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Stress-testing
banks' corporate credit portfolio

Olivier de Bandt, Nicolas Dumontaux, Vincent Martin, Denys Médée*

[*] Autorité de Contrôle Prudentiel, Direction des Etudes. Contact : olivier.debandt@acp.banque-france.fr; the authors are grateful to C. Gourieroux , A. Frachot , Ph Mathouillet and Ph. Trainar for detailed comments on a preliminary draft. All remaining errors are the authors' sole responsibility.

SECRETARIAT GENERAL DE L'AUTORITE DE CONTROLE PRUDENTIEL
DIRECTION DES ÉTUDES

STRESS-TESTING BANKS' CORPORATE CREDIT PORTFOLIO

Olivier de Bandt, Nicolas Dumontaux, Vincent Martin, Denys Médée

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Abstract

The paper describes the methods used by the French Banking Supervision Authority (ACP) to run stress tests for the corporate credit portfolio, through credit migration matrices (or transition matrices). This approach is currently used for "top-down" stress tests exercises. Developed for Basel II, it is still relevant under the Basel III framework. It includes sufficient flexibility to accommodate the severe crisis period observed recently. The paper introduces the basic model underlying the approach, largely based on Merton's model; it then describes carefully the different steps for its practical implementation, providing hints on how it can be extended to other banking sectors. Finally the paper comments a few outputs of a stress testing exercise.

Key words: credit risk, corporate, stress tests, migration matrices
JEL: G21, G28, G32, E44

Mise en œuvre de stress tests sur les crédits aux entreprises

Résumé

Le papier décrit les méthodes utilisées par l'Autorité de Contrôle Prudentiel (ACP) pour réaliser des stress tests sur le portefeuille de crédit aux entreprises, par l'intermédiaire de matrices de migration du risque de crédit. Cette approche est actuellement utilisée pour la réalisation d'exercices de stress « top down » par le superviseur. Développé sous le régime de Bâle II, il est parfaitement transposable sous Bâle III. Le modèle est suffisamment flexible pour s'adapter à la période de crise marquée qui a été observée récemment. Le papier introduit le modèle de base fondant cette approche, largement inspiré par le modèle de Merton ; il décrit en suite pas-à-pas les différentes étapes de sa mise en œuvre, en fournissant des pistes pour l'étendre à d'autres systèmes bancaires. Finalement, le papier commente certains résultats des stress tests.

Mots clés : risque de crédit, entreprises, stress tests, matrice de migration
JEL : G21, G28, G32, E44

Stress-testing banks' corporate credit portfolio

Olivier de Bandt, Nicolas Dumontaux, Vincent Martin, Denys Médée[1]

Since the recent financial crisis, which drew unprecedented attention to the stress testing of financial institutions, stress-tests exercises have become a central risk management tool to assess the potential impact of extreme events on banks' P&L and balance sheets structures.

Stress tests are viewed as complementary to traditional risk measurement metrics such as Value-at-Risk, as they are an important mechanism for detecting weaknesses both of a single financial institution as well as threats to financial stability. Nowadays, financial institutions are required to perform regular exercises within Pillar II of the regulatory framework of the Basel Accord in order to assess the global impact of adverse events or changes in market conditions on banks' capital adequacy. Supervisory authorities as well are used to leading such exercises: the International Monetary Fund (IMF) with its regular Financial Sector Assessment Program, the European Banking Authority (EBA) with its European "bottom-up" stress tests including a disclosure step, and national supervisory authorities which all have built dedicated tools, especially for regular top-down exercises. The scope of stress-testing includes traditional credit risks, market risks, operational risks, interest rate risks, and since the 2007 and beyond financial crisis, liquidity risks.

Stress testing corporate credit risk, also known as "wholesale credit risk" as opposed to "retail credit risk", is a key component of stress testing for global institutions. Credit risk in itself (i.e. including retail credit risk) is indeed one of the major sources of risk for banks judging by the extent of banks' credit risk-weighted assets (RWA),[2] and, accordingly, may have a major impact on the solvency of financial institutions. The paper examines stress testing for credit risk, focusing on risks arising from corporate loans and other credit exposures.[3] It aims at introducing a Basel II-type modelling framework to perform credit stress test scenarios through credit migration matrices (or transition matrices), which has been implemented by French authorities and which is currently used as a tool for top-down stress tests exercises. This approach is still relevant under the Basel III framework, since nothing new has been introduced in the Basel III framework with respect to the assessment of credit risks of banks' corporate portfolios.

The paper is organized as follows: section 1 briefly depicts the model our stress test framework rests on, which is largely based on the Merton's model; section 2 introduces the way this framework is implemented to conduct top-down stress test exercises. Section 3 comments a few outputs of the stress tests.

1. Model specification

Several models are available for quantifying credit risk, this risk stemming either directly from actual defaults of credit exposures, or indirectly from migrations of credit ratings, taking into account their prudential treatment.[4] These models maybe either structural (modelling of firms'

[1] Autorité de Contrôle Prudentiel, Direction des Etudes. Contact : olivier.debandt@acp.banque-france.fr

[2] In the case of France, credit risk RWAs represents more than 75% of total RWAs.

[3] Exposures from structured credit products or from Over-the-Counter (OTC) derivatives exposures are not covered here.

[4] Prudential filters may sometimes dampen the effect of marked-to-market gains or losses.

value and capital structure) or reduced forms, where credit events are exogenous to the firms. Here we rely on the latter approach, with credit events triggered by macroeconomic shocks, and focus on credit migrations (see ECB, 2007, for alternative industry credit models).

The model we introduce in this section relies on the basic idea that the evolution of rating transitions can easily be linked to a synthetic credit indicator. The main hypothesis is that the underlying asset value of a firm evolves over time, through a geometric Brownian motion, and that default is triggered by a drop in firm's asset value below the value of its callable liabilities. In the Merton framework,[5] shareholders actually hold a call option on the asset value of the firm, while debt holders hold a put option.

The model relies on the general assumption that all firms log asset value exhibit a uniform pairwise correlation ρ. As a consequence, under the assumption that changes in a firm's log asset value ($\Delta \log A_i$) is normally distributed with variance 1, it is always possible to decompose this variable for each firm i into the weighted sum of a standard normal systemic factor (Z, thereafter referred to as the credit index) and an standard normal idiosyncratic factor (ε_i), as follows:

$$\Delta logA_{i,t} = -\sqrt{\rho}Z_t + \sqrt{1-\rho}\varepsilon_{i,t} + c_i,$$

where c_i is the long-run growth of firm i's asset value. All firms are supposed to have identical characteristics (e.g. their correlation to Z) with respect to their credit rating, which then leads to identify i as a credit class rather than an entity.

The most important – though rather innocuous – assumption here is that the idiosyncratic factor ε_i is normally distributed, while the normality of the systemic factor – which might be challenged on empirical grounds, in particular during the 2007-8 financial crisis – is not essential for the derivation of the model. Indeed, we use it to compute probabilities of transition and default conditional on the realization of this systemic factor (see below). While we use a classical framework including normal distribution of log asset values' variations and a uniform correlation matrix to rigorously derive the above equation (thus implying a standard Gaussian systemic factor), we might just as well, for the purpose of this work, have assumed the form of the above relation, and relaxed the constraint of a normal standard Z_t. This would not have changed the remainder of this paper, except for minor calibration details.

The default probability may then be expressed as the probability of a standard normal variable falling below a critical value, defined with respect to the different ratings (with a total of n rating classes, with $n=8$). Similarly, thresholds[6] can be set up for rating migrations, as graphically represented in figure 1.

[5] Merton, R. (1974)

[6] To be complete, note that the thresholds also depend on the level of firms' liabilities

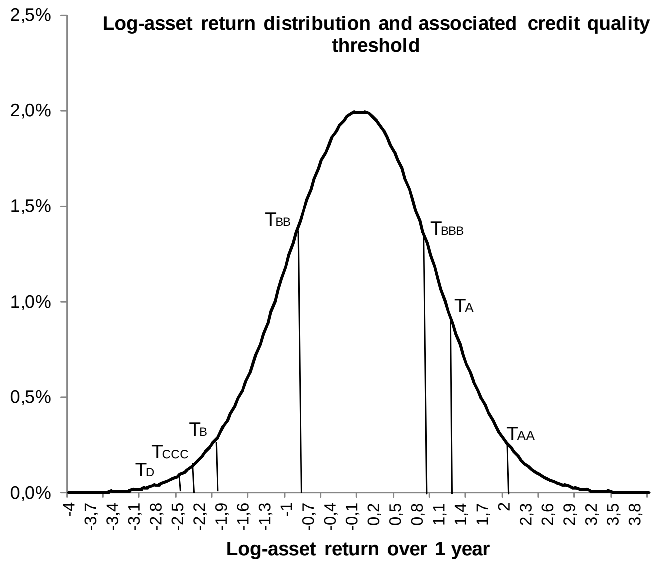

Figure 1: log-asset return probability density function and associated credit quality thresholds for BBB issuers

This framework, on which is also based the Basel II Asymptotic Single Risk Factor (ASRF)[7] model, results in the following relations with respect to default probabilities:

$$PD_i \left| \left\{ Z_t = \Phi^{-1}(\alpha) \right\} = \Phi \left[\frac{\Phi^{-1}(\bar{p}_i) + \sqrt{\rho}\, \Phi^{-1}(\alpha)}{\sqrt{1-\rho}} \right] \right.,$$

where $PD_i\ |\{\ldots\}$ is the (conditional) default probability in state Z, Φ is the Gaussian cumulative distribution function, $\bar{p}_i$ is the long term average probability of default (PD) of class i (i=1,…n) (or unconditional PD) and α is the probability that the value Z (or below) occurs. The closer α is to 0, the less frequent is the crisis and the greater its severity: in the Basel II framework, α is equal to 0.1% (this corresponds to the regulatory confidence level of 99.9%, which is the required confidence level to compute regulatory capital requirements under Basel II & III frameworks). In our application we use n=8 risk classes, where class 8 stands for the default class. Default probabilities along with rating migrations depend on a sole parameter (Z_t, the credit latent index discussed below), meaning that migrations matrices can be modeled through one macroeconomic factor (ASRF: Asymptotic Single Risk Factor). This framework can perfectly accommodate several risk factors[8] (industry, region, country for example). Note that we only consider shocks on the default probability (PD). Shocks on Loss –Given-Default (LGD), may be calibrated exogenously, but we do not consider their link with the macroeconomic environment (ie possible changes in LGD over the business cycle). This is reserved for future work.

To compute the probability of transition from rating class i to rating class j, the above formula, expanded to every component of the nxn (here 8x8) transition matrix, yields:

$$P_{ijt} = \Phi \left[\frac{\Phi^{-1}(\bar{P}_{i8} + \ldots + \bar{P}_{ij}) + \sqrt{\rho}\, Z_t}{\sqrt{1-\rho}} \right] - P_{i8t} - \ldots - P_{i,j+1,t}$$

[7] Vasicek, O. “Limiting Loan Loss Probability Distribution”, KMV Corporation

[8] Feng, Gourieroux, Jasiak (2008); Gagliardini, Gourieroux (2005a). For the estimation of ρ_i (ρ in our case), see 2.4, as well as Foulcher, Gourieroux and Tiomo (2006) and Gagliardini, Gourieroux (2005b). when relying on a complete data set of credit exposures’ default or survival trajectories, rather than rating migration matrices

This approach, which aims at representing transition matrices by a single parameter, was firstly studied by Belkin, Suchower and Wagner (1998). They follow the *CreditMetrics* framework proposed by Gupton, Finger and Bhatia (1997).

The modelling assumption of a uniform correlation matrix, although debatable, is justified by the desirable property of having the best possible forecasting power, within a simple and tractable analytical framework relying only on transition matrices for calibration. Using empirical asset correlations would have needed to modify it to some extent, as actual data does not confirm the uniform correlation hypothesis. This shortcoming is not a major drawback as the correlation parameter as a degree of freedom to best fit historical transition matrices (see below for calibration issues).

Note that such a framework can accommodate various scenarios, but relies mostly on the correlation between a systemic factor that we measure by aggregate corporate defaults (see below) and the ratings in a corporate portfolio. It can replicate either the average link between ratings and the business cycle, or particular features of the 2007-8 crisis, namely the origination of the crisis in the AAA segments of the portfolio. This last property stems from the fact that a feature of the model is to perform the calibration of the sensitivity to the systemic factor on a dataset which includes crisis investment grade transition matrices

2. The stress testing framework

The following framework is based on a relationship between the latent credit index and the macro-economic situation. This link is indeed fundamental since most stress test exercises start with the choice of a set of macroeconomic stress scenarios. Those stress scenarios are then linked to risk parameters – default rates, loss rates, regulatory PDs, regulatory LGDs, transition matrices – which will, in the end, affect banks' solvency.
Our approach is based on an intermediary variable, namely the aggregate default rate.[9] First, we measure the link between GDP (or the macroeconomic scenario) and the default rate, then between the default rate and the latent credit index, in order to compute the stressed transition matrix.

The different steps described here are based on relationships uncovered for France, in particular regarding the link between credit risk and the macroeconomy but we explain how they may be replicated for other institutions/countries.

2.1. The data

Adequate data are of course necessary for calibrating the model. We show how to rely on S&P transition matrices, a method that can be to some extent transferable to other institutions/countries as long as they have a similar global portfolio of corporate loans.

As aforementioned, our framework takes advantage of the S&P CreditPro database, which contains issuer ratings history for 15 726 obligors over the 1981-2011 period, of which 2 127 ended in default. The obligors are mainly large corporate institutions - sovereigns and

[9] According to S&P definition. Historical data provided by Standard & Poor's in its database Credit Pro.

municipals are excluded - and pools include both US and non-US industrials, utilities, insurance companies, banks and other financial institutions, and real estate companies.[10]

Over the past two decades, three major crisis periods can be distinguished: (i) the recession that took place in the wake of the 1st Gulf war at the beginning of the 1990s (GDP growth dropped to -0,3% in 1991 in the US and to -0,7% in 1993 in France); (ii) the burst of the Internet bubble (GDP growth dropped in the US to 1,1% in 2001 from over 4% in the previous years while in France it GDP growth declined to 0,9% in 2002 and 2003 from over 3,5% in 2000); (iii) and the 2007 and beyond subprime crisis (both American and French GDP growth dropped to less than -3% in 2009).

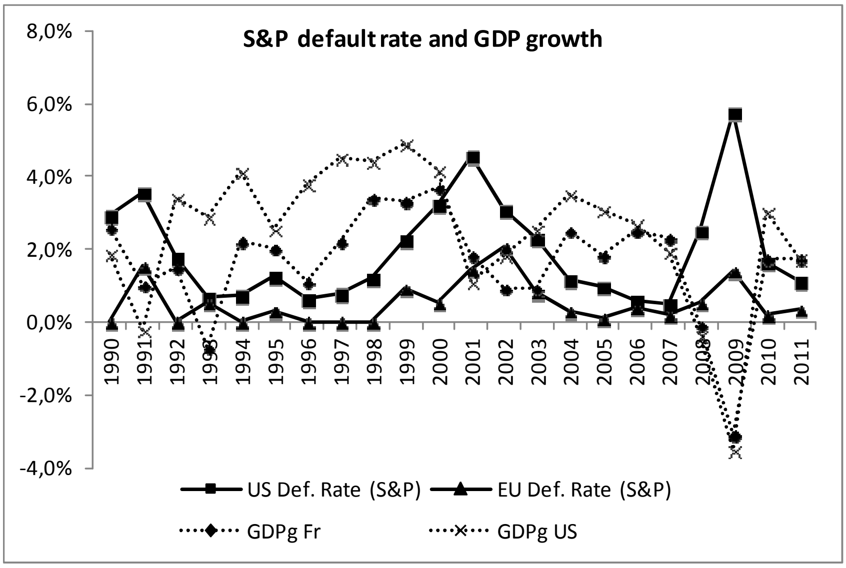

Figure 2: Default rate according to S&P and GDP growth over the 1990-2011 period

During each of these periods, default rates surged both in Europe and the US (see Figure 2). It is especially striking after the burst of the Internet bubble during which 1) the default rate of American corporates reached the level of 4.5% (2% in Europe) and 2) the total amount of debt defaulting was historically high due to failures of major companies (Enron, Worldcom, Parmalat, etc.). Default rates during the subprime crisis surged even higher in the US (5.7% in 2009).

[10] The structure of the corporate portfolio of French banks, dominated by international groups allows to use such a reference sample to calibrate our stress testing framework. It could therefore be replicated to other global banks, once we are ready to assume that all global banks tap the same markets, in terms of risk characteristics, but differ in terms of portfolio composition.

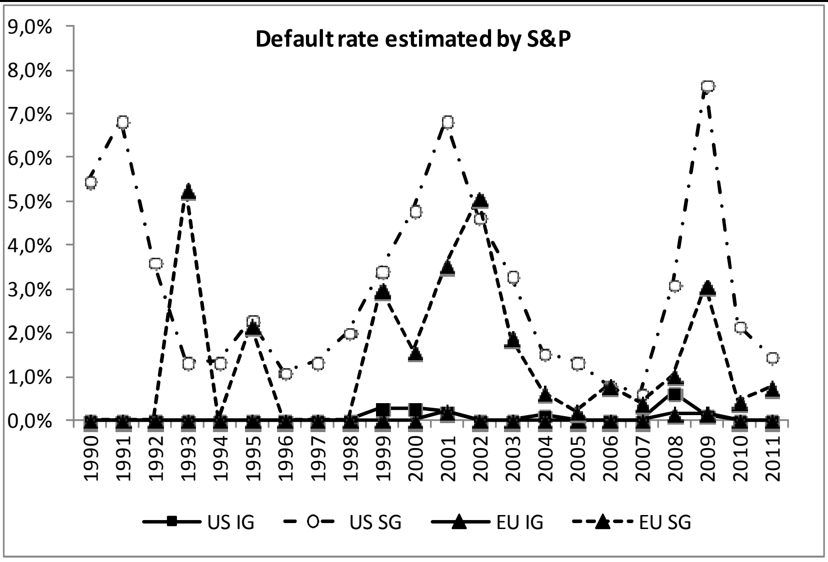

Figure 3: Default rate according to S&P over the 1990-2011 period for both speculative grades (SG) and investment grades (IG) obligors

If default rates, and more globally credit migrations, are therefore clearly linked to the economic context, it turns out that default events mainly involve speculative grade obligors (rated BBB and below). Investment grade obligors are much less sensitive to the business cycle, underlining two different dynamics for investment grade corporates on the one hand, and for speculative grade (which is actually the main driver of global default rate) on the other hand (see figure 3).

In Table 1, annual S&P transition matrices, displaying probabilities to move (or migrate) from one rating to another, are based on a (quasi) "static pool approach". Credit migrations rates are computed by comparing ratings on the first day and on the last day of the year to construct the migration rates. Rating movements within the year are accordingly not counted. This estimation approach, based on average behaviour, does not actually capture rare events such as back and forth transitions or series of consecutive downgrades within the year. Default is considered to be an absorbing risk class: if a recovery from default may be observed, it is extremely rare. Usually, firms having defaulted are excluded from the pool the following year, which prevents the recovery trajectory to ever be caught in a one year transition matrix.

	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCC,CC,C**	**D**
AAA	90,2%	8,9%	0,5%	0,2%	0,0%	0,0%	0,2%	0,0%
AA	0,3%	89,3%	9,8%	0,6%	0,0%	0,0%	0,0%	0,0%
A	0,0%	2,1%	92,3%	5,1%	0,2%	0,1%	0,0%	0,0%
BBB	0,0%	0,1%	4,2%	91,2%	3,4%	0,7%	0,1%	0,2%
BB	0,0%	0,1%	0,4%	5,1%	86,9%	6,4%	0,5%	0,6%
B	0,0%	0,0%	0,2%	0,5%	7,1%	82,7%	4,8%	4,7%
CCC,CC,C	0,0%	0,0%	0,3%	0,6%	1,1%	13,0%	58,1%	27,1%
D	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	100,0%

Table 1: S&P average credit rating transition matrix (1990-2011)

As mentioned before, migration matrices may be driven by a systematic factor (-almost- without losing any information). Migrations are then not depicted by migration rates but through a latent dynamic variable Z_t and its position with respect to the thresholds characterizing the ratings. This is the dynamics of Z_t which is driving the dynamics of migration rates.

As an example, we suppose that an issuer is currently rated A. Table 2 and Figure 4 show the migration probability from the current A rating to any of the other 8 ratings, together with the corresponding threshold from a standard normal distribution.

	AAA	AA	A	BBB	BB	B	CCC,CC,C	D
Migration rates	0,0%	2,1%	92,3%	5,1%	0,2%	0,1%	0,0%	0,0%
Score bins	]3,61 ; +∞[	]2,03 ; 3,61]	]-1,60 ; 2,03]	]-2,64 ; -1,60]	]-2,88 ; -2,64]	]-3,25 ; -2,88]	]-3,30 ; -3,25]	]-∞ ; -3,30]

Source: Standard & Poor's

Table 2: Migration rates and scores for an 'A' rated issuer (1990-2011)

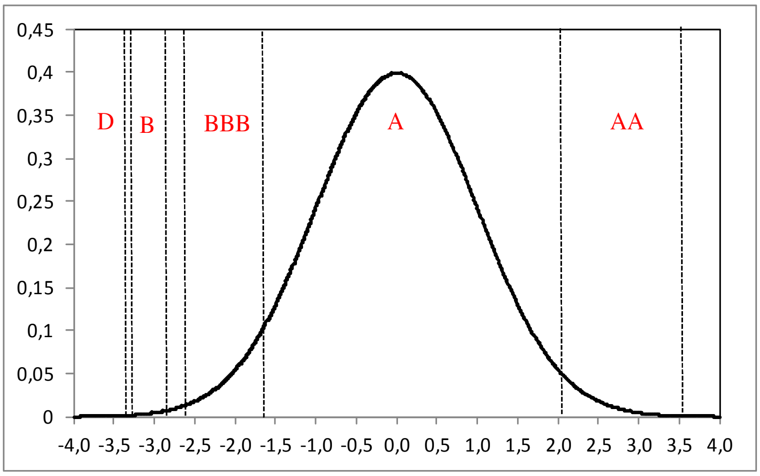

Figure 4: Rating thresholds and transition probability density for an 'A' rated issuer

Under adverse economic conditions, the normal distribution of rating migration would shift to the left, implying worse ratings levels (see Figure 5) meaning that the probability of downgrade and default increases. As the whole credit migration matrices are driven by a single parameter Z, which depicts the average financial health of corporate institutions (credit index), this shift corresponds to a simple change in the value of Z.

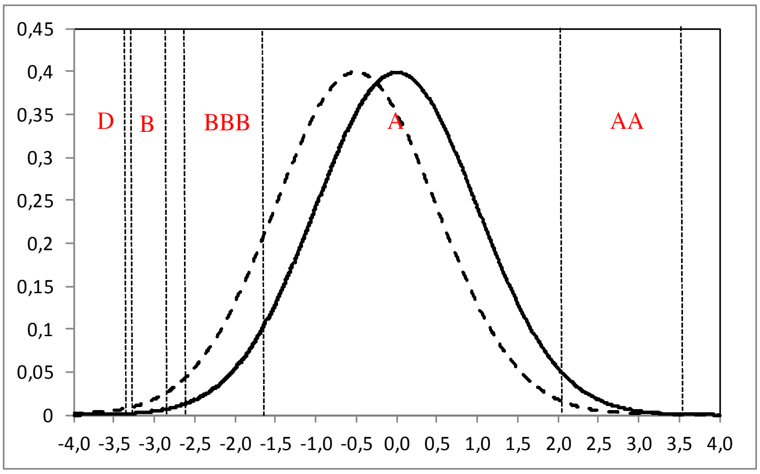

Figure 5: Rating thresholds and transition probability density for an 'A' rated issuer after a shift to the left (-0.5)

2.2. The economic situation: linking GDP to the aggregate default rate

In order to calibrate the model one needs to measure the link between the macroeconomic environment and defaults. The results presented here are specific to the French situation, hence would need to be re-estimated to implement it on other countries. However the method is quite general and can be replicated, following the same steps. This implies:

- defining an economic situation scale, as a percentage deviation from maximum defaults;

- linking defaults to (domestic) macroeconomic determinants;
- mapping the aggregate default rate into the latent credit index and we offer a numerical method for doing that.

The model we present is mainly designed to compute Risk Weighted Assets (RWAs), through PDs linked to the macroeconomic environment. The impact of stressed scenarios on P&Ls, including credit losses, is computed through another model.[11]

2.2.1. The economic situation scale

Our economic situation index is the aforementioned S&P corporate annual default rate (DR) since the latter is both highly correlated to macro-variables like the GDP and directly linked to the situation of corporate institutions.

In our stress test framework, the state of the economy (or the intensity of the crisis) is accordingly measured on the following scale:

$$\lambda_t = \frac{\widehat{DR}_t - \overline{DR}}{DR_{crisis} - \overline{DR}}$$

where $\widehat{DR}_t$ is the default rate forecast at t and $\overline{DR}$ is the average default rate over the sample period and DR_{crisis} is the DR reached during the worst crisis observed over the period under observation (in our example, this is computed as a mean of yearly DR for the years 1991, 2001, 2002, 2009). λ_t equals 0 on average over the business cycle, 1 when the reference crisis is reached. If λ_t equals 0.33 for example, the economic situation would be one third of the maximum historical deviation from the average default rate.[12] This scale is unbounded so as to suit stress scenarios which never occurred previously.

2.2.2. Forecasting the default rate in a stress testing exercise

We provide now a few alternative specifications for the link between the default rate (DR) and the economic situation, using different macroeconomic variables. Obviously a full investigation of the issue is beyond the scope of the present paper and reserved for future research. These equations can be used in order to project the default rate over the simulation horizon of the stress tests (each scenario would consist of time series of GDP, inflation, interest rates, etc) that would be fed into the equation to get a default rate scenario. The equations are estimated by Ordinary Least Squares. These equations should be re-estimated for implementing our model to other countries.[13]

<u>*Equation 1:*</u>

$$\begin{cases} DR_t = \underset{(0.0003)}{2.565}^{***} + \underset{(0.0038)}{0.579}^{***} DR_{t-1} - \underset{(0.0010)}{0.379}^{***} GDPg_t - \underset{(0.0021)}{0.887}^{***} INFL_{t-1} \\ R^2 = 0.67, DW = 2.17 \end{cases}$$

Where *DR* is the default rate, *GDPg* is GDP growth, INFL is the inflation rate.

[11] See Coffinet J., Lin S (2010)

[12] Actually, it is one third of the deviation between the average default rate and DR_{crisis} (the mean of yearly DRs for the years 1991, 2001, 2002, 2009)

[13] Notice that there is a common practice to compute a transformation of the LHS variable, in order to avoid running a regression on a variable bounded in the [0,1] interval, as it might raise econometric problems. The most frequent transformation is the logit transformation or the logarithm of the odd ratio log(p/1-p). In our case, in order to get a clearer interpretation of parameters, we chose to model directly the default probability. We checked that the PD distribution is indeed Gaussian without accumulation at 0 or 1. For illustrative purposes, we provide one alternative calibration with the logit transformation (cf. Equation 5).

Equation 2:

$$\begin{cases} DR_t = \underset{(0.0019)}{2.673}^{***} - \underset{(0.0077)}{0.372}^{***} GDPg_t - \underset{(0.0345)}{0.705}^{**} INFL_{t-1} \\ R^2 = 0.39, DW = 1.05 \end{cases}$$

Equation 3:

$$\begin{cases} DR_t = \underset{(0.0077)}{3.584}^{***} + \underset{(0.0286)}{0.478}^{**} DR_{t-1} - \underset{(0.0032)}{0.348}^{***} GDPg_t - \underset{(0.0074)}{0.798}^{***} INFL_{t-1} - \underset{(0.3278)}{0.125} UR_t \\ R^2 = 0.70, DW = 2.15 \end{cases}$$

Where UR stands for the unemployment rate at t.

Equation 4:

$$\begin{cases} DR_t = \underset{(0.0012)}{2.4512}^{***} + \underset{(0.0055)}{0.5718}^{***} DR_{t-1} - \underset{(0.0017)}{0.3744}^{***} GDPg_t - \underset{(0.0050)}{0.8508}^{***} INFL_{t-1} + \underset{(0.6097)}{0.0507} SPREAD_t \\ R^2 = 0.68, DW = 2.12 \end{cases}$$

Where *SPREAD* is the spread between the interest rates on the ten-year French Treasury note and the three-month Euribor.

Equation 5 (Logit Transformation):

$$\begin{cases} \ln\left(\dfrac{DR_t}{1-DR_t}\right) = -\underset{(0.0135)}{2.686}^{**} - \underset{(0.0135)}{0.457}^{**} GDPg_t - \underset{(0.0277)}{0.998}^{**} INFL_{t-1} \\ R^2 = 0.36, DW = 1.03 \end{cases}$$

Among the prominent points highlighted by these equations:

(i) *The inertia of the default rate*. The default rate inertia, i.e. the autoregressive coefficient in equation (1), (3) and (4), is both strong and significant: past values of the default rate provide good forecasting results.

(ii) *The indicators of the state of the economy*. According to the range of econometric tests, led by the ACP, the most relevant economic variables with respect to default rate forecasting are GDP growth, the inflation rate and the unemployment rate.

(iii) *The financial environment*. The spread between long term interest rates (10y) and short term ones (3m) used in equation (4) is both classical and relevant. However, its impact is often mild and positive.[14]

(iv) Using a Logit transformation we get consistent results with a negative impact of GDP growth on the logarithm of the odd ratio.

In addition, it might be interesting to integrate feedback effects between defaults and the business cycle (see Bruneau, de Bandt and El Amri, 2012)

2.3. The "conversion" function: mapping the aggregate default rate into the latent credit index to generate the stressed transition matrix

The final step is to map our time series of defaults (more precisely of our economic situation scale, which is a simple transformation of defaults, as indicated in 2.1) into our latent macroeconomic systemic factor on which the transition matrices are based. We provide here a numerical method for doing that, which could be easily replicated. We thus define a second

[14] There is a vast literature on the forecasting properties of the slope of the yield curve.

conversion scale in order to convert the crisis percentage into the corresponding stressed transition matrix.

For that purpose we consider the actual transition matrix observed when the economy enters into the recession periods mentioned above (namely in 1991, 2001-2002 and 2009).[15] We compare that matrix to the unconditional transition matrix over the sample period. The latter matrix can be viewed as a "through the cycle" transition matrix.

More precisely, we define a scale based on the couple $Z_{0\%\ crisis}$ and $Z_{100\%\ crisis}$ which are the two credit indexes that respectively best fit: i) the "through the cycle" transition matrix (TM_{TTC}) as the average of transition matrices observed over the period and ii) the "crisis" transition matrix (TM_{crisis}) as an average of the transition matrices observed in 1991, 2001, 2002 and 2009. Note that we chose a simple average transition matrix conditional on exceeding the probability of 82% (which would be equivalent to the 4 worst observations in a 22 years sample). However, one could have used variable weights over time to compute an average transition matrix highlighting different crisis episodes (banking, or industry related crisis). In addition, as noted above, we can accommodate the particular features of the 2008-9 crisis which originated among AAA-rated assets, by concentrating on the investment grade portfolio.

From the comparison of the observed and asymptotic single risk factor (ASRF) transition matrices we derive the value of the latent macroeconomic systemic factor corresponding respectively to "normal times" and "crisis", given that we are mainly interested by worst case scenarios:

$$\begin{cases} \hat{Z}_{0\% crisis} = \arg\min_{Z_t} \left| TM_{TTC} - TM(Z_t) \right| \\ \hat{Z}_{100\% crisis} = \arg\min_{Z_t} \left| TM_{crisis} - TM(Z_t) \right| \end{cases}$$

Several kinds of matrix norms can be used: we chose the Euclidian norm, typically used in linear optimization problems. Assuming the relationships $Z_t = Z_{0\%\ crisis} \Leftrightarrow \lambda_t = 0$ and $Z_t = Z_{100\%\ crisis} \Leftrightarrow \lambda_t = 1$ between the state of the economy and the credit index, the final step is to compute the value of the macroeconomic systemic risk which comes from the following 'scaling' function:

$$Z_t = \left[\left(Z_{100\% crisis} - Z_{0\% crisis} \right) \times \lambda_t + Z_{0\% crisis} \right].$$

Here, λ_t is defined as a crisis intensity scaling factor, depending on the long term average default rate, the average crisis default rate estimated as the mean of the 1991, 2001, 2002 and 2009 default rates, and the default rate forecast over the stress horizon. The stressed transition matrix is then used to compute the level of RWAs (under the large corporate parameters of the Basel II formula, using regulatory PDs).

Let us mention that in our stress test framework, the transition matrices do not depend on one latent variable Z but on a couple of latent variables $(Z_{investment\ grade}; Z_{speculative\ grade})$. This allows i) to stick to a simple approach – we could have as many credit indices as notches within S&P transition matrices – and ii) to reflect the two distinct regimes followed by investment grade and speculative grade obligors. Indeed, as depicted above in the paper, global default rates are, on average, largely driven by the credit quality of speculative grade

[15] Notice that we consider the recession dates in the US since the database we used is based on a sample of US and European firms, also assuming that large corporates are global companies significantly affected by the US business cycle. In the practical implementation of stress tests, however, we assume that this calibration also holds for the portfolio of corporate assets held by French banking groups. Such an assumption is imposed by the data constraints (ratings on corporate assets as provided in the Banque de France FIBEN database are only available with a lag, preventing their use in real-time stress testing).

counterparties, so that the identified crisis periods are periods of crisis for speculative grade obligors, rather than for investment grade ones. Practically, this means that two sets of parameters ($Z_{0\%\,crisis}$, $Z_{100\%\,crisis}$) have been estimated, each on the investment (resp. speculative) sub-part of the TTC and crisis matrices.

2.4. Estimation of the correlation factor

The uniform correlation factor ρ between all obligors has been estimated in order to obtain the best possible fit of historical data by the model. This is performed by minimizing the total distance, on the complete 1981-2011 sample, between empirical transition matrices and matrices stressed by our model, using, for each year, the observed default rate to compute λ_t.

Other possibilities for calibrating correlations are using stock returns, as is done by Moody's KMV commercial model, or empirical default observations on a loan database, as in Gagliardini & Gourieroux (2005).

3. Numerical application

We present now a few outputs of the model for stress testing. We provide first more detailed information on necessary inputs, namely banks' exposures. We then use the model presented in section 2 to compute ratings migration on banks' portfolio, hence to compute the level of risk weighted assets (RWAs) under stress. We present the aggregate results for our sample of 5 of the largest French banks in a baseline and a stressed scenario.

3.1. Prudential Common Reporting (COREP)[16]

Our framework basically requires information on the structure of banks' portfolios by types of rating. They are available from banks' COREP reporting. COREP reporting is a set of European harmonized reporting on solvency issues (own funds adequacy, credit risks RWAs, market risks RWAs, operational risks RWAs) handled by the EBA. It intends to enhance the level of harmonization of the supervisory reporting. A specific COREP template is dedicated to credit risk of IRB corporate portfolios, in which the regulatory PDs for each class of risk are reported to the French authorities by banks in the quarterly prudential COREP templates.

This information is actually combined with mappings (provided by on-site inspections division), which convert the internal rating system of each bank into the S&P rating scale. This step is facilitated by the fact that most banks use such a conversion scale to compute, when possible, a distance between their internal rating and agency ratings, as an indicator for assessing the performance of their internal models. This is a necessary step in order to stress banks portfolio by using S&P transition matrices.

3.2. Composition of French large banks' corporate credit portfolio and evolution over time of exposures at Default (EAD)

An initial portfolio is made up of corporate credit exposures of the 5 largest French banks. Information on exposures and risk profile[17] is available in the aforementioned COREP reports.

[16] For any further information: http://www.eba.europa.eu/Supervisory-Reporting/COREP.aspx

[17] The breakdown by rating is given by S&P equivalent of internal rating.

Banks' portfolios are relatively diversified in terms of sectors. Most exposures are investment grade and are mainly located in Europe and North America.

Based on this information, one needs to compute the evolution over time of exposures at Default.

Let us assume that the horizon of the following exercise is 2 years: so, starting from end-year 0, the stress test ends year 2.

Starting with a portfolio of assets in different rating categories, one computes how the portfolio changes over time following a shock. This implies computing a stressed transition (or migration) matrix with the probability of moving from one rating category to another. Technically, this is a Markov chain matrix, meaning that the probability distribution at time t+1 depends only of the distribution a time t.

Furthermore, banks' balance-sheets are supposed to be *static* (as opposed to *dynamic*) meaning that the total amount of non-defaulted exposures remain stable over the stress period. The assumptions made in our example could very well be modified.

3.3. Calculation of risk weighted assets and capital requirements for credit risk.

Formally, considering an initial portfolio with a given risk structure $EAD_{0,i}$ at time *0* with *i=AAA, AA,…,D,* the dynamic behaviour of the portfolio has the following form:

$$EAD_{t,i} = EAD_{t-1,i}[TM_t^{stress}(Z_{IG};Z_{SG}) + \Delta EAD_t]$$

The portfolio risk structure depends on the credit migration matrix which is a function of macroeconomic and financial factors X_t . ΔEAD_t is the amount of new loans; it is adjusted so as to comply with the static balance sheet constraint. Regulatory capital requirements are then calculated according to the Basel II formula.

We consider the two hypothetical following scenarios:

- A baseline scenario based on GDP growth projections from the IMF's World Economic Outlook (WEO); and
- An adverse scenario which is supposed to lead to a maximum cumulated deviation from baseline of two standard deviations of GDP growth for 2012-2013.

The table below shows the main key macroeconomic factors that drive our 2 scenarios.

	Baseline		*Adverse*	
	2012	*2013*	*2012*	*2013*
Inflation	1,7%	1,5%	1,3%	0,2%
GDP real growth (%)	0,5%	1,0%	-1,9%	0,0%

Table 3: Macro-economic variables' forecast used for a stress test simulation

The main outcomes under these 2 scenarios, in terms of risks parameters (regulatory PDs) and capital requirements (RWAs levels), which are the main final output of our stress testing framework, are displayed in Table 4. Changes in RWAs in the table are computed as the sum of changes in RWAs over 5 of the largest French banks.

	Baseline		*Adverse*	
	2012	*2013*	*2012*	*2013*
Stressed regulatory PDs (annual rate of change)	+2%	+12%	+15%	+11%
RWAs (annual rate of change)	+2.6%	+5.3%	+12.9%	+5.9%

Table 4: Main outcomes under the baseline and adverse scenarios

Table 4 shows the outcome of this sensitivity analysis in which regulatory PDs[18] and migrations rates were stressed over the 2012-2013 period. It is worth noticing that other regulatory parameters like LGD and correlations have not been stressed in this particular exercise. The results in Table 4 illustrate the existence of a smoothing effect of the stressed scenario on RWAs, due to the negative relationship between regulatory PDs and the correlation with the credit index (ρ), as assumed in the internal ratings-based (IRB) model.

The outcome of this simulation shows that the sensitivity of IRB minimum capital requirements to increases in regulatory PDs and credit migrations is significant. Indeed, an increase of PD by 15% in 2012 (resp. 13% in 2013) in the adverse scenario raises capital requirements by about 11% (resp. 5.9% in 2013). Moreover, the initial shock, a deep recession in 2012, raises both risk parameters and capital requirements at least up to 2013. This is consistent with our expectations since Basel II formulas rely on "through-the-cycle" PDs, which tend to smooth the impact of the shock at the very beginning of the stress but makes it last longer.

Conclusion

Credit risk remains one of the most important risks faced by commercial banks. This paper provides a stress-testing framework for banks' corporate credit portfolios, a framework which is currently used by French authorities to perform biannual top-down exercises.

This framework is therefore appropriate for data available at a supervisory authority level and intends to reach the best trade-off between simplicity and robustness. Our stress test framework takes advantage of the quarterly prudential COREP templates and of the S&P CreditPro database, which provides statistics over the past two decades regarding credit migration of more than 10 000 American and European companies.

The calibrations proposed – namely AR models for observed default rates which assume stationary explanatory variables as well as mean reversion dynamics – are consistent with the Basel II and III frameworks which rely on through-the-cycle risk parameters (PDs). This framework is therefore fully relevant for benchmarking bottom-up exercises. Furthermore, from a regulatory point of view, this framework is a realistic approach to how bank compute their RWAs: regulatory parameters, such as PDs and LGDs, are estimated as through-the-cycle parameters, possibly with an add-on coefficient for prudence (taking into account downturn economic conditions for LGD). As a consequence, they tend to become less and less sensitive to a given stress period, given that they are based on ever increasing historical datasets. It is

[18] Regulatory PDs, consistently with Basel II regulations, are estimated as a long term moving average of observed default rates; a stressed default rate, which is produced by our model, is therefore included in the new time window at the end of each year, thus yielding a stressed regulatory PD.

indeed important in our view that the stress testing framework is as close as possible to the actual regulation governing the computation of RWAs.

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AUTORITÉ
DE CONTRÔLE
PRUDENTIEL
BANQUE DE FRANCE

61, rue Taitbout
75009 Paris
Téléphone : 01 49 95 40 00
Télécopie : 01 49 95 40 48
Site internet : www.acp.banque-france.fr