CoMargin

Jorge A. Cruz Lopez[1] Jeffrey H. Harris

Christophe Hurlin Christophe Pérignon

August 1, 2013

Abstract: We present a new collateral framework, called CoMargin, for derivatives exchanges. CoMargin depends on both the tail risk of a given market participant and its interdependence with other participants. This collateral system aims at internalizing market interdependencies and enhancing the stability of the financial system. CoMargin can be estimated using a model-free and scenario-based methodology, validated using formal statistical tests, and generalized to any number of market participants. We assess and illustrate our methodology using proprietary data from the Canadian Derivatives Clearing Corporation (CDCC). Our data set includes daily observations of the actual trading positions, margin requirements and profits-and-losses of all forty-eight CDCC clearing members from 2002 to 2009. We show mathematically and empirically that CoMargin outperforms existing margining systems by minimizing the occurrence and economic shortfall of simultaneous financial distress events, particularly when trading similarity across clearing members and comovement among underlying assets increases.

JEL Classification: G13

Keywords: Collateral, Counterparty Risk, Derivatives Markets, Extreme Dependence

---

[1] Cruz Lopez is at the Bank of Canada, Harris is at Syracuse University, Hurlin is at the University of Orléans, France, and Pérignon is at HEC Paris, France. Emails: jcruzlopez@bankofcanada.ca; jhharr03@syr.edu; christophe.hurlin@univ-orleans.fr; perignon@hec.fr. Contact Author: Christophe Pérignon, Finance Department, HEC Paris, 1 Rue de la Libération, 78351 Jouy-en-Josas, France. Tel: (+33) 139 67 94 11. We thank the Canadian Derivatives Clearing Corporation (CDCC) for providing us with the data used in the empirical section of this paper. Special thanks to Nicholas Labelle St-Pierre. We are grateful to Andrew Barton, Mark Chambers, and Nicholas Linder, seminar participants at the Bank of England, Banque de France and the Reserve Bank of Australia, as well as to participants at of the 2013 Annual Workshop of the Dauphine-Amundi Chair in Asset Management, Bank of Canada Financial Intermediation and Market Dynamics Workshop, the 24th Australasian Finance and Banking Conference, and the QUT-Princeton Frontiers in Financial Econometrics Workshop for their valuable comments.

1. Introduction

How much margin should a given market participant post against its derivatives positions? In this paper, we argue that margin requirements should increase with both (1) the variability and (2) the interdependence of each participant's future profits and losses (P&Ls). We show that commonly used collateral methods, such as the Standard Portfolio Analysis of Risk (SPAN) or the Value-at-Risk (VaR) system, can fail to properly allocate collateral requirements because they disregard the interdependence across the P&Ls of different market participants. Therefore, we propose a new margining system, called CoMargin, which explicitly internalizes these interdependencies. Our methodology is a model-free, scenario based approach that can be generalized to any number of market members. In addition, CoMargin requirements can be backtested and are less procyclical than those estimated with alternative methods.

We focus on clearing houses in derivatives exchanges because these institutions concentrate a significant amount of counterparty risk in the financial system (Pirrong, 2009). However, our collateral approach and backtesting methodology is general enough to be applied to any context where counterparty risk needs to be managed. Examples include, but are not limited to, banks and lending institutions, over-the-counter (OTC) securities dealers, newly-proposed swap execution facilities (SEFs), and insurance companies.

In a derivatives exchange, the clearing house conducts the clearing function, which consists on confirming, matching, and settling all trades. Clearing houses operate with a small number of members, referred to as clearing firms, who are allowed to clear their own trades (i.e., conduct proprietary trading), those of their customers, and those of non-clearing firms. Through the process of novation, the clearing house becomes the counterparty to every contract and guarantees their performance; thus, reducing the counterparty risk faced by its members. In the process of providing this service, however, the clearing house concentrates a significant amount of default risk, which is primarily managed through the use of margining systems.[2]

A clearing house margining system requires members to post funds (or liquid assets) as collateral in a margin account. These funds are used to guarantee the performance of clearing

[2] Other common default risk management tools include capital requirements for clearing firms, default funds, private insurance arrangements, and strict segregation between customer and house margin accounts (see Jones and Pérignon, 2013).

members' positions over a period of time, usually one day, such that the clearing house is protected against the losses and potential default of its counterparties. However, clearing firms sometimes experience losses that exceed their posted collateral, leaving them with a negative balance in their margin accounts. These clearing firms may delay their payments or in some cases default; thus, creating a shortfall in the market. In either case, the clearing house has to cover this shortfall with its own funds in order to compensate the counterparties who profited from taking opposite trading positions. Usually, financing the shortfall of a single member over a limited period of time does not impose a hefty financial burden on the clearing house. However, when two or more large clearing firms have a negative margin balance simultaneously, the consequences tend to be more severe. In this case, if the clearing members only delay their payments temporarily, the resulting shortfall tends to be short lived, but it can significantly affect market liquidity, particularly during volatile periods. If on the other hand, the clearing members default, the shortfall tends to be long-lived or even permanent which can erode the resources of the clearing house to the point of financial distress or even failure.

While clearing house failures are rare events, the cases of Paris in 1973, Kuala Lumpur in 1983 and Hong Kong in 1987 (Knott and Mills, 2002) demonstrate that these extreme scenarios are not only possible, but also very economically significant.[3] In addition, recent consolidation of clearing facilities through economic integration and mergers and acquisitions, as well as the strong pressure from governments and market participants to facilitate, or force, OTC derivatives to be cleared by central counterparties, has dramatically increased the systemic importance of these institutions (see, for instance, Acharya et al., 2009; US Congress' OTC Derivatives Market Act of 2009; US Department of Treasury, 2009; Duffie, Li, and Lubke, 2010; Duffie and Zhu, 2010).[4] Therefore, it is increasingly necessary to devise appropriate risk management systems that enhance the stability and resiliency of clearing facilities.

Current margining systems employed by derivatives exchanges set the margin level of a derivatives portfolio based on a coverage probability or a target probability of a loss in excess of

[3] Default of clearing firms are of course much more frequent. Recent examples on the CME include Refco in 2005, Lehman in 2008, and MF Global in 2011 (see Jones and Pérignon, 2012).
[4] The Chicago Mercantile Exchange (CME), Intercontinental Exchange (ICE), EUREX, Euronext Liffe, and LCH-Clearnet have each recently created clearing facilities for Credit Default Swaps.

posted collateral (Figlewski 1984; Booth et al. 1997; Cotter 2001).[5] However, by focusing only on individual firm portfolios, these systems ignore the fact that clearing firms sometimes face homogenous risk exposures that make them highly interdependent and causes them to exceed their posted margin simultaneously. As a consequence, the clearing house may experience sudden and extreme shortfalls that could undermine its stability and resiliency.

The level of risk homogeneity across clearing firms increases with trade crowdedness and underlying asset comovement. Trade crowdedness refers to the similarity of clearing firms' trading positions. When member portfolios are very similar, they tend to have equivalent exposures and returns, regardless of how underlying assets behave. Underlying asset comovement refers to underlying assets returns moving in unison. When underlying assets experience high levels of comovement, clearing firms tend to have similar risk exposures because, regardless of their individual trading decisions, all securities in all portfolios tend to move in the same direction.[6]

Both dimensions of risk homogeneity are related; however, the first one is directly influenced by the individual trading behaviour of clearing firms, while the second one is determined by aggregate market behaviour. Similar trading positions, or crowded trades, tend to arise among large clearing firms because they share a common (and superior) information set. This informational advantage leads them to pursue similar directional trades, arbitrage opportunities and hedging strategies.[7] On the other hand, underlying assets tend to move in the same direction during economic slowdowns or during periods of high volatility, both of which are rarely the result of individual market participants' actions.[8]

---

[5] For example, Kupiec (1994) shows the empirical performance of the SPAN system for selected portfolios of S&P 500 futures and futures-options contracts and he finds that, over the period 1988-1992, the historical margin coverages exceed 99% for most considered portfolios.

[6] The importance of asset comovement has been identified in previous studies. For example, in an early attempt to analyze the default risk of a clearing house, Gemmill (1994) highlights the dramatic diversification benefit from combining contracts on uncorrelated or weakly correlated assets.

[7] Much of the proprietary trading activity on derivatives exchanges consists of arbitraging futures and over-the-counter or cash markets (e.g. cash-futures arbitrage of the S&P 500 index, eurodollar-interest rate swap arbitrage, etc.).

[8] Extreme dependence and contagion across assets is discussed in Longin and Solnik (2001), Bae, Karolyi and Stulz (2003), Longstaff (2004), Poon, Rockinger and Tawn (2004), Boyson, Stahel and Stulz (2010), and Harris and Stahel (2011), among others.

In this paper, we depart from the traditional view of setting margin requirements based on individual member positions. Instead, we account for their interdependence by computing the margin requirement of a clearing firm conditional on one or more firms being in financial distress. By adopting this approach, we obtain a system that allows the margin requirements of a particular member to increase when it is more likely to experience financial distress simultaneously with others.

Our method builds on the CoVaR concept introduced by Adrian and Brunnermeier (2011) which is defined as the VaR of the system (i.e., banking sector) conditional on a given institution being in financial distress. The core of their analysis is the so-called delta CoVaR that measures the marginal contribution of a particular institution to the overall risk in the system; i.e., the difference between the VaR of the system conditional on a given institution being in distress and the VaR of the system in the median state of the institution. There are some key differences between the CoVaR and CoMargin methodologies, however. First, the objective of CoMargin is not to measure systemic risk. Instead, it is used to estimate margin requirements that account for the interdependence of market participants. Thus, we do not consider the VaR of the system but the VaR of a firm conditional on one or several other firms being in financial distress. Second, CoMargin is not applied to bank stock returns but to the trading P&Ls of clearing firms. Moreover, we only focus on a sub-set of these firms because by construction their aggregate P&Ls in a derivatives exchange is zero. Finally, the estimation of CoMargin is non-parametric and much simpler than that used for CoVaR, which requires a quantile regression technique.

The CoMargin estimation process starts by taking the trading positions of all clearing members at the end of the trading day as given. Then, we consider a series of one-day-ahead scenarios based on changes in the price and volatility of the underlying assets. For each scenario, we mark-to-model each firm's portfolio and obtain its hypothetical P&L. Based on these hypothetical P&L calculations we compute margin requirements that minimize the probability of joint financial distress. We show that the CoMargin system enhances financial stability because it reduces the likelihood of several clearing members being in financial distress simultaneously. In addition, we also show that this method increases financial resiliency because it actively adjusts the allocation of collateral as a function of market conditions. As a result, the average magnitude of the margin shortfall given simultaneous financial distress is minimized relative to other collateral systems. Both of these conditions greatly reduce systemic risk concerns.

The rest of the paper is organized as follows. In Section 2, we describe how margin requirements are currently estimated under the SPAN and VaR margining systems. In Section 3 we define a list of properties needed to achieve a sound margining system. We present the theoretical foundations of the CoMargin system in Section 4 and examine its effectiveness through an empirical analysis in Section 5. Finally, Section 6 concludes.

2. Standard Margin Systems

2.1. Derivatives Market

Consider a derivatives exchange with N clearing firms and D derivatives securities (futures, options, credit default swaps, etc.) written on U underlying assets. Let $w_{i,t}$ be the number of contracts in the derivatives portfolio of clearing firm i, for $i = 1, \dots, N$, at the end of day t:

$$w_{i,t} = \begin{bmatrix} w_{1,i,t} \\ \vdots \\ w_{D,i,t} \end{bmatrix} \tag{1}$$

The performance bond, $B_{i,t}$, is the margin or collateral requirement imposed by the clearing house on clearing firm i at the end of day t. This performance bond depends on the outstanding trading positions of the clearing firm at the end of day t, $w_{i,t}$. The variation margin, $V_{i,t}$, represents the aggregate P&L of clearing firm i on day t.

In derivatives markets, margins are collected to guarantee the performance of member obligations and to guard the clearing house against default. Therefore, we are interested in situations when trading losses exceed margin requirements; i.e., when $V_{i,t} \leq B_{i,t-1}$. In these cases, we say that firm i is in financial distress. Identifying firms in this state is important because they have an incentive to default on their positions or to delay payment on their obligations, which generates a shortfall in the system that needs to be covered by the clearing house.

2.2. SPAN Margin

The most popular margining system around the world is the Standard Portfolio Analysis of Risk (SPAN) system. This system was introduced by the CME in 1988 and as it is illustrated in Table 1, it is currently used by more than 50 derivatives exchanges. SPAN is a scenario based system that is applied on a firm by firm basis; however, it is not a comprehensive portfolio margining system. Instead, it divides the portfolio into contract families, which are defined as groups of contracts that share the same underlying asset. Thus, in a market with U underlying assets, there are U different contract families. SPAN sets the margin requirements for these families independently, and the collateral level for the entire portfolio is then computed by aggregating the margin requirements of all contract families according to the aggregation rules set by the clearing house.

To compute the margin level for a derivatives portfolio, the SPAN system simulates potential one-day changes in the value of each contract using sixteen scenarios that vary the value (ΔX) and the volatility ($\Delta \sigma_X$) of the underlying assets, as well as the time to expiration of the derivatives products. The potential price movements for each underlying asset are defined in terms of a price range, which is derived from historical data. In most cases, the price range is selected to cover 99% of the historical one-day price movements observed in the calibration window. A similar approach is adopted for the volatility range.

Every day following the market close, the clearing house applies each scenario to each one of the D derivatives securities traded on the exchange. The price changes of non-linear instruments, such as options, are obtained by using numerical valuation methods or option pricing models. A risk array with sixteen gain or loss values is created for each contract (i.e., each maturity and each strike price will have its own array). Using these arrays, the predicted losses across contracts are computed to find the scenario that creates the worst-case loss for the contract family as a whole. This worst-case value is then used to determine the margin requirement for that contract family.

The overall portfolio margin requirement is computed by aggregating the margin requirements of different contract families. However, since SPAN allows different futures and options months within a family to offset each other, the aggregation across contract families is adjusted with intermonth spread charges. Similarly, inter-commodity credits are given to account for inter-commodity spreads. It is important to note that the magnitudes of these charges and credits are left to the discretion of the margin committee of the clearing house, so they may not be consistent across commodities, market conditions or clearing houses. As a consequence, the actual coverage probability of the SPAN system may vary through time or across markets.

As an illustration, we display in Figures 1 and 2 the daily SPAN margins and P&L for all sixty nine clearing firms in the CME between January 1 and December 31, 2001. More precisely, these figures correspond to the house trading account (i.e., proprietary trading) of these clearing firms, and as such, do not reflect customer trading. The most striking feature of the data is the segmentation of the market between extremely large (i.e., systemically-important) clearing firms and smaller ones. The top-10 largest clearing firms (the brokerage units of Morgan Stanley, Goldman Sachs, Credit Suisse, etc.) account on average for approximately 80% of all collateral collected. We see in Figure 1 that the posted margin for a single firm can be close to $3 billion and the daily trading gain or loss can exceed $1 billion.

Figure 2 displays the ratio of daily P&L to posted SPAN margin ($V_{i,t}/B_{i,t-1}$) for all of the clearing firms in the sample. This graph illustrates two important features of the SPAN margining system. First, SPAN margins are frequently exceeded by trading losses. In our sample, 30 days (out of 251) show a negative balance in at least one member's margin account (i.e. these accounts ended the trading day *under-water*). In addition, 14 different clearing firms experienced at least one margin-exceeding loss event during this one year sample. Second, margin deficiencies tend to cluster. This feature of the data is particularly salient and concerning from a risk management perspective for the largest clearing firms. The ten most extreme losses as a proportion of posted collateral that affected the ten largest clearing firms occurred on two different trading days.

2.3. VaR Margin

VaR is defined as a lower quantile of a P&L distribution. It is the standard measure used to assess the aggregate risk exposure of banks (Berkowitz and O'Brien, 2002; Berkowitz, Christoffersen and Pelletier, 2011), as well as their regulatory capital requirements (Jorion, 2007). VaR can also be used to set margins on a derivatives exchange. In this case, the margin requirement corresponds to a given quantile of a clearing firm's one-day-ahead P&L distribution.

Definition 1: The VaR margin, B_i, corresponds to the $\alpha\%$ quantile of the P&L distribution:

$$\Pr\left(V_{i,t+1} \leq -B_{i,t}\right) = \alpha \tag{2}$$

Like the SPAN system, the VaR collateral method is applied on a firm by firm basis using a scenario analysis (Cruz Lopez, Harris and Pérignon, 2011). We consider S scenarios and use them to evaluate each clearing firm's entire portfolio. The hypothetical P&L or variation margin of each clearing member is computed by *marking-to-model* their positions in each scenario. Thus, for each clearing member and each date t, we obtain a simulated sample of $V_{i,t+1}$ denoted $\left\{v_{i,t+1}^{s}\right\}_{s=1}^{S}$ that can be used to estimate the VaR collateral requirement as follows:

$$\hat{B}_{i,t} = percentile\left(\left\{v_{i,t+1}^{s}\right\}_{s=1}^{S}, 100\alpha\right) \tag{3}$$

Compared to market risk VaR (Jorion, 2007), the estimation of VaR margin is much simpler. In general, the quantile of the return at time t, needed for market risk VaR, cannot be estimated without making some strong assumptions about the underlying distribution because for each date, there is only one P&L observation available for each firm. For example, the historical simulation approach broadly used by financial institutions for market risk VaR estimations assumes that the P&Ls of each firm are independently and identically distributed over time. Under these assumptions, the unconditional VaR is stationary and it can be estimated from the historical path of past P&Ls. The estimation of more refined measures, such as conditional VaR also require some particular assumptions regarding the dynamics of the P&L quantiles. For instance, the CAViaR approach proposed by Engle and Manganelli (2004), assumes an autoregressive process for the P&L quantile.

In our context, however, the situation is quite different and much simpler because we have S simulated observations of the P&L of each clearing firm at time t. This is an ideal situation from an econometric point of view because the quantile of the P&L distribution can be directly implied without making any assumptions regarding its behavior over time. Thus, $\hat{B}_{i,t}$, which represents the empirical quantile based on the S simulated observations (see equation 3), is a consistent estimate of the VaR when S tends to infinity.

3. Characteristics of a Sound Margining System

Surprisingly, there is very little guidance in the literature regarding the properties that a sound margining system should satisfy. Nevertheless, this is a fundamental issue that needs to be addressed in order to understand the relative merits of different margining methodologies. Therefore, in this section we attend to this issue by proposing five main properties that any well designed margining system must satisfy. These properties and the rationale behind them are explained below.

i. Margins must increase with P&L variability

Let $\sigma_{i,t}$ be a measure of the variability of the P&L of clearing member i at time t:

$$\text{If } \sigma_{i,t}^1 \geq \sigma_{i,t}^2 \text{ then } B_{i,t}\left(w_{i,t}, \sigma_{i,t}^1\right) \geq B_{i,t}\left(w_{i,t}, \sigma_{i,t}^2\right) \tag{4}$$

This basic property is at the heart of existing margining methods (see Table 2). Intuitively, it means that since riskier trading portfolios (as measured by their variability) tend to have larger potential losses, more collateral must be collected to guarantee their performance. Or in simple words, riskier clearing firms should post higher margins. The SPAN and VaR methods comply with this property because both the simulated loss of the SPAN system and the quantile that determines the VaR margin increase with the variability of P&Ls.[9]

[9] Artzner et al. (1999) define a coherent risk measure using four axioms: *monotonicity* (if portfolio 1 (P1)'s returns are always lower than portfolio 2 (P2)'s returns, then P1 is riskier than P2), *translation invariance* (adding $K in cash to P1 reduces its risk by the same amount), *homogeneity* (increasing the size of P1 by a factor S increases its risk by the same factor), and *subadditivity* (risk measures need to account for diversification). Conceptually, with non-subadditive margins, it may be optimal for participants to breakdown their trading portfolio into smaller sub-portfolios in order to reduce their total margin requirements. However, in practice, clearing houses prevent financial institutions from having multiple clearing firms. Furthermore, netting rules allow clearing members to post considerably less margin than they would if they had dislocated portfolios.

ii. Margins must increase with loss dependence

Let $\delta_{i,t}$ be a measure of dependence between the losses of market participant i and those of other market participants at time t. Loss dependence can originate from similarities in trading positions, correlated asset prices, or both:

$$\text{If } \delta_{i,t}^{1} \geq \delta_{i,t}^{2} \text{ then } B_{i,t}\left(w_{i,t}, \delta_{i,t}^{1}\right) \geq B_{i,t}\left(w_{i,t}, \delta_{i,t}^{2}\right) \tag{5}$$

The intuition behind this property is that a sound margining system should prevent (or minimize) the occurrence of joint-financial distress across market participants. As shown in Section 2, both the SPAN and VaR margin methods set margins on a firm-by-firm basis and hence completely disregard loss dependence across clearing firms.

iii. Margins should not be excessively procyclical

When margins are procyclical, market downturns and excess volatility can lead to higher initial margins and more frequent margin calls. This situation can adversely affect funding conditions and market liquidity, and it can force traders to close out their positions simultaneously; thus, intensifying market declines. Brunnermeier and Pedersen (2009) explain and model this sequence of reinforcing events which they refer to as a "margin spiral". Current margining requirements are prone to trigger these spirals because they are only a function of expected price and volatility changes. In addition, discretionary parameters, such as the ranges and aggregation rules used in the SPAN system, are usually modified after significant or persistent market shocks. Therefore, margin requirements often exhibit significant jumps, as opposed to smooth transitions, which can be very destabilizing for the market.

iv. Margins must be robust to outliers

Erratic margin swings due to outliers should be prevented as much as possible as they may lead to severe operational problems, such as sudden margin calls. Since SPAN margins are based on the maximum simulated loss and not on a quantile, they are much more sensitive to outliers than VaR margins.

v. **Margins must be testable ex-post**

The only way to systemically measure the accuracy and effectiveness of a margining system is by backtesting it. Backtesting aims at identifying misspecified models that lead to margin requirements that are either too high or too low. Therefore, if a margining system cannot be backtested using formal statistical methods, then we cannot identify its potential shortcomings and fine tune it to meet its objectives.

VaR margins can be easily backtested because they are defined by the quantile of the P&L distribution. The intuition behind the backtesting procedure is that the actual trading losses of a given clearing firm should only exceed its VaR margin α% of the time. Well known VaR validation tests can be found in Jorion (2007) and a more refined approach can be found in Hurlin and Pérignon (2012). On the other hand, backtesting SPAN margin requirements is extremely challenging because they are based on the minimum of a simulated distribution, which is very hard to identify. Therefore, the validation tests cannot be performed without making strong assumptions about the P&L distributions of clearing firms.

As it can be seen from the discussion above, the SPAN system only complies with the first key property, whereas the VaR margining system complies with properties one, four and five. Table 2 summarizes these findings. It is very interesting to notice, however, that existing margining techniques are unable to account for the loss dependence across participants and to generate margin requirements that are not highly procyclical.

4. CoMargin

4.1. Concept

As shown in Section 3, VaR and SPAN collateral systems only focus on firm specific risk; that is, the unconditional probability of financial distress of each individual member. By adopting either system, the clearing house guards itself from unique or independent financial distress occurrences, but it leaves itself exposed to simultaneous distress events. These events, however, tend to be more economically significant because they place a more substantial burden on the resources of the clearing house, which may exhaust its funds and eventually default.

Consider firms i and j. Using the VaR collateral system, their probability of joint financial distress is given by:

$$\begin{aligned} &\Pr\left[\left(V_{i,t+1} \leq -B_{i,t}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right] \\ &= \Pr\left(V_{i,t+1} \leq -B_{i,t} | V_{j,t+1} \leq -B_{j,t}\right) \times \Pr\left(V_{j,t+1} \leq -B_{j,t}\right) \end{aligned} \tag{6}$$

Equation 6 shows that joint financial distress events tend to happen more frequently not only when firm specific risk increases (i.e., when $\Pr\left(V_{j,t+1} \leq -B_{j,t}\right)$ increases), but also when risk homogeneity increases (i.e., when $\Pr\left(V_{i,t+1} \leq -B_{i,t} | V_{j,t+1} \leq -B_{j,t}\right)$ increases). In the first case, firms are more likely to experience losses that exceed their collateral levels in all situations. In the second case, firms are more likely to experience these losses when other firms are in financial distress, either because they hold similar positions (i.e., trade crowdedness is high) or because underlying assets have a tendency to move together (i.e., underlying asset comovement is high). However, VaR and SPAN systems completely disregard risk homogeneity and its potential effect on financial distress and market stability. In the case of the VaR system, risk managers only target unconditional distress probabilities by setting a coverage level for each clearing member. In the case of the SPAN system, risk managers do not have direct control over the unconditional distress probabilities, so the clearing house is potentially left even more vulnerable to simultaneous distress occurrences.

Now, consider a fully orthogonal market; that is, a market that has firms with orthogonal trading positions and orthogonal underlying asset returns. In this case, firms have orthogonal risk exposures and their probabilities of financial distress are independent. Therefore,

$$\Pr\left(V_{i,t+1} \leq -B_{i,t} | V_{j,t+1} \leq -B_{j,t}\right) = \alpha \tag{7}$$

and

$$\Pr\left[\left(V_{i,t+1} \leq -B_{i,t}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right] = \alpha^2 \tag{8}$$

Equation 8 shows that a fully orthogonal market minimizes the probability of joint financial distress across clearing members. Given a common coverage probability, α, a fully orthogonal market provides the best possible level of market stability, regardless of the collateral system being adopted by the clearing house. Therefore, a fully orthogonal market can be seen as a conceptual construct that provides a common benchmark for all margining systems.

With this in mind and in the spirit of the CoVaR measure of Adrian and Brunnermeier (2011), we propose a new collateral system, called CoMargin, which enhances financial stability by taking into account the risk homogeneity of clearing firms. Our starting point is the framework used to estimate VaR margin requirements, which was described in the previous section. Once we establish the S scenarios for each underlying asset, we jointly evaluate the portfolios of firms i and j and compute their associated hypothetical P&Ls or variation margins, $V_{i,t+1}$ and $V_{j,t+1}$ respectively, such that for each date t, we obtain a panel of simulated P&Ls, denoted $\left\{v_{i,t+1}^s, v_{j,t+1}^s\right\}_{s=1}^S$. Thus, the CoMargin of firm i, denoted $B_t^{i|j}$, conditional on the realisation of an event affecting firm j is:

$$\Pr\left(V_{i,t+1} \leq -B_t^{i|j} | \mathbf{C}\left(V_{j,t+1}\right)\right) = \alpha \tag{9}$$

The conditioning event that we consider in this case is an extreme loss in the portfolio of firm j, which is defined as a loss that exceeds its α% VaR, or equivalently, a loss that exceeds its VaR margin; i.e., $\mathbf{C}\left(V_{j,t+1}\right) = \left\{V_{j,t+1} \leq -B_{j,t}\right\}$.

Definition 2: The CoMargin, $B^{i|j}$, corresponds to the α% conditional quantile of their joint P&L distributions:

$$\Pr\left(V_{i,t+1} \leq -B_t^{i|j} | V_{j,t+1} \leq -B_{j,t}\right) = \alpha \tag{10}$$

Through Bayes theorem we know that:

$$\Pr\left(V_{i,t+1} \leq -B_t^{i|j} | V_{j,t+1} \leq -B_{j,t}\right) = \frac{\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right]}{\Pr\left(V_{j,t+1} \leq -B_{j,t}\right)} \tag{11}$$

where the numerator represents the joint probability of i exceeding its CoMargin requirement and j experiencing an extreme loss. From Definitions 1 and 2, we can see that the CoMargin of firm i is defined as the margin level $B_t^{i|j}$ such that:

$$\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right] = \alpha^2 \tag{12}$$

Notice from equation 10 that the CoMargin system starts by defining a threshold level or extreme loss as the α% VaR of the P&L of a conditioning firm j. This threshold, which corresponds to that firm's VaR margin, accounts for firm specific risk in the CoMargin calculation. Risk homogeneity is then incorporated by directly targeting the conditional probability of financial distress of firm i, such that it behaves as if the market was fully orthogonal when firm j experiences an extreme loss. This means that when the market is indeed fully orthogonal, the CoMargin and VaR collateral systems are equivalent and produce the same results. When the market is not fully orthogonal, any differences between the collateral requirements of these two systems can be attributed to risk homogeneity. Thus, the CoMargin of firm i, $B_t^{i|j}$, can be interpreted as the margin level that guarantees that firm i remains solvent at an optimal level when firm j experiences an extreme loss. The optimal level of solvency corresponds to that seen in a fully orthogonal market, in the sense that, given that firm j experiences an extreme loss, firm i will have enough funds in its margin account to cover its potential losses $1 - \alpha^2$% of the time. Therefore, the CoMargin system greatly enhances financial stability.

4.2. Illustration

4.2.1. Properties

We consider a simple case with two firms that have normally-distributed P&Ls. For simplicity, we consider an unconditional distribution, with respect to past information, and consequently neglect the time index t:

$$(V_1, V_2)' \sim N(0, \Sigma)$$

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

In this setting, the CoMargins for both members, denoted ($B^{1|2}$, $B^{2|1}$), are defined by:

$$\Pr\left(V_i \leq B^{i|j} | V_j \leq -B_j\right) = \alpha \tag{13}$$

For $i = 1,2$, where $B_i = -\sigma_i \Phi^{-1}(\alpha)$ denotes the unconditional VaR of firm i and $\Phi(\cdot)$ the cdf of the standard normal distribution. The conditional distribution of V_i given that $V_j < c$, $\forall c \in \mathbb{R}$ is a skewed distribution (Horace, 2005) and is denoted by $g(\cdot)$. The CoMargin for the firm i is the solution to:

$$\int_{-\infty}^{-B^{i|j}} g(u; \sigma_i, \sigma_j, \rho) du = \alpha \tag{14}$$

$$g\left(u; \sigma_i, \sigma_j, \rho\right) = \frac{1}{\alpha\sigma_i} \times \phi\left(\frac{u}{\sigma_i}\right) \times \ \Phi\left(\frac{-B_j/\sigma_j - \rho u/\sigma_i}{\sqrt{1-\rho^2}}\right) \tag{15}$$

where $\phi(\cdot)$ denotes the pdf of the standard normal distribution (Arnold et al., 1993). Using the expression of CoMargin in equation 14, we can illustrate some of its properties:

i. The CoMargin of firm i increases with the variability of its P&L:

$$\frac{\partial B^{i|j}}{\partial \sigma_i} > 0 \tag{16}$$

See Appendix A1 for the proof.

ii. When there is no correlation between the P&Ls of firms i and j, CoMargin and VaR margin converge. In our case, the dependence is fully captured by the correlation coefficient, ρ:

$$B^{i|j} = B_i \text{ when } \rho = 0 \tag{17}$$

Notice, however, that this result is not specific to the normal case. When there is no dependence (linear or not) between the P&L of the two firms, the CoMargin simply reduces to the VaR margin. See Appendix A2 for the proof.

iii. The CoMargin of firm i increases with the dependence between its P&L and that of other firms (i.e., firm j). See Appendix A3 for the proof:

$$\frac{\partial B^{i|j}}{\partial \rho} > 0 \tag{18}$$

iv. When the correlation between the P&Ls of firm i and firm j approaches one, the CoMargin converges towards the VaR margin defined with an α^2% coverage probability, $B_i(\alpha^2)$:

$$\lim_{\rho \to 1} B^{i|j} = B_i(\alpha^2) \tag{19}$$

This property shows that the CoMargin is not explosive when the correlation becomes very large. See Appendix A4 for the proof.

v. The CoMargin of firm i does not depend on the variability of the P&L of firm j:

$$\frac{\partial\, B^{i|j}}{\partial \sigma_j} = 0 \tag{20}$$

See Appendix A5 for the proof.

4.2.2. Performance

In order to illustrate the performance of the CoMargin system, we now consider with the case of four firms, where two of them (firms 1 and 2) have correlated P&Ls:

$$V_t \sim N(0, \Sigma)$$

where $V_t = \left(V_{1,t}, V_{2,t}, V_{3,t}, V_{4,t}\right)'$ and:

$$\Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Firms 1 and 2 have P&Ls with correlations increasing from 0 to 0.8. As it was explained before, the increasing correlation in the P&Ls of these firms can reflect an increase in their trading similarity or the comovement across underlying assets. In Table3, we present the margins of each clearing firm computed using both a VaR approach and a CoMargin approach. Here, the conditioning event in the CoMargin definition is that at least one of the other three firms is in financial distress.

We see in Table 3 that both types of margins are equal when $\rho = 0$ and that the CoMargin is larger than the VaR margin when $\rho > 0$. We also report a so called Budget-Neutral (BN) margin that is designed to collect as much aggregate collateral as the CoMargin system, but the funds collected over and above the aggregate level of VaR margin are obtained only from the independent firms (i.e., firms 3 and 4), for which CoMargin is always equal to VaR Margin. Thus, the BN margin scheme redistributes the allocation of collateral across firms. [10]

Figure 7 reports the probability of having, respectively, one (p_1), two (p_2), three (p_3), and four (p_4) firms in financial distress, i.e., firms suffering a margin-exceeding loss. We see that the CoMargin system clearly outperforms its VaR counterpart as it systematically leads to smaller probabilities of having firms in financial distress and, consequently, it leads to higher probabilities of not having firms in financial distress (p_0).

Next, we check whether the better performance of the CoMargin system is due to (1) a better allocation of margins across firms or to (2) the fact that it collects higher margins. In order to

[10] An alternative budget-neutral margin scheme would be to redistribute all of the additional collateral across all firms.

do this assessment, we neutralize the second effect by using BN margins. Given the symmetric nature of the market structure – two dependent firms and two independent firms (see Table 3) – p_0 is the same for CoMargins and BN margins. The key difference between the two curves shown in Figure 7 is that joint-financial distresses happens less frequently with CoMargins than they do with BN margins. Notice that this effect is only the result of a better allocation of margins across firms, and not the result of additional funds collected under the CoMargin system.

4.3. Scenario Generation

One common feature of all margin methods is that they are all scenario based. As a consequence, generating meaningful scenarios is a crucial stage in the margin setting process. We consider a series of scenarios based on potential shocks in the value of some state variables. Depending on the derivative securities included the portfolio, one can consider changes in the derivatives prices (e.g., for futures), changes in the price and volatility of an underlying asset (e.g., for options), changes in the time to expiration (e.g., for any derivatives contract), or default (e.g., for CDS).

The shocks associated with a particular scenario can be of two types: historical or simulated from a parametric multivariate distribution. There are two types of multivariate scenarios: unconditional and conditional ones. One type of information that we suggest should be used to condition scenarios is the trading positions of systemically important market participants. The reason for suggesting this approach is that when market participants have large and similar trading positions, they are likely to have a non-trivial effect on the future value and volatility of the derivatives that they hold. In addition, since these participants tend to operate under a relatively homogeneous information set, they are likely to have analogous responses to changing market conditions. For example, they may decide to close out their positions at the same time during a downturn, which could trigger large price swings and excess volatility. This feed-back effect from trading positions to volatility is an important source of endogenous risk in the system.

There are two important implications of using scenarios that are conditional on trading positions: (1) Joint financial distress occurrences are less likely to occur as collateral requirements increase with trading similarities and (2) margins will tend to be less procyclical.

As noted by Brunnermeier and Pedersen (2009), under standard collateral systems, margins increase with current volatility, which sometimes leads to margin spirals. Thus, having scenarios that account for trading positions can lead to higher margins prior to episodes of high volatility.

Besides conditional scenarios, another way to control for endogenous risk is to define margin requirements such that they account for trading similarities. This is exactly the approach followed by the CoMargin system. As a consequence, CoMargins are less procyclical than SPAN and VaR margins, even when computed from unconditional scenarios.

4.4. Estimation

Given the simulated path $\left\{v_{i,t+1}^{s}, v_{j,t+1}^{s}\right\}_{s=1}^{S}$, conditional on $B_t^{i|j}$, a simple estimate of the joint probability $\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right]$, denoted $P_t^{i,j}$, is given by:

$$\hat{P}_t^{i,j} = \frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(v_{i,t+1}^{s} \leq -B_t^{i|j}\right) \times \mathbf{I}\left(v_{j,t+1}^{s} \leq -B_{j,t}\right) \tag{21}$$

where $v_{i,t+1}^{s}$ and $v_{j,t+1}^{s}$ correspond to the s^{th} simulated P&L of firms i and j, respectively. Given this result, we can now estimate $B_t^{i|j}$. For each time t and for each firm i, we look for the value $B_t^{i|j}$, such that the distance $\hat{P}_t^{i,j} - \alpha^2$ is minimized:

$$\hat{B}_t^{i|j} = \arg\min_{\left\{B_t^{i|j}\right\}} \left(\hat{P}_t^{i,j} - \alpha^2\right)^2 \tag{22}$$

Thus, for each firm i, we end up with a time series of CoMargin requirements $\left\{\hat{B}_t^{i|j}\right\}_{t=1}^{T}$ for which confidence bounds can be bootstrapped.

4.5. Backtesting

Just like VaR margin, CoMargin allows us to test the null hypothesis of an individual member exceeding its margin requirement. More importantly, however, is the fact that we can also test the conditional probability of financial distress defined by the CoMargin of firm i, $B_t^{i|j}$. The null hypothesis in this case becomes:

$$H_0: \Pr\left(V_{i,t+1} \leq -B_t^{i|j} | V_{j,t+1} \leq -B_{j,t}\right) = \alpha \tag{23}$$

Since the null implies that $E\left[\mathbf{I}\left(V_{i,t+1} \leq -B_t^{i|j}\right) \times \mathbf{I}\left(V_{j,t+1} \leq -B_{j,t}\right)\right] = \alpha$, then a simple likelihood-ratio (*LR*) test can also be used (Christoffersen, 2009). To assess the conditional probability of financial distress, we use the historical paths of the P&Ls for both members i and j; i.e., $\left\{v_{i,t+1}\right\}_{t=1}^{T}$ and $\left\{v_{j,t+1}\right\}_{t=1}^{T}$. The corresponding LR test statistic, denoted $LR_{i|j}$ takes the same form as LR_i:

$$LR_{i|j} = -2\ln[(1-\alpha)^{T-N_{i|j}}\alpha^{N_{i|j}}] + 2\ln\left[\left(1-\frac{N_{i|j}}{T}\right)^{T-N_{i|j}} \frac{N_{i|j}}{T}^{N_{i|j}}\right] \tag{24}$$

except that in this case $N_{i|j}$ denotes the total number of joint past violations observed for both members i and j; that is, $N_{i|j} = \sum_{t=1}^{T} \mathbf{I}\left(v_{i,t+1} \leq -B_t^{i|j}\right) \times \mathbf{I}\left(v_{j,t+1} \leq -B_{j,t}\right)$.

4.6. Extension to n Conditioning Firms

Consider now that the conditioning event depends on two firms denoted j and k. In this case, the CoMargin of firm i, denoted by $B_t^{i|j,k}$, is defined as follows:

$$\Pr\left(V_{i,t+1} \leq -B_t^{i|j,k} | \mathbf{C}\left(V_{j,t+1}, V_{k,t+1}\right)\right) = \alpha \tag{25}$$

$$\frac{\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j,k}\right) \cap \mathbf{C}\left(V_{j,t+1}, V_{k,t+1}\right)\right]}{\Pr\left[\mathbf{C}\left(V_{j,t+1}, V_{k,t+1}\right)\right]} = \alpha \tag{26}$$

The conditioning event that we consider is either firm j or firm k, or both, being in financial distress; i.e., $\mathbf{C}\left(V_{j,t+1}, V_{k,t+1}\right) = V_{j,t+1} \leq -B_{j,t} \; or \; V_{k,t+1} \leq -B_{k,t}$. In this case, the probability of the conditioning event is equal to 2α only if the financial distress events of firms j and k are mutually exclusive. In the general case, we have:

$$\begin{aligned}\Pr\left[\mathbf{C}\left(V_{j,t+1},V_{k,t+1}\right)\right] &= \Pr\left[\left(V_{j,t+1}\leq -B_{j,t}\right)\text{ or }\left(V_{k,t+1}\leq -B_{k,t}\right)\right]\\ &= \Pr\left(V_{j,t+1}\leq -B_{j,t}\right)+\Pr\left(V_{k,t+1}\leq -B_{k,t}\right)\\ &\quad -\Pr\left[\left(V_{j,t+1}\leq -B_{j,t}\right)\cap\left(V_{k,t+1}\leq -B_{k,t}\right)\right]\\ &= 2\alpha-\Pr\left[\left(V_{j,t+1}\leq -B_{j,t}\right)\cap\left(V_{k,t+1}\leq -B_{k,t}\right)\right]\end{aligned} \tag{27}$$

Hence, CoMargin $B_t^{i|j,k}$ satisfies the following condition:

$$\frac{\Pr\left[\left(V_{i,t+1}\leq -B_t^{i|j,k}\right)\cap\mathbf{C}\left(V_{j,t+1},V_{k,t+1}\right)\right]}{2\alpha-\Pr\left[\left(V_{j,t+1}\leq -B_{j,t}\right)\cap\left(V_{k,t+1}\leq -B_{k,t}\right)\right]}=\alpha \tag{28}$$

Given this result, we proceed to estimate CoMargin $B_t^{i|j,k}$. First, notice that the probability $\Pr\left[\left(V_{j,t+1}\leq -B_{j,t}\right)\cap\left(V_{k,t+1}\leq -B_{k,t}\right)\right]$, denoted $P_t^{j,k}$, does not depend on the CoMargin level $B_t^{i|j,k}$; thus, it can simply be estimated by:

$$\hat{P}_t^{j,k}=\frac{1}{S}\sum_{s=1}^{S}\mathbf{I}\left(v_{j,t+1}^{s}\leq -B_{j,t}\right)\times\mathbf{I}\left(v_{k,t+1}^{s}\leq -B_{k,t}\right) \tag{29}$$

Second, conditional on $B_t^{i|j,k}$, the joint probability in the numerator of equation 28, denoted $P_t^{i,j,k}$, becomes:

$$\begin{aligned}P_t^{i,j,k} &= \Pr\left[\left(V_{i,t+1}\leq -B_t^{i|j,k}\right)\cap\mathbf{C}\left(V_{j,t+1},V_{k,t+1}\right)\right]\\ &= \Pr\left[\left(V_{i,t+1}\leq -B_t^{i|j,k}\right)\cap\left[\left(V_{j,t+1}\leq -B_{j,t}\right)\text{ or }\left(V_{k,t+1}\leq -B_{k,t}\right)\right]\right]\\ &= \Pr\left[\left(V_{i,t+1}\leq -B_t^{i|j,k}\right)\cap\left(V_{j,t+1}\leq -B_{j,t}\right)\right.\\ &\quad\left.\text{ or }\left(V_{i,t+1}\leq -B_t^{i|j,k}\right)\cap\left(V_{k,t+1}\leq -B_{k,t}\right)\right]\\ &= \Pr\left[\left(V_{i,t+1}\leq -B_t^{i|j,k}\right)\cap\left(V_{j,t+1}\leq -B_{j,t}\right)\right]\\ &\quad+\Pr\left[\left(V_{i,t+1}\leq -B_t^{i|j,k}\right)\cap\left(V_{k,t+1}\leq -B_{k,t}\right)\right]\end{aligned} \tag{30}$$

$$-\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j,k}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right) \cap \left(V_{k,t+1} \leq -B_{k,t}\right)\right]$$

Thus, a simple estimator of this probability is given by:

$$\begin{aligned}
\hat{P}_t^{i,j,k} &= \frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(v_{i,t+1}^s \leq -B_t^{i|j,k}\right) \times \mathbf{I}\left(v_{j,t+1}^s \leq -B_{j,t}\right) \\
&+ \frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(v_{i,t+1}^s \leq -B_t^{i|j,k}\right) \times \mathbf{I}\left(v_{k,t+1}^s \leq -B_{k,t}\right) \\
&- \frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(v_{i,t+1}^s \leq -B_t^{i|j,k}\right) \times \mathbf{I}\left(v_{j,t+1}^s \leq -B_{j,t}\right) \times \mathbf{I}\left(v_{k,t+1}^s \leq -B_{k,t}\right)
\end{aligned} \tag{31}$$

and the CoMargin $B_t^{i|j,k}$ can be estimated by:

$$\hat{B}_t^{i|j,k} = \arg\min_{\left\{B_t^{i|j,k}\right\}} \left(\frac{\hat{P}_t^{i,j,k}}{2\alpha - \hat{P}_t^{j,k}} - \alpha\right)^2 \tag{32}$$

Following a similar argument, CoMargin can be generalized to n conditioning firms, with $n < N - 1$. In this case, the conditioning event is that at least one of the n clearing members is in financial distress (see Appendix for details).

5. Empirical Analysis

5.1. Data

In this section we evaluate and compare the empirical performance of the SPAN, VaR and CoMargin systems. We conduct this assessment using proprietary data from the Canadian Derivatives Clearing Corporation (CDCC). The data set includes daily observations of the actual trading positions, margin requirements and profits-and-losses of all forty-eight clearing members of the CDCC from January 2002 to March 2011.

The data set reports the daily net open interest of each clearing member on the S&P/TSX 60 Index Standard Futures (SXF), the ten-year Government of Canada Bond Futures (CGB), and the three-month Canadian Bankers' Acceptance Futures (BAX). These open interest figures are reported for each delivery date available for each contract. The SXF, CGB and BAX contracts account for almost 75% of the trading volume of all exchange-traded derivatives in Canada and for over 80% of the net total margin requirements collected by the CDCC. Table 4 summarizes the specifications of each one of these contracts. Our data set also includes the daily net SPAN margin requirements, net clearing fund contributions and net P&L for the customer, house and market-maker accounts of each clearing member.

Using the daily positions reported for each clearing member, we estimate their net SPAN, VaR and CoMargin collateral requirements and assess the number and magnitude of financial distress occurrences during the sample period. We show that, relative to other methods, the CoMargin system minimizes the number of joint financial distress events and the shortfalls associated with them. In addition, we show that the CoMargin methodology reduces procyclicality and abrupt changes in margin requirements because it incorporates information that is ignored by its SPAN and VaR counterparts. More specifically, by accounting for risk homogeneity, CoMargin is able to mitigate the effects of increased asset correlations and drastic price changes caused by clearing members exiting the market simultaneously.

Because of the proprietary nature of the data set used for this section, at this point, we are unable to publicly disclose our results in detail. However, in the next few days, we expect to be granted permission by the CDCC and its clearing members to disclose our findings. At that time, we will update this section of the paper.

6. Conclusion

In this paper, we have presented a new margin system, called CoMargin, for derivatives exchanges. CoMargin depends on both the tail risk of a given market participant and the interdependence between this participant and others participants. CoMargin can be estimated by a model-free and scenario-based methodology, backtested using formal statistical tests, and generalized to any number of market members. We show that the CoMargin outperforms alternative margin systems when the level of trading similarity and the comovement among underlying assets increase.

Appendix A: Proofs for CoMargin Properties (Section 4.2.1)

[Proof A1]: Let $H(B^{i|j},\sigma_i)$ be a function such that:

$$H(B^{i|j},\sigma_i) = \int_{-\infty}^{-B^{i|j}} g(u,\sigma_i)du - \alpha = 0 \quad \text{(A1)}$$

Note that we simplified the notation of the pdf $g(u;\sigma_i)$ compared to equation 15. Then, the CoMargin can be defined as an implicit function $B^{i|j} = h(\sigma_i)$. By the Implicit Functions Theorem, we have:

$$\frac{\partial B^{i|j}}{\partial \sigma_i} = -\frac{H_{\sigma_i}(B^{i|j},\sigma_i)}{H_B(B^{i|j},\sigma_i)} \quad \text{(A2)}$$

The derivative $H_B(B^{i|j},\sigma_i)$ can be expressed as follows:

$$H_B\left(B^{i|j},\sigma_i\right) = -g\left(-B^{i|j};\sigma_i\right) < 0 \quad \text{(A3)}$$

and is negative since $g(u;\sigma_i)$ is a pdf. Thus, the sign of $\frac{\partial B^{i|j}}{\partial \sigma_i}$ is given by the sign of $H_{\sigma_i}(B^{i|j},\sigma_i)$:

$$H_{\sigma_i}\left(B^{i|j},\sigma_i\right) = \frac{\partial}{\partial \sigma_i}\left(\int_{-\infty}^{-B^{i|j}} g(u;\sigma_i)\,du - \alpha\right) = \int_{-\infty}^{-B^{i|j}} \frac{\partial g(u;\sigma_i)}{\partial \sigma_i} du \quad \text{(A4)}$$

For simplicity, let us consider the case where $\rho = 0$:

$$\frac{\partial g(u;\sigma_i)}{\partial \sigma_i} = \frac{\partial}{\partial \sigma_i}\left(\frac{1}{\sigma_i}\phi\left(\frac{u}{\sigma_i}\right)\right) = -\frac{1}{\sigma_i^2}\phi\left(\frac{u}{\sigma_i}\right) - \frac{u}{\sigma_i^3}\phi'\left(\frac{u}{\sigma_i}\right) \quad \text{(A5)}$$

Since $\phi'(x) = -x\,\phi(x)$, we have:

$$\frac{\partial g(u;\sigma_i)}{\partial \sigma_i} = -\frac{1}{\sigma_i^2}\phi\left(\frac{u}{\sigma_i}\right)\left(1 - \left(\frac{u}{\sigma_i}\right)^2\right) \quad \text{(A6)}$$

For any value of u such that $u < -\sigma_i$, we have $\partial g(u;\rho)/\partial \sigma_i > 0$. This condition is satisfied when $u \in \left]-\infty, -B^{i|j}\right]$ since $-B^{i|j} = \sigma_i \Phi^{-1}(\alpha) = -\sigma_i \Phi^{-1}(1-\alpha)$ and $\Phi^{-1}(1-\alpha) > 1$ for most of the considered coverage rates (e.g. 1%, 5%). Consequently, the integral AX8 is also positive and $H_{\sigma_i}\left(B^{i|j},\sigma_i\right) > 0$. Then we conclude that:

$$\frac{\partial B^{i|j}}{\partial \sigma_i} = -\frac{H_{\sigma_i}(B^{i|j},\sigma_i)}{H_B(B^{i|j},\sigma_i)} > 0 \quad \text{(A7)}$$

A similar result can be obtained when we relax the assumption.

[Proof A2]: If $= 0$, the last term in equation 15 becomes $\Phi\left(-B_j/\sigma_j\right) = \Phi(\Phi^{-1}(\alpha)\,) = \alpha$ since $B_i = -\sigma_i \Phi^{-1}(\alpha)$. Consequently, the CoMargin of firm i is the solution of the following integral:

$$\int_{-\infty}^{-B^{i|j}} \frac{1}{\sigma_i} \times \phi(\frac{u}{\sigma_i})\, du = \alpha \tag{A8}$$

By properties of the normal distribution, we have $B^{i|j} = -\sigma_i \Phi^{-1}(\alpha) = B_i$.

[Proof A3]: Let $F(B^{i|j}, \rho)$ be a function such that:

$$F(B^{i|j}, \rho) = \int_{-\infty}^{-B^{i|j}} g(u;\rho)\,\mathrm{du} - \alpha = 0 \tag{A9}$$

Note that we simplified the notation of the pdf $g(u;\rho)$ compared to equation 15. Then, the CoMargin can be defined as an implicit function $B^{i|j} = f(\rho)$. By the Implicit Functions Theorem, we have:

$$\frac{\partial B^{i|j}}{\partial \rho} = -\frac{F_\rho(B^{i|j}, \rho)}{F_B(B^{i|j}, \rho)} \tag{A10}$$

where $F_\rho(.)$ and $F_B(.)$ denote respectively the first derivative of the F function with respect to ρ and B. For any function $H(x)$ defined as:

$$H(x) = \int_{-\infty}^{-b(x)} h(t)\, dt \tag{A11}$$

we have $H'(x) = h(b(x)) \times \partial b(x)/\partial x$. Consequently, the derivative $F_B(B^{i|j}, \rho)$ can be expressed as follows:

$$F_B\left(B^{i|j}, \rho\right) = -g\left(-B^{i|j}; \rho\right) < 0 \tag{A12}$$

and is negative since $g(u;\rho)$ is a pdf. Thus, the sign of $\partial B^{i|j}/\partial \rho$ is given by the sign of $F_\rho(B^{i|j}, \rho)$:

$$F_\rho\left(B^{i|j}, \rho\right) = \frac{\partial}{\partial \rho}\left(\int_{-\infty}^{-B^{i|j}} g(u;\rho)\, du - \alpha\right) = \int_{-\infty}^{-B^{i|j}} \frac{\partial g(u;\rho)}{\partial \rho} du \tag{A13}$$

Given the expression of the pdf $g(u;\rho)$ we have:

$$\begin{aligned} \frac{\partial g(u;\rho)}{\partial \rho} &= \overbrace{-\frac{1}{\alpha\sigma_i} \times \phi\left(\frac{u}{\sigma_i}\right) \times \phi\left(\frac{-B_j/\sigma_j - \rho u/\sigma_i}{\sqrt{1-\rho^2}}\right)}^{A} \\ &\quad \times \left(\frac{-u/\sigma_i\sqrt{1-\rho^2} - (B_j/\sigma_j + \rho u/\sigma_i)\rho(1-\rho^2)^{-1/2}}{1-\rho^2}\right) \\ &= A \times \left(\frac{1}{1-\rho^2}\right)^{3/2} \times \left(\frac{u}{\sigma_i} + \frac{\rho B_j}{\sigma_j}\right) \end{aligned} \tag{A14}$$

This function is positive for any value of u such that $u \leq \rho B_i = -\rho\sigma_i\Phi^{-1}(\alpha)$ with $-\rho\sigma_i\Phi^{-1}(\alpha) > 0$. Since $B^{i|j} \geq 0$ by definition, this condition is satisfied for the interval $\left]-\infty, -B^{i|j}\right]$ and $F_\rho\left(B^{i|j}, \rho\right) > 0$. Then we conclude that:

$$\frac{\partial B^{i|j}}{\partial \rho} = -\frac{F_\rho(B^{i|j},\rho)}{F_B(B^{i|j},\rho)} > 0 \tag{A15}$$

[Proof A4]: For $\rho = 1$, the pdf $g\big(u;\sigma_i,\sigma_j,\rho\big)$ in equation 15 is degenerated. However, when ρ tends to one, we have:

$$\lim_{\rho\to 1} \Phi\left(\frac{-\mathrm{B_i}/\sigma_j - \rho u}{\sqrt{1-\rho^2}}\right) = 1 \tag{A16}$$

as long as $u < \frac{-\mathrm{B_i}}{\sigma_j} = \Phi^{-1}(\alpha)$. If we assume that the standardized CoMargin for i is larger than the standardized VaR margin for i, i.e., $-B^{i|j}/\sigma_i \leq B_j/\sigma_j$, then we have:

$$\lim_{\rho\to 1} g(u) = \frac{1}{\alpha\sigma_i} \times \phi\left(\frac{u}{\sigma_i}\right) \tag{A17}$$

And consequently the CoMargin corresponds to the VaR margin defined for a coverage rate α^2 since:

$$\lim_{\rho\to 1} \int_{-\infty}^{-B^{i|j}} \frac{1}{\sigma_i} \times \phi\left(\frac{x}{\sigma_i}\right) dx = \alpha^2 \tag{A18}$$

$$\lim_{\rho\to 1} B^{i|j} = -\,\sigma_i \Phi^{-1}(\alpha^2) \tag{A19}$$

We can check that condition $-B^{i|j}/\sigma_i \leq B_j/\sigma_j$ is satisfied since $\Phi^{-1}(\alpha^2) \leq \Phi^{-1}(\alpha)$.

[Proof A5]: Since $B_j = -\sigma_i \Phi^{-1}(\alpha)$, the pdf $g(.)$ in equation 15 can be rewritten as:

$$g\big(u;\sigma_i,\sigma_j,\rho\big) = \frac{1}{\alpha\sigma_i} \times \phi(\frac{u}{\sigma_i}) \times \ \Phi\left[\frac{\Phi^{-1}(\alpha) - \rho u/\sigma_i}{\sqrt{1-\rho^2}}\right] \tag{A20}$$

As $g(.)$ does not depend on σ_j, $\partial B^{i|j}/\partial \sigma_j = 0$.

Appendix B: CoMargin with *n* Firms

With n conditioning firms, $n < N - 1$, the conditioning event of the CoMargin is that at least one of the n clearing members is in financial distress. Thus, the definition of CoMargin becomes:

$$\frac{\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|n}\right) \cap \mathbf{C}\left(V_{1,t+1},..,V_{n,t+1}\right)\right]}{\Pr\left[\mathbf{C}\left(V_{1,t+1},..,V_{n,t+1}\right)\right]} = \alpha \tag{B1}$$

where the probability to observe the conditioning event is:

$$\Pr\left[\mathbf{C}\left(V_{1,t+1},\ldots,V_{n,t+1}\right)\right] = \Pr\left[\left(V_{1,t+1} \leq -B_{1,t}\right) \text{ } or \text{ } \ldots or \text{ } \left(V_{n,t+1} \leq -B_{n,t}\right)\right] \tag{B2}$$

Using Poincaré's formula for the probability of the union of events, we can see that:

$$\begin{aligned}
\Pr\left[\mathbf{C}\left(V_{1,t+1},..,V_{n,t+1}\right)\right] = &\sum_{\boldsymbol{j=1}}^{\boldsymbol{n}} \Pr\left[\left(V_{j,t+1} \leq -B_{j,t}\right)\right] \\
&- \underbrace{\sum_{1 \leq j_1 < j_2 \leq n}^{\boldsymbol{n}} \Pr\left[\left(V_{j_1,t+1} \leq -B_{j_1,t}\right) \cap \left(V_{j_2,t+1} \leq -B_{j_2,t}\right)\right]}_{\boldsymbol{2\ events}} \\
&+ \underbrace{\sum_{1 \leq j_1 < j_2 < j_3 \leq n}^{\boldsymbol{n}} \Pr\left[\left(V_{j_1,t+1} \leq -B_{j_1,t}\right) \cap \left(V_{j_2,t+1} \leq -B_{j_2,t}\right) \cap \left(V_{j_3,t+1} \leq -B_{j_3,t}\right)\right]}_{\boldsymbol{3\ events}} \\
&\ldots \underbrace{+(-1)^{\boldsymbol{n-1}} \Pr\left[\left(V_{1,t+1} \leq -B_{1,t}\right) \cap \ldots \cap \left(V_{n,t+1} \leq -B_{n,t}\right)\right]}_{\boldsymbol{n\ events}}
\end{aligned} \tag{B3}$$

Thus, the probability of the conditioning event can be rewritten as follows:

$$\Pr\left[\mathbf{C}\left(V_{1,t+1},..,V_{n,t+1}\right)\right] = n\alpha - P_t^n \tag{B4}$$

where P_t^n denotes the sum of the probabilities of all common events (for two events, three events, etc.). An estimator of this value, $\hat{P}_t^n$, can be obtained from the simulated path $\left\{V_{1,t+1}^s,..,V_{n,t+1}^s\right\}_{s=1}^S$. When the financial distress events of the conditioning firms are mutually exclusive, however, the probability of the conditioning events simplifies to $n\alpha$. Therefore, an estimator of the CoMargin of firm i conditional on n clearing firms, $B_t^{i|n}$, is the solution of the program:

$$\hat{B}_t^{i|n} = \arg\min_{\left\{B_t^{i|n}\right\}} \left(\frac{\hat{P}_t^{i,n}}{n\alpha - \hat{P}_t^n} - \alpha\right)^2 \tag{B5}$$

where $\hat{P}_t^{i,n}$ denotes the estimator of $\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|n}\right) \cap \mathbf{C}\left(V_{1,t+1},..,V_{n,t+1}\right)\right]$, which is obtained by generalizing equation 30 conditioning on $B_t^{i|n}$.

References

Acharya, V. V., Engle, R., Figlewski, S., Lynch, A., and Subrahmanyam M., 2009, Centralized Clearing for Credit Derivatives. In *Restoring Financial Stability: How to Repair a Failed System*, edited by V. Acharya and M. Richardson (John Wiley & Sons).

Adrian, T., and Brunnermeier, M. K., 2011, CoVaR. Working Paper, Princeton University and FRB of New York.

Arnold, B., Beaver, R., Groeneveld, R., and Meeker W., 1993, The Nontruncated Marginal of a Truncated Bivariate Normal Distribution. *Psychometrika*, 58, 471-488.

Artzner, P., Delbaen, F., Eber, J.-M., and Heath, D., 1999, Coherent Measures of Risk. *Mathematical Finance*, 9, 203-228.

Bae, K. H., Karolyi, A., and Stulz, R. M., 2003, A New Approach to Measuring Financial Contagion. *Review of Financial Studies*, 16, 717-763.

Berkowitz, J., Christoffersen, P. F., and Pelletier, D., 2011, Evaluating Value-at-Risk Models with Desk-Level Data. *Management Science*, forthcoming.

Berkowitz, J., O'Brien, J., 2002, How Accurate Are Value-At-Risk Models at Commercial Banks? *Journal of Finance*, 57, 1093-1111.

Booth, G. G., Broussard, J. P., Martikainen, T., and Puttonen, V., 1997, Prudent Margin Levels in the Finnish Stock Index Futures Market. *Management Science*, 43, 1177-1188.

Boyson, N., Stahel, C. W., and Stulz, R. M., 2010, Hedge Fund Contagion and Liquidity Shocks. *Journal of Finance*, 65, 1789-1816.

Brunnermeier, M. K. and Pedersen L. H., 2009, Market Liquidity and Funding Liquidity. *Review of Financial Studies*, 22, 2201-2238.

Chicago Mercantile Exchange, 2009, CME Clearing Financial Safeguards.

Christoffersen, P. F., 2009, Backtesting. In Encyclopedia of Quantitative Finance, R. Cont (Editor), Wiley.

Cruz Lopez, J. A., Harris J. H., and Pérignon, C., 2011, Clearing House, Margin Requirements, and Systemic Risk. *Review of Futures Markets*, 19, 39-54.

Duffie, D., Li, A., and Lubke, T., 2010, Perspectives on OTC Derivatives Market Infrastructure. *Federal Reserve Bank of New York Staff Reports Policy No. 424.*

Duffie, D., and Zhu, H., 2011, Does a Central Clearing Counterparty Reduce Counterparty Risk? *Review of Asset Pricing Studies*, 1, 74-95.

Engle, R. F., and Manganelli, S., 2004, CAViaR: Conditional Autoregressive Value at Risk by Regression Quantiles. *Journal of Business and Economic Statistics* 22, 367-381.

Figlewski, S., 1984, Margins and Market Integrity: Margin Setting for Stock Index Futures and Options. *Journal of Futures Markets*, 4, 385-414.

Harris, J. H. and Stahel, C. W., 2011, Funding Constraints and Liquidity Contagion in U.S. Equity and Treasury Markets. George Mason University Working Paper.

Horrace, W. C., 2005, Some Results on the Multivariate Truncated Normal Distribution. *Journal of Multivariate Analysis*, 94, 209-221.

Hurlin, C. and Pérignon, C., 2012, Margin Backtesting.*WReview of Futures Markets*, 20, 179-194.

Jones, R. A., and Pérignon, C., 2013, Derivatives Clearing, Default Risk, and Insurance. *Journal of Risk and Insurance*, 80, 373-400.

Jorion, P., 2007, *Value at Risk: The New Benchmark for Managing Financial Risk*. McGraw-Hill, 3rd Edition.

Kupiec, P., 1994, The Performance of S&P 500 Futures Product Margins under the SPAN Margining System. *Journal of Futures Markets*, 14, 789-811.

Kupiec, P., 1995, Techniques for Verifying the Accuracy of Risk Measurement Models. *Journal of Derivatives*, 3, 73-84.

Knott, R. and Mills, A., 2002, Modelling Risk in Central Counterparty Clearing Houses: A Review. *Bank of England Financial Stability Review*, December, 162-174.

Longin, F., and Solnik, B., 2001, Extreme Correlation of International Equity Markets. *Journal of Finance*, 56, 649-676.

Longstaff, F., 2004, The Flight-to-Liquidity Premium in U.S. Treasury Bond Prices. *Journal of Business*, 77, 511-526.

Pirrong, C., 2009, The Economics of Clearing in Derivatives Markets: Netting, Asymmetric Information, and the Sharing of Default Risks through a Central Counterparty. *Working Paper, University of Houston*.

Poon, S. H., Rockinger, M., and Tawn, J., 2004, Extreme Values Dependency in International Stock Markets: Diagnostics, Models, and Financial Implications. *Review of Financial Studies*, 17, 581-610.

US Department of Treasury, 2009, Financial Regulatory Reform: A New Foundation. Washington D.C.

Table 1: Major Clearinghouses

Clearinghouses	CME	Eurex	LCH.Clearnet	Nymex	OCC
Markets cleared	CME, CBOT	Eurex	Euronext.liffe, ICE, LME, Powernext	Nymex, Comex	AMEX, CBOE
Number of clearing firms	86	90	77	40	120
Average daily volume	9 million	6 million	4 million	1 million	10 million
Average daily turnover	$3,000 billion	$550 billion	$2,500 billion	N/A	$10 billion
Margining system	SPAN	SPAN	SPAN	SPAN	SPAN
Aggregate margin	$47 billion	$39 billion	N/A	N/A	$77 billion
Default fund	$1 billion	$0.9 billion	$2.6 billion	$0.2 billion	$2.9 billion
Default insurance	-	-	$0.4 billion	$0.1 billion	-
Other guarantees	Membership value and assessment power	Deutsche Boerse, SWX	-	Protection scheme for retail customers	-
Total default protection	$53 billion	$41 billion	N/A	N/A	$80 billion

Notes: This table presents some descriptive statistics about the major clearinghouses in the world. We list the major derivatives markets that they clear, the number of clearing firms, the average daily volume in million of contracts, the average daily turnover (notional value), the margining system they use, the total aggregate margin or collateral collected from clearing firms for both customer trading and proprietary trading, the size of the default fund, the policy limit of the default insurance (if any), any other protections against default, as well as the total default protection, i.e. margin + default fund + default insurance + other guarantees. Source: Clearinghouses websites and annual reports (as of 2007).

Table 2: Margin Properties

Properties	SPAN Margin	VaR Margin	CoMargin
Reflect P&L variability	**Yes**	**Yes**	**Yes**
Reflect P&L dependence across participants	No	No	**Yes**
Exhibit low procyclicality	No	No	**Yes**
Be robust to outliers	No	**Yes**	**Yes**
Can be backtested	No	**Yes**	**Yes**

Notes: This table presents a list of properties the three margin systems (SPAN, VaR, and CoMargin) comply with, respectively violate.

Table 3: Margin Values

	VaR Margin	CoMargin	Budget-Neutral Margin
Correlation: ρ = 0			
Member 1	1.6449	1.6449	1.6449
Member 2	1.6449	1.6449	1.6449
Member 3	1.6449	1.6449	1.6449
Member 4	1.6449	1.6449	1.6449
[Total Margin]	*[6.5794]*	*[6.5794]*	*[6.5794]*
Correlation: ρ = 0.2			
Member 1	1.6449	1.7956	1.6449
Member 2	1.6449	1.7956	1.6449
Member 3	1.6449	1.6449	1.7956
Member 4	1.6449	1.6449	1.7956
[Total Margin]	*[6.5794]*	*[6.8809]*	*[6.8809]*
Correlation: ρ = 0.4			
Member 1	1.6449	1.9811	1.6449
Member 2	1.6449	1.9811	1.6449
Member 3	1.6449	1.6449	1.9811
Member 4	1.6449	1.6449	1.9811
[Total Margin]	*[6.5794]*	*[7.2519]*	*[7.2519]*
Correlation: ρ = 0.8			
Member 1	1.6449	2.3736	1.6449
Member 2	1.6449	2.3736	1.6449
Member 3	1.6449	1.6449	2.3736
Member 4	1.6449	1.6449	2.3736
[Total Margin]	*[6.5794]*	*[8.0370]*	*[8.0370]*

Notes: This table presents the margins computed according to three margin systems: VaR Margin (equation 2), CoMargin (equation 10), and Budget-Neutral Margin. The later margin scheme is designed to collect as much collateral as the CoMargin system but the extra collateral is collected only from independent firms, for which CoMargin always equal VaR Margin. We consider four firms with normally distributed P&L, $V_t \sim N(0, \Sigma)$, $V_t = \left(V_{1,t}, V_{2,t}, V_{3,t}, V_{4,t}\right)'$ and:

$$\Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

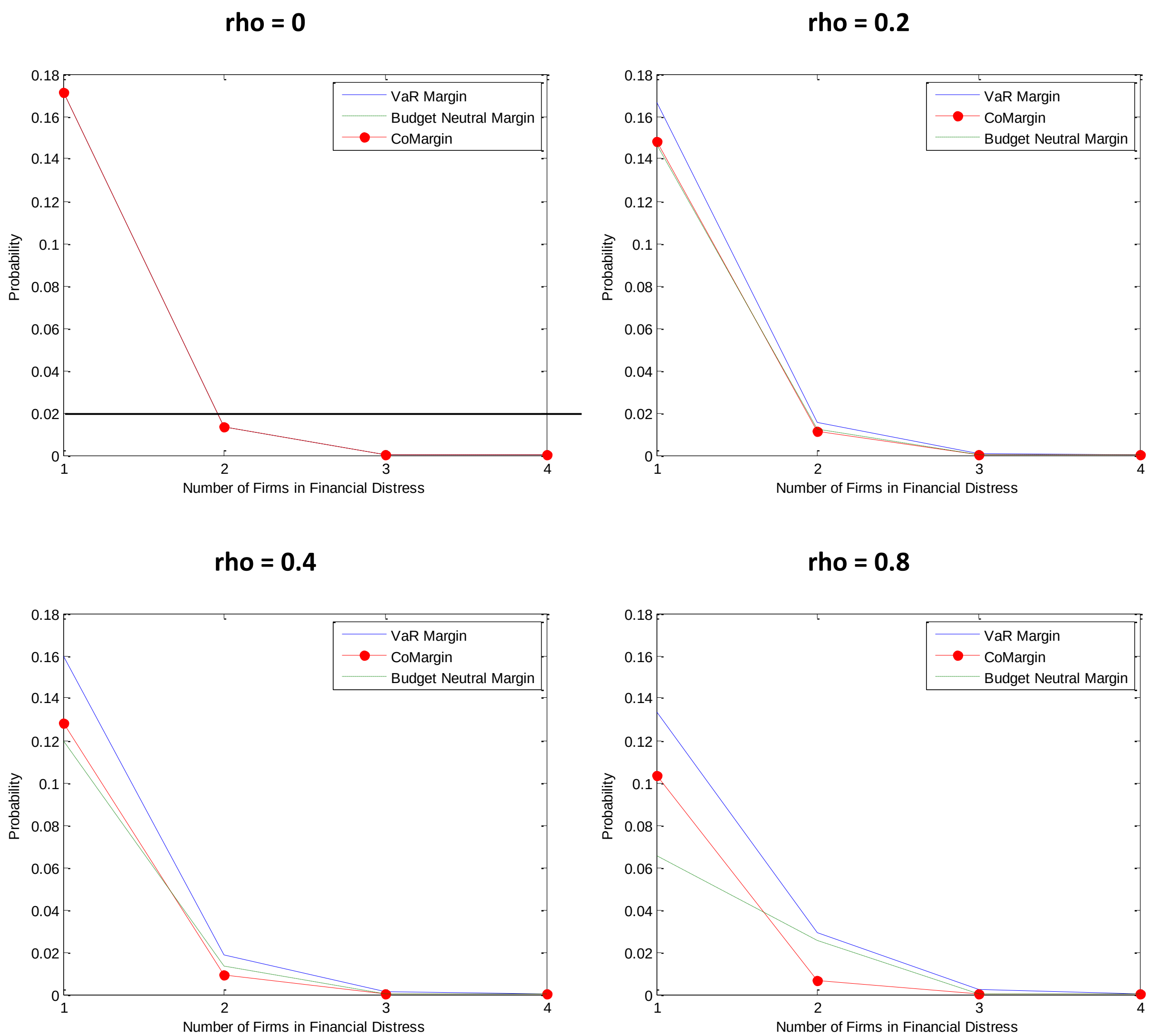
Table 4: Specifications of the Contracts Included in the CDCC Dataset

	S&P/TSX 60 Index Standard Futures (SXF)	Three-Month Canadian Bankers' Acceptance Futures (BAX)	Ten-Year Government of Canada Bond Futures (CGB)
Underlying	The S&P/TSX 60 Index		
Trading Unit	C$200 times the S&P/TSX 60 index futures value	C$1,000,000 nominal value of Canadian bankers' acceptance with a three-month maturity.	C$100,000 nominal value of Government of Canada Bond with 6% notional coupon.
Contract Months	March, June, September and December.	March, June, September and December plus two nearest non-quarterly months (serials).	March, June, September and December.
Price Quotation	Quoted in index points, expressed to two decimals.	Index : 100 minus the annualized yield of a three-month Canadian bankers' acceptance.	Par is on the basis of 100 points where 1 point equals C$1,000.
Last Trading Day	Trading ceases on the trading day prior to the Final Settlement Day.	Trading ceases at 10:00 a.m. (Montréal time) on the 2nd London (Great Britain) banking day prior to the 3rd Wednesday of the contract month or if a holiday, the previous bank business day.	Trading ceases at 1:00 p.m. (Montréal time) on the 7th business day preceding the last business day of the delivery month.
Final Settlement Day	The 3rd Friday of the contract month or if a holiday, the preceding day.	The final settlement price is based on the average of the three-month Canadian bankers' acceptance bid rates as quoted on CDOR page on the last trading day at 10:15 a.m. (Montréal time), excluding the highest and lowest values.	
Contract Type	Cash settlement.	Cash settlement.	Physical delivery of eligible Government of Canada Bonds.
Price Fluctuation	0.10 index points for outright positions. 0.01 index points for calendar spreads	0.005 = C$12.50 per contract for the nearest three listed contract months, including serials. 0.01 = C$25.00 per contract for all other contract months.	0.01 = C$10
Reporting Limit	1,000 futures contracts on the S&P/TSX 60 Index gross long and short in all contract months combined.	300 contracts.	250 contracts.

Price Limits	A trading halt will be invoked in conjunction with the triggering of "circuit breaker" in the underlying stocks.	None	None
Trading Hours (Montréal time)	Early session*: 6:00 a.m. to 9:15 a.m. Regular session: 9:30 a.m. to 4:15 p.m. * A trading range of -5% to +5% (based on previous day's settlement price) has been established only for this session.	Early session: 6:00 a.m. to 7:45 a.m. Regular session: 8:00 a.m. to 3:00 p.m. Extended session*: 3:09 p.m. to 4:00 p.m. * There is no extended session on the last trading day of the expiring contract month.	Early session: 6:00 a.m. to 8:05 a.m. Regular session: 8:20 a.m. to 3:00 p.m. Extended session*: 3:06 p.m. to 4:00 p.m. * There is no extended session on the last trading day of the expiring contract month.

Source: TMX Montreal Exchange (http://www.m-x.ca).

Figure 1: Financial-Distress Probabilities



Notes: This figure presents the probability of having, respectively, one (p_1), two (p_2), three (p_3), and four (p_4) firms in financial distress (i.e., firms suffering a margin-exceeding loss). We consider four firms with normally distributed P&L, $V_t \sim N(0, \Sigma)$, $V_t = \left(V_{1,t}, V_{2,t}, V_{3,t}, V_{4,t}\right)'$ and:

$$\Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Figure 2: Daily P&L

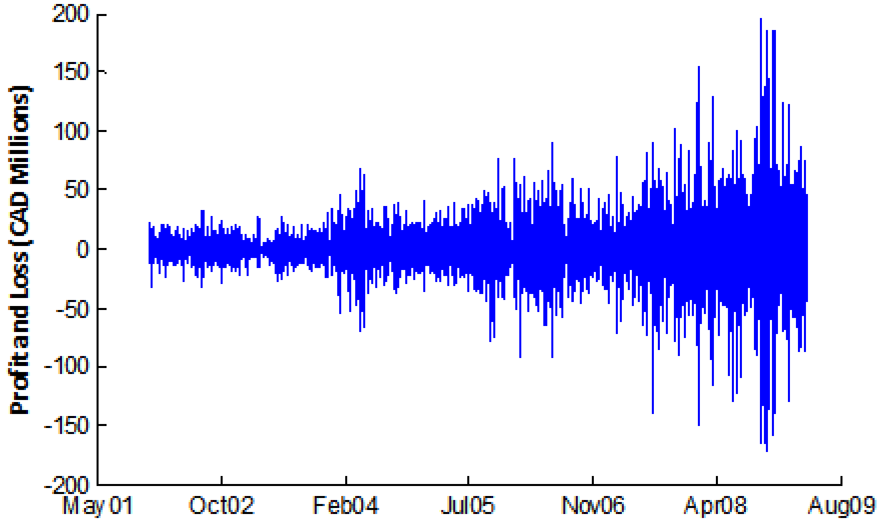


Figure 3: SPAN Margins

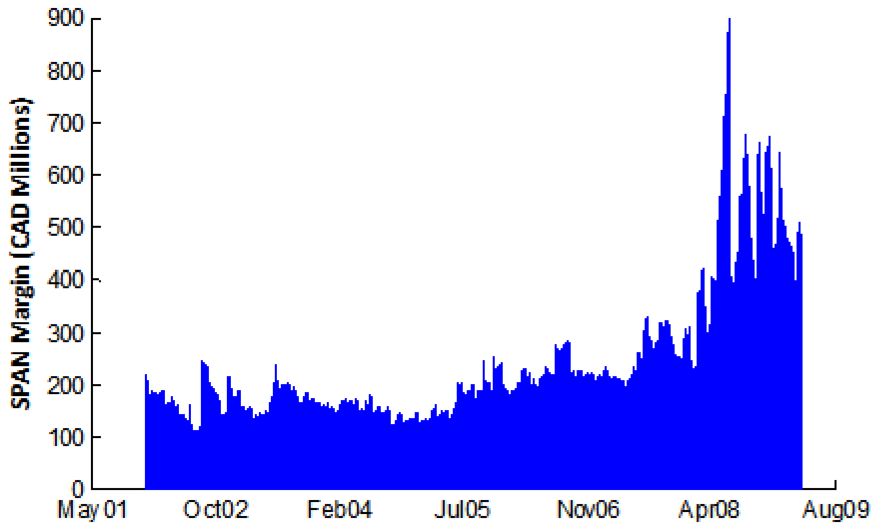


Figure 4: Daily Profit and Loss and Margins of CDCC Clearing Firms from January 2002 to April 2009

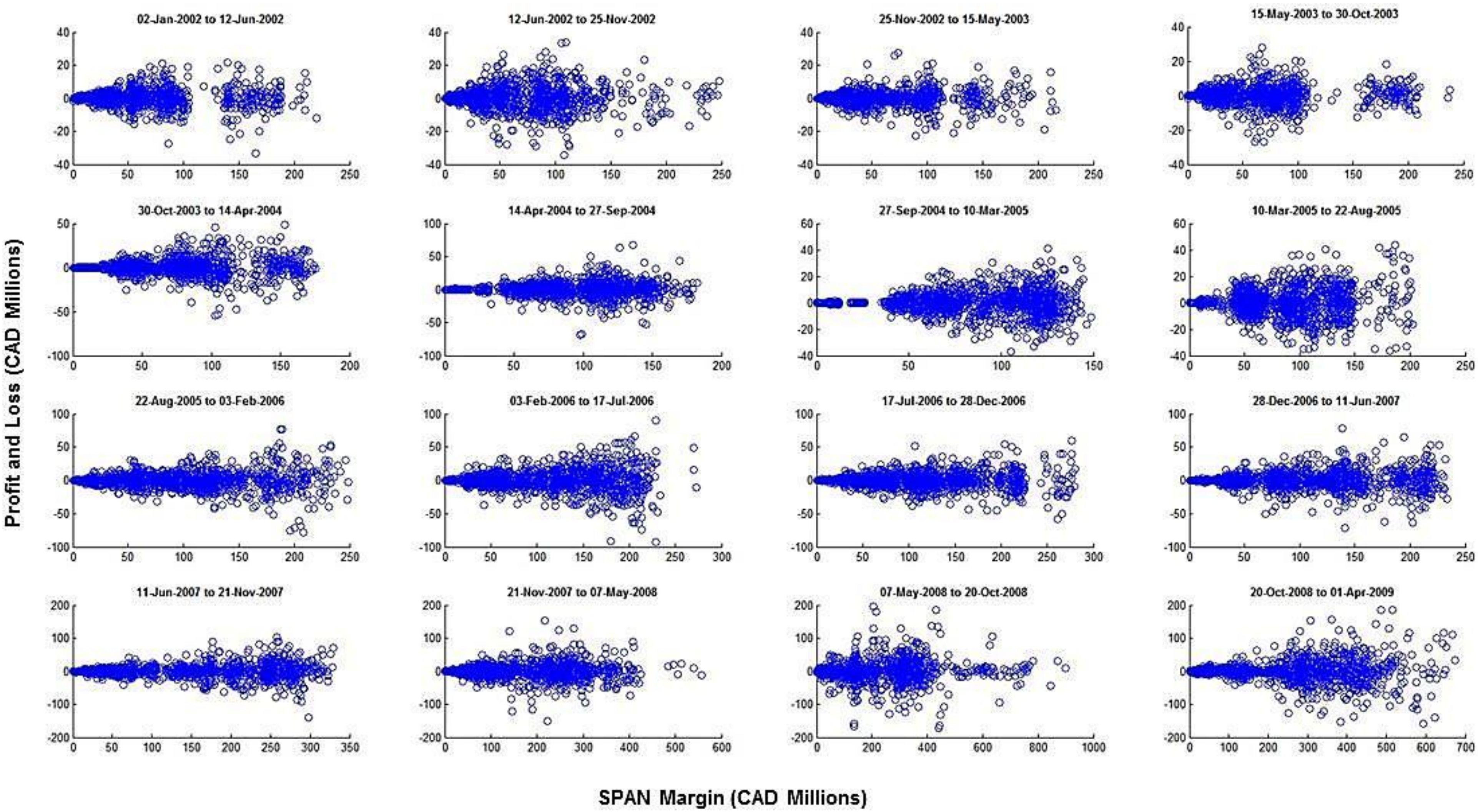


Figure 5: Ratio of the Daily P&L and SPAN Margin