Mass lapse scenario in insurance, the use of a dynamic contagion process

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Outline

A word on the surrender risk

FIRSTLY: what is the SURRENDER RISK?

Why is the insurer interested in understanding this risk?

1. For the design of new products,
   - to set up assumptions on the average surrender rate,
   - because it has a straight impact on A.L.M. and E.E.V.
2. To understand the behaviours,
   - main discriminating factors of surrenders? CART
   - behaviour risk essential (adapt product features to gain market shares).
3. Predictions: risk segmentation and management.
   - better assess the surrender risk at underwriting process,
   - be able to quantify the surrender probability whenever we would like during the contract lifetime.

Our aim is to extract triggers, classify and predict surrenders.

Some key-points

1. Business lines specificities:
    - In Savings, key-point. Yearly lapse rate $\sim$[5%,15%],
    - In Protection, secondary since surrender is not allowed on collective treaties (70% of the business).
      $\rightarrow$ Main pb: adverse selection and moral hazard...
2. Typical aspects in France:
    - Fiscality constraints VS duration (peak 9th year),
    - Penalties : capped at 5% of the capital saved over a 10-year period.
    - Policy options : partial surrenders...

Keeping discriminant policyholder's characteristics as well as contract features is thus important $\Rightarrow$ regression framework! (see (MMDL11))

Existing literature in modelling surrenders

The 2 main historical explanations for surrenders are liquidity needs (Out90) and rise of interest rates (obvious...)

There are 4 different approaches to model surrenders:

- Finance (TKC02), (Kue05), (BBP08) : price surrender option, optimal and rational behaviours assumption,
- Statistics (RH86), (FLP07): collective. Empirical data allow to calibrate surrender functions like:

$$r_d = r_0 * (1 - a * ln(d) * (ln(d+1) - b))$$

- Economics (FLP07): microeconomy, expected utility theory, rational behaviour. But is it really like this?
- Econometrics (CL06), (Kim05), (Kag05): individual.
  - segmentation models to define risk classes,
  - GLM to quantify the impact of risk factors,
  - intensity models (see also prepayment for mortgages).

Threats for Life insurers (rachats conjoncturels)

Remaining questions:

- how did the financial crisis impact the surrender rates?
- do financial markets strongly impact policyholders' behavior? Very difficult to answer...

Current context:

- never experienced such low interest rates,
  $\Rightarrow$ underwriting of new business has been done with these abnormal low rates since 4 years,
- Surrenders could be forbidden by the regulatory
  $\Rightarrow$ could lead policyholders to invest differently, and makes life insurance investments become less attractive.

Major issue: face massive surrenders due to a sudden increase of rates (how to adapt contracts, market shares...).

Intuitions

The surrender behaviours are mainly driven by two risk classes: fiscality constraints and financial markets $\rightarrow$ sources from two different areas:

- *endogenous* or *idiosynchratic* factors: surrender fees profile, tax relief, contract options, distribution channel, customer segment, cross-selling $\rightarrow$ structural surrenders;
- *exogenous* or *environmental* factors: financial markets, reputation risk and bankruptcy fear, regulatory changes $\rightarrow$ temporary surrenders.

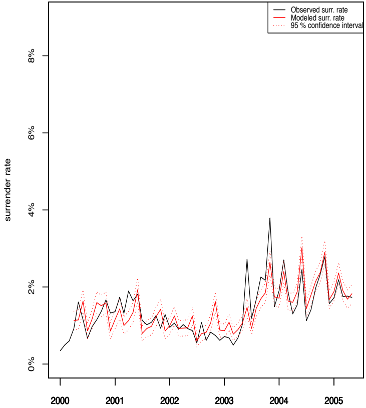
From our experience,

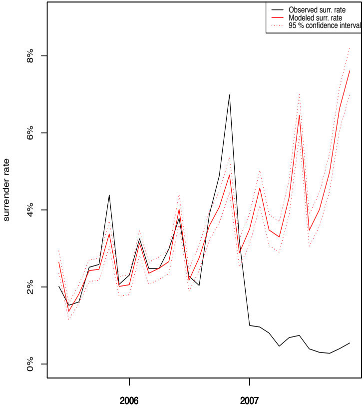
$\Longrightarrow$ GLM fail into modelling such a complex dependence, especially because of exogenous factors (see next slide).

$\Longrightarrow$ Survival models do not catch the heterogeneity of the data, even if we use frailty models.

Ex.: dynamic logit on Spanish endowments

Model statistically significant, predictions dramatically bad!





→ Correct model as long as everything remain stable, but...
→ Financial crisis not captured by the model (although financial variables input as covariates). Correlation (??).

Tables summing up descriptive statistics

Usually, insurance companies try to fill this kind of tables using their own experience. Here, these are yearly structural lapse rates for 9 segments.

	Bank	Agent	Direct
$[0,4]$ years	5%	8%	15%
$[4,8]$ years	3%	10%	15%
>8 years	10%	19%	20%

This means that they segment their population with a priori discriminant risk factors to deal with their structural lapses...

Associated important assumptions:

- independence between policyholders,
- requires to have a minimum number of observations in each segment to apply the law of large numbers.

CEIOPS (EIOPA) - QIS 5 - Calibration papers

Recommendations for capital requirements are computed in TWO steps (cf p.105).

(1) Shocks up / down applied to structural lapse rate:

Following empirical studies in 2003 in UK on individual with-profit life insurance policies (and also in Poland), the structural lapse rate should be shocked in the following way:

$$LR_{up} = \min(100\%, 150\% \times LR),$$

$$LR_{down} = \min(0, \max(50\% \times LR, LR - 20\%)).$$

$\rightarrow$ Should cover misestimation or permanent changes of LR.
$\rightarrow$ Incorporates temporary lapses!

CEIOPS (EIOPA) - QIS 5 - Calibration papers

(2) Mass lapse event:

Corresponds to the deterioration of a financial position of the undertaking, reputation $\sim$ "bank run" (Northern Rock!), "catastrophe type event"...

$\rightarrow$ Empirical basis to calibrate mass lapse event is very poor (or never observed...);
$\rightarrow$ It is advised to consider a loss of 30% of the sum of positive surrender strain over the portfolio;
$\rightarrow$ It should be adjusted to the type of life insurance policy, e.g with-profit contract usually have higher persistency...

$\rightarrow$ **Also incorporates temporary lapses!**

We keep as required capital the maximum capital to reserve corresponding to all these scenarii ((1) and (2)).

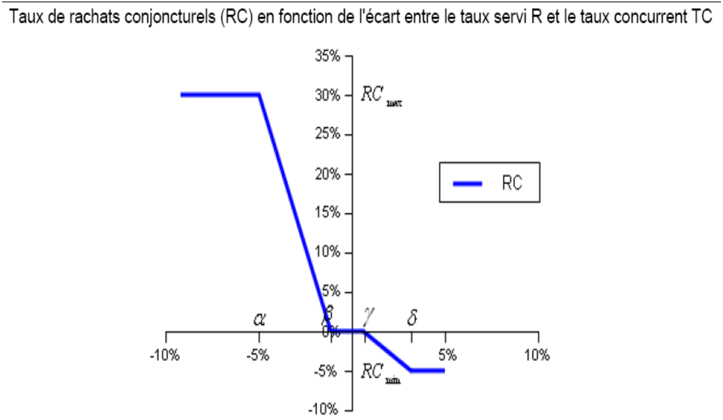
Other suggestion for managing lapse rates: Orientations nationales complémentaires du QIS 5

Temporary lapses in addition to structural lapses by:

$$\text{RC} = \begin{cases} RC_{max} & \text{si} \quad TS - TA < \alpha \\ RC_{max} \times \frac{TS-TA-\beta}{\alpha-\beta} & \text{si} \quad \alpha \leq TS - TA < \beta \\ 0 & \text{si} \quad \beta \leq TS - TA < \gamma \\ RC_{min} \times \frac{TS-TA-\gamma}{\delta-\gamma} & \text{si} \quad \gamma \leq TS - TA < \delta \\ RC_{min} & \text{si} \quad TS - TA \geq \delta \end{cases}$$

Paramètres loi de rachat conjoncturel proposé		
	Plafond min	*Plafond max*
α	-6%	-4%
β	-2%	0%
γ	1%	1%
δ	2%	4%
RC_min	-6%	-4%
RC_max	20%	40%

Temporary lapses : the dynamic function



Indications retirées depuis car l'ACPR promeut le dvp de modèles internes adaptés au risque propre de la compagnie.

Orientations nationales complémentaires du QIS 5

At the end, we consider

$$LR_{shocked} = \min(1, \max(0, RS + RC)),$$

where

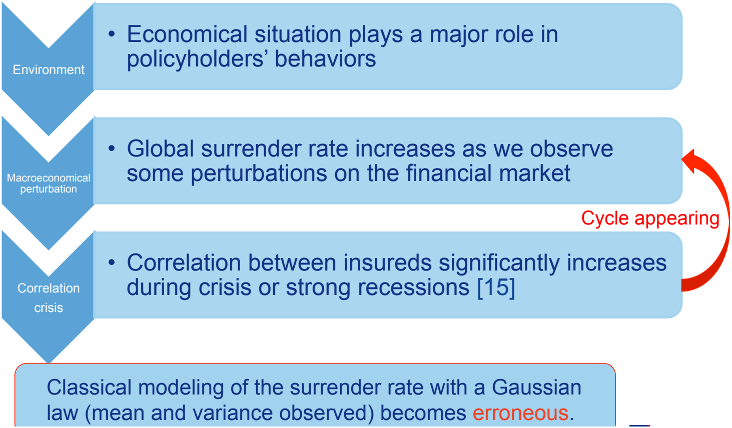
- RS : structural lapses,
- RC : temporary lapses (max. is still 30%, so integrates the mass lapse event as defined previously).

→ Looking at the spread between the contract credited rate and some competitive rate seems to be the right approach.
→ Easy to implement for companies.
→ However, it still lacks the consideration of potential correlated behaviours in case of financial distress...

Main idea

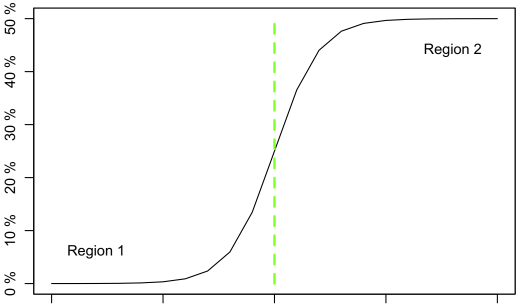
A vicious circle can originate from bad economic conditions...

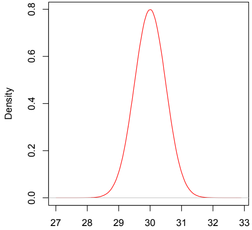
This could lead to big surprise in a Solvency II framework!

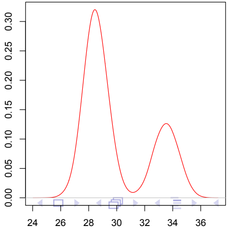


(Surrender rate VS spread): Arctan function

Low level determined empirically/ high plateau by an expert.

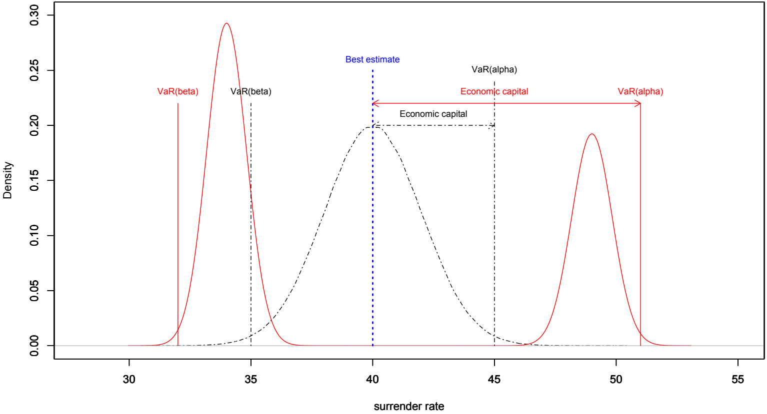






First attempt: a basic mixture model (Region 2)

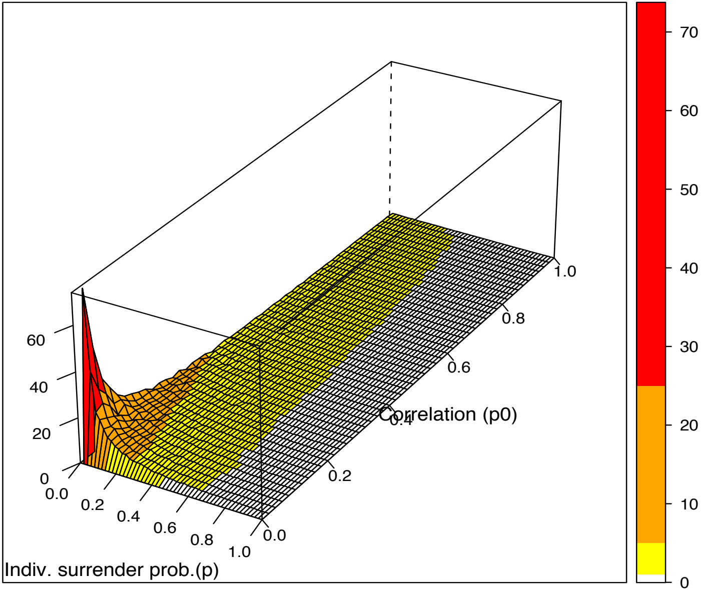
Common shock model (LM11): $I_k = J_k I_0 + (1 - J_k) I_k^{\perp}$



Lesson: correlation makes the EC become much higher!

However, this is a static model...(does not depend on time t)

ΔVaR : worst situation when the crisis appears...



New approach: Hawkes processes (see (HO74))

Intensity models (almost classical duration models) are also used in mortgage prepayments.

Let $N = (N_t)_{t \geq 0}$ be a point process with intensity $\lambda = (\lambda_t)$.

N describes the surrenders in an insurance portfolio with an intensity λ following the piecewise deterministic dynamics, i.e.

$$\lambda_t = \lambda_\infty + (\lambda_0 - \lambda_\infty) e^{-\beta t} + \alpha \int_0^t e^{-\beta(t-s)} dN_s,$$

with α, β, λ_∞ and λ_0 being some positive constants.

$\rightarrow$ the surrender intensity is stochastic,
$\rightarrow$ "internal" source of excitation,
$\rightarrow$ path-dependent $\rightarrow$ depends on its history!

Extension : the dynamic contagion process

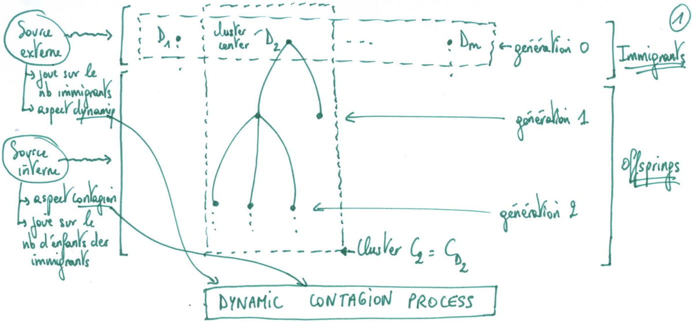
$\rightarrow$ Slightly modified version of Hawkes processes (DZ11).

The mathematical expression for surrender intensity follows

$$\lambda_t = \lambda_\infty + (\lambda_0 - \lambda_\infty) e^{-\beta t} + \alpha_1 \int_0^t e^{-\beta(t-s)} dN_s + \alpha_2 \int_0^t e^{-\beta(t-s)} d\hat{N}_s$$

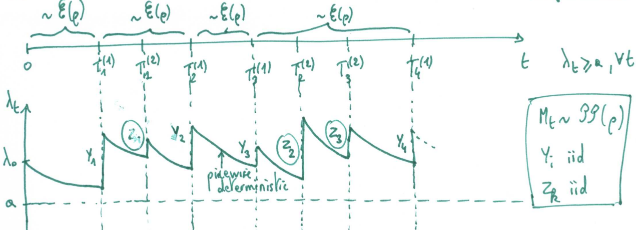
Notice that the surrender intensity depends on...

1. the initial level of surrender intensity λ_0,
2. λ_∞ (for structural surrenders): constant, realistic if the portfolio composition remains similar over time)...
3. endogenous factors: history of N_t $\rightarrow$ contagion, internal;
4. exogenous shocks: history of $\hat{N}_t$ $\rightarrow$ dynamic dependence, external source of excitation.



N_t = nb de rachats total global ≡ on compte tous les points du graphe ci-dessous.

$$\lambda_t = \underbrace{a + (\lambda_0 - a)e^{-\delta t}}_{\text{Cox}} + \underbrace{\sum_{i \geq 1} Y_i \, e^{-\delta(t - T_i^{(1)})} \mathbb{1}_{\{T_i^{(1)} \leq t\}}}_{\text{externally-excited}} + \underbrace{\sum_{k \geq 1} Z_k \, e^{-\delta(t - T_k^{(2)})} \mathbb{1}_{\{T_k^{(2)} \leq t\}}}_{\text{self-excited}}$$



Exogenous shocks in our context I

Consider a contract with guaranteed return $\underline{r} \geq 0$ and let $r = (r_t)_{t \geq 0}$ be the interest rate with GBM dynamics

$$r_t = r_0 \, e^{X_t} \quad \text{and} \quad X_t = (\mu - \frac{\sigma^2}{2}) \, dt + \sigma W_t,$$

where $\mu, \sigma > 0$ and W_t a standard brownian motion.

How the surrender decision is affected by the level of r_t?

$\rightarrow$ A rational policyholder will surrender as soon as the quantity $\Delta r_t := \frac{r_t - \underline{r}}{\underline{r}}$ becomes high enough.

$\rightarrow$ Assume that the insurance company incorporates this feature in its internal risk model by adjusting the credited rate $\underline{r}$ depending on market interest rates level.

Exogenous shocks in our context II

$\rightarrow$ Policyholders exercise their option to surrender at time $\hat{T}_1$ (first time the spread Δr_t hits a constant barrier $m > 0$).

$\rightarrow$ At that time, the insurer adjusts the guaranteed return $\underline{r} = r_{\hat{T}_1} \rightarrow$ "new" spread given by $\Delta^1 r_t = (r_t - r_{\hat{T}_1})/r_{\hat{T}_1}$.

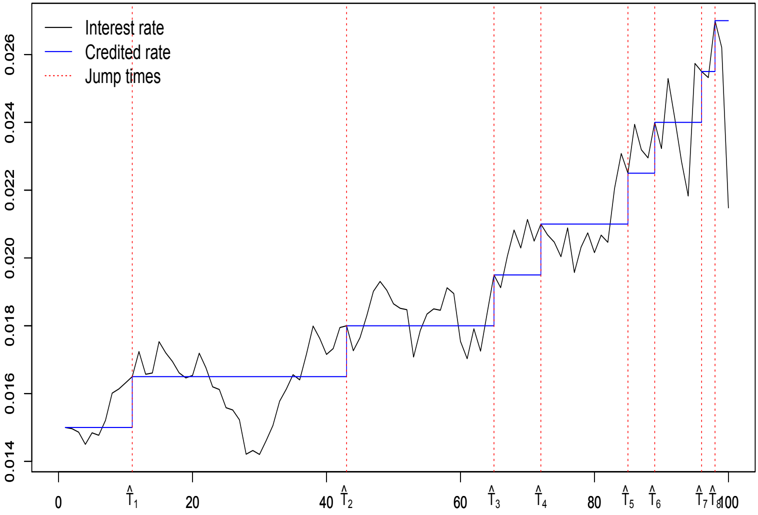
$\rightarrow$ Next adjustment will be operated whenever the new spread will go beyond the same fixed threshold m.

The sequence $\hat{T}_j$, for $j = 1, 2, \cdots$ characterizes these events:

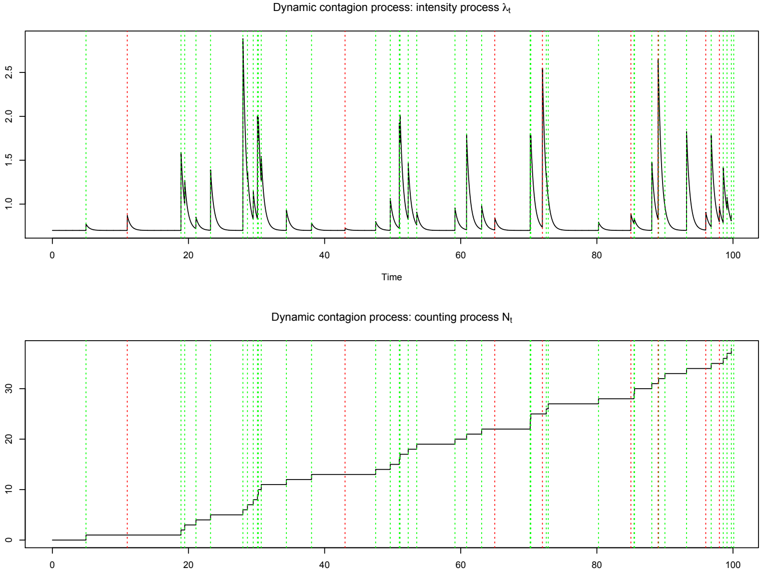
$$\hat{T}_{j+1} = \inf\{t \geq \hat{T}_j,\ \Delta^j r_t \geq m\},$$

with $\Delta^j r_t = \frac{r_t - r_{\hat{T}_j}}{r_{\hat{T}_j}}$ and under the assumption $\hat{T}_0 = 0$.

Adjustments of the credited rate



Surrender intensity and counts trajectories



An algorithm for simulating such trajectories

The algorithm that was originally used (but does not work in our case) embeds the following steps:

1. simulate the first internal jump time,
2. simulate the first external jump time,
3. take the minimum,
4. restart at step 1 (from this jump time) to draw the next one by simulating following interarrival jump times.

Should work for i.i.d. external inter arrival jump times.

Surrender intensity and risk indicators

Recall the discretized expression of the surrender intensity:

$$\lambda_t = \lambda_\infty + (\lambda_0 - \lambda_\infty) e^{-\beta t} + \sum_{i \geq 1} Y_i \, e^{-\beta(t - T_i)} \mathbf{1}_{\{T_i \leq t\}} + \sum_{j \geq 1} Z_j \, e^{-\beta(t - \hat{T}_j)} \mathbf{1}_{\{\hat{T}_j \leq t\}},$$

Note that

- here the size of the jumps is a random variable,
- constraints for the process stationarity: it decays (via β) faster than the expected internal jump size,
- inter arrival times of externally-excited jumps are i.i.d ??
- these inter arrival times are (close to be) inverse gaussian, with intensity h.

Risk indicators related to the intensity

Thanks to the piecewise deterministic Markov theory (see (Dav84)), we can derive the expression of the infinitesimal generator corresponding to this process.

This leads to (more or less) simple expressions for some key risk indicators, such as:

- conditional expectation of the intensity

$$\mathbb{E}[\lambda_t|\lambda_0] = \left(\lambda_0 - \frac{\beta\lambda_\infty}{\beta - 1/\gamma}\right) e^{-(\beta-\frac{1}{\gamma})t} + \frac{\beta\lambda_\infty}{\beta - 1/\gamma} + \frac{1}{\delta} e^{-(\beta-\frac{1}{\gamma})t} \int_0^t h(s; M, \mu, \sigma) \tag{1}$$

- (un)conditional variance of the intensity : too long...
- unconditional expectation ($L = \frac{\log(1+m)^2}{\sigma^2}$, $M = \frac{\log(1+m)}{\mu - 0.5\sigma^2}$):

$$\mathbb{E}[\lambda_t] = \frac{\beta\lambda_\infty}{\beta - 1/\gamma} + \frac{1}{\delta(\beta - 1/\gamma)} \frac{L}{2M^2},$$

Risk indicators related to the counting process

It is also possible to derive such expressions for the counting process N_t.

For instance,

- expectation of N_t
- variance of N_t
- probability generating function of N_t : the expression should show that the distribution of N_t is multimodal.

The last quantity could allow us to define risk management strategies in terms of managing massive lapses due to financial distress.

Still to do

What we are now going to look at is:

- compare this approach with others (Solvency II, Guru-type model, Gaussian approximations, ...) on several key indicators in a (in)finite time horizon:
  - expectation,
  - variance,
  - probability generating function.
- introduce the impacts on some risk measures (VaR, TVaR and others)
- assess the corresponding solvency constraints and economic capital requirements;
- optimization for ORSA purpose.

Key messages

1. Integrate only main risk factors, consider correlation.
2. Incorporating risk factors must be done separately depending on their source :
   - exogenous factors,
   - endogenous factors.

   [Seems realistic because of the nature of their impact.]
3. In my opinion, stress tests as considered in most of companies are clearly underestimated.

Perspectives

1. Hidden Markov models,
2. Mixtures of survival distributions.

Thank you very much for your attention...

...and all comments/questions/ideas are obviously welcome!

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