $p_s - R_s$, we get

$$p_s - R_s = \frac{\lambda \left[-b - \frac{K_s^* - E}{S'(p_s)} \right]}{1+\lambda}.$$

We need to establish that the gradient of the profit function $B(k) = b\mathbb{E}[k_s]$ is above the gradient of the constraint $H(k) = e\mathbb{E}[p_s]$ (see Figure 4). Now

$$\frac{\partial B}{\partial k_s} = b\pi_s \quad \text{and} \quad \frac{\partial H}{\partial k_s} = \pi_s(p_s - R_s + b).$$

Since S is log-concave and $p_h > p_l$, we know that

$$\frac{S(p_h)}{S'(p_h)} > \frac{S(p_l)}{S'(p_l)} \quad \text{and} \quad \frac{K_h^* - E}{K_h^*} > \frac{K_l^* - E}{K_l^*}.$$

Thus by multiplying these inequalities we obtain

$$\frac{K_h^* - E}{S'(p_h)} > \frac{K_l^* - E}{S'(p_l)},$$

which establishes the desired result, namely

$$\frac{\partial B/\partial k_h}{\partial B/\partial k_l} > \frac{\partial H/\partial k_h}{\partial H/\partial k_l}.$$

■

An important comment is in order at this stage. The regulation that we propose in Proposition 5 (and which allows to correct the constrained inefficiency of the competitive equilibrium) is **in the spirit** of counter-cyclical capital regulations that are currently considered by the Basel committee and several domestic regulators: It amounts to impose a cap on bank lending during booms. However what regulators propose are **classical** capital ratios, based on **interim** levels of bank capital, i.e.

$$e_h \geq \rho k_h. \tag{20}$$

As we show below, such a regulation is inefficient in our model: Through contingent contracts signed at $t = 0$, banks can guarantee themselves a high level of capital e_h in the boom (the counterpart being a low level of capital e_l in the recession), which still leads to a highly pro-cyclical lending policy, in spite of the regulator's attempt to stabilize credit fluctuations, through the counter-cyclical capital requirement.

In contrast with constraint (18), which directly limits the volume of lending in the boom, constraint (20) can largely be by-passed by banks through contingent financing contracts. To effectively limit the volume of lending in the boom, with a "classical" capital ratio, the regulator must choose a very high value for this ratio ρ which distorts banks' capital allocation and indirectly reduces the volume of lending in the recession.

To illustrate the inefficiency of "classical" capital regulation, we now characterize the set of contingent credit allocations that can be implemented by such a regulation, allowing ρ to be chosen arbitrarily by the regulator. Suppose $K = (K_l, K_h)$ is such an allocation. The corresponding capital prices are $p_s = F'(1 - K_s)$, $s = h, l$. We have to check that, confronted with p, p_h and p_l, an individual bank with initial capital e selects the individual allocation $\frac{e}{E}K$. By linearity, we can also study the "representative" bank that owns the entire capital of the banking sector ($e = E$) but behaves competitively. In other words we only have to check that K maximizes the representative bank's expected profit under the regulatory constraint (20) and the usual financing constraints:

$$
\begin{aligned}
& \max b\mathbb{E}[K_s] \\
\text{s.t.} \quad & K_h \leq \frac{E_h}{\rho}, \\
& K_l \leq \frac{E_l}{1 - \frac{R_l - b}{p_l}}, \\
& \mathbb{E}[p_s(E_s - E)] \leq 0.
\end{aligned}
$$

It is easy to see that all the constraints are binding at the solution. By eliminating E_h and E_l, one obtains a unique constraint:

$$\pi_h p_h \rho K_h + \pi_l (p_l + b - R_l) K_l \leq \mathbb{E}[p_s]E. \tag{21}$$

The solution can only be interior if

$$p_h \rho = p_l + b - R_l, \tag{22}$$

which determines ρ. In this case, constraint (21) becomes

$$(p_l + b - R_l)(\pi_h K_h + \pi_l K_l) \leq \mathbb{E}[p_s]E. \tag{23}$$

Conversely, if K satisfies (23) (with $p_s = F'(1 - K_s)$, $s = h, l$), it can be implemented by regulation (20) by choosing

$$\rho = \frac{p_l + b - R_l}{p_h} = \frac{F'(1 - K_l) + b - R_l}{F'(1 - K_h)}.$$

Thus we have established

Proposition 6

A contingent credit allocation $K = (K_l, K_h)$ can be implemented by a "classical" capital ratio such as (20) if and only if two conditions are satisfied:

$$J(K) \equiv (F'(1 - K_l) + b - R_l)\mathbb{E}[K_s] - \mathbb{E}[F'(1 - K_s)]E \leq 0, \tag{24}$$

and the participation constraint of investors:

$$G(K) = \mathbb{E}[(F'(1 - K_s) + b - R_s)K_s - EF'(1 - K_s)] \leq 0. \tag{25}$$

An immediate corollary of Proposition 6 is that a "classical" capital ratio such as (20) is inefficient. In the relevant region (south-east of the competitive equilibrium, i.e. $(K_h < K_h^c$, $K_l > K_l^c)$, the curve defined by (24) is strictly below the curve defined by binding the participation constraint of investors:

$$G(K) = \mathbb{E}[(F'(1 - K_s) + b - R_s)K_s - EF'(1 - K_s)] = 0.$$

Indeed

$$J(K) - G(K) = \pi_h K_h [F'(1 - K_l) - R_l - F'(1 - K_h) + R_h].$$

In the relevant region ($K_h < K_h^c$, $K_l > K_l^c$) we have

$$F'(1 - K_l) - F'(1 - K_h) > F'(1 - K_l^c) - F'(1 - K_h^c) = R_l - R_h.$$

Thus in that region $J(K) > G(K)$ as was to be established. By contrast, in the north-west quadrant of the competitive equilibrium, the market imposed ratio is more restrictive than the regulatory ratio, which means that $J(K) < G(K)$. Thus the feasible set, which is characterized by $\max(J(K), G(K)) \leq 0$, has a kink at the competitive equilibrium.

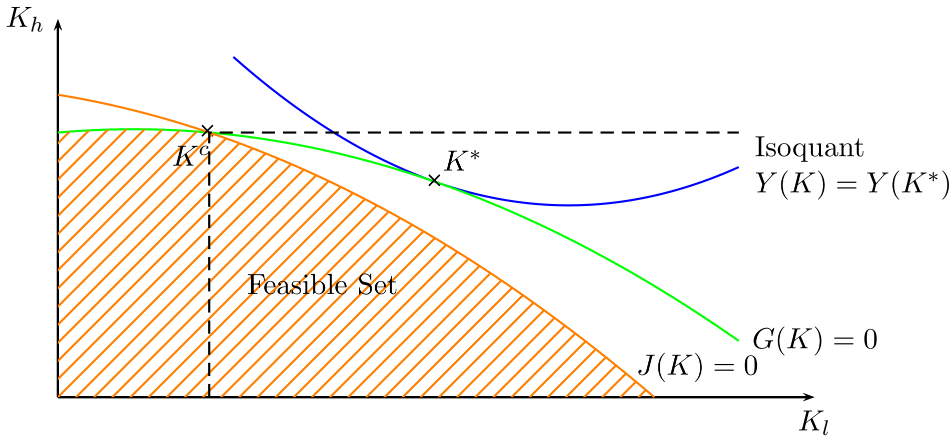

Figure 5: The feasible set with a "classical" capital ratio. The constrained efficient allocation K^* does not belong to this set.

Figure 5 illustrates the inefficiency of "classical" capital ratios: the second best optimum K*, which can be implemented by a capital regulation based on the bank's **initial equity**, as established in Proposition 5, cannot be implemented by a "classical" capital ratio, based on the bank's **interim equity**.[18]

5.4 Robustness checks

This section relaxes two assumptions, namely the binary support of the macro-shock s and the constant friction parameter b. Suppose that s is a real number and can take an arbitrary number of values (with the convention that a higher s corresponds to a higher return R_s) and the friction parameter b_s varies with s. If markets are complete, a bank having initial equity e will choose its contingent lending plans k_s so as to maximize $\mathbb{E}[b_s k_s]$ under the participation constraint of investors:

$$\mathbb{E}[k_s(p_s + b_s - R_s) - p_s e] \leq 0. \tag{26}$$

An interior solution is only possible when b_s is proportional to $p_s + b_s - R_s$. Denoting by b and p the expectations of b_s and p_s, this implies that

$$p_s = R_s - (R - p)\frac{b_s}{b}. \tag{27}$$

Thus the equilibrium price in state s is a convex combination of the return on banks' assets R_s and of the pledgeable income $R_s - b_s$. If b_s is lower in good states and higher in bad states

[18] In practice, such an approach requires that bank equity capital ratios for trend growth of GDP are determined and regulatory capital requirements are based on these data.

(thus b_s and R_s move in opposite directions, i.e. b_s decreases in s) then instability is reinforced: even in absolute terms (i.e. not dividing by expectations), capital prices fluctuate more than fundamentals. However the opposite is true if b_s and R_s move together, for example if they are positively proportional, an assumption often made in the literature on financial frictions.

We can establish the existence and uniqueness of a competitive equilibrium like in Proposition 2. The equilibrium value of p, which we denote p^c is the unique value of p that equalizes demand and average supply:

$$\mathbb{E}[\frac{b_s}{b}S(R_s - (R-p)\frac{b_s}{b})] = \frac{E}{1-\frac{R-b}{p}}. \tag{28}$$

The variance of b_s and its covariance with R_s have an impact on the equilibrium value of p. As an illustration, consider the case where $S(p) = p$ and b_s proportional to R_s. Easy computations show that

$$p^c = R - b + \frac{R^2 E}{\mathbb{E}[R_s^2]}. \tag{29}$$

Since $R^2 < \mathbb{E}[R_s^2]$, we see that in this case, more banking capital is needed to stabilize the economy than in the case where frictions are constant.

Proposition 4 can be extended to this more general set-up. The competitive equilibrium is generically constrained inefficient: the equilibrium allocation of capital does not maximize expected welfare $Y(K)$ under the participation constraint of investors to the financing of the banks, i.e.

$$G(K) = \mathbb{E}[(F'(1-K_s) + b_s - R_s)K_s - EF'(1-K_s)] \leq 0. \tag{30}$$

Of course, in this more complex set-up, it is not possible anymore to implement the second best optimum by a simple (uniform) capital ratio. But when S is log concave and b_s is decreasing in s (more frictions in bad times), it can be established that, starting from the competitive allocation, social welfare can be improved by forcing banks to reduce lending in good states (i.e. when s is greater than some threshold s^*). Banks react by increasing lending in the other states, in such a way that the participation constraint of investors remains binding.

6 Simple Extensions of the Model

6.1 Chained Financial Frictions

In the basic version, we have assumed that bankers cannot pledge all future revenues to investors, but they can lend frictionlessly to entrepreneurs. The model, however, can easily be extended to circumstances when bankers themselves face the problem that entrepreneurs cannot fully pledge their output[19]. Suppose, e.g., that entrepreneurs can secretly divert part of the assets by an (inefficient) technology. If the entrepreneur chooses to divert assets, he obtains b^{en} ($b^{en} > 0$) consumption goods per unit of assets diverted. The banker, therefore, has to set the lending rate in such a way that the entrepreneur obtains at least b^{en} per unit of loan granted to him in order to avoid diversion. Hence, bankers receive $R_s - b^{en}$. Bankers themselves could divert the assets, thereby earning b^{ba} ($b^{ba} > 0$) per unit of assets diverted. Hence, in total, the maximal pledgeable income the bankers can offer to investors is $R^s - b^{en} - b^{ba} = R^s - b$, where $b := b^{en} + b^{ba}$ reflects the total intensity of financial frictions.

All of our preceding results can be applied to this chain of financial frictions, by interpreting b as the total intensity of financial frictions. Counter-cyclical capital ratios of the form $e \geq \rho^* K_h$ with $\rho^* = \frac{E}{K_h^*}$ can improve welfare, as outlined in Proposition 5. Entrepreneurs and bankers will obtain the expected rents $b^{en}\mathbb{E}[K_s^*]$ and $b^{ba}\mathbb{E}[K_s^*]$, respectively.

6.2 Stabilization of Wage Fluctuations

The preceding analysis has focused on the case of a single input for production. However, the model is easily reformulated with more standard macroeconomic production functions where capital and labor are inputs. More specifically, suppose that there is an additional continuum of workers endowed with an aggregate amount of labor $\overline{L}$, which is normalized to $\overline{L} = 1$.[20] $\overline{L}$ is supplied inelastically in the labor market. The traditional sector is characterized by a production function with constant returns to scale

$$\tilde{F}(1 - K, L)$$

[19] This is in the spirit of Holmström-Tirole (1997) and Meh-Moran (2010).

[20] The equilibrium would be the same if investors are endowed with labor and no separate group of workers is present.

where $1-K$ and L are the inputs of capital and labor respectively. $\tilde{F}(1-K, L)$ is assumed to fulfill the standard assumptions of positive but decreasing marginal products of capital and labor $\left(\frac{\partial F}{\partial(1-K)} > 0, \frac{\partial^2 F}{\partial(1-K)^2} < 0, \frac{\partial F}{\partial L} > 0, \frac{\partial^2 F}{\partial L^2} < 0\right)$. The textbook example is the Cobb-Douglas production function

$$\tilde{F}(1-K, L) = A(1-K)^{\alpha} L^{1-\alpha}$$

with $0 < \alpha < 1$, $A > 0$. The technology in the traditional sector is operated by a representative firm that acts competitively in the markets for capital, labor and consumption goods. We observe that profit maximization of the representative firm and market clearing in the labor and capital market in state s imply

$$\frac{\partial \tilde{F}(1-K_s, 1)}{\partial(1-K_s)} = p_s, \tag{31}$$

$$\frac{\partial \tilde{F}(1-K_s, 1)}{\partial L} = w_s. \tag{32}$$

In the case of Cobb-Douglas production functions, we obtain

$$\begin{aligned} \alpha A(1-K_s)^{\alpha-1} &= p_s, \\ (1-\alpha)A(1-K_s)^{\alpha} &= w_s. \end{aligned}$$

In this case, workers and investors share the output in the traditional sector according to

$$\begin{aligned} p_s(1-K_s) &= \alpha A(1-K_s)^{\alpha}, \\ w_s\overline{L} &= (1-\alpha)A(1-K_s)^{\alpha}. \end{aligned}$$

We also note that all preceding considerations continue to hold when we define

$$F(1-K_s) = \tilde{F}(1-K_s, 1).$$

Hence, we obtain

Proposition 7
If capital supply is log-concave, the capital requirement $k_h \leq \frac{e}{\rho^*}$ *where* $\rho^* = \frac{E}{K_h^*}$ *reduces the fluctuations of wages in the economy.*

Proposition 7 follows from Proposition 5 and the equilibrium condition (32) for wages together with the properties of $\tilde{F}(\cdot, \cdot)$.

Proposition 7 shows that imposing a cap on lending in booms by capital requirements has indirect effects on other factor prices in the economy. This property has implications beyond the results derived so far, if workers are risk-averse and lack access to contingent markets (or asset markets which allow the same type of trades). In such cases, counter-cyclical capital regulations partially insure workers against wage fluctuations and thus partially substitute their lack of access to advanced financial markets.

6.3 Distributing the Gains from Macroprudential Policies

6.3.1 Forms of Redistribution

So far, we have focused on how bank capital requirements can increase aggregate output. From a utilitarian perspective, therefore, imposing stricter capital ratios in booms is welfare-improving, as all individuals are risk-neutral. Besides this traditional macroeconomic focus, one might also be interested in the distributional consequences of macroprudential policies and in how such consequences might be altered. This is in line with recurrent monetary policy debates of the welfare costs of inflation and their distribution within society. Like inflation - or its absence -, macroprudential policy generates winners, but it may also hurt some segments of society, depending on the dispersion of factors of production and the organization of the economy. In our most simple set-up with only investors and bankers, we face a particular form of distribution of welfare gains. The gains accrue solely to bankers, and the expected consumption of investors declines.[21]

There are at least three ways how gains of bankers from macroprudential policies are or can be distributed to the other agents in the economy, thereby ensuring a Pareto improvement. For this purpose, the simple model in the preceding sections has to be embedded into a broader context. We briefly outline the first two channels and then provide a detailed analysis of the third channel.

The first way are transfers that occur in multi-member households. For instance, let us consider the economy in the last section and suppose that a typical household is composed of two individuals: a banker (with endowment e_1 of the capital good) and a worker (with endowment e_2 of the capital good and labor endowment l). Household members stay together, as they

[21] At the competitive equilibrium a social welfare function $\beta\{\mathbb{E}[(F(1-K_s)] + \mathbb{E}[(R_s - b)K_s]\} + (1-\beta)b\mathbb{E}[K_s]$ is maximized for some value of $\beta \in (0,1)$. Imposing capital requirements in the boom increases $\mathbb{E}[K_s]$ at the expense of the expected consumption of investors.

benefit from group externalities or they may use household formation to pool resources. Total endowment of the household that the banker can use as capital in his bank is $e = e_1 + e_2$. Then, the expected amount of consumption goods of the household increases when capital requirements are imposed in booms. Moreover, both household members gain if they allocate income according to their contributions and e_2 is sufficiently high, or if they Nash bargain over consumption using exit and income shares determined by law as threat point, or simply allocate consumption in the household according to some sharing rule.

A second type of redistribution is possible in a monetary version of our model, when cash has to be used to buy consumption goods and financial frictions are caused by cash diversion. In such circumstances, bankers who divert cash have to go to the marketplace to buy consumption goods. Then, it is possible to tax such purchases of consumption goods and tax revenues can be distributed to investors. The important point is that bankers cannot avoid such taxation - whether they divert cash or receive rents from their lending activities. Hence, the intensity of financial frictions does not increase when consumption goods purchases are taxed. As a consequence, when capital regulation is introduced, the gains of bankers can be collected by introducing (or increasing) sale taxes and the tax revenues can be redistributed to investors.

In the next subsection, we outline in detail how taxation can engineer a Pareto improvement in the context of multiple consumption goods, which represents a third type of redistribution of gains from capital regulation.

6.3.2 Taxation of Complementary Consumption Goods

If we embed the economy in a broader context with multiple consumption goods, the gains of bankers can be distributed by taxing consumption goods that cannot be completely substituted by the non-pledgeable output in the banking sector. In such circumstances, taxation does not, or only moderately increases the intensity of financial frictions, i.e. the non-pledgeable share of the output. Tax revenues are distributed to investors (and workers if present).[22] We illustrate the working of such a redistribution with a simple example deliberately designed to preserve the structure and the results of our model in sections 1 to 5.[23] We focus on financial frictions

[22] This is related to the classical observation of e.g. Arnott and Stiglitz (1990) that models with moral hazard have to be put into a broader context with multiple consumption goods, to be able to derive robust conclusions. In our context, however, the purpose of taxation is not to alleviate moral hazard, but to redistribute the gains from macroprudential policies.

[23] The same procedure can be applied to the variant of the model in section 6.2.

generated by asset diversion or inalienability of human capital as those forms are the easiest to illustrate.

We assume that all agents in the model variant in sections 1 to 5 or section 6.2 derive utility from two goods in $t = 2$. Their preferences are expressed by Bernoulli utility functions

$$u(c_1^s, c_2^s) = c_1^s + \mu \ln c_2^s$$

where c_1^s (resp. c_2^s) is the first (resp. second) good consumed in state s and agents maximize their expected utilities $\mathbb{E}[u(c_1^s, c_2^s)]$. The first good is the output in the traditional and in the banking sector and we call it car. The second good called bread can be produced in $t = 2$ according to a linear production function

$$Z = \gamma X \tag{33}$$

where $\gamma > 0$ and X is the amount of the first good that serves as input to this production. Typically, transportation equipments can be used for both purposes, consumption and production of other consumption goods.

The bread sector is operated by a continuum of (passive) entrepreneurs which will make zero profit in equilibrium. The price of bread in terms of cars is denoted by p_Z. Due to the linear technology the equilibrium price p_Z^c will be equal to γ.

We first derive the competitive equilibrium without regulation. With asset diversion or inalienability of human capital, in $t = 2$, a banker receives bk_s of the first consumption good in state s. Hence, he solves the following problem

$$\begin{aligned} &\max_{\{c_1^s, c_2^s\}} \{c_1^s + \mu \ln c_2^s\} \\ \text{s.t.} \quad & c_1^s + p_Z c_2^s = bk_s \end{aligned}$$

which yields in equilibrium[24]

$$\begin{aligned} c_1^s &= bk_s - \mu, \\ c_2^s &= \frac{\mu}{p_Z} = \frac{\mu}{\gamma}. \end{aligned}$$

We note that c_2^s is independent of the realization of the macroeconomic shock s.

[24] Throughout this section, we assume interior solutions ($c_1^s > 0$, $c_2^s > 0$).

Anticipating their behavior in $t = 2$ and the state independent equilibrium price $p_Z = \gamma$, the expected utility of the banker in $t = 0$ can be rewritten as

$$\mathbb{E}[c_1^s] + \mathbb{E}[\mu \ln c_2^s] = \mathbb{E}[bk_s - \mu] + \mu \ln(\frac{\mu}{p_Z}) = \mathbb{E}[bk_s] - \mu + \mu \ln(\frac{\mu}{\gamma}). \tag{34}$$

Hence, the problem of the banker in $t = 0$ is

$$\max_{\{k_s\}} \{\mathbb{E}[bk_s] - \mu + \mu \ln(\frac{\mu}{\gamma})\}$$
$$\text{s.t.} \quad \mathbb{E}[k_s(p_s + b - R_s) - p_s e] \leq 0.$$

As μ and γ are constants, we obtain

Proposition 8
The competitive equilibrium (K_h^c, K_l^c) in the car-bread economy coincides with the competitive equilibrium in the one-consumption good economy.

We next observe that we can apply the preceding results on capital regulation to the first consumption good.

Corollary 1
If capital supply is log-concave, the aggregate amount of the first consumption good can be increased by imposing the capital ratio $k_h \leq \frac{e}{\rho^*}$ with $\rho^* = \frac{E}{K_h^*}$ in the boom.

We next describe how the increase of the expected aggregate consumption good can be redistributed to investors in order to engineer a Pareto improvement. For this purpose, we impose a sales tax τ ($\tau > 0$) on buying bread. As the equilibrium producer price stays at γ, agents demanding bread have to pay $\gamma(1 + \tau)$ per unit of bread. We observe

Lemma 1
For asset diversion or non-alienability of human capital, the payment to the banker is b per unit of assets and is independent of the tax rate τ.

We show the Lemma for the case of asset diversion. Suppose that bread is taxed at tax rate τ, and thus $p_Z = (1 + \tau)\gamma$. If the banker has received capital goods k_s, he can divert secretly bk_s units of cars. He will buy bread in $t = 2$, facing the price $p_Z = (1 + \tau)\gamma$. Using the expression derived in (34), the banker's expected utility upon diversion is

$$\mathbb{E}[bk_s] - \mu + \mu \ln\left(\frac{\mu}{(1 + \tau)\gamma}\right).$$

Hence, by paying at least b per unit of capital invested, asset diversion is avoided, independently of the tax rate τ.

Finally, we can determine the maximal tax rate, denoted by τ^{max}, that can be levied on bankers. In the aggregate, bankers are equally well off in the unregulated economy as in the economy with capital regulation and taxation of bread if

$$\mathbb{E}[bK_s^c] - \mu + \mu \ln(\frac{\mu}{\gamma}) = \mathbb{E}[bK_s^*] - \mu + \mu \ln\left(\frac{\mu}{(1+\tau^{max})\gamma}\right)$$

which yields

$$\ln(1+\tau^{max}) = \frac{b}{\mu}\left\{\mathbb{E}[K_s^*] - \mathbb{E}[K_s^c]\right\}.$$

Tax revenues in this case from bankers amount to $\tau^{max} c_2^s = \dfrac{\mu}{1+\tau^{max}}\tau^{max}$, which are independent of the state of the world. If μ is not too small, those tax revenues are sufficiently large to engineer a Pareto improvement.

7 A Fully Dynamic Model

Our benchmark model is essentially static. However, if we maintain the assumption of complete financial markets (admittedly more restrictive in a dynamic set-up), this model can easily be made fully dynamic. Consider indeed an infinitely repeated version of the benchmark model. At date $t = 0$ complete contingent markets are opened for all future periods $t \geq 1$ and all future states of the world s_t. At any future date $t \geq 1$, a random state s_t is drawn from a continuous distribution with a (time invariant) compact support. This state determines the productivity $R(s_t)$ of SMEs and the production function $a(s_t)F(1-K_t)$ in the traditional sector[25]. Bankers and investors receive at each period the same initial endowments[26] e (E on aggregate) and $1-E$, respectively. We assume full depreciation of capital at each period. The price of capital at date t is equal to its marginal productivity in the traditional sector: $p_t = a(s_t)F'(1-K_t)$. The intensity of frictions $b(s_t)$ also depends on s_t. The assumptions we need to maintain the basic properties of the benchmark model are:

$$\text{(A)} \quad a(\cdot) \nearrow, \quad \frac{R(\cdot)}{a(\cdot)} \nearrow, \quad \text{and} \quad b(\cdot) \searrow .$$

[25]Thus we relax our simplifying assumption that the traditional sector is not affected by the macro shocks.

[26]This assumption is not crucial: e (and E) could vary with s_t.

Thus, all the firms are more productive when s is high (good times), but the SMEs' relative productivity (vis-à-vis the traditional sector) also increases in s. Moreover, financial frictions are (weakly) less intense in good times.

Each bank forms a sequence $(k_t)_t$ of lending plans that can be conditional on history $s^t = (s_1, \ldots, s_t)$ at each date. Investors accept to participate if and only if:

$$\sum_{t\geq 1} \beta^t \mathbb{E}\Big[\{p_t + b(s_t) - R(s_t)\}k_t - p_t e\Big] \leq 0,$$

where $\beta < 1$ is the discount factor, supposed to be the same for bankers and investors. The objective function of a typical banker is

$$\sum_{t\geq 1} \beta^t \mathbb{E}\Big[b(s_t)k_t\Big].$$

In any interior equilibrium the coefficients of k_t in this objective function and in the participation constraint of investors must be proportional:

$$\exists \quad \lambda \quad | \quad b(s_t) \equiv \lambda\{p_t + b(s_t) - R(s_t)\}.$$

Thus

$$p_t = R(s_t) - \Big(1 - \frac{1}{\lambda}\Big)b(s_t).$$

Aggregate credit K_t is determined by the capital supply function:

$$K_t = S\Big(\frac{R(s_t)}{a(s_t)} - \big(1 - \frac{1}{\lambda}\big)\frac{b(s_t)}{a(s_t)}\Big).$$

Finally, λ is determined by binding the aggregate participation constraint of investors (which we multiply by λ for convenience):

$$\sum_{t\geq 1} \beta^t \mathbb{E}\Big[b(s_t)S\Big(\frac{R(s_t)}{a(s_t)} - \big(1 - \frac{1}{\lambda}\big)\frac{b(s_t)}{a(s_t)}\Big) - \big\{\lambda R(s_t) + (1-\lambda)b(s_t)\big\}E\Big] = 0.$$

Since $0 < b(s_t) < R(s_t)$ and $S(\cdot) \nearrow$, the left-hand side of this condition is a decreasing function φ of λ. Now

$$\varphi(1) = \sum_{t\geq 1} \beta^t \mathbb{E}\Big[b(s_t)S\Big(\frac{R(s_t)}{a(s_t)}\Big) - R(s_t)E\Big],$$

which is positive for E small enough. Note that φ tends to $-\infty$ when $\lambda \to \infty$. Thus the equation above has a unique solution $\lambda^E > 1$. By contrast, if E is large enough so that

$\varphi(1) < 0$, the competitive equilibrium is such that $\lambda = 1$, $p(s_t) = R(s_t)$ and the first-best allocation is attained. Thus we have

Proposition 9

There is a unique competitive equilibrium. When

$$E < \frac{\sum_{t\geq 1} \beta^t \mathbb{E}\Big[b(s_t) S\big(\frac{R(s_t)}{a(s_t)}\big)\Big]}{\sum_{t\geq 1} \beta^t \mathbb{E}\Big[R(s_t)\Big]},$$

this equilibrium entails positive spreads

$$R(s_t) - p_t = \big(1 - \frac{1}{\lambda^E}\big) b(s_t) > 0.$$

Prices only depend on the current state s_t (Markov property) and spreads are decreasing in s_t (anti-cyclicality). Bank lending is procyclical and Markovian:

$$K_t^E = S\Big(\frac{R(s_t)}{a(s_t)} - \big(1 - \frac{1}{\lambda^E}\big)\frac{b(s_t)}{a(s_t)}\Big).$$

The property of generic inefficiency of the competitive equilibrium still holds:

Proposition 10

The competitive equilibrium is (generically) constrained inefficient. Moreover, when $\log S$ is concave, social welfare can be increased by a small variation of bank credit around $K^E : \triangle K_t = \dfrac{\alpha(s_t)}{b(s_t)}\varepsilon$, where $\varepsilon > 0$ and $\alpha(s_t)$ are weights that are negative for s large (good times) and positive for s small (bad times).

Proof of Proposition 10:

Social welfare equals

$$W = \sum_{t\geq 1} \beta^t \mathbb{E}\big[a(s_t) F(1 - K_t) + R(s_t) K_t\big].$$

A small variation around K^E gives:

$$\begin{aligned}
\triangle W &= \sum_{t\geq 1} \beta^t \mathbb{E}\Big[\big(R(s_t) - p_t^E\big) \triangle K_t\Big] \\
&= \sum_{t\geq 1} \beta^t \mathbb{E}\Big[\big(1 - \frac{1}{\lambda^E}\big) b(s_t) \triangle K_t\Big].
\end{aligned}$$

Given the expression $\triangle K_t = \frac{\alpha(s_t)}{b(s_t)}\varepsilon$, this gives:

$$\triangle W = \sum_{t \geq 1} \beta^t \big(1 - \frac{1}{\lambda^E}\big)\mathbb{E}\big[\alpha(s_t)\big]\varepsilon.$$

Thus, $\triangle W > 0$ iff $\sum_{t\geq 1} \beta^t \mathbb{E}\big[\alpha(s_t)\big] > 0$. Now the participation constraint of investors is

$$\sum_{t\geq 1} \beta^t \mathbb{E}\Big[\big\{p_t(K_t) + b(s_t) - R(s_t)\big\}K_t - p_t(K_t)E\Big] = 0.$$

To be satisfied to the first order after the variation of K^E it must be that

$$\sum_{t\geq 1} \beta^t \mathbb{E}\Big[\big\{p_t'(K_t)(K_t - E) + p_t^E + b(s_t) - R(s_t)\big\} \triangle K_t\Big] = 0$$

or

$$\sum_{t\geq 1} \beta^t \mathbb{E}\left[\left\{a(s_t)\frac{S(\frac{p_t^E}{a(s_t)})}{S'(\frac{p_t^E}{a(s_t)})}\left(1 - \frac{E}{K_t}\right) + \frac{b(s_t)}{\lambda^E}\right\} \triangle K_t\right] = 0.$$

Now $\triangle K_t = \frac{\alpha(s_t)}{b(s_t)}\varepsilon$, thus it must be that

$$\sum_{t\geq 1} \beta^t \mathbb{E}\left[\alpha(s_t)\left\{\frac{a(s_t)}{b(s_t)}\frac{S(\frac{p_t^E}{a(s_t)})}{S'(\frac{p_t^E}{a(s_t)})}\left(1 - \frac{E}{K_t}\right) + \frac{1}{\lambda^E}\right\}\right] = 0.$$

By log concavity of S, $\frac{S}{S'}(\frac{p_t^E}{a(s_t)})$ increases in s_t. The same is true for $\frac{a(s_t)}{b(s_t)}$ and $1 - \frac{E}{K_t}$. Since $\alpha(s)$ is positive for s small (and the term between brackets is small) and negative for s large (and the term between brackets is large) the above condition implies that

$$\sum_{t\geq 1} \beta^t \mathbb{E}\big[\alpha(s_t)\big] > 0,$$

which gives the desired result that $\triangle W > 0$.

■

In the dynamic model with continuous states of the world, it is not possible anymore to implement the constrained optimum through a simple countercyclical capital ratio. However, such a ratio still increases welfare, as long as it does not change too dramatically the equilibrium allocation. Suppose indeed that banks' lending plans (k_t) have to satisfy

$$k_t \leq \frac{e}{\rho^*} \quad \text{for} \quad s_t \geq s^*,$$

where ρ^* is an equity capital ratio that is binding for $s_t \geq s^*$. By the same reasoning as before, the linearity of the objective and the constraints implies that the banker chooses a corner solution

$$\begin{cases} k_t = \frac{e}{\rho^*} & \text{for} \quad s_t \geq s^* \\ p_t = R(s_t) - \big(1 - \frac{1}{\lambda}\big) b(s_t) & \text{for} \quad s_t < s^* \end{cases}$$

where λ is the Lagrange multiplier associated with the investors' participation constraint.

The competitive equilibrium with regulation (hence the upper index R) is now characterized by

$$p^R(s) = \begin{cases} R(s) - \big(1 - \frac{1}{\lambda^R}\big) b(s) & \text{for } s < s^* \\ R(s^*) - \big(1 - \frac{1}{\lambda^R}\big) b(s^*) \equiv p^* & \text{for } s \geq s^* \end{cases}$$

where λ^R is obtained by binding the participation constraint of investors:

$$\begin{aligned} \varphi^R(\lambda, s^*) =& \mathbb{E}\big[\mathbb{1}_{s<s^*}\big\{b(s)S\Big(\frac{R(s)}{a(s)} - \big(1 - \frac{1}{\lambda}\big)\frac{b(s)}{a(s)}\Big)\big\} - \{\lambda R(s) + (1-\lambda) b(s)\}E\big] + \\ & P[s > s^*]\big[\big\{b(s^*)S\Big(\frac{R(s^*)}{a(s^*)} - \big(1 - \frac{1}{\lambda}\big)\frac{b(s^*)}{a(s^*)}\Big)\big\} - \{\lambda R(s^*) + (1-\lambda) b(s^*)\}E\big] = 0. \end{aligned}$$

It is easy to see that this equation has for all s^* a unique solution $\lambda^R(s^*)$ which characterizes the new (regulated) equilibrium. When $s^* = s_{max}$ (the upper bound of s) the probability $P[s > s^*]$ is zero and we are back to the competitive equilibrium: $\lambda^R(s_{max}) = \lambda^E$.

When $s^* = s_{max} - \varepsilon$, with $\varepsilon > 0$ and small, the competitive equilibrium is only slightly perturbed: $\triangle K_t = \triangle K(s_t)$ is small and satisfies the conditions of Proposition 10: it is positive[27] for $s < s^*$ and negative for $s > s^*$. Thus $\triangle W > 0$, as was to be established.

8 Discussion and Conclusion

Our paper develops a very simple model where financial frictions generate excessive credit fluctuations that can be dampened by counter-cyclical regulation of banks' capital. The source of this inefficiency is a distortion of the allocation of borrowing capacity between good and bad times. Interestingly, in this model, regulation does not impose any cost on the banking industry,

[27] This can be seen by contradiction: $\triangle K(s) < 0$ for $s > s^*$ because the capital ratio binds. If $\triangle K(s)$ was also negative for $s < s^*$, the participation constraint of investors would not be binding, hence a contradiction.

but instead, it works as a coordination device: banks collectively gain from the imposition of a counter-cyclical capital ratio.

We note that our model does not involve defaults of banks. We thus next sketch how defaults of banks and the possibility of banking crises can be introduced into our model. Suppose that in addition to our sophisticated investors, there is a continuum of retail depositors. Those individuals do not trade in contingent markets for capital, but can offer their endowment in $t = 0$ to banks, in the form of deposits that are guaranteed by the government. Possible bail-outs of banks would be financed by lump sum taxation at date $t = 2$. Let us first consider the unregulated economy. If the amount of deposits the banks receive in $t = 0$ is sufficiently large, banks may be unable to raise new funds from sophisticated investors in the bad state at $t = 1$ if the negative shock is sufficiently severe. In such circumstances, banks violate the market-imposed solvency ratio in the bad state in $t = 1$. Furthermore, they may be unable to pay back depositors in $t = 2$ and default. By imposing a stricter capital ratio in the good state, banks will allocate more equity and thus more borrowing capacity to the bad state. This might avoid default. In such cases, counter-cyclical capital regulation may simultaneously reduce credit fluctuations and banking crises.

Suppose, however, that optimal counter-cyclical capital regulation cannot avoid bank default, e.g. because banks are exposed to additional idiosyncratic risk in the bad state. Then, a trade-off arises between the reallocation of borrowing capacity to the bad state and the limitation of the costs of banking crises. Then, microprudential regulation aimed at limiting individual bank lending in relation to equity in bad states – when the risk of default is high – tends to be in conflict with macroprudential regulation that aims at expanding lending in bad states. Therefore, the optimal mix of micro- and macroprudential policy must strike a balance between the costs of possible defaults of a share of banks and the benefits from higher borrowing capacity in bad states.

Numerous extensions deserve further scrutiny. Two avenues for future research are particularly valuable. First, a complementary rationale for counter-cyclical capital regulation may be that such regulation reduces the likelihood of banking crises. As sketched above, counter-cyclical capital regulation might simultaneously moderate credit fluctuations and lower the likelihood of banking crises. Full-fledged extensions of the model may provide the conceptional foundation of such a rationale.

Appendix 1

How different forms of financial frictions generate similar leverage constraints

Asset Diversion

Suppose that managers can secretly divert assets like in Gertler-Karadi (2009) and Gertler-Kiyotaki (2011). The diversion technology is inefficient and only gives to the managers a return $b < R$ per unit of assets diverted. Inefficient asset diversion is avoided if and only if managers get (at least) an expected payment of b per unit of capital invested.

Moral Hazard

Suppose, like Holmström and Tirole (1997) that the banks' assets are risky (they default with probability π) and that bankers can secretely select an inferior technology, characterized by a higher probability of default $\pi + \Delta\pi$, but that provides bankers with a private benefit B per unit of investment. In order to avoid the choice of this inferior technology, bankers must be promised a bonus of $\frac{B}{\Delta\pi}k$ in case of success, which means that the non pledgeable income of investors is at least $b \equiv \pi\frac{B}{\Delta\pi}$ per unit of capital.

Inalienability of Human Capital

Suppose, like Hart and Moore (1994) or Diamond and Rajan (2001), that bank managers can threaten to walk away from their jobs, in which case investors have to replace them by new, less efficient managers that generate lower returns $(1-\theta)R_s$. To keep the initial managers, investors must promise them a payment of at least $b_s = \theta R_s$ per unit of capital.

Haircuts and limits to arbitrage

Suppose, like Gromb and Vayanos (2010), that investors are only ready to lend a fraction of the value of the assets they finance (haircut). A simple case arises when these investors are infinitely risk-averse. Then bankers will only offer them debt that is completely riskless. If by contrast bank loans are risky, the maximum borrowing capacity of the bank in state s (per unit of capital) is the (minimum) amount that can be recovered in case of default R_s^{min}, which is strictly less than R_s.

Appendix 2

Calibration of the Model

We propose here a very simple calibration of our model with complete markets based on a capital ratio $CR = 1 - \frac{R-b}{p}$ of 2%, a ratio of financial frictions $FF = \frac{b}{R}$ of 30% and an elasticity of capital supply $\epsilon^s = \frac{pS'(p)}{S(p)}$ of 1. The competitive equilibrium is thus the same whether or not financial markets are complete.

We measure “volatilities” of random variables by their dispersion coefficients:

$$\sigma_R = \frac{R_h - R_l}{R}, \sigma_p = \frac{p_h - p_l}{p}, \sigma_Y = \frac{Y_h - Y_l}{\mathbb{E}[Y]} \quad \text{and} \quad \sigma_K = \frac{K_h - K_l}{\mathbb{E}[K]}.$$

We assume that these “volatilities” are small and use first-order Taylor expansions:

$$\sigma_K \sim \frac{S(p_h) - S(p_l)}{S(p)} \sim \frac{pS'(p)}{S(p)} \left(\frac{p_h - p_l}{p} \right) = \epsilon^s \sigma_p.$$

Similarly $Y_s = F(1 - K_s) + R_s K_s$. Thus

$$\begin{aligned} Y_h - Y_l &\sim -F'(1-K)(K_h - K_l) + (R_h - R_l)K + R(K_h - K_l) \\ &\sim (R-p)(K_h - K_l) + K(R_h - R_l), \end{aligned}$$

so that

$$\begin{aligned} Y_h - Y_l &\sim (R_h - R_l)[(R-p)S'(p) + K], \\ \text{and} \quad \sigma_Y = \frac{Y_h - Y_l}{Y} &\sim \frac{RK}{Y}[(\frac{R}{p} - 1)\epsilon^s + 1]\sigma_R. \end{aligned}$$

Finally, we can compute the volatility of (the value of) bank credit pK:

$$\sigma_{pK} = \sigma_p + \sigma_K = (1 + \epsilon^s)\sigma_p.$$

Recall that $p_s = R_s - R + p$ where $p = \mathbb{E}[p_s]$. Thus $\sigma_p = \dfrac{p_h - p_l}{p} = \dfrac{R_h - R_l}{p}$.

Volatility of capital prices

$$\frac{\sigma_p}{\sigma_R} = \frac{R}{p} = \frac{1-CR}{1-FF} = \frac{98}{70} = 1.4.$$

Thus capital prices fluctuate 40% more than returns.

Volatility of credit to GDP ratio

$$\frac{\sigma_{pK}}{\sigma_Y} = \frac{\sigma_p + \sigma_K}{\sigma_Y} \sim \frac{(1+\epsilon^s)R/p}{1+(\frac{R}{p}-1)\epsilon^s}\frac{Y}{RK} = \frac{(1+\epsilon^s)}{1+(\frac{R}{p}-1)\epsilon^s}\frac{Y}{pK}.$$

Thus if we take an average credit to GDP ratio $\frac{pK}{Y}$ of 1, the volatility of this ratio is

$$\frac{\sigma_{pK}}{\sigma_Y} = \frac{2}{1.4} \approx 1.4.$$

Again, credit fluctuates 40% more than GDP.

Appendix 3

Proof of Proposition 3:

It results from a Taylor expansion around $p^0 \equiv \mathbb{E}[p_s^0]$ and $R \equiv \mathbb{E}[R_s]$ of the following equation, which is equivalent to (5):

$$S(p_s^0)[p_s^0 + b - R_s] = p_s^0 E.$$

We obtain for $s = h, l$

$$(p_s^0 - p^0)[S'^0)(p^0 + b - R) + S(p^0)] - (R_s - R)S(p^0) \sim (p_s^0 - p^0)E.$$

The approximation is accurate when the macro shock is small. Now take the difference between these two equations:

$$(p_h^0 - p_l^0)[S'^0)(p^0 + b - R) + S(p^0) - E] \sim (R_h - R_l)S(p^0).$$

Now $p_h^c - p_l^c = R_h - R_l$ and $p^c \sim p^0$ when σ_R is small. Dividing the above relation by $p^0 S(p^0)$ we obtain

$$\sigma_p^c = \frac{p_h^c - p_l^c}{p^c} \sim \frac{R_h - R_l}{p^0} \sim \sigma_p^0 \left[\frac{S'^0)}{S(p^0)}(p^0 + b - R) + 1 - \frac{E}{S(p^0)} \right].$$

As the macro shock is small, $p^c + b - R = p^c \frac{E}{S(p^c)}$ holds in equilibrium with contingent markets and $p^c \sim p^0$, we obtain $p^0 + b - R \sim p^0 \frac{E}{S(p^0)}$. Therefore, we have finally established that

$$\sigma_p^c \sim \sigma_p^0 \left[1 - \frac{E}{S(p^0)} \left(1 - \frac{p^0 S'^0)}{S(p^0)} \right) \right].$$

Thus when $\frac{p^0 S'^0)}{S(p^0)} < 1$, and σ_R is small, $\sigma_p^c < \sigma_p^0$.

■

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