Strategic Selection of Risk Models and Bank Capital Regulation*

Jean-Edouard Colliard[†]

April 20, 2015

Abstract

Using banks' internal models for regulatory purposes, while aimed at making capital requirements more accurate, invites regulatory arbitrage. I develop a framework to study the strategic selection of risk models, and derive predictions in line with recent empirical evidence. I also study optimal regulations. Penalizing banks with low risk-weights when they suffer abnormal losses is a powerful tool, currently used for market risk models, but needs to be amended when model uncertainty bears on tail risks. Recent regulatory reforms using non-risk based ratios are counter-productive in this framework, because the issue is not model risk but a "hidden model" problem.

Journal of Economic Literature Classification Number: D82, D84, G21, G32, G38.

Keywords: Basel risk-weights, internal risk models, leverage ratio, tail risk.

*I am grateful for helpful comments and suggestions to Toni Ahnert, Christophe Chamley, Alexandre de Cornière, Amil Dasgupta, Gabrielle Demange, Thierry Foucault, Alfred Galichon, Denis Gromb, Ian Harrison, Peter Hoffmann, David Martimort, Eric Monnet, Tuomas Peltonen, Joel Peress, Craig Pirrong, Jean-Charles Rochet, Arthur Silve, Elu von Thadden, Emmeline Travers, Larry Wall, Xiaopu Zhang, Balazs Zsamboki, Gabriel Zucman, participants to the HEC Workshop on "Finance, Banking, Real Economy and Trade", the 2013 BCBS RTF Workshop, the Atlanta Fed 2013 Financial Markets Conference, the RBNZ conference on "Stability and efficiency of financial systems", the 60th AFSE Congress, the 26th EEA Congress, the 11th SAET Conference, the 28th GdRE Symposium on Money, Banking and Finance, and seminar audiences at the Bank of England, the ECB, École Polytechnique, ESSEC, HEC Paris, Paris School of Economics, Sciences Po Paris, the universities of Amsterdam, Mannheim, Oxford, and Zürich.

[†]HEC Paris, Finance Department. Address: HEC Paris, 1 rue de la Liberation, 78351 Jouy-en-Josas, France. E-mail: `colliard@hec.fr`. Phone: (33) 1 39 67 72 90.

1 Introduction

Risk models have increasingly become a strategic variable for financial institutions, as their output is used to communicate risk measures to investors and regulators. This external use may affect the way models are chosen and implemented, leading to biased risk estimates. Dowd *et al.* (2008) study the example of Goldman Sachs, which according to its internal (Gaussian) models suffered several "25-sigmas events" in a row in August 2007, although they were supposed to occur once every 10^{135} years.

A clear rationale for the "strategic" use of internal models is banking regulation. To compute risk-based capital requirements, the Basel approach heavily relies on the banks' self-reported risk measures, which, according to a growing number of empirical studies, suffer from serious anomalies and inconsistencies (see, e.g., Behn, Haselmann, and Vig (2014) or Plosser and Santos (2014)). The reliability of internal risk measures has become one of the key issues in a heated debate on capital regulation, especially with the implementation of Basel III in the United States in 2014, with many voices calling for simpler rules and the use of non risk-based tools such as leverage ratios.[1]

Two questions are of particular importance in this debate: (i) How can we test whether the observed discrepancies between a bank's self-reported risk measures and its actual losses come from honest mistakes (model risk), or from strategic misreporting (a "hidden model" problem)? (ii) If strategic misreporting is the issue, what is the optimal regulatory answer, and how far from the optimum are current reforms?

This paper offers a framework to answer these questions. First, I derive and compare different mechanisms ensuring that risk measures are reported truthfully, taking into account several important practical constraints, such as the fact that uncertainty on risk measures often bears on tail risks, and the impossibility to "punish" banks already in default. Second, the model nests as a particular case the current regulatory framework and recent reforms such as leverage ratios. This approach gives implications on how banks' reported measures react to market or regulatory changes under strategic misreporting, giving guidance on how to empirically separate this hypothesis from alternative explanations.

I consider a regulator who wants to use a bank's internal risk estimates, which are private

[1] See "The dog and the frisbee", speech given by Andrew Haldane at the 36th economic policy symposium in Jackson Hole; or the joint press release of the Federal Reserve Board, the FDIC and the OCC on July 9, 2013: http://www.federalreserve.gov/newsevents/press/bcreg/20130709a.htm. Bundesbank's former Vice-President S. Lautenschläger expressed a more skeptical view of such tools: http://www.bundesbank.de/Redaktion/EN/Reden/2013/2013_10_21_lautenschlaeger.html.

information, and thus faces a hidden information problem.[2] I study solutions to this problem in increasingly constrained environments. The first-best outcome can be obtained with a mechanism imposing a penalty on a bank when observed losses are unlikely given its internal model, thus reducing incentives for using optimistic models. I then introduce the possibility that internal models differ mainly in their predictions about tail risk. In such a case, a bank reporting a very optimistic model can be allowed a high leverage. The regulator cannot learn that a model was too optimistic unless losses in the tail materialize, in which case the highly leveraged bank is in default and can no longer be punished. The first-best outcome can still be reached if the regulator commits to bailing out troubled banks that reported conservative risk measures, but not the ones that reported low risk weighted assets and are likely to have misreported. If such bail-outs are not feasible, the second-best solution is either to have capital requirements so high that all types of banks can be punished, or to offer a "reward" to banks reporting conservative risk measures, which then enjoy an informational rent. The optimal solution depends on the regulator's prior information about risk models and on the cost of public funds.

While penalty mechanisms have been used since the 1996 Amendment of the Basel Accord for the regulation of market risk,[3] and could be extended to other types of risk, it is also important to consider current reforms that attempt to solve the problem without using penalties. I first derive the optimal regulation without penalties. Capital requirements should be sensitive to reported risk measures. However, if this sensitivity is too high compared to the costs of misreporting, then banks have incentives to use optimistic risk models. To avoid this outcome, second-best capital requirements should react less to a bank's report. Interestingly, the second-best regulation does *not* complement risk-based capital requirements with a leverage ratio, as Basel III does. I study the impact of using such a tool, and show in particular that it can lead more banks to use optimistic models because of a market equilibrium effect. Indeed, a leverage ratio restricts the supply of credit by safer banks, for which it is binding. As a result, the interest rate on loans increases, so that bypassing regulatory constraints to lend more becomes more profitable. Riskier banks, for which the ratio is not binding, then report low risk measures and lend more. The total impact on risk

[2] Although the manipulation of risk models is what seems to worry regulators the most, there may be in addition a moral hazard problem: banks can invest in riskier assets if they can misrepresent the risk ex post. The framework of this paper can be adapted to the case of moral hazard, as discussed in footnote 15. Moreover, some mechanisms derived in the paper implement the first-best without leaving any rents to banks, which suppresses such moral hazard problems as the regulator can learn the quality of banks' assets at no cost).

[3] Under the "traffic light" approach, banks are required to report one-day 99% value-at-risk measures. Additional capital charges are imposed if these measures are exceeded more than 5% of the time over a given quarter, and the penalty increases with the number of exceedances.

may not be the one expected by the regulator. This is a simple yet important caveat for current reforms such as leverage ratios as complements to capital requirements, the Collins amendment in the United States, or floors based on Basel II's standardized approach.

If the problem with banks' risk estimates comes from model risk, possible solutions include a leverage ratio (Kiema and Jokivuolle (2014)) or a more robust approach to computing the estimates (Boucher *et al.* (2014)). These solutions are ineffective or even counter-productive when strategic selection of risk models is the real problem, it is thus important to derive predictions so that empirical studies can tell these two theories apart. Analyzing the equilibrium choice of risk models under current regulatory arrangements delivers testable implications. For instance, an exogenous increase in the demand for banks' loans causes more banks to select optimistic models; tightening regulatory constraints through leverage ratios, increasing capital requirements from Basel II to Basel III levels, or increasing the risk-sensitivity of capital requirements has the same impact. Importantly, if poor measures of risk come from honest modeling mistakes, none of these variables should have an impact on the choice of a particular risk model.

The analysis can be applied to all types of risks that are regulated using internal models in Basel II and III, but credit risk is particularly relevant. Credit risk models are extremely difficult to backtest due to their time horizon (typically one year) and the scarcity of available data. More generally, the "hidden model" framework developed here can also be used to study other situations in which an agent may strategically use internal models: For instance stress-testing exercises, the assessment of a pool of loans by a credit rating agency, the compensation of a trader or trading desk based on risk estimates.[4]

Related literature. There is a growing empirical literature on banks' internal risk measures. The most recent studies aim at showing that the discrepancies observed between the ratings given by different banks are compatible with a regulatory arbitrage hypothesis (e.g., Behn, Haselmann, and Vig (2014) or Plosser and Santos (2014)). My paper is complementary to this literature as it offers a framework to derive precise implications of this hypothesis. I discuss this literature at length in 5.1, show that the results of the model are compatible with the stylized facts that emerge from this literature, and suggest additional testable hypotheses.

On the theoretical side, this paper fits in the literature studying bank regulation as an asymmetric information problem. Although the seminal papers in this area proposed to use deposit

[4]The "London Whale" is a good illustration, see for instance "JP Morgan manipulated VAR and CRM models at London whale unit - Senate report" by D. Wood, *Risk Magazine*, 15.03.13.

insurance premia as a tool to reveal the bank's information (e.g., Chan, Greenbaum, and Thakor (1992) and Giammarino, Lewis, and Sappington (1993)), international regulatory standards use different tools, in particular self-reported risk measures with various mechanisms to control the incentives to misreport. I study optimal regulatory arrangements taking as given the families of mechanisms available in the Basel toolbox, and show that there are in principle enough tools to implement the first-best, without using deposit insurance premia.[5]

The previous stages of the Basel framework generated a few papers on the regulatory use of internal models, such as Kupiec and O'Brien (1995), Lucas (2001), Prescott (2004), Cuoco and Liu (2006), and Marshall and Prescott (2006), the latter two being the most related. Cuoco and Liu (2006) study the optimal reaction of a bank to the penalty mechanism actually used for backtesting market risk models, and use numerical methods to show that the threat of future increases in capital requirements has a powerful disciplining effect. The penalties imposed on a misreporting bank in my model can be seen as the static counterpart of such a mechanism. Marshall and Prescott (2006) study optimal penalty mechanisms to make banks report risk truthfully and also resort to a numerical analysis, due to the complexity of the problem (both adverse selection and moral hazard). Prescott (2004) is the only paper studying how to optimally audit risk models so as to deter strategic behavior.[6]

The approach of my paper is quite different from this literature. I study a more stylized and tractable framework in order to obtain analytical results on the optimal regulation. I derive optimal mechansims under a variety of realistic constraints that were neglected in previous studies, and compare these mechanisms to different regulatory options that are currently contemplated. Moreover, I analyze the equilibrium reaction of banks to realistic but non-optimal mechanisms, such as constraints on leverage ratios, in order to derive empirical predictions that can be confronted to the empirical literature.

An additional novelty of this paper is that uncertainty is not only about a single number (e.g., the probability that a bank survives) but about the entire distribution of a bank's possible losses (a "risk model"). This approach allows me to introduce asymmetric information about tail risk

[5]Note that demand for loans is inelastic in Chan, Greenbaum, and Thakor (1992), so that higher capital ratios do not imply fewer loans, a concern that seems currently important. Giammarino, Lewis, and Sappington (1993) study a moral hazard problem in which the regulator can check the quality of a bank's assets ex post at no cost, an assumption that applies less well to the issue of internal risk models.

[6]Some papers have also studied the choice between Basel's standardized (SA) and internal ratings based (IRB) approaches, such as Repullo and Suarez (2004), Hakenes and Schnabel (2011) or Feess and Hege (2012), but have not looked at the strategic choice of risk models inside the IRB approach.

in a model of bank regulation, and let it interact with limited liability.[7] Taking into account this important feature identifies some limits to the penalty-based mechanisms studied in prior literature. For instance, I show that if banks cannot be bailed-out then tail-risk uncertainty will make penalty-based mechanisms inefficient. Second-best mechanisms typically distort capital requirements in addition to defining penalties. These results can be compared to Blum (2008), who concludes that penalties must be combined with a leverage ratio constraint in a setup with two types of banks and two levels of losses. In a framework with an entire distribution of losses, penalties are insufficient when the different models differ only for loss levels so high that the bank is in default (a setup with two levels of losses mechanically features this implicit assumption). Moreover, the proper remedy is in general to make capital requirements less sensitive to a bank's reported risk measure, which is equivalent to a leverage ratio constraint only with two types of banks. I thus reach very different conclusions regarding the desirability of such a tool.

This paper finally contributes to the general question of understanding how economic incentives can affect the decision of agents who develop or rely on statistical models. As markets become more sophisticated, internal models are increasingly used to communicate information to other market participants.[8] However, the theoretical literature typically neglects this strategic dimension of model selectio,. One exception, in a very different context, is Ghosh and Masson (1994), who suggest that governments could in fact pretend to believe in economic models they know to be false, so as to gain in bargaining power when meeting with other countries' representatives.

The remainder of the paper is organized as follows: Section 2 develops the general framework; I study the optimal regulation using ex-post penalties in 3, and then regulatory schemes without penalties in 4, a particular case being leverage ratios. Section 5 discusses the empirical implications of the model and compares the different regulatory options derived in the paper.

[7]A few papers have also looked at the regulation of tail risks specifically, but under moral hazard, in particular Biais *et al.* (2010), and Perotti, Ratnovski, and Vlahu (2011).

[8]See for instance Rajan, Seru, and Vig (2010) and Rajan, Seru, and Vig (2014) on the use of risk estimates when selling securitized loans.

2 Framework

Three elements are needed to study the strategic choice of models: Regulated financial intermediation, model uncertainty, and asymmetric information between the intermediaries and the regulator.

Agents and assets. *Borrowers* need to finance risky projects that can either succeed or fail. If a measure L of projects are financed, the gross return on the marginal project, if it is successful, is $\rho(L)$, $\rho(.)$ being a strictly decreasing function. A random proportion t of projects will fail, following a distribution defined below. Failed projects yield 0, leading to the borrower's default.

Depositors can either invest in a safe asset yielding the risk-free rate $r_0 = 1$ with certainty, or lend to financial intermediaries, but not directly to borrowers. They are fully insured and provide an elastic supply of deposits at rate r_0.

A regulated *intermediary* (or bank) owns a fixed amount of equity E and can borrow D from depositors at r_0.[9] The intermediary is a monopolist and chooses a quantity L to lend to borrowers, which determines the gross interest rate $r = \rho(L)$. He can also invest in the safe asset at rate r_0. The intermediary is protected by limited liability and has to comply with the regulatory framework in order to collect deposits. Alternatively, he can opt out of the regulation and receive an exogenous payoff $\bar{\pi}$.

The regulatory framework is designed by a benevolent *regulator* maximizing a weighted sum of the payoffs to depositors, borrowers, the deposit insurance fund, and the intermediary. A female pronoun will refer to the regulator, a male pronoun to the intermediary.

There are many depositors and borrowers, so that the bank is intermediating between two price-taking groups of agents. Borrowers demand funds until the marginal payoff in case of success ρ is equal to the repayment r, thus generating a demand for loans equal to $q(r) = \rho^{-1}(r)$. I assume $q(1) = +\infty$ and $\lim_{+\infty} q(r) = 0$.

Model uncertainty. A family of cdfs $\{F(., \tilde{s}), \tilde{s} \in [\underline{s}, \overline{s}]\}$, with support over $[0, 1]$, represent the set of plausible risk models to describe the distribution of t, the proportion of defaulting borrowers. This family of cdfs indexed by $\tilde{s}$ can be interpreted as one model with different parameterizations, or models from different families. Denote $\{f(., \tilde{s})\}$ the corresponding pdfs.

The correct risk model, s, is randomly selected by nature in $[\underline{s}, \overline{s}]$ according to some cdf $\Psi(.)$, with associated density $\psi(.)$. The actual proportion t of defaulting borrowers then follows the distribution $F(t, s)$, continuously differentiable in both arguments. Moreover, for some results it

[9] The assumption of fixed equity simplifies the exposition. As discussed below, it is enough to assume that some frictions (e.g., informational) make the issuance of equity costly for a bank.

is useful to assume, as is customary in the mechanism design literature, that the family $\{F(., \tilde{s})\}$ satisfies the monotone likelihood ratio property, which implies in particular that models with a low s give risk estimates unambiguously more favourable than models with a high s:[10]

$$\forall t_0, t_1, s_0, s_1 \text{ with } t_1 \geq t_0,\ s_1 \geq s_0,\ \frac{f(t_1, s_1)}{f(t_1, s_0)} \geq \frac{f(t_0, s_1)}{f(t_0, s_0)}. \tag{MLRP}$$

Regulation under complete information. As a benchmark, assume that the regulator knows s and can directly choose the loan volume L. In addition, she can impose a transfer $T(s,t)$ from the intermediary to the public budget, conditional on t defaults realizing. The regulator's objective is to maximize total welfare. The agent (here the bank) has a weight $1-\lambda < 1$. The welfare of insured depositors is constant and can thus be neglected. The choice of L determines $r = \rho(L)$. There is no point in having the intermediary borrowing at r_0 and investing at the same rate, so that $D = L - E$. The proportion of losses θ above which the intermediary defaults, or default point, is determined by $rL(1-\theta) - r_0(L-E) = 0$. Finally, denoting $\mathbb{E}_s$ an expectation according to the distribution $F(., s)$, we can write the regulator's objective function for a given s as:

$$\begin{aligned}\max_{L,T} \mathcal{V}(L,T,s) \quad &= \quad \underbrace{\mathbb{E}_s(1-t)\left(\int_0^L \rho(u)du - rL\right)}_{\text{Borrowers}} + \underbrace{(1-\lambda)\int_0^\theta (rL(1-t) - r_0(L-E) - T(s,t))f(t,s)dt}_{\text{Intermediary}} \\ &+ \quad \underbrace{\int_\theta^1 (rL(1-t) - r_0(L-E))f(t,s)dt}_{\text{Deposit insurer}} + \underbrace{\int_0^\theta T(s,t)f(t,s)dt}_{\text{Public budget}}. \end{aligned} \tag{1}$$

To implement the optimal L, even with perfect information, the regulator faces two constraints: (i) limited liability (LL) - she cannot transfer more than what the intermediary has earned; (ii) individual rationality (IR) - an intermediary must receive more than his outside option $\bar{\pi}$. In this context, the outside option would typically be to opt for Basel's SA, not use any internal model and earn a profit that is higher if loans are less risky (low s). The outside option is then type-dependent (Jullien (2000)) and denoted as $\bar{\pi}(s)$, with $\bar{\pi}' \leq 0$.[11]

If the regulator knows s, she can choose transfers $T(s,t)$ such that the bank of type s receives exactly $\bar{\pi}$ in expectation. Maximizing $\mathcal{V}(L,T,s)$ then reduces to maximizing $r_0(W+E-L)+\mathbb{E}_s(1-$

[10] Families of exponential and Poisson distributions (restricted to the interval $[0,1]$) satisfy this assumption, as well as normal distributions with different means but the same standard deviation. The assumption will be needed only for tail levels. For lower levels it is enough to have first-order stochastic dominance.

[11] I discuss this assumption in more details at the end of this section.

$t) \int_0^L \rho(u)du$ and the optimal L is such that the interest rate is equal to the break-even rate $r^e(s)$:

$$\rho(L) = r^e(s) = \frac{r_0}{\mathbb{E}_s(1-t)}. \tag{2}$$

As the intermediary is a monopolist, he may have an incentive to restrict lending so as to increase the interest rate. To focus on the interesting case in which the first-best and second-best levels of lending are below the profit-maximizing value for the intermediary, I make the following assumption:

$$\rho(E) > r^e(\bar{s}) - E\rho'(E). \tag{H1}$$

$$\forall L \in [E, q(r^e(\underline{s}))], (L\rho(L))'' \geq -\bar{\varepsilon}, \tag{H2}$$

where the value of $\bar{\varepsilon}$ is given in the Appendix A.1. Assumption (H1) means that E is small enough enough for the interest rate to be very high if the bank only lends E. This implies that the bank always wants to lend more than E. Moreover, the bank's own capital is too low to cover the first-best volume of loans, even when the most conservative model is true, so that the regulator would like the bank to have some leverage. Assumption (H2) means that the bank's revenue is not too concave in L. Limited liability makes the bank's profit convex in L and gives incentives to lend more than the first-best, whereas monopoly power gives the opposite incentive. The assumption implies that the former effect dominates, so that the analysis can focus on the regulation of a bank engaged in risk-shifting rather than on the different problem of curbing monopoly power.[12]

These simplifying assumptions are not strictly necessary. If there exists s such that $\rho(E) < r^e(s)$, this type of bank would be given 100% capital requirements and would invest the optimal amount of loans. (H1) allows to neglect this extreme and less interesting case. If (H2) does not hold, some types of banks may want to lend less than in the first-best, so that capital requirements are not binding and there are no incentives to report optimistic models, a case also of less interest.

Under (H1) and (H2), choosing the optimal L becomes equivalent to choosing capital requirements α, as the bank picks $L = E/\alpha$. The choice of α then defines the interest rate, an intermediary's

[12] Alternatively, it is possible to consider the regulation of a competitive banking sector, in which case this assumption is not needed. However, the monopoly case permits to remain close to the traditional principal-agent framework.

default point and the economic surplus when $\lambda = 0$, denoted $\mathcal{W}(\alpha, s)$, as follows:

$$
\begin{aligned}
r(\alpha) &= \rho(E/\alpha), \ \theta(\alpha) = 1 - \frac{r_0}{r(\alpha)}(1-\alpha), &(3)\\
\mathcal{W}(\alpha, s) &= \mathbb{E}_s(1-t)\int_0^{\frac{E}{\alpha}} \rho(u)du - r_0\frac{E}{\alpha}. &(4)
\end{aligned}
$$

Rewriting (1), the program determining the first-best can be expressed as:

$$
\max_{\alpha,T} \int_{\underline{s}}^{\bar{s}} \mathcal{W}(\alpha(s), s) - \lambda \int_0^{\theta(\alpha(s))} \left(\frac{E}{\alpha(s)}[r(\alpha(s))(1-t) - r_0(1-\alpha(s))] - T(s,t)\right) f(t,s)dt \quad (5)
$$

$$
s.t. \ \forall s, \int_0^{\theta(\alpha(s))} \left(\frac{E}{\alpha(s)}[r(\alpha(s))(1-t) - r_0(1-\alpha(s))] - T(s,t)\right) f(t,s)dt \geq \bar{\pi}(s) \quad \text{(IR)}
$$

$$
\forall s, t, \frac{E}{\alpha(s)}[r(\alpha(s))(1-t) - r_0(1-\alpha(s))] \geq T(s,t). \quad \text{(LL)}
$$

This program delivers the following result, proven in the Appendix A.1:

Lemma 1. *Under (H1) and (H2), the first-best can be implemented through capital requirements $\alpha^*(s)$ and transfers $T^*(s,t)$ such that:*

-1. For any s, $r(\alpha^*(s)) = r^e(s)$, so that α^* is increasing in s.

-2. $T^*(s,t)$ leaves no rent to the bank:

$$
\int_0^{\theta(\alpha^*(s))} \left(\frac{E}{\alpha^*(s)}[r(\alpha^*(s))(1-t) - r_0(1-\alpha^*(s))] - T^*(s,t)\right) f(t,s)dt = \bar{\pi}(s). \quad (6)
$$

Given that the optimal amount of loans depends on the true level of risk, capital requirements should be risk-sensitive and thus, in this framework, model-sensitive. α is interpreted as a capital requirement, but equivalently $\alpha(s)L$ times some constant can be seen as risk-weighted assets, with risk-weights computed according to the bank's internal model. We thus obtain a first-best regulation relying on risk-sensitive capital requirements, as in the Basel approach.

Discussion. The role for regulation in the model fits the classical "representation hypothesis" (Dewatripont and Tirole (1994)): Due to limited liability and the absence of market discipline exercised by depositors, banks borrow and lend too much, as they do not take into account the losses to the deposit insurer. The regulator represents the interests of the deposit insurance fund, itself a substitute for uninformed and scattered depositors who could not efficiently exercise market discipline.

Although the welfare function $\mathcal{V}(L, T, s)$ is natural in this economy, similar results would be obtained in the paper for any criterion leading to a function $\mathcal{W}(\alpha, s)$ with $\mathcal{W}_{12} > 0$.[13] The assumption that a weight $1 - \lambda < 1$ is given to the payoff of the regulated entity is customary in the principal-agent literature and ensures that the regulator minimizes the cost of the mechanism (see for instance Baron and Myerson (1982)). The mechanisms of the next section that implement the first-best outcome are also optimal if $\lambda = 0$, but other optimal mechanisms can be obtained by simply making arbitrary lump sum transfers to the bank. A small but positive λ is enough to solve this indeterminacy.

For parsimony, I abstract from a number of elements that also affect strategic model selection and can be integrated into the framework of this paper without affecting the results qualitatively:

Banks in the model do not pay a risk-based deposit insurance premium. In the relatively few countries in which premia are risk-based, they depend on risk-weighted assets and thus only strengthen the incentives to use optimistic models. It is also possible to model banks with uninsured creditors, e.g., wholesale funding. Even assuming that uninsured creditors can detect overoptimistic risk estimates, in a competitive equilibrium banks maximize the total payoff to shareholders and uninsured creditors and neglect the losses to the deposit insurance fund. As a result, there is still an incentive to take too much risk in this case, and use optimistic models to bypass regulatory constraints.

Bank capital E is exogenously fixed in the model. This assumption reflects the fact that, under the current regulatory framework, higher capital requirements have a negative impact on bank lending (see, e.g., Aiyar, Calomiris, and Wieladek (2014) for recent evidence of this effect). Studying whether a general overhaul of the Basel framework would reduce this effect and produce a better outcome is beyond the scope of this paper. Notice that with the mechanism derived in Proposition 1 the first-best can be implemented for any E, so that a fixed E is not a restrictive assumption. Otherwise, the Internet Appendix shows how the model can be extended by assuming that additional capital can be levied, although at a cost, due for instance to asymmetric information.

Banks are assumed to have an outside option that depends on the actual riskiness of their loans. This constraint reflects the necessity to encourage different types of banks to adopt the IRB approach and played an important role in the design of Basel II. Another concern is that assets will move from the regulated to the unregulated sector, an equilibrium effect studied in Harris, Opp, and Opp (2014). Respecting the type-dependent participation constraint ensures that the

[13] More weight could be given to the deposit insurance fund, or the goal of the regulator could be to ensure that any type of bank has a default probability below a certain bound.

regulation does not have this unwanted effect. Relaxing this assumption does not affect the analysis of Section 4 in which participation constraints are not binding, and reinforces the point of Section 3 that transfers are a powerful tool.

It is of course possible to take into account traditional countervailing forces to risk-shifting such as random deposit withdrawals, risk aversion or a dynamic charter-value effect for instance. As long as these forces are not strong enough to make the regulation non-binding for all banks, the main economic mechanisms at work in the model would not be affected.

3 Optimal regulation with hidden model

The purpose of using a bank's internal model for determining capital ratios is to rely on its supposedly better knowledge of risk models. This is by definition a situation of asymmetric information: The regulator wants to set capital requirements based on better risk measures as in Lemma 1, but this gives banks incentives to strategically misreport more favourable models so as to increase leverage. The goal of the regulator is to ensure that the bank truthfully reports its model s or, more formally, its type.

There is a natural limit to misreporting: Reporting a wrong model requires time to convince the supervisor that it can be relied upon, may lead to some wrong decisions, and so on. To model this aspect in a simple way, I assume the presence of a "falsification cost" (as in, e.g., Lacker and Weinberg (1989)): If the true model is s and the intermediary reports model $s' < s$, he incurs a non-pecuniary cost $c(s', s)$, increasing in the distance between s and s'.[14] I thus assume that for any (s', s), with $s' \leq s$, we have $c(s, s) = 0, c_1(s, s) \leq 0, c_{11}(s', s) \geq 0$. There is never an incentive to report $s' > s$. Observe that if the cost is high enough the intermediary always reports truthfully, even if the first-best regulation is implemented. To avoid discussing this case, I assume from now on that there is always an incentive to misreport at least a little:

$$\forall s, \bar{\pi}'(s) - c_1(s, s) \leq 0. \tag{H3}$$

It is easy to relax this assumption, in which case some models may always be reported truthfully and the regulator does not need to be concerned about them.

The goal of this section is to derive the optimal second-best regulation when the intermediary

[14] It is possible to assume a pecuniary cost, but this complexifies the analysis as it impacts the bank's default probability.

must be given incentives to report the correct model. The current regulatory framework makes use of a number of tools that can all be included in the following representation of the game between the regulator and the intermediary:

-**Period 0.** The regulator specifies a formula linking any s to a capital ratio $\alpha(s)$, as well as penalties $T(s',t)$ to be paid if model s' is used and t defaults realize.

-**Period 1.** The true model s is drawn from $\Psi(.)$ and observed by the intermediary. He can report a model $s' \in [\underline{s}, \overline{s}]$ at a cost $c(s',s)$, or opt out of the regulation and receive $\bar{\pi}(s)$.

-**Period 2.** If he did not opt out, the intermediary supplies $L = E/\alpha(s')$ loans. r is determined by $q(r) = L$.

-**Period 3.** A proportion t of borrowers default, drawn from $F(.,s)$. Payoffs are realized, the intermediary pays the penalties $T(s',t)$.

This game incorporates most tools actually available to the regulator. The penalties T are a generalization of the penalty mechanism that is already in use for market risk models. The cost c reflects how demanding the regulator is when approving the model. The capital ratio $\alpha(.)$ links a bank's model to capital requirements, as for instance in the regulatory formulas used for credit risk and market risk. $\alpha(.)$ can incorporate additional measures, such as floors, regulatory multipliers or add-ons. Moreover, a prohibitively high $\alpha(s')$ for a particular model s' means that it fails to get supervisory approval, based on a comparison with "industry standards", required assumptions of the model, performance on historical data, and so on. The approval decision is thus embedded in the definition of $\alpha(.)$.

I now analyze under which conditions the regulator can use ex post penalties, and show specific difficulties associated with this mechanism in the presence of tail risk. It is useful to denote $u(\alpha, r, t)$ the profit before transfers of an intermediary facing capital requirements α when the interest rate is r and t defaults realize. We can then define $\pi(s', s)$ the expected profit before transfers of an intermediary who reports model s' and gets capital requirements $\alpha(s')$ when the true model is s:

$$
\begin{aligned}
u(\alpha, r, t) &= (E/\alpha)\left(r(1-t) - r_0(1-\alpha)\right), & (7)\\
\pi(s', s) &= \int_0^{\theta(\alpha(s'))} u(\alpha(s'), r(\alpha(s')), t) f(t, s) dt. & (8)
\end{aligned}
$$

As the regulator cannot observe s, the mechanism must be incentive-compatible. An intermediary must be better off revealing his type s and paying the transfers $T(s,.)$ than misreporting:

$$\forall s, s',\ \pi(s,s) - \mathbb{E}_s(T(s,t)) \geq \pi(s',s) - \mathbb{E}_s(T(s',t)) - c(s',s). \tag{IC}$$

The regulator's program is then to maximize in $\alpha(.)$ and $T(.,.)$ the following objective function, under the constraints (LL), (IR), and (IC):

$$\mathbb{E}\left(\mathcal{W}(\alpha(s),s) - \lambda\left[\int_0^{\theta(\alpha(s))} u(\alpha(s), r(\alpha(s)), t) f(t,s)dt - \mathbb{E}_s(T(s,t))\right]\right). \tag{9}$$

where the first expectation is taken over all values of s. The idea behind such a mechanism is that the regulator offers a profile of transfers $T(s,t)$ such that an intermediary reporting s is heavily taxed if the realized level of defaults was relatively unlikely given the model announced.[15]

3.1 Reaching the first-best when models are easy to distinguish

Consider first an example with only two types s_1 and $s_2 > s_1$, two possible realizations of defaults $\underline{t}$ and $\bar{t} > \underline{t}$, and $\Pr(t = \underline{t}|s_i) = p_i$, $p_1 > p_2$. To satisfy (IC) and bind (IR), the regulator can choose transfers contingent on how many defaults materialize: A bank reporting the optimistic model s_1 receives $\bar{\pi}(s_1)/p_1$ if the low losses $\underline{t}$ realize, but 0 in the case of high losses; a bank reporting the more conservative model s_2 receives $\bar{\pi}(s_2)$ irrespective of the realization. By definition, (LL) is met. (IC) for type s_2 gives:

$$c(s_1, s_2) \geq \bar{\pi}(s_1)\left(\frac{p_2}{p_1} - \frac{\bar{\pi}(s_2)}{\bar{\pi}(s_1)}\right). \tag{10}$$

Even if $c(s_1, s_2) = 0$, it is possible to reach the first-best if the right-hand side is negative, that is if the likelihood ratios of the two states under both models are very different, while the outside options of both types are close. Put differently, when profit decreases quickly in s and the two models do not give very different predictions, a rent has to be left to the regulated bank if the cost c is not high enough. The following assumption of "distinguishable models" generalizes the idea of giving different predictions to a continuum of types:

$$\forall t \in [0,1],\ \forall s \in [\underline{s}, \bar{s}],\ \frac{F(t,s)}{\bar{\pi}(s)} \text{ is log-concave in } s \text{, and decreasing in } s \text{ when } t \to 0. \tag{DM}$$

[15] Assuming moral hazard instead of hidden information would yield a similar role for contingent penalties. Imagine that s reflects the riskiness of the bank's portfolio and this riskiness is achieved at a cost $e(s)$. The regulator wants to implement some level s^* and has to respect an incentive compatibility constraint of the form $\pi(s^*, s^*) - \mathbb{E}_{s^*}(T(s^*, t)) - e(s^*) \geq \pi(s^*, s') - \mathbb{E}_{s'}(T(s^*, t)) - e(s')$, for any s'. Penalizing the bank in states t that are likely given s' but unlikely given s^* will be a typical outcome of such a constraint.

(DM) means that $F(., s)$ is more log-concave in s than $\bar{\pi}$, i.e., it decreases more quickly in s. Moreover, the probability to have less than t defaults must be more sensitive to the model chosen than the outside option, for small enough values of t. When this assumption holds, we have:

Proposition 1. *With distinguishable models (DM), for any function c and menu of capital requirements $\alpha(.)$, the transfers $T(.,.)$ below satisfy (IC) and (LL), and bind (IR) for every s:*

$$T(s,t) = \begin{cases} \max(0, u(\alpha(s), r(\alpha(s)), t)) & \text{if } t > a(s) \\ u(\alpha(s), r(\alpha(s)), t) - \frac{\bar{\pi}(s)}{F(a(s),s)} & \text{if } t \leq a(s) \end{cases}. \tag{11}$$

with $a(s)$ increasing and such that:

$$\frac{F_2(a(s), s)}{F(a(s), s)} = \frac{\bar{\pi}'(s)}{\bar{\pi}(s)}. \tag{12}$$

With the proposed menu, an intermediary reporting model s receives a constant payoff $\frac{\bar{\pi}(s)}{F(a(s),s)}$ as long as the realized level of defaults is less than $a(s)$, and zero otherwise. By definition, such a mechanism satisfies (LL). Moreover, if he reports truthfully the intermediary receives exactly $\bar{\pi}(s)$ in expectation, thus (IR) is binding. The Appendix A.2 shows that under (DM) it is possible to find a function $a(.)$ satisfying (12), which ensures (IC) even in the worst case in which the cost $c(s', s)$ is always null. It is thus possible to implement the first-best $\alpha^*(.)$ without leaving rents to intermediaries. Moreover, due to (MLRP), $a(.)$ is increasing: Intermediaries announcing a low s receive a high payoff if the level of defaults is low, intermediaries announcing a higher s receive a lower payoff more often. Notice that other menus may also implement the first-best in this environment. It is possible for instance to give the bank a positive payoff only in the states of the world that are the most likely given the model he reported. The general point made by the proposition is that conditional transfers that reward the bank when its forecasts were accurate are a powerful tool to encourage the adoption of the best possible models.

Fig. 1 gives an example.[16] The top panel shows the expected payoff an intermediary receives if the true parameter is s and he reports s'. The mechanism is designed so that the maximum payoff is obtained for $s' = s$. The bottom panel shows how this is achieved with a plot of the payoff an intermediary obtains when he reports different models and t defaults realize.

[16]The parameters for all figures are given in A.7. They are meant for illustrative purposes.

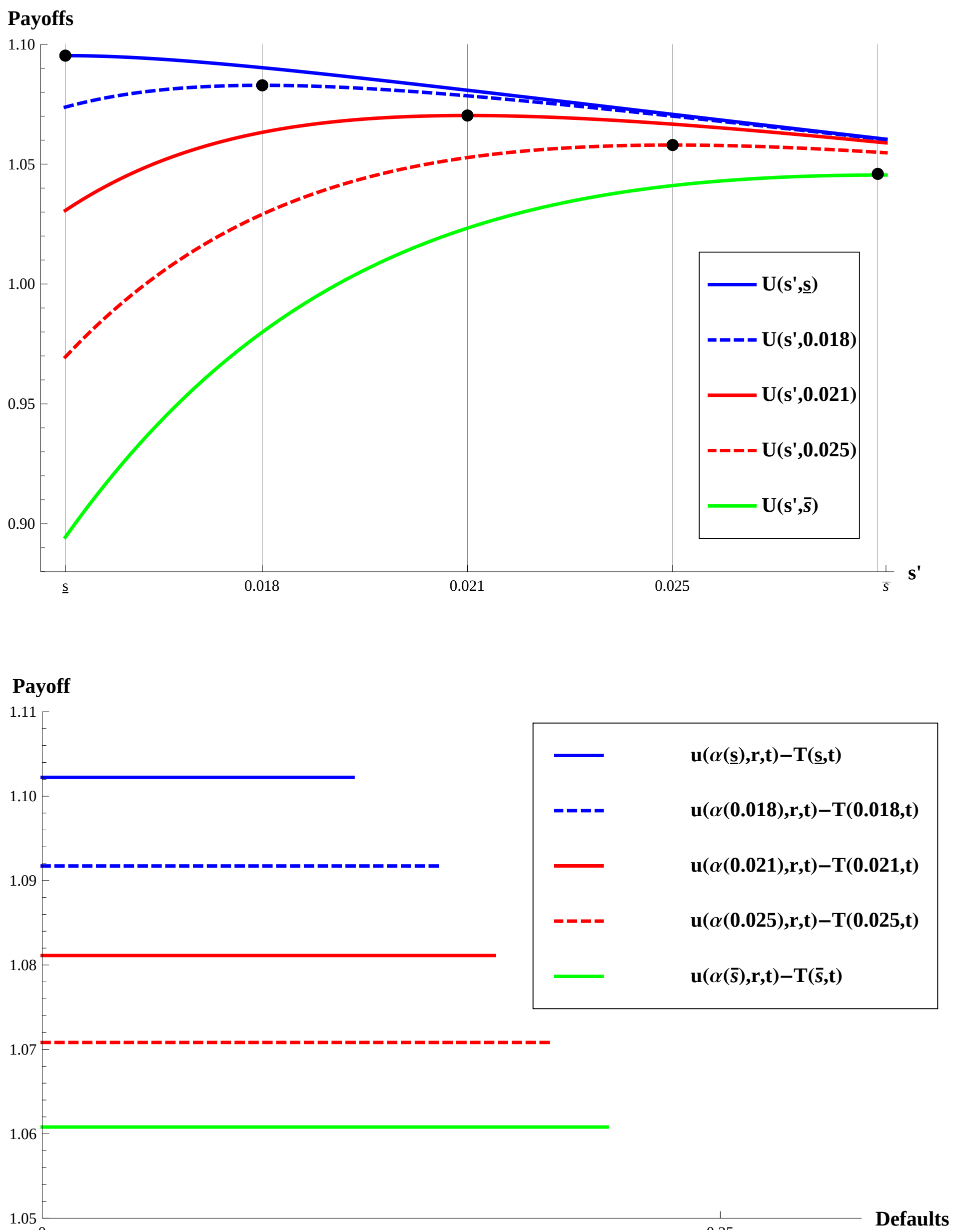

Figure 1: Incentive-compatible transfers. The top panel plots $U(s', s) = \pi(s', s) - \mathbb{E}_s(T(s', t))$ as a function of s', for different values of s. $U(s', s)$ is the average payoff obtained by a bank of type s that reports model s', by construction it is maximized in $s' = s$. The bottom panel plots $u(\alpha(s'), r, t) - T(s', t)$ as a function of t, for different values of s'. These are the payoffs that any type of bank will obtain if it reports s', conditionally on t defaults realizing.

3.2 Uncertainty on tail risk, bail-outs, and second-best regulation

Bail-outs and incentives. The optimal menu just derived may include transfers for levels of losses above those at which the bank itself defaults. Due to limited liability, the regulator cannot impose penalties then. It may thus be necessary to *subsidize* defaulting banks that had announced high risk measures.

This case arises with the mechanism of Proposition 1 when $a(s) > \theta(\alpha(s))$, but can happen more generally with any revealing mechanism, in particular when model uncertainty is focused on tail risk. For low levels of risk there is a lot of historical data to calibrate different models, such that they tend to deliver similar predictions, while for extreme levels data is much more sparse. This fact is at the same time a reason why the regulator would like to use the bank's expertise. We can model this situation in a stylized way by assuming that the different models are perfectly equivalent up to a given level of defaults:

$$\exists \tau \in [0,1] \ s.t. \ \forall (s,s') \in [\underline{s},\overline{s}]^2, \ \forall t < \tau, \ f(t,s) = f(t,s'). \tag{UM}$$

Assumption (UM) is a violation of assumption (DM): Instead of being easy to distinguish, the different models are undistinguishable below a threshold level of losses τ. They thus differ only in the tail. The Appendix A.3 proves the following:

Proposition 2. *Under (UM), if $\theta(\alpha^*(\underline{s})) < \tau$, then any revealing mechanism implementing the first-best, respecting limited liability, and leaving no rents to intermediaries involves bail-outs: For some (s,t) we have $u(\alpha^*(s), r(\alpha^*(s)), t) < 0$ and $T(s,t) < 0$.*

Indeed, imagine that the correct model is $\underline{s}$, the most optimistic one. If the regulator implements the first-best capital requirements, a bank will default for $t > \theta(\alpha^*(\underline{s}))$. α^* is increasing and r decreasing in s, so that $\theta(\alpha^*(\underline{s}))$ is the lowest θ implemented in the first-best. If it is below τ, a bank reporting $\underline{s}$ is already in default when the realized t gives the regulator information about which models are more likely. It is then impossible to "punish" the use of such an optimistic model ex post. Instead, one needs to "reward" banks suffering high losses when they reported high risk measures, which automatically involves bailing out truthful but unlucky banks.

Second-best with a no bail-out constraint. The use of bank bail-outs to ensure the truthful revelation of risk models may conflict with other unmodeled regulatory objectives. Proposition 2 implies that a constraint not to bail-out defaulting banks, while potentially necessary to foster market discipline for instance, comes at a cost: Either rents will have to be left to intermediaries, or capital requirements will be different from their first-best values. The proof of the proposition actually shows the following:

Corollary 1. *When models are undistinguishable below τ (UM), if the regulator implements capital requirements $\alpha(.)$ with $\alpha' \geq 0$ and $\theta(\alpha(\underline{s})) < \tau$ but cannot bail out defaulting banks, then a mechanism satisfying (IC), (IR), and (LL) leaves to any type s an informational rent at least equal to* $\max_{s', \theta(s') \leq \tau} \bar{\pi}(s') - c(s', s) - \bar{\pi}(s)$.

Indeed, in the particular case in which $c(s', s)$ is almost always zero for instance, all intermediaries necessarily receive a payoff close to the highest outside option among all types $\bar{\pi}(\underline{s})$, because this payoff can always be obtained by reporting the most optimistic risk model. As it is impossible to achieve first-best capital requirements without leaving rents to intermediaries, the regulator needs to find a second-best solution trading off rents and efficiency.

The trade-offs underlying the second-best solution can be best understood by considering the special case of two types $\{\underline{s},\ \overline{s}\}$ realizing with prior probabilities ψ and $1 - \psi$. For brevity, denote $\theta^*(s) = \theta(\alpha^*(s))$ and $\theta^{**}(s) = \theta(\alpha^{**}(s))$ the first-best and second-best values of θ for a bank of type s. Assume that $\tau \in (\theta^*(\underline{s}), \theta^*(\overline{s}))$, and that $c(\underline{s}, \overline{s}) < \bar{\pi}(\underline{s}) - \bar{\pi}(\overline{s})$, as otherwise there are no incentives to misreport in the first place. It is straightforward to adapt the objective function (9) to this special case. The second-best solution maximizes (9) under (IC), (IR), (LL) and the no bail-out constraint:

$$\forall s \in \{\underline{s}, \overline{s}\},\ u(\alpha(s), r(\alpha(s)), t) < 0 \Rightarrow T(s, t) = 0. \tag{NBO}$$

Proposition 3. *The second-best regulation when models are undistinguishable below τ (UM) and no bail-outs are allowed (NBO) can be of two types:*

1. High capital requirements and no rents. *$\alpha^{**}(\underline{s})$ is such that $\theta^{**}(\underline{s}) \geq \tau$. No intermediary defaults for $t < \tau$. An upper bound on welfare in this case is achieved if $\theta^{**}(\underline{s}) = \tau$, $\alpha^{**}(\overline{s}) = \alpha^*(\overline{s})$, and transfers can be found such that both types have no rent.*

2. Compensating the high-risk type. *$\alpha^{**}(\underline{s}) = \alpha^*(\underline{s})$ and $\alpha^{**}(\overline{s}) = \alpha^*(\overline{s})$, type $\underline{s}$ has no rent, type $\overline{s}$ has a positive rent of $\bar{\pi}(\underline{s}) - \bar{\pi}(\overline{s}) - c(\underline{s}, \overline{s})$.*

A sufficient condition for Solution 2 to be preferred is:

$$\psi[\mathcal{W}(\alpha^*(\underline{s}), \underline{s}) - \mathcal{W}(\alpha_\tau, \underline{s})] \geq \lambda(1 - \psi)[\bar{\pi}(\underline{s}) - \bar{\pi}(\overline{s}) - c(\underline{s}, \overline{s})]. \tag{13}$$

The proof is in the Appendix A.4. The first option is to increase $\alpha(\underline{s})$ so much that no intermediary defaults for $t < \tau$. It may then be possible to find a mechanism similar to the one of Proposition 1 and leave no rents to the agent, at the cost of capital requirements higher than necessary for the low-risk type. If some models are relatively unlikely but not very costly to misreport, then it is optimal to associate high capital requirements with them so that misreporting can be detected and punished ex post. The second option is to incentivize the high-risk type to reveal his information by transferring much less money from him than from the low-risk type, thus leaving a rent.

Which solution is optimal is affected by several parameters. There is first a distortion effect: Models that can be distinguished only further in the tails (τ is higher) make it more costly in terms of welfare to use the first approach. This distortion is traded off against the cost in terms of rents: A lower heterogeneity in the outside options $\bar{\pi}(\underline{s}) - \bar{\pi}(\bar{s})$ and a higher misreporting cost $c(\underline{s}, \bar{s})$ make it less costly to use the second approach. A higher ψ makes it more likely to face a low-risk type rather than a high-risk type, thus reinforcing the distortion associated with the first approach and reducing the cost of the second one.

Notice that, with more than two models, solution 1 would not be equivalent to a floor $\underline{\alpha}$ on risk-weights such that $\theta(\underline{\alpha}) = \tau$. This can be seen by extending the problem to three models $s_0 < s_1 < s_2$, with $\theta^*(s_1) < \tau$. Assume that the costs for both types s_1 and s_2 to report model s_0 are higher than $\bar{\pi}(s_0)$, which is the best payoff they could receive from reporting this model. Then no type has an incentive to misreport s_0. There is thus no cost for the regulator to choose $\alpha^{**}(s_0) = \alpha^*(s_0)$ and transfers such that type s_0 receives no rent. The problem then reduces to the two-models problem of Proposition 3. It is possible in particular if $\psi(s_1)$ is small that the regulator chooses $\theta^{**}(s_1) = \tau$ while $\theta^{**}(s_0) < \tau$.[17]

4 Optimal regulation without penalties

While a penalty mechanism has been used since the 1996 Amendment to the Basel capital accord for the estimation of market risks, and could be extended, the current reforms go in a different direction and use restrictions on attainable risk-weights, i.e., they modify the function α. This section first derives the optimal capital requirements when penalties cannot be used at all, that is how α should optimally be modified. I then adopt a positive approach, study the impact of introducing leverage ratios and derive empirical implications about which models banks choose

[17] A leverage ratio is thus not optimal in this framework, contrary to the findings in Blum (2008) with a two-types model. What matters is actually not to set a floor on capital requirements per se, but to reduce the sensitiveness of capital requirements to the reported model.

under the current regulatory regime.

4.1 Second-best regulation

In this section, denote $\pi(\alpha(s'), s)$ the profit of an intermediary reporting s' when the true model is s. Due to the absence of transfers, it is convenient to make two additional assumptions. First, given the absence of transfers there is no possibility anymore to ensure the participation of a bank for which $\pi(\alpha^{**}(s), s) \leq \bar{\pi}(s)$ with a negative transfer. I assume that a bank opting out of the regulation is not allowed to use leverage at all: $\bar{\pi}(s) = \pi(1, s)$. This assumption ensures that any capital requirements satisfy (IR).[18] Second, since there is no other tool to make transfers between the bank and the public budget, the first-best α^* is no longer independent of λ. I assume that $\lambda = 0$, so that α^* is then the same with and without the possibility of transfers, which facilitates the comparison with Section 3. All the results of this section are still valid without this assumption, only the definition of the objective function changes.

The regulator cannot use ex post penalties and can only choose the function $\alpha(.)$. The revelation principle still applies; in order for each type of intermediary to report truthfully, the second-best α^{**} must be such that $\pi(\alpha^{**}(s'), s)$ is maximized at $s' = s$, for each s. The regulator wants to maximize the expected value of welfare under this incentive compatibility constraint. (LL) is automatically satisfied. A necessary condition for incentive compatibility is that each type of intermediary has no incentive to deviate from $s' = s$, at least locally, which writes as:

$$\forall s \in [\underline{s}, \overline{s}], \alpha'(s)\pi_1(\alpha(s), s) - c_1(s, s) \geq 0. \tag{14}$$

As c_1 and π_1 are negative, this constraint means that $\alpha(.)$ must be sufficiently flat. An extreme case is $\alpha' = 0$, in which case capital requirements do not depend at all on the model reported, so that no bank has an incentive to misreport. More generally, the second-best capital requirements satisfy the following proposition:

Proposition 4. *In the absence of penalties, second-best capital requirements $\alpha^{**}(.)$ satisfy:*

-1. For any s, either $\alpha^{**}(s) = \alpha^*(s)$ or $0 \leq \alpha^{**\prime}(s) \leq \alpha^{*\prime}(s)$: Capital requirements increase in the reported risk measure, but less than in the first-best.

-2. $\alpha^{**}(\underline{s}) \geq \alpha^*(\underline{s})$ and $\alpha^{**}(\overline{s}) \leq \alpha^*(\overline{s})$.

[18] The IRB approach of Basel II was explicitly designed to give lower capital requirements than the standardized approach. This new assumption fits such a situation in which IRB risk-weights are lower than the standardized ones.

The first point is proved in the Appendix A.5. The second point is more direct. There are two reasons why capital requirements may be distorted compared to the first-best: Lower capital requirements for type s give him a higher profit if he reports his type truthfully, reducing incentives to misreport. But it also gives other types of banks more incentives to pretend their type is s. Higher capital requirements on the contrary ensure that a model is not reported for strategic reasons. Since $\underline{s}$ already faces the lowest capital requirements, he has no incentives to misreport and only the second distortion plays a role. Conversely, there is nothing to gain by misreporting model $\overline{s}$ and facing the highest capital requirements, so that only the first distortion applies.

Points 1 and 2 rely on the same intuition: The intermediaries' incentives to misreport stem from a more optimistic model allowing to increase leverage. The strength of this effect is measured by the derivative of $\pi(\alpha(s'), s)$ with respect to s'. If this derivative taken in s is more negative than the derivative of the cost to misreport, then the intermediary has an incentive to report a wrong model. As the regulator cannot use penalties, the only possibility left is to decrease the gains from misreporting by making capital requirements less sensitive to the reported model s'. Second-best capital requirements are thus less reactive to the internal measure of risk than in the first-best. This property is true both locally, point 1 of the proposition, and globally, point 2. Fig. 2 shows a numerical example comparing the first-best and the second-best capital requirements. It also shows that higher misreporting costs reduce the discrepancy between first-best and second-best capital requirements.

4.2 The use of leverage ratios

The regulatory answer to suspicions that internal models can be used for regulatory arbitrage has been to complement internal risk measures with leverage ratios or floors on capital requirements. In this setup, a leverage ratio constraint is a particular way to make capital requirements less risk-sensitive, in line with the intuition of Proposition 4. However, a critical difference is that a leverage ratio constraint is in general not incentive-compatible. Moving from the normative perspective of second-best regulations to a positive analysis of current reforms, I show in this section some dangers associated with leverage ratios. From a policy perspective, it is important to prove that simple floors cannot substitute for a deeper solution of the information problems associated with internal risk models. Moreover, the modeling of the current regulatory framework allows to derive testable implications, discussed in 5.1.

To model leverage ratios, imagine that instead of implementing α^{**} as in Proposition 4, the

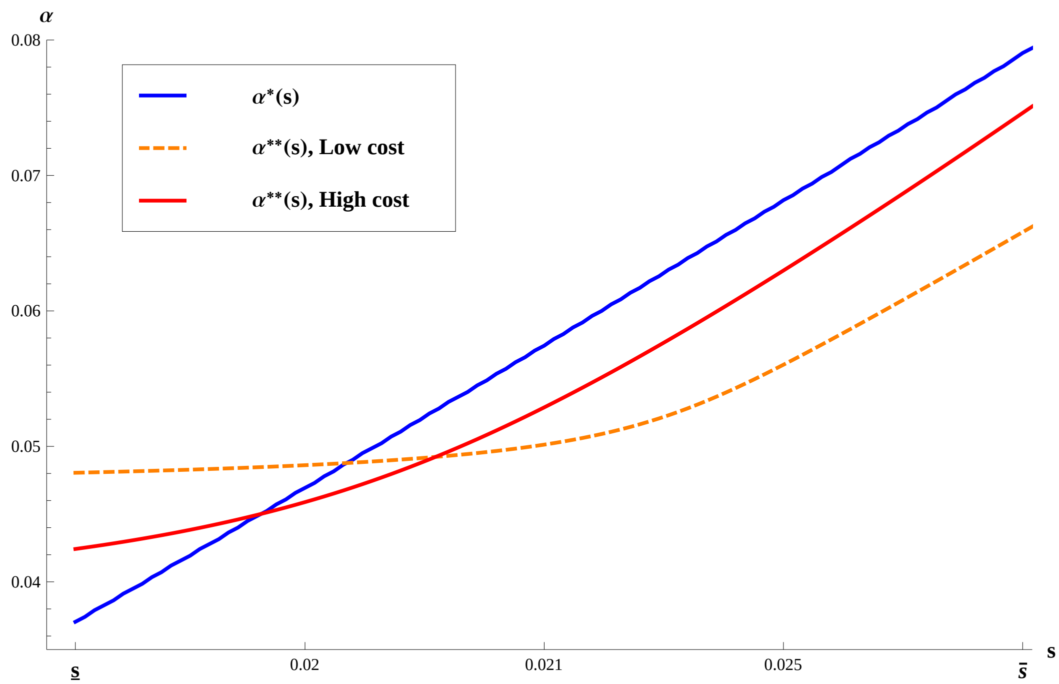

Figure 2: Second-best capital requirements, without penalties. The blue line corresponds to the first-best capital requirements $\alpha^*(s)$. The orange dotted line represents the second-best capital requirements $\alpha^{**}(s)$ studied in Proposition 4, when the misreporting costs $c(s', s)$ are low. The red line shows how $\alpha^{**}(s)$ is modified when misreporting costs are increased.

regulator tries to implement α^* and adds a leverage ratio constraint imposing that $E/L \geq \underline{\alpha}$ (a special case is the absence of leverage ratio, $\underline{\alpha} = 0$). The intermediary then faces capital requirements equal to $\hat{\alpha}(s) = \max(\underline{\alpha}, \alpha^*(s))$.[19] Consider the most general case in which $\alpha^*(\underline{s}) < \underline{\alpha} < \alpha^*(\overline{s})$, so that the leverage ratio constraint is binding for the lowest types but not for the highest types, and there exists $s_{\underline{\alpha}}$ such that $\alpha^*(s_{\underline{\alpha}}) = \underline{\alpha}$.

An intermediary then optimally selects s' so as to maximize $\pi(\hat{\alpha}(s'), s) - c(s', s)$. As regulation may not be incentive-compatible, an intermediary of type s may not obtain the capital requirements foreseen by the regulator but *manipulated* capital requirements $\hat{\alpha}(s')$.

A single intermediary. This case is easier to understand by looking at Fig. 3. On the top panel I plot the model s' reported by each type s of intermediary. The 45° line corresponds to truthful reporting, the dashed line assumes that $\underline{\alpha} = 0$ (no leverage ratio), the red line corresponds to a leverage ratio with $\underline{\alpha} = 0.05$. On the bottom panel I plot the manipulated capital requirements obtained by each type. Without a leverage ratio (dashed line), all types of intermediaries use a

[19] The results would be the same with any increasing α instead of α^*.

model more optimistic than the one they believe in, and those with the lowest s all report the most optimistic model (flat part of the curve).

How is this modified when we introduce $\underline{\alpha} > 0$? Intermediaries become partitioned into three groups. Since $\hat{\alpha}$ is increasing, all the types in $[\underline{s}, s_{\underline{\alpha}}]$ have no incentive to misreport: By reporting truthfully they face the lowest capital requirements $\underline{\alpha}$, which are higher than in the first-best. Riskier intermediaries, in particular those with s close but above $s_{\underline{\alpha}}$, strategically report overoptimistic risk measures but are constrained by the leverage ratio. Since they receive the same capital requirements $\underline{\alpha}$ for reporting any model $s' \leq s_{\underline{\alpha}}$, they choose to report $s_{\underline{\alpha}}$, the model giving capital requirements $\underline{\alpha}$ with the lowest misreporting cost. Third, there are intermediaries with $s > s_{\underline{\alpha}}$ who choose to report $s' > s_{\underline{\alpha}}$, so that they are unaffected by the leverage ratio.

The outcome is that some intermediaries with intermediate risk measures do not misreport too much, and face capital requirements that are below the first-best, but above those they would face in the absence of a leverage ratio. This comes at the cost of too high capital requirements for intermediaries with low risk. Depending on how plausible are the different models, the result may be worse than choosing $\underline{\alpha} = 0$, i.e., not using a leverage ratio at all. The logic is the same as in Proposition 3: If the s below $s_{\underline{\alpha}}$ are very likely, while the types s above this threshold and for whom the constraint is binding are on the contrary unlikely, then the costs of using a leverage ratio outweigh the benefits.

Extension - several intermediaries. Although imposing a leverage ratio constraint can decrease welfare, in the case just studied it always reduces misreporting. I show in the following extension that this property does not hold anymore with several intermediaries instead of only one.

Assume that instead of a single intermediary we have a continuum $[0, 1]$ of intermediaries, who each draw an independent model s so that there is a measure $\psi(s)ds$ of intermediaries for whom the correct model is s. To keep the model close to the single intermediary case, assume that all the intermediaries lend to the same pool of borrowers, and the differences in riskiness come from the way they monitor the borrowers afterwards, hedge their risk etc., so that all intermediaries choose how much to lend, taking the same interest rate r as given.[20]

The only difference with Section 3 is that $\alpha(s)$ does not determine the interest rate r through the market-clearing condition $q(r) = E/\alpha(s)$. There is no aggregate uncertainty and a single r

[20] This assumption fits the case of syndicated loans studied by Firestone and Rezende (2013). They note that different banks participating in a loan to the same firm may genuinely report different risk measures, because they may not be equally good at recovering losses in case of a default. It is also possible to assume that the heterogeneity in s comes from different pools of borrowers, in which case one needs a more complex model of differentiated competition. The equilibrium effect highlighted in this section would still obtain in this more complex setup.

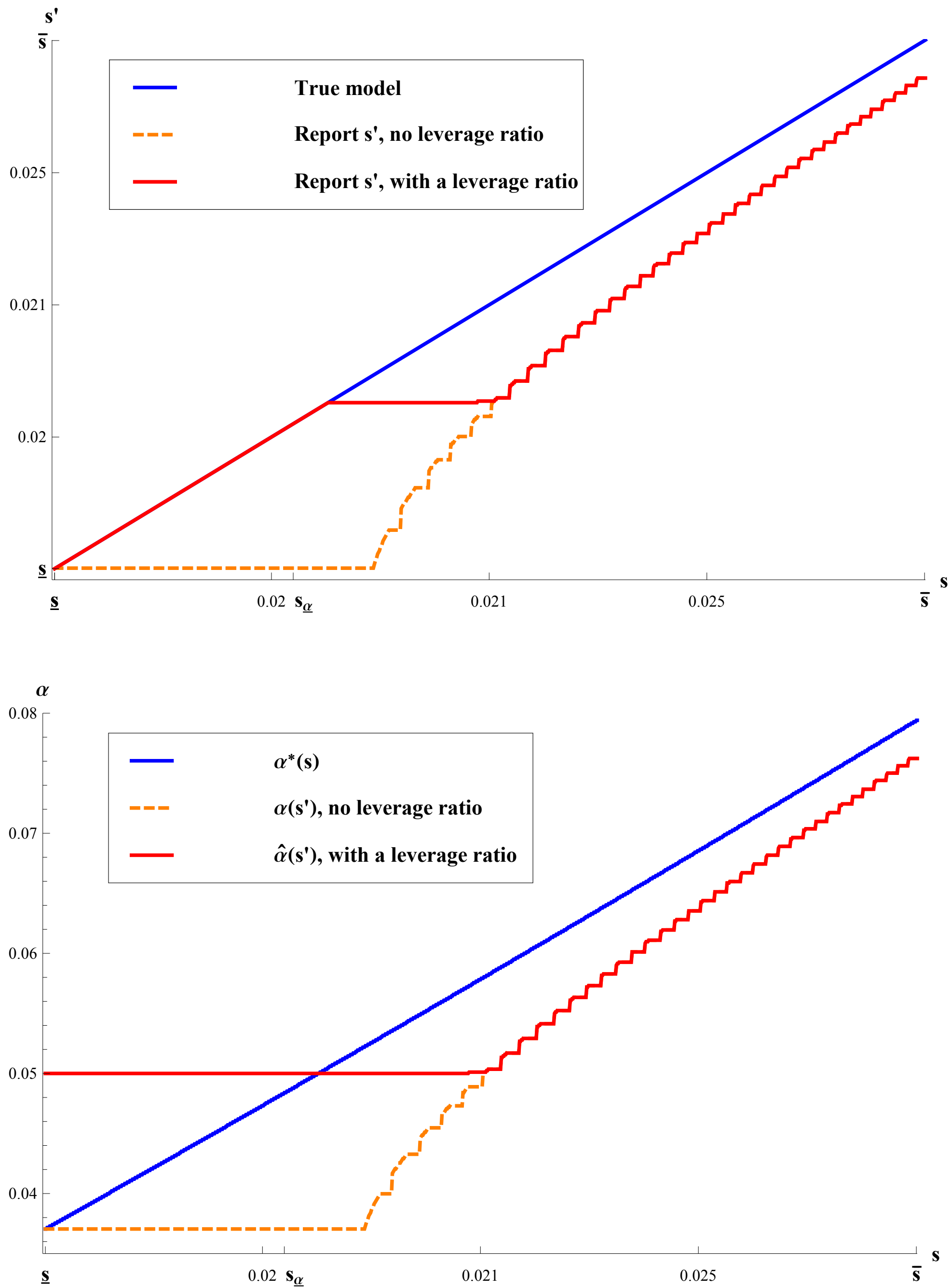

Figure 3: Manipulated models and capital requirements. The top panel plots the model s' chosen by each type of bank s, and the bottom panel reports the associated capital requirements. The blue lines correspond to truthful reporting without a leverage ratio, so that capital requirements are equal to the first-best $\alpha^*(s)$. The dotted orange lines represent the case in which $\underline{\alpha} = 0$ and banks choose their risk model strategically. The red line introduces a leverage ratio constraint $\underline{\alpha} = 0.05$.

equates aggregate supply and demand. Instead of maximizing the expected welfare with respect to each $\alpha(s)$, the regulator will maximize with respect to r and $\alpha(.)$ the following:

$$\int_{\underline{s}}^{\overline{s}} \mathcal{W}(\alpha(s), r, s)\psi(s)ds \text{ s.t. } \int_{\underline{s}}^{\overline{s}} \frac{E}{\alpha(s)}\psi(s)ds = q(r). \tag{15}$$

All the second-best solutions studied above can be adapted by first solving for the second-best for any possible r with the new market clearing constraint, and then pick the r achieving the highest welfare. While this case introduces some new trade-offs, it does not fundamentally change the nature of the different second-best options. However, for a regulation relying on leverage ratios, unwanted effects of the regulation appear.

Assume again that the regulator uses capital requirements $\hat{\alpha}(s)$ including a leverage ratio. Denote $\pi(\alpha, r, s)$ the profit of an intermediary of type s facing capital requirements α and interest rate r. Each intermediary s reports its profit-maximizing model, denoted $\sigma(\underline{\alpha}, r, s)$:

$$\sigma(\underline{\alpha}, r, s) = arg \max_{s' \geq s_{\underline{\alpha}}} \pi(\hat{\alpha}(s'), r, s) - c(s', s). \tag{16}$$

While the convexity of profit still implies that an intermediary choosing to lend $L > 0$ uses as much leverage as he can, in this new setup with several banks he may be better off choosing $L = 0$ and investing E in the safe asset. For given $\underline{\alpha}$ and r, denote $s_0(\underline{\alpha}, r)$ the highest type s that lends a positive amount (s_0 can be equal to $\overline{s}$). In equilibrium each type $s \leq s_0(\underline{\alpha}, r)$ reports $s' = \sigma(\underline{\alpha}, r, s)$, and the market clearing condition gives us:

$$\int_{\underline{s}}^{s_0(\underline{\alpha}, r)} \frac{E}{\hat{\alpha}(\sigma(\underline{\alpha}, r, s))}\psi(s)ds = q(r). \tag{17}$$

Imagine now that the regulator increases the leverage ratio $\underline{\alpha}$ (possibly from 0 to some positive number). For a given r, this does not affect the incentives of intermediaries with types above $s_{\underline{\alpha}}$ to misreport. Intermediaries with $\hat{\alpha}(\sigma(\underline{\alpha}, r, s)) = \underline{\alpha}$ are more constrained than before and lend less, while some intermediaries now face a binding constraint whereas before they did not. Thus for any given r the supply of loans, or the left-hand side of (17), decreases. As a result of the increased leverage ratio, the supply curve moves to the left, so that the equilibrium interest rate necessarily increases. This higher interest rate increases $s_0(\underline{\alpha}, r)$, and also has an impact on $\sigma(\underline{\alpha}, r, s)$ for each s. The Appendix A.6 shows the following:

Proposition 5. *If the regulator uses capital requirements with a leverage ratio* $\hat{\alpha}(s) = \max(\underline{\alpha}, \alpha^*(s))$*, an increase in the leverage ratio constraint* $\underline{\alpha}$ *increases the interest rate. An intermediary of type* s *not constrained by the leverage ratio surely chooses a more optimistic model if his default point* θ *satisfies* $f(\theta, s)/F(\theta, s) \leq 1$.

The quantity $(f(x, s)/F(x, s))dx$ is the probability to observe between $x - dx$ and x defaults in the bank's portfolio, knowing that there are less than x defaults. The default point θ should in general be a tail loss, with $F(\theta, s)$ close to 1 and $f(\theta, s)$ close to zero. This condition is sufficient for a higher leverage ratio to translate into the choice of a more optimistic model.

Fig. 4 and 5 illustrate this proposition. On the bottom panel of Fig. 4 we have for two different interest rates the manipulated capital requirements chosen by each type of bank. For the lower interest rate, the riskiest banks actually choose not to lend at all (vertical line). These choices for each interest rate determine the supply curve on the top panel. The interaction with demand then determines the equilibrium interest rate, and thus the models reported. In particular, a higher demand leads to a higher interest rate, more misreporting by banks and lower capital requirements.

On the top panel of Fig. 5, I plot for the same interest rate $r = 1.085$ the manipulated capital requirements chosen by banks with and without a leverage ratio. The higher capital requirements in the latter case translate into a shift of the supply curve to the left, as observed in the bottom panel. The equilibrium interest rate increases from 1.085 to 1.103. This feeds back into a choice of more optimistic models and lower capital requirements, so that the red line on the top panel is below the blue line.

A leverage ratio makes the banks for which it is binding by definition safer. But this is equivalent to reducing the supply of credit by these banks, thus increasing the interest rate r. Intermediaries with a high risk and for whom the leverage ratio is not binding then have incentives to lend more, which can be done by choosing more optimistic risk models. Because of this simple equilibrium effect, a higher leverage ratio thus makes safe banks use less leverage and risky banks use more leverage, which may not be the intended effect.

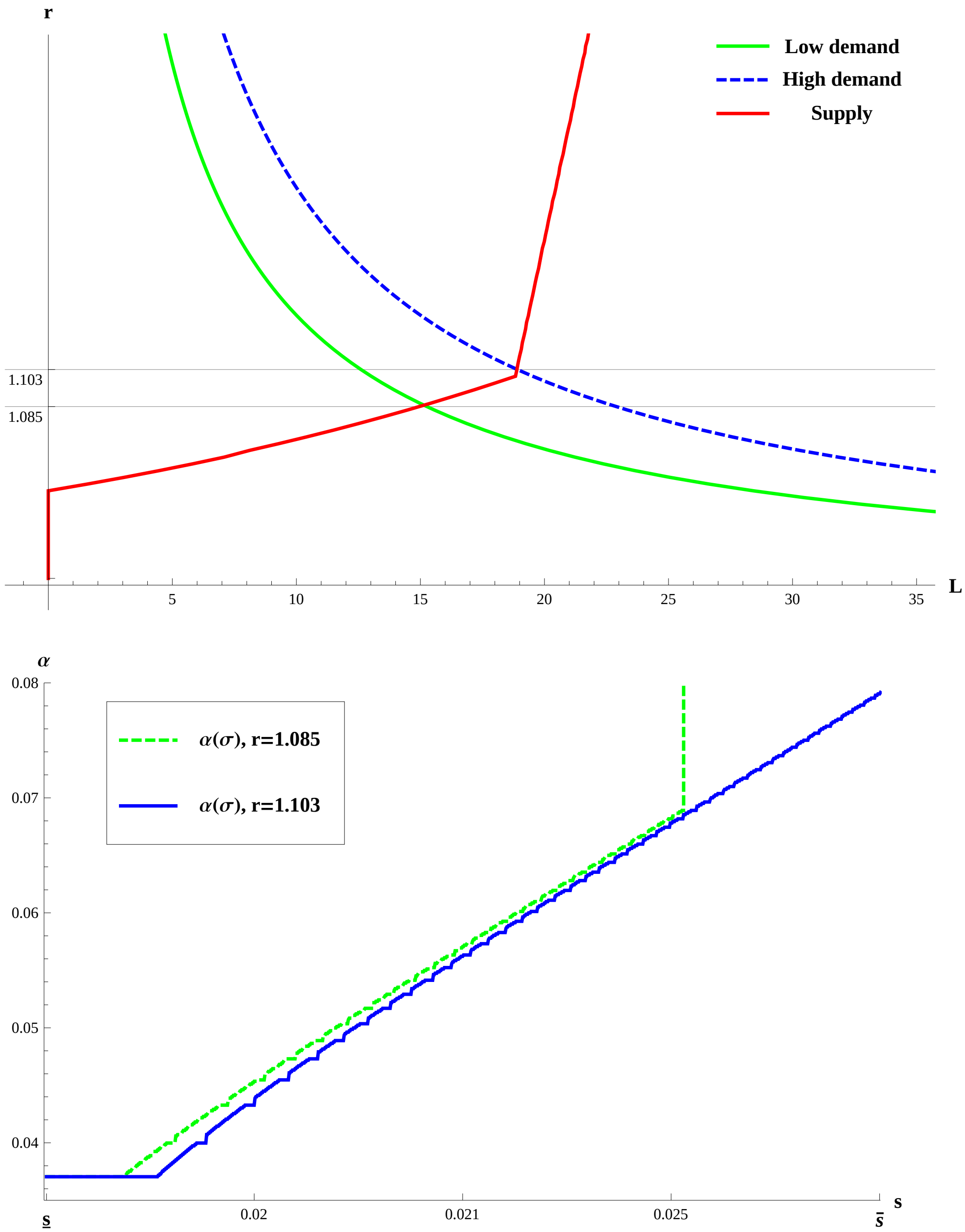

Figure 4: Market equilibrium and loan demand. The top panel plots the loan supply function derived from the banks' strategic behavior, two different demand functions, and the associated equilibrium interest rates, defined by (17). The bottom panel shows the manipulated capital requirements $\hat{\alpha}(\sigma(\bar{\alpha}, r, s))$ obtained by a type s who strategically reports model $\sigma(\bar{\alpha}, r, s)$, for the two different equilibrium values of r. The difference between the two curves illustrates the impact of loan demand on strategic model selection.

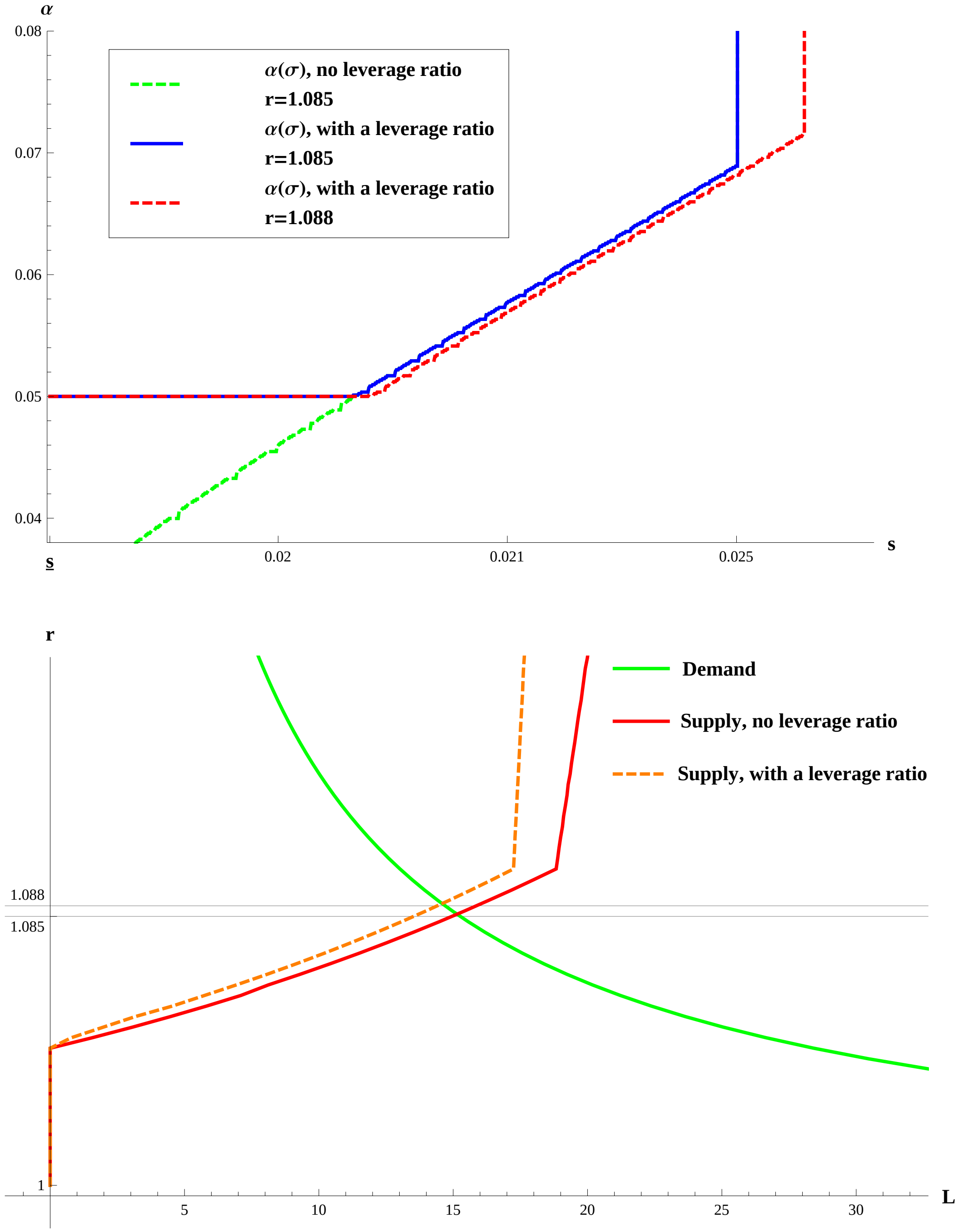

Figure 5: Market equilibrium and leverage ratio. The top panel reports the manipulated capital requirements $\hat{\alpha}(\sigma(\bar{\alpha}, r, s))$ obtained by a type s who strategically reports model $\sigma(\bar{\alpha}, r, s)$. The bottom panel plots the loan demand function and the supply functions obtained with and without a leverage ratio, in dashed orange and solid red, respectively. The green line on the top panel represents capital requirements in the absence of a leverage ratio at the equilibrium interest rate of 1.085. Introducing a leverage ratio generates the blue line if the interest rate is constant. However, the leverage ratio increases the equilibrium interest rate to 1.088, which gives the dotted red line.

5 Discussion

5.1 Empirical implications

While it is difficult to derive predictions from an analysis of the optimal regulation, the regulatory framework of 4.2 is a stylized but realistic representation of the incentives banks currently face when choosing their risk models. This approach gives new predictions about the use of internal risk models by regulated financial institutions and their risk weights:

Implications. *A bank has more incentives to report overoptimistic risk measures when:*

-1. The capital ratio of the bank α is lower.

-2. The demand for loans $q(.)$ is higher.

-3. The bank's loans are less risky, i.e., s is lower.

-4. The cost of misreporting $c(s', s)$ is lower.

-5. Capital requirements react more strongly to internal risk measures, i.e., $\alpha'(.)$ is higher.

-6. A floor on capital requirements is imposed, or increased, but is not binding for the bank.

There is a growing empirical literature on banks' internal risk estimates. Studies by the Basel Committee and the European Banking Authority (BCBS (2013a), BCBS (2013b), EBA (2013)) show the dispersion of risk weights across banks, which is of critical importance in the wider debate on model-based regulation. Firestone and Rezende (2013) exploit data on syndicated loans to show that banks report different risk parameters (in particular LGDs) for loans to a same firm, leading to different capital requirements, but they also note that the variation they observe could be entirely legitimate. Going beyond dispersion, studies on market risk models show that VaRs reported for market risk were biased before the crisis, and actually too conservative, see Berkowitz and O'Brien (2002) and Pérignon and Smith (2010); the latter paper hypothesizes that this behavior may come from the penalty mechanism used for market risk.

However, even systematic biases in risk estimates may come from mistakes: It is in the nature of tail events that their probability is underestimated when only small samples are available. This paper offers a new avenue to test the hypothesis that the selection of risk models reacts to regulatory incentives. Under the null assumption that these incentives play no role, internal risk measures should be uncorrelated with market conditions or with regulatory changes. Conversely, observing correlations with the signs suggested in Implications 1-6 is difficult to explain if models are not strategically chosen. Recent empirical papers have explored the determinants of risk estimates heterogeneity, with results broadly consistent with the implications of the model.

Implication 1 derives from the convexity of profit in α, which itself derives from the bank's limited liability (see the Appendix A.6): obtaining a marginal decrease in capital requirements is more profitable when the bank is highly leveraged and likely to default. Several papers find evidence for this effect, such as Mariathasan and Merrouche (2014), Begley *et al.* (2014), Behn, Haselmann, and Vig (2014), and Plosser and Santos (2014).

Implication 2 suggests to look at shocks on the demand for loans. Industries or housing markets experiencing a boom would be typical scenarios in which banks can be expected to face more incentives to misreport risk so as to extend more loans. Implication 3 states that misreporting incentives also depend on the true quality of the loans, as the incentives to issue more loans remain stronger when their quality (as perceived by the bank) is higher. Implication 4 can be tested by considering proxies for the strength of supervisory oversight, as more effort will be needed to get a stronger supervisor to approve an optimistic model. Mariathasan and Merrouche (2014) provide evidence of this effect: They show that banks switching to the IRB approach are more likely to improve their capital ratios when they are less tightly supervised, as measured by indices of supervisory independence.

Supporting Implication 5 (and also in line with Implication 3), Behn, Haselmann, and Vig (2014) find evidence of strategic behavior for portfolios of loans with low PDs, precisely those for which capital requirements react a lot to PD estimates. Interestingly, Plosser and Santos (2014) find the opposite result using syndicated loan data from the U.S. market. They note that larger biases in reported PDs are necessary to achieve the same reduction in risk-weights when loans are riskier. The evidence in Behn, Haselmann, and Vig (2014) suggests that the costs of misreporting depend on the PDs, while the evidence in Plosser and Santos (2014) suggests that they depend on the achieved risk-weights. Different banking supervisors may well have different auditing practices in this regard, explaining these different results.

Finally, Implications 5 and 6 suggest to look at changes in the regulatory environment itself: Changes in the formula linking risk estimates to capital requirements, imposition of higher capital requirements through the implementation of Basel III or other changes have an impact on internal risk measures if models are chosen strategically. In line with this logic, Hendricks *et al.* (2013) show that U.S. mortgage-servicing banks reacted to the announcement of Basel III by manipulating regulatory inputs.

5.2 Policy implications: Regulatory options

The regulatory community has only recently reacted to the possibility that internal models are used by undercapitalized banks to bypass regulatory constraints (see, e.g., BCBS (2013c), p. 15). Many of the policy options currently discussed to improve the credibility of Basel's pillar I amount to an increase in $\underline{\alpha}$ in Section 4.2: Higher capital requirements under Basel III, "Basel I floors", the Collins amendment in the United States, provisions for model risk, higher regulatory multipliers and leverage ratios, for instance.[21] This paper allows to model the impact of such measures and compare them to several "second-best" benchmarks, which constitute other regulatory options.

1. Leverage ratios and non-risk based constraints. According to Section 4.2, a leverage ratio is a binding constraint only for the banks whose risk is already at the lower end of the spectrum. Banks with intermediate risk measures indeed misreport less, because the benefits to misreporting are limited by the constraint on the leverage ratio. The cost is that banks with the lowest risk lend less than would be optimal, so that the trade-off depends on how likely it is ex ante that the bank's risk is low or intermediate. Proposition 5 shows an additional weakness of leverage ratios: When more banks adopt optimistic models and use a high leverage, the interest rate on loans decreases and bypassing the regulation is less profitable. On the contrary, imposing a leverage ratio restricts the loan supply, which invites riskier banks to extend more loans by reporting more optimistic models. More generally, such measures are not a natural response to an asymmetric information problem.

2. Penalty mechanisms with bail-outs. A powerful tool to punish the use of overoptimistic models is to apply penalties to banks reporting low risk measures when high losses realize. A low-risk bank is ready to pay high penalties if high default levels realize, because this event is unlikely. A high-risk bank prefers paying transfers even when few defaults realize, but without further penalties for high default levels, which is a likely event. Similar transfers are already used for market risk models. They could be applied more generally at the level of a banking entity with penalties, levies or restrictions on dividends when banks with low risk-weighted assets suffer high losses. Proposition 1 shows a simple implementation of such a mechanism, as well as a limitation due to the possibility for banks to opt out of the mechanism.

Proposition 2 raises a second problem: In the presence of tail risk, one may learn that a bank was too optimistic only when it is in default and cannot be punished anymore. A good example would be Dexia, which had good risk-weighted Tier 1 capital ratios before suffering the losses that led to its nationalization. How to impose penalties on a dead bank? A possibility is to keep the

[21] Lesle and Avramova (2012) give a useful overview of the policy options and associated trade-offs.

bank alive with a bail-out, and adjust the shareholders' payoffs to the risk estimates previously reported. The rationale for a bail-out is to give incentives to truthfully report high risk measures: Shareholders of a defaulting bank receive some residual payment if the regulator was warned about risks in advance, but nothing otherwise (see Harris and Raviv (2014) for a similar argument). More generally, this solution consists in adjusting the resolution of a bank to the conservativeness of its self-reported risk measures.

3. Penalty mechanisms without bail-outs. Bail-outs may have other undesirable consequences. Proposition 3 studies second-best options if the bank cannot be kept alive. The first possibility is to impose capital requirements high enough for all types of banks to face the threat of being punished while alive. This avoids leaving informational rents to the regulated banks, at the cost of inefficiently reducing the credit supply of low risk banks. The second possibility is to compensate banks reporting high risk measures instead of punishing misreporting banks, but this can be very costly. Which solution is optimal depends on a trade-off between the inefficiency of the first solution, and the costs of the second solution. A key parameter is how well capitalized a bank must be in order to survive even tail events, which determines how distortive it is to ensure that a misreporting bank can be punished.

4. Capital requirements without penalties. If penalties contingent on the realized level of default cannot be used at all, the only solution left is to adjust capital requirements so that the misreporting costs are enough to discipline the bank. The cost of misreporting is fixed, whereas the benefit depends on how much capital requirements can be decreased by reporting an optimistic estimate. If capital requirements are more "flat", the incentives to misreport are lower. In practice, this means amending the regulatory formula linking internal risk estimates to capital requirements so that the latter are less sensitive to the estimates. As a consequence, second-best capital requirements are higher than in the first-best for low-risk banks, and lower for high-risk banks.

The different regulatory options studied in this paper are easy to rank in terms of welfare: Option 2 implements the first-best, option 3 gives a second-best solution under an additional constraint but surely does better than option 4, since it can use more tools. Option 1 is surely worse than 4 and is dominated by all the others. While it is sometimes argued that internal models are simply not sufficiently reliable, this section suggests different solutions to the strategic selection of risk models, depending on the constraints imposed on the regulator. Proposals to move away from the regulatory use of internal models should thus be traded off against more ambitious reforms trying to restore their credibility by addressing the asymmetric information problem.

Other options can be studied, and are more fully developed in the Internet Appendix:

Market observations. The regulator could use the observation of market prices to detect misreporting banks. The model can be extended to feature junior creditors whose claims are priced on the market, and a regulator trying to use their market price to infer a bank's type. I show in the Internet Appendix that no equilibrium exists when model uncertainty is too high because market participants anticipate the regulator's intervention, as in Bond, Goldstein, and Prescott (2010).

Benchmarking. Another solution for the regulator is to use reports from different banks as part of the Pillar 2 processes. This tool is not useful here because there is either a single bank or banks with uncorrelated models, but can be used under the opposite assumption that several banks share a common model. In practice, such a mechanism risks to discourage banks from developing unbiased but innovative models that may by definition give different risk measures than their competitors'.

6 Conclusion

A model-based regulation exploits banks' better information about their own risks to compute capital ratios. However, this information is private, and financial intermediaries cannot be expected to develop unbiased models if they face incentives to do otherwise. The process of adoption of new models may then be biased towards more "profitable" models.

This paper gives new predictions on how model selection reacts to market and regulatory changes. In particular, current reforms using risk-weight floors or leverage ratios tend to restrict the loan supply by banks for which these constraints are binding. As a result, other banks have more incentives to also adopt similar models and reach the new regulatory constraints, a substitution which can offset the intended effect.

Moreover, such instruments make capital requirements less risk-sensitive, thus penalizing banks whose risk is genuinely low. A more ambitious avenue would be to keep using internal models while giving the regulators tools to solve the associated asymmetric information problem. The framework of this paper allows to study several policy options under a variety of realistic constraints, such as restrictions on bail-outs and uncertainty on tail risks.

Finally, the strategic use of models can take place in other instances. Internal models are also used to measure the performance of employees, convey information inside firms, or to rating agencies and shareholders. In many cases "hidden model" problems can be as challenging as model risk and call for different solutions.

A Appendix - Proofs

A.1 Proof of Lemma 1

Conditions for capital requirements to be binding. For a given L, the bank's profit is:

$$\pi(L,s) = \int_0^{\theta(L)} [\rho(L)L(1-t) - r_0(L-E)]f(t,s)dt, \text{ with } \theta(L) = 1 - \frac{r_0}{\rho(L)}\left(1 - \frac{E}{L}\right). \tag{A.1}$$

Taking the first two derivatives with respect to L, we have:

$$\begin{aligned}
\pi_1(L,s) &= \int_0^{\theta(L)} [\rho(L)(1-t) - r_0]f(t,s)dt + \rho'(L)L\int_0^{\theta(L)}(1-t)f(t,s)dt, && \text{(A.2)}\\
\pi_{11}(L,s) &= \theta'(L)f(\theta(L),s)r_0\frac{L\rho'(L)(L-E) - \rho(L)E}{L\rho(L)} + \int_0^{\theta(L)}(1-t)f(t,s)dt \times (L\rho''(L) + 2\rho'(L)) \\
&= f(\theta(L),s)\frac{r_0(E\rho(L) - L\rho'(L)(L-E))^2}{(L\rho(L))^2} + \int_0^{\theta(L)}(1-t)f(t,s)dt \times (L\rho(L))''. && \text{(A.3)}
\end{aligned}$$

Notice that if $(L\rho(L))'' \geq 0$ we have $\pi_{11}(L,s) \geq 0$. With a price-taking bank for example we would directly have $(L\rho(L))'' = 0$ and a convex profit. Otherwise, we need $(L\rho(L))'' \geq -\bar{\varepsilon}$, with:

$$\bar{\varepsilon} = \min_{L\in[E,q(r^e(\underline{s}))],s\in[\underline{s},\overline{s}]} \frac{f(\theta(L),s)}{\int_0^{\theta(L)}(1-t)f(t,s)dt} \times \frac{r_0}{L\rho(L)} \times \frac{E^2}{L^2}, \tag{A.4}$$

where the upper bound for L comes from the fact that the regulator will never implement an interest rate r higher than $r^e(\underline{s})$. Under (H2), profit is thus convex in L. For $L = E$ we have:

$$\pi_1(E,s) = \mathbb{E}_s(1-t)[\rho(E) - r^e(s) + E\rho'(E)]. \tag{A.5}$$

Under (H1), $\pi_1(E,s) \geq 0$, so that the bank lends up to the constraint imposed by the regulator.

Proof of the Lemma. Since the regulator knows s, she can always find transfers that bind (IR) while respecting (LL). As a result, $\alpha(s)$ appears only in $\mathcal{W}(\alpha(s),s)$ in equation (5) and the optimal $\alpha(s)$ maximizes $\mathcal{W}(\alpha(s),s)$. Differentiating with respect to the first argument gives:

$$\mathcal{W}_1(\alpha,s) = \frac{-E}{\alpha^2}\left(\mathbb{E}_s(1-t)r(\alpha) - r_0\right). \tag{A.6}$$

Clearly if α is too small the derivative is positive, while we have $\mathcal{W}_1(1,s) \leq 0$ due to $\rho(E) > r^e(\overline{s})$ (as implied by (H1). The optimum is thus given by the interior solution $r(\alpha) = r^e(s)$. Assumption (MLRP) implies that a distribution f with a larger s dominates a distribution with a smaller s, in the sense of first-order stochastic dominance. $\mathbb{E}_s(1-t)$ thus decreases in s, so that $r^e(s)$ increases. As $r(\alpha)$ increases in α, α^* also increases with s.

A.2 Proof of Proposition 1

It is clear that if we show the proposed mechanism to work when $c(s', s)$ is identically null, then it works a fortiori for any positive misreporting cost. Let us thus assume $c(s', s) = 0$ for any (s', s). Denoting $U(s', s)$ the payoff of an intermediary reporting s' when the true model is s, and using log-derivatives, we have:

$$\begin{aligned} U(s', s) &= F(a(s'), s)\frac{\bar{\pi}(s')}{F(a(s'), s')}, \\ \frac{\partial \ln U(s', s)}{\partial s'} &= a'(s')\left(\frac{f(a(s'), s)}{F(a(s'), s)} - \frac{f(a(s'), s')}{F(a(s'), s')}\right) + \frac{\bar{\pi}'(s')}{\bar{\pi}(s')} - \frac{F_2(a(s'), s')}{F(a(s'), s')}, \end{aligned} \tag{A.7}$$

where F_2 denotes the derivative of F with respect to its second argument. Incentive compatibility requires to find for every s an $a(s)$ such that:

$$U_1(s, s) = 0 \Leftrightarrow \frac{F_2(a(s), s)}{F(a(s), s)} = \frac{\bar{\pi}'(s)}{\bar{\pi}(s)}. \tag{A.8}$$

Does such a function $a(.)$ exist? Under (DM) we have, for any s, $F_2(t, s)/F_t(s) < \bar{\pi}'(s)/\bar{\pi}(s)$ when $t \to 0$. Conversely, $F(1, s) = 1$ for any s, so that when $t \to 1$ we have the opposite inequality. Under (MLRP), $F_2(t, s)/F(t, s)$ is increasing in t, so that there exists a unique value $a(s)$ satisfying (A.8). Moreover, (DM) implies that the left-hand side of (A.8) decreases more in s than the right-hand side, while the left-hand side increases in a, so that $a(s)$ is increasing.

We finally have to show that the second-order condition is met. We can use (A.8) to replace $F_2(a(s'), s')$ in (A.7). After some rearrangements this gives us:

$$U_1(s', s) \geq 0 \Leftrightarrow \frac{f(a(s'), s)}{f(a(s'), s')} \geq \frac{F(a(s'), s)}{F(a(s'), s')}. \tag{A.9}$$

This condition is equivalent to $s' \leq s$ as a consequence of (MLRP), hence reporting s globally maximizes $U(., s)$.

A.3 Proof of Proposition 2

To reduce the notational burden, denote $\alpha = \alpha(s)$, $r = r(\alpha)$, $\theta = \theta(\alpha)$, and use the same notations with a prime for the same functions taken in s'. By contradiction, assume that the regulator wants to implement an increasing $\alpha(.)$ such that $\underline{\theta} < \tau$ without resorting to any bail-out. An intermediary of type s who reports model s' such that $\theta' < \tau$ receives:

$$\int_0^{\theta'} \max[u(\alpha', r', t) - T(s', t), 0] f(t, s) dt - c(s', s). \tag{A.10}$$

As $\theta' < \tau$, $f(t, s)$ can be replaced by $f(t, s')$ in the integral. The deviation payoff of type s is thus the payoff of type s' when this type reports truthfully, minus the misreporting cost $c(s', s)$. By assumption (IR) holds for type s', so that by reporting s', type s receives $\bar{\pi}(s') - c(s', s)$. An intermediary with type s thus reports truthfully only if he receives a payoff at least as large as the maximum of $\bar{\pi}(s') - c(s', s)$ over the s' such that $\theta(s') \leq \tau$. Note that the derivative of $\bar{\pi}(s') - c(s', s)$ with respect to s' and evaluated in s is strictly negative, by assumption (H3). Misreporting at least a little is thus profitable, whereas reporting s brings $\bar{\pi}(s)$. The intermediary thus earns a strictly positive rent equal to $\max_{s', \theta(s') \leq \tau} \bar{\pi}(s') - c(s', s) - \bar{\pi}(s)$.

A.4 Proof of Proposition 3

For brevity denote $\overline{\alpha} = \alpha(\overline{s})$, $\overline{r} = r(\overline{\alpha})$, $\overline{\theta} = \theta(\overline{s})$, and use the same notations underlined for $\underline{s}$.

Solution 1: If the regulator does not want agents to have rents she has to implement $\underline{\alpha}$ so that $\underline{\theta} \geq \tau$, otherwise Corollary 1 applies. A higher bound on welfare with this solution is obtained for $\underline{\alpha} = \alpha_\tau$ such that $\underline{\theta} = \tau$, $\overline{\alpha} = \alpha^*(\overline{s})$, and no rents are left to any type. This higher bound is:

$$\bar{\mathcal{V}}_1 = \psi \left[\mathcal{W}(\alpha_\tau, \underline{s})\right] + (1 - \psi) \left[\mathcal{W}(\alpha^*(\overline{s}), \overline{s})\right] - \lambda[\psi \bar{\pi}(\underline{s}) + (1 - \psi)\bar{\pi}(\overline{s})]. \tag{A.11}$$

Solution 2: In the first-best we have $\alpha^*(\overline{s}) > \alpha^*(\underline{s})$. As the regulator can always choose $\overline{\alpha} = \underline{\alpha}$ and then leave the minimum possible rents to agents, we must have $\overline{\alpha} \geq \underline{\alpha}$ in the second-best. This implies that $\overline{r} \geq \underline{r}$ and $\overline{\theta} \geq \underline{\theta}$. As the problem here is to prevent type $\overline{s}$ from misreporting $\underline{s}$ and receive lower capital requirements, (IR) is binding for $\underline{s}$ and (IC) for $\overline{s}$, which gives:

$$\begin{aligned} \int_0^{\underline{\theta}} [u(\underline{\alpha}, \underline{r}, t) - T(\underline{s}, t)] f(t, \underline{s}) dt &= \overline{\pi}(\underline{s}), & \text{(A.12)} \\ \int_0^{\overline{\theta}} [u(\overline{\alpha}, \overline{r}, t) - T(\overline{s}, t)] f(t, \overline{s}) dt &= \int_0^{\underline{\theta}} [u(\underline{\alpha}, \underline{r}, t) - T(\underline{s}, t)] f(t, \underline{s}) dt - c(\underline{s}, \overline{s}). & \text{(A.13)} \end{aligned}$$

(A.12) and (A.13) allow us to express the expected transfers as functions of $\overline{\alpha}$ and $\underline{\alpha}$ only, so that the regulator's objective (9) can be written as:

$$\bar{\mathcal{V}}_2 = \psi\,[\mathcal{W}(\underline{\alpha},\underline{s})] + (1-\psi)\,[\mathcal{W}(\overline{\alpha},\overline{s})] - \lambda[\psi\bar{\pi}(\underline{s}) + (1-\psi)(\bar{\pi}(\underline{s}) - c(\underline{s},\overline{s}))]. \tag{A.14}$$

As $\underline{\alpha}$ and $\overline{\alpha}$ appear only as arguments in the welfare function $\mathcal{W}$ there is no incentive for the regulator to distort the capital requirements from their first-best values. We thus have $\alpha^{**}(\underline{s}) = \alpha^{*}(\underline{s})$ and $\alpha^{**}(\overline{s}) = \alpha^{*}(\overline{s})$. Comparing $\bar{\mathcal{V}}_1$ and $\bar{\mathcal{V}}_2$ then immediately gives the condition (13).

A.5 Proof of Proposition 4

Calculus of variations can give us more information on the shape of α^{**}. As the regulator wants to maximize the expected welfare under constraint (14), her program writes as:

$$\max_{\alpha}\int_{\underline{s}}^{\overline{s}}[\mathcal{W}(\alpha(s),s)\psi(s) + \mu(s)\left(\alpha'(s)\pi_1(\alpha(s),s) - c_1(s,s)\right)]ds, \tag{A.15}$$

where $\mu(s)$ is the Lagrange multiplier associated with the incentive compatibility constraint (14). The Euler-Lagrange equation of this program gives us the following necessary condition:

$$\begin{aligned} & [\mathcal{W}_1(\alpha(s),s)\psi(s) + \mu(s)\alpha'(s)\pi_{11}(\alpha(s),s) = \tfrac{d}{ds}[\mu(s)\pi_1(\alpha(s),s)] \\ \Leftrightarrow \qquad & \mathcal{W}_1(\alpha(s),s)\psi(s) = \mu'(s)\pi_1(\alpha(s),s) + \mu(s)\pi_{12}(\alpha(s),s). \end{aligned} \tag{A.16}$$

There are two possibilities for the behavior of α^{**}. If on a given interval the constraint (14) is slack we have $\mu(s) = 0$ on this interval, thus $\mu'(s) = 0$ and (A.16) gives us $\alpha^{**}(s) = \alpha^{*}(s)$. This is possible only if $\alpha^{*\prime}(s)\pi_1(\alpha^{*}(s),s) \geq c_1(s,s)$, that is if the first-best capital requirements are sufficiently flat. If this is not the case, then $\alpha^{**\prime}$ is determined by (14), and as $\pi_1 \leq 0$ we obtain that $\alpha^{**\prime} \leq \alpha^{*\prime}$. In both cases $\alpha^{**\prime} \geq 0$.

A.6 Proof of Proposition 5

It is convenient to denote $\theta(\alpha, r)$ the default point of a price-taking intermediary. We have:

$$\begin{aligned}
\pi(\alpha,r,s) &= \frac{E}{\alpha}\int_0^{\theta(\alpha,r)}[r(1-t)-r_0(1-\alpha)]f(t,s)dt. && \text{(A.17)}\\
\pi_1(\alpha,r,s) &= -\frac{E}{\alpha^2}\int_0^{\theta(\alpha,r)}[r(1-t)-r_0(1-\alpha)]f(t,s)dt+\frac{r_0E}{\alpha}F(\theta(\alpha,r),s)+\theta_1(\alpha,r)\times 0\\
&= -\frac{E}{\alpha^2}\int_0^{\theta(\alpha,r)}[r(1-t)-r_0]f(t,s)dt. && \text{(A.18)}\\
\pi_{11}(\alpha,r,s) &= \frac{E}{\alpha^3}\int_0^{\theta(\alpha,r)}[r(1-t)-r_0]f(t,s)dt-\frac{E}{\alpha^2}\theta_1(\alpha,r)(r(1-\theta(\alpha,r))-r_0)f(\theta(\alpha,r),s)\\
&= \frac{E}{\alpha^3}\int_0^{\theta(\alpha,r)}[r(1-t)-r_0]f(t,s)dt+\frac{r_0E}{\alpha}\theta_1(\alpha,r)f(\theta(\alpha,r),s). && \text{(A.19)}
\end{aligned}$$

As shown in A.1, profit is always increasing in L in the case of a price-taking intermediary, so that $\pi_1(\alpha,r,s) \leq 0$ and thus the integral in (A.18) is positive. The same integral appears in (A.19). Since $\theta_1(\alpha,r) \geq 0$, the profit π is convex in α, which is due to the bank's limited liability. We want to show that $\pi_{12}(\alpha,r,s) \leq 0$. We have:

$$\pi_{12}(\alpha,r,s) = -\frac{E}{\alpha^2}\int_0^{\theta(\alpha,r)}(1-t)f(t,s)dt+\frac{r_0^2E(1-\alpha)}{r^2\alpha}f(\theta(\alpha,r),s). \tag{A.20}$$

As $\pi_1(\alpha,r,s) \leq 0$ we have the following inequality:

$$r\int_0^{\theta(\alpha,r)}(1-t)f(t,s)dt \geq r_0F(\theta(\alpha,r),s). \tag{A.21}$$

Plugging into (A.20), we have that $\pi_{12}(\alpha,r,s) \leq 0$ if $rF(\theta(\alpha,r),s) \geq r_0\alpha(1-\alpha)f(\theta(\alpha,r),s)$, which is certainly true if $f(\theta(\alpha,r),s)/F(\theta(\alpha,r),s) \leq 1$.

A.7 Parameters used in the figures

All the figures rely on simulations using the following baseline parameters. $\{F(t,s)\}$ is a family of Beta distributions with parameters $a = 3.5$ and $b \hookrightarrow \mathcal{U}([35,70])$. s is equal to $1/b$. $\mathbb{E}_s(1-t)$ thus ranges from 4.8% to 9.1%. $q(r) = \gamma(r-1)^{-\eta}$, with $\gamma = 1$ and $\eta = 1.1$, and E is set to 1. The interval $[\underline{s}, \overline{s}]$ is discretized in a set of 1000 models indexed from $i = 1$ to $i = 1000$. The cost to report model i' when the true model is i is $c(i',i) = c_0|i'-i| + c_1(i'-i)^2$, with $c_0 = 10^{-4}$ and $c_1 = 5.10^{-4}$. $\bar{\pi}(s)$ is assumed to be proportional to $\mathbb{E}_s(1-t)$.

On Fig. 2 the high cost scenario assumes $c_0 = 2.10^{-4}$. On Fig. 3 $\hat{\alpha}(s) = \alpha^*(s)$ without a leverage ratio, and $\hat{\alpha}(s) = \max(0.05, \alpha^*(s))$ otherwise. On Fig. 4 the high demand scenario assumes $\gamma = 1.5$. Fig. 5 again assumes a leverage ratio at 0.05.

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