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How different is the regulatory capital from the economic capital: the case of business loans portfolios held by the major banking groups in France

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Abstract:
There is a growing concern about the differences between risk weighted assets (RWAs) amounts across banks and across countries. This paper provides new evidence on this issue by using French Credit Register data and firms' ratings histories of more than 160.000 French firms, including a large proportion of Small and Medium sized firms, to compute capital requirements in business loans portfolios of French banking groups. Using Credit Register information and ratings provided by the Banque de France rating system allows computing capital requirements by using a single common credit risk metrics and actual empirical rates of default at the bank's exposure level. Using this information, capital ratios are computed for each banking group operating in the French business loans market. The paper addresses the issue of the ability of Basel 2 Internal Rating Based (IRB) formulas to hedge portfolio's credit risk. Here, the analysis relies on an extension of the asymptotic single risk factor model (ASFR), which was used for the calibration of Basel II regulatory formulas. Therefore, a multifactor portfolio's credit risk model is implemented to compute economic capital requirements taking account of potential credit risk concentration in business loans portfolios. The paper compares the required capital ratios provided with this model with the one required by the regulation. Our main findings are firstly that regulatory capital ratios do not underestimate credit risk: Basel II regulatory capital requirements are larger than the economic capital requirements. Secondly, the single risk factor regulatory model does not capture potential diversification effects in business loans portfolios. In the regulatory model, firms' heterogeneity is only captured by their ratings. The introduction in the portfolio credit risk modeling of additional systematic risk factors - which are here size and sector – show that managing large portfolios composed of borrowers of different size or sector helps to produce diversification effects. Such situations lead in most cases to a decrease of the capital level required to cover future unexpected losses.
Keywords: Credit Risk, economic capital, regulatory capital, business loans
JEL Classifications : G21, G28, G32

De combien le capital réglementaire s'écarte t-il du capital économique: le cas des prêts aux entreprises par les grands groupes français

Résumé :
Il existe aujourd'hui un débat sur la qualité des actifs des banques européennes et une éventuelle sous-évaluation du risque de crédit dans l'utilisation des formules réglementaires par les banques. Ce papier apporte des éléments de réponse à ce débat en mesurant les exigences en capital réglementaires sur les portefeuilles de crédits aux entreprises résidentes des six premiers groupes bancaires opérant en France et en les comparant aux exigences en capital économique mesurées notamment en utilisant un modèle multifacteur de risque de crédit de portefeuille. Ce modèle permet de tenir compte de l'hétérogénéité des entreprises emprunteuses en les distinguant selon la taille ou le secteur. Il permet aussi de détecter d'éventuels effets de concentration de portefeuille liés à l'existence de situations de défauts corrélés. Le papier exploite les encours de crédit et l'historique des ratings de quelques 160.000 entreprises, incluant une fraction importante de PME, disponibles via la Centrale des Risques et le système de cotation des entreprises de la Banque de France pour la période 2000-2011. Ces données permettent de calculer les exigences en capital en utilisant le même critère objectif de défaut et le même système d'évaluation du risque de crédit individuel des entreprises pour toutes les banques analysées. Le premier résultat est que les formules d'exigences réglementaires ne sous-estiment pas les le risque de crédit de portefeuille. Les exigences réglementaires calculées sont supérieures aux exigences économiques dans une très large majorité de segments de portefeuille construits à partir des critères de taille et de secteur. Le papier montre aussi que le modèle réglementaire surestime la sensibilité des emprunteurs au cycle et sous-estime le potentiel de diversification de portefeuille. Le potentiel est mis en évidence dans ce papier dès lors que les facteurs de risque additionnels associés à la différenciation des entreprises sont intégrés dans l'analyse. Au total, les exigences en capital économique sont tirées à la baisse par des effets de diversification induits par l'hétérogénéité des emprunteurs.
Mots-clés : Risque de crédit, capital économique, capital réglementaire, prêts aux entreprises
Code JEL : G21, G28, G32

Contents

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Michel DIETSCH[1] and Henri FRAISSE[2]

1. Introduction

There is a growing concern about the differences between risk weighted assets (RWAs) amounts across banks and across countries. EBA (2012) as well as the Basel Committee (2012) have recently launched new working groups to assess the extent of the differences and to deliver explanations of their origins. Some international banks have expressed doubts about the reliability of banks' RWAs and the consistency and comparability of capital requirements. Such doubt about the reliability of banks' RWAs could have major consequences. In particular, investors could disregard regulatory capital ratios and require higher capital to compensate for the low perceived reliability of the capital ratio's denominator. Then the risk is that they restrict lending to banks for which they have doubts about reported capital adequacy.

Previous papers have provided an overview of the concerns surrounding the differences of risk-weighted assets (RWAs) across banks and jurisdictions and how this might undermine the Basel III capital adequacy framework. They have proposed assessments of the key drivers behind these differences, drawing upon samples of important banks in Europe, North America, and Asia Pacific. Among the main drivers which have been proposed to explain such discrepancies are the differences in regulatory environment, accounting rules, the position of the country in the economic cycle which influence the level of the probabilities of default, and, finally, the differences in banks' business models across regions of the world (Le Leslé and Avramova, 2012, Cannata, Casellina and Guidi, 2012, and several notes coming from financial analysis departments of international banks).

However, the ability of RWAs to reflect bank portfolios' credit risk could be questioned, at least for two reasons. The first reason deals with the the modelling of dependency across obligors. As emphasized by the Basel Committee (BCBS, 2008), this issue is a main challenge of portfolio credit risk measurement. Asset correlations quantify this dependency. Asset correlations measure the common sensitivity of borrowers to systematic risk factors, which are macroeconomic, industry or geographic factors. If the

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correlation is high, this sensitivity to systematic risk factors is high, and in the case where extreme values of the systematic factors append, losses will climb to very high levels. Asset correlation reflects the uncertainty associated to events which can produce extremes losses. Thus, asset correlations are a crucial metrics in portfolio's credit risk measurement. As input parameters into a credit risk model, correlations affect the credit portfolio Value-at-Risk (VaR) which measures credit risk in a portfolio. Thus, the modeling of individual asset correlations has a strong impact on VaR for credit portfolios of heterogeneous borrower size, suggesting that the omission of individual dependencies can substantially reduce the VaR estimate[3]. That said, to reflect portfolio's credit risk, asset correlations should be computed using internal data. Credit ratings alone do not reflect the uncertainty associated with forecasting tail credit loss events. However, in the regulatory formulas defining RWAs, asset correlation R is entirely determined by the PDs.

The second reason deals with potential concentration in loans portfolios. Concentration is another main driver of credit risk in a portfolio. Concentration risk in loan portfolios could come from name concentration (the incomplete diversification of idiosyncratic borrower risk) and sector concentration (the existence of multiple systematic risk factors, generally related to industry or geographic effects). Correlated defaults can be attributed to the dependency of credit exposures to common factors that are specific to some segments of the portfolio or to particular banks' clienteles. If firm heterogeneity is defined as heterogeneity of risk factor loadings across firms, it characterizes loan portfolios due to differences in size[4], sector or localization of borrowers. Thus, taking account for potential concentration effect implies to consider borrowers' heterogeneity. BCBS (2006) underlines that "concentration of credit risk in asset portfolios has been one of the major causes of bank distress". However, the calibration of the IRB formulas was allegedly chosen to match the economic risk in a credit portfolio that should be very-well diversified across industries. Consequently, regulatory formulas do not take into account borrowers' heterogeneity and potential concentration effects coming from potentially correlated defaults across borrowers belonging to the same portfolio's segment and whose financial situation is driven by systematic risk factors which are specific to their group. Taking account explicitly for concentration phenomena implies to use multifactor framework. Departures from the underlying assumptions of the single factor model, i.e. perfect granularity and a single source of systematic risk could result in substantial deviations of economic capital requirements from regulatory capital requirements.

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[3] Recent studies in the credit risk literature (Tarashev and Zhu, 2007, Heitfield, 2008, Coval and ali. 2009) show that credit risk models main sources of errors generally come from a misspecification of default dependencies. To compute credit risk in a loans portfolio, it is necessary to characterize the entire joint distribution of payoffs for the loans pool.

[4] One should note that IRB formulas actually differ with the turnover of the firms and the size of the exposure. The RWA are computed differently whether the business is classified in the retail portfolio, the SME portfolio or the corporate portfolio. However, the differences in the formulas applied to these portfolios do not stem from the single risk factor theoretical model underlying the regulatory framework.

The aim of this study is to evaluate the ability of the regulatory capital requirements formula to hedge the portfolio credit risk. To this aim, we compare the level of capital requirements computed by using regulatory Basel 2 formula to the level of capital computed by using a model of portfolio credit risk which take into account multiple sources of risk as well as borrowers' heterogeneity. Therefore, we have extended the asymptotic single risk factor (ASRF) model to a multifactor framework which takes account for additional systematic risk factors, such as size or sector factors. The first advantage of this approach is that results obtained in a multifactor framework are consistent with results provided by the regulatory approach, allowing direct comparisons of economic and regulatory capital requirements. The second advantage is that taking additional risk factors into account allows detecting potential diversification benefits in banks' loans portfolios, or on the contrary, potential credit risk concentration due to correlated defaults. Indeed, credit risk concentration could be defined as a situation of strong correlated defaults in a portfolio's segment, what induces a larger number of defaults and higher losses. In that perspective, taking account for concentration implies to decompose the portfolio in segments according to the choice of additional risk factors. For instance, following this logic, in this paper, we segment the portfolio of each banking group in four size segment. Then, the concentration measurement relies on the computation of the marginal contribution of each segment to the portfolio's total losses. If a segment's contribution to losses is high, that means that losses are concentrated in this segment, requiring more capital to cover unexpected losses. On the other hand, if the contribution is weak, there is a great chance that this segment contributes to portfolio's diversification.

To conduct our quantitative study, we use information about business loans portfolios contained in the French Credit Register ("Fichier Central des Risques") on a quarterly basis over the 2000-2011 period. This database includes all loans of all kinds (short term, long term, leasing) with an amount over 25 000 Euros provided by French banks to their business customers. As a matter of fact, the bulk of business loans portfolios is built up by loans to SME. We consider the potential for correlated defaults inside the portfolio of large banking groups lending to businesses operating in France, taking successively size and sector as additional systematic risk factors.

We use this information to compute capital requirements in each of the six major banking groups operating in the French business loans market. We compare three capital ratios: a) the regulatory ratio using the Basel II IRB formulas, b) the economic multifactor ratio computed by using a multifactor model which takes into account firm size and firm sector as additional risk sources, and finally c) an economic single factor ratio, which uses the standard ASRF model to compute asset correlations, and replaces correlations computed by using regulatory formulas by asset correlations computed in this way.

Results show firstly, that the single risk factor regulatory model does not success to capture potential concentration or diversification effects in strongly granular business loans portfolios. In this model, firms' heterogeneity is only captured by their ratings. The introduction in the portfolio credit risk modeling of additional systematic risk factors - which are here firm size and firm sector – show that situations of strongly correlated defaults could exist in certain segments of the portfolios or, on the contrary, that some segments could produce a diversification effect. Such situations determine an increase or a decrease of the capital level required to cover future unexpected losses, as compared to the regulatory level, depending of the case. Secondly, on average, Basel II regulatory capital requirements are larger than the economic capital requirements, either in the single or in the multifactor approach. In other words, our results demonstrate that present RWAs formulas do not under-estimate portfolio credit risk, at least when considering French business loans portfolios, and that whatever the banking group.

Section 2 presents the database. Section 3 presents and justifies the use of the multifactor credit risk model as a benchmark. Section 4 shows comparisons of three measures of the capital ratio to treat the issue of RWAs' consistency and reliability. Section 5 concludes.

2. The data

In this paper, we exploit the diversity of the portfolios composition across banking groups. To compute regulatory and economic capital requirements, we use two sources of information. The first one is the French Credit Register, (Fichier Central des Risques, FCR), which includes all loans of all kinds (short term, long term, leasing) with an amount over 25 000 Euros provided by French banks to businesses. We have extracted from the FCR all loans supplied by the six large French banking groups. We have retained loans to industrial and commercial sectors, excluding financial sector and state or municipal services sector. The period of the study covers years from 2000 to 2011, including the 2008-2009 financial crisis period. The FCR provides also information about firms' characteristics, such as size, industry, geographical location.

The second source of information is the Banque de France (BDF) ratings system ("Cotation BDF"), for which the BDF was recognized as an External Credit Assessment Institution (ECAI)[5]. The BDF ratings system provides ratings for quite all firms whose turnover is over 0.75 M€. However, even if the system provides ratings for micro-businesses (very small firms with turnover lower than 0.75 M€), if their

5 The firm's credit risk assessment provided by the Banque de France Credit Register and ratings system is very safe, due to the fact that the database is very representative, the 25.000 € threshold level guaranteeing a quasi-exhaustive coverage of the French businesses population. Another benefit of the rating system is its permanent updating, what allows an instantaneous evaluation of risk. In addition, the Banque de France operates a close monitoring of firms knowing financial difficulties, what provides a solution to the issue of missing accounting information for such firms. Finally, these databases provide the exclusive possibility to estimate risk in the real portfolios of the French banks accordingly to the same rating system.

exposure's amount is greater than 0.35 M€, we do not consider the latter population in this study[6]. The BDF ratings system includes twelve ratings grades. Among them, two refer to default states: i) legal failure, which is bankruptcy, and ii) bank default, which corresponds in the BDF ratings system to severe banking problems –"incidents bancaires sérieux". Taking together these two criterions of default help to catch a set of default situations which is quite close to the set of default situations using the Basel II default criterion, especially in the small businesses population. So, using this database, it is possible to compute annual rates of quasi-bank default and to distinguish them by ratings grade. In this study, these default rates were computed distinctively for four size classes: a) very small firms, with turnover between M€ 0.75 and M€ 1.5, b) small firms, with turnover between M€ 1.5 and M€ 10, c) medium-sized firms, with turnover between M€ 10 and M€ 50, and d) intermediate size firms and large firms, with turnover over M€ 50. These empirical rates of default were used as proxies for probabilities of default (PDs) in the Basel II capital requirements formulas.

The perimeter of this study is the population of French firms which fulfill four conditions: i) they have exposures in the Credit Register, ii) the BDF rating department gives them a rating (including default grades), iii) they get loans from at least one or several of the six major banking groups operating in the French loans to businesses market, and iv) their annual turnover is over 0.75 M€. The population contains a very large number of French firms (more than 160.000 firms on average each year). Table 1 shows the number of in the sample by size classes. All types of business loans are included in the total amount, whatever the maturity or the object of loans. The loans amount of the sample's firms represents a total of more than 650 billion Euros on average over the period. The market share of the six studied groups in the French business is around 70% during the period. In addition, business loans exposures of the studied banking groups represent on average around 40% of the groups' total assets. The sample does not only reflect the reality of the business loans market but it is also very representative of the French businesses population.

[6] The main reason is that the Banque de France ratings system covers only that part of this population which is composed of firms with exposures amounts higher than 350.000 euros. Consequently, reliable information about individual credit risk is missing for most of the micro businesses population. Another reason comes from the fundamental heterogeneity in terms of credit risk of the micro businesses population, which mixes personal affairs – such as doctors, lawyers,, - with very small firms operating on competitive markets..

Table 1: Number of firms by size in the sample (6 banking groups)

	2006	2007	2008	2009	2010	2011
Very small firms	69 639	71 427	68 082	71 369	70 230	68 165
Small firms	77 392	78 465	74 965	77 712	76 726	74 928
Medium-sized firms	21 395	21 105	20 008	20 620	20 274	19 737
Intermediate and large firms	4 482	4 520	4 318	4 448	4 387	4 318
Total number of firms	172 908	175 517	167 373	174 149	171 617	167 148
Total exposures of the six groups (bn €)	639	673	665	648	671	665
Share of the six groups in total exposures (in%)	71.6	71.5	71.5	70.8	69.4	67.5

Source: ACP-BDF and Authors' computations.

Table 2 shows the distribution of size and sector composition of loans portfolios across banking groups. Significant differences across groups appear, especially in what concerns the share of the very small firms or the largest firms (intermediate size and large size firms). Significant differences in industry composition across groups also appear. In particular, some groups are characterized by a high share of those sectors which are closest to the final consumers (retailing, services to households) while others lend more to manufacturing and construction and real estate sectors.

Table 2: Distribution of size and industry composition of business loans portfolios across banking groups (year 2011)

	min	mean	max
Size classes			
Very small	16.2%	28.1%	36.1%
Small	14.5%	24.2%	28.8%
Medium-sized	8.3%	19.1%	26.1%
Intermediate & large	9.0%	28.6%	61.0%

Industries			
Agriculture	0.5%	1.6%	3.8%
Construction & real estate	14.9%	32.7%	42.4%
Manufacturing	14.3%	19.8%	22.9%
Retail	7.3%	10.9%	21.7%
Wholesale	8.0%	11.1%	15.5%
Transport	4.8%	7.1%	10.4%
Service to business	4.0%	5.2%	12.5%
Services to households	5.6%	11.7%	14.9%

Source: ACP-BDF and Authors' computations

Note: this table reproduces the minimum, the mean and the maximum fraction of exposures across banking groups and by portfolio segment. For illustration, in term of exposures, the banking group the less exposed to the Agrigulture sector hods 0.5% of its total exposure on this segment.

To compute capital ratios, we have used the Basel II formulas in the Internal Ratings Based Foundation (IRBF) approach, what means using the “other retail” capital requirements formula when the exposure’s amount is lower than 1 million Euros and the corresponding borrower’ turnover is below 50 million Euros, and using the “corporate” capital requirements formula, taking account for size adjustment when the firm’s turnover is lower than 50 million Euros, when the exposure’s amount is higher than 1 million Euros.

In this paper, we do not use the banks’ regulatory expected PDs but instead the observed empirical rates of default at the one year horizon as proxies for PDs to compute regulatory as well as economic capital requirements. These default rates are computed as the number of firms going to default state during the year relative to the total number of firms in safe condition at the beginning of the same year. Table 3 presents average annual rates of default at the one year horizon by firm size, giving a first view of the credit risk structure in the sample under study. The table shows that the level of the default rates tends to decrease with firm size. It shows also that credit quality tends to vary with the business cycle, with a significant downgrade in 2009.

Table 3: Average observed rates of default at the one year horizon by size in the sample (in %)

	2006	2007	2008	2009	2010	2011
Very small businesses	1.36	1.25	1.27	2.25	1.87	1.82
Small firms	1.11	1.01	1.06	1.95	1.63	1.51
Medium-sized firms	0.70	0.64	0.66	1.10	0.76	0.81
Intermediate and large size firms	0.31	0.45	0.41	0.49	0.31	0.38

Source: ACP-BDF and Authors’ computations

As mentioned before, in this paper, we compare capital requirements in business loans portfolios at the banking group’s level. Therefore, we compute capital requirements at the level of each large banking group’s portfolio (the French Credit Register allows to distinguish banks’ portfolios) and we express capital requirements in terms of capital ratios.

Now, to assess the ability of regulatory capital requirements to cover portfolio credit risk, we need to use other measures of capital requirements as benchmarks. As argued before, our choice is to use a structural credit risk multifactor model to compute capital requirements in an economic perspective, taking account for multiple sources of risk.

3. The methodology

To assess the existence of a potential bias in the estimation of capital charges associated with the previous regulatory capital requirements formulas, we compare regulatory capital requirements with capital requirements computed by using a more comprehensive economic approach provided by a multifactor portfolio credit risk model.

In a multifactor framework, we have to determine the risk factors. In a first step, we include firm size as additional systematic risk factor and in a second step we consider firm sector as additional factor. The choice of these factors rely on recent research that shows that concentration exists in business loans portfolios and that credit risk varies in portfolios according to their industry and size composition (Carling, Ronnegard and Roszbach, 2004, Dietsch and Petey, 2004, Duellmann and Scheule, 2003, Heitfield, Burton and Chomsisengphet, 2006, Duellmann and Masschelein, 2006). Recall that regulatory formulas do not consider such factors. However, for the comparison, we compute regulatory capital requirements and economic capital requirements at the same disaggregated level of portfolio's size or sector segments we use to implement the multifactor model.

In what follows, first, we give a short presentation of the methodology. A more detailed presentation is in the appendix of this paper. Then we explain why the capital requirements measures derived from a multifactor framework can be used as benchmarks.

3.1. A short view of the multifactor model

The multifactor model belongs to the class of structural credit risk models[7]. It is in fact an extended version of the standard asymptotic single risk factor ASRF model. The extension consists to introduce additional factors varying across groups of borrowers. We have expanded the model by adding new latent factors of systematic risk that can be linked to observable characteristics of borrowers. Such an extension to a multi-factor model improves substantially the computation of the dependency structure (asset correlations) across exposures in a typical loans portfolio. Using this approach permits in particular to compare the credit risk in groups of borrowers getting their loans from different banking groups.

The extension of the ASRF framework allows taking account for potential credit risk concentration which is linked to borrowers' heterogeneity. In small portfolios of large exposure concentration risk comes from name concentration. But in large portfolios of business loans, which are highly granular,

7 See appendix for a complete presentation of the model.

concentration risk arises from correlated defaults among groups of borrowers. Then, measurement of concentration risk needs to proceed to an appropriate portfolio's segmentation able to reflect borrowers' heterogeneity. Here, we adopt as criterion of segmentation the belonging of an exposure to the business loans portfolio of one of the five French banking groups we consider in this study.

To compute economic capital in this framework, we proceed in two steps. The first step is devoted to calculus of portfolios' main risk parameters, and in particular the dependence structure among exposures measured by asset correlations. The second step uses Monte-Carlo simulation to build the probability distribution function of losses, determine the total portfolio VaR and compute the level of capital requirements associated to each additional systematic factor which are in this study specific to banking groups and their business model and lending policy.

Let consider briefly the first step. As econometric specification of the multifactor credit risk model, following Frey and Mc Neil, 2003, and McNeil and Wendin, 2006) we use the methodology of generalized linear mixed models (GLMM). Thus, takings firms' credit ratings histories to build time series of rates of default by portfolio's segment, we get estimates of portfolio's credit risk parameters in a multi-factor context. The GLMM model implements in a coherent way the Merton latent factor default modeling approach, in which the default occurs when the value of the firm's assets become smaller than the value of its debt, that is, because firm's assets values are difficult to observe, when the value of a latent variable describing the financial situation of the firm - which depends on the realization of a set of risk factors - crosses an unobservable threshold which determines the default.

In this framework, the default rate is modeled as:

$$P\left(default \middle| \gamma_t\right) = \Phi\left[x'_{ti}\, \mu_r + z'_{ti}\, \gamma_t\right]$$

in which the default rate depends on i) a fixed effect measured by the borrower's internal rating (μ_r), and ii) random effects (γ_t), which are related to a general latent factor (the state of the economy), augmented by a set of factors corresponding to a given segmentation of the portfolio.

The GLMM model produces estimates of default thresholds considered as fixed effects and covariance matrixes of a set of latent random effects corresponding to the set of systematic factors. The estimation of such parameters allows computing economic capital as buffer of losses in portfolios exposed to different systematic risk factors.

Let consider now the second step. In the structural credit risk framework, measuring concentration risk calls for allocating economic capital between segments of borrowers, i.e. to compute marginal contributions of different segments to portfolios' total losses. A portfolio's segmentation is built by identifying groups of borrowers with the same observable characteristics which expose them to the same risk factors. In a multi-factor context, capital allocation can be implemented at the segment level such that it is possible to investigate the heterogeneity in capital allocation induced by the various risk factors. Thus, a single factor homogeneous framework could induce a misrepresentation of the concentration risk even in large portfolios of retail exposures. While they are calibrated using a single factor framework, Basel 2 IRB regulatory formulas of capital requirements could be of limited interest in allocating capital. The computation of the portfolio's value-at-risk (VaR) and marginal risk contributions are made by using a methodology proposed by Tasche (2009), which grounds on an importance sampling based simulation of expected conditional losses. This methodology has the advantage to take into account the impact of borrowers' heterogeneity on economic capital charges and capital allocation.

3.2. The multi-factor model as benchmark for capital requirements measurement

At this stage, it is important to note that there is a relationship between regulatory capital requirements and economic capital requirements derived from a multifactor model. In fact, the Basel II risk weight formulas were calibrated using a simplified version of a portfolio credit risk model, the Asymptotic Single Risk Factor (ASRF) model. In this framework (see Gordy, 2003), bank's total capital requirements is computed by using two parameters which refer to firm's individual risk, which are the probability of default (PD) and the loss given default (LGD), and a third parameter - the asset correlation R – which measures the sensitivity of borrowers to a common single systematic risk factors, which is a macroeconomic undetermined risk factor. The asset correlation reflects the fact that default rates are volatile and that this volatility depends on their sensitivity to a systematic risk factor. If the correlation is high, this sensitivity is strong and, in case of realizations of extreme unfavorable value of the systematic risk factor, losses will climb to higher levels. Thus, more generally, asset correlations reflect the potential for joint defaults in a portfolio. Following this approach, in Basel II risk weighting formulas, under the IRB approach, RWAs depend upon these three credit risk parameters.

So, regulatory RWAs are consistent measures of credit risk. However, as mentioned before, two calibration choices determine potential differences between regulatory capital requirements and economic capital requirements, what justifies to compare the two types of measures. Firstly, in the regulatory formulas of Basel II, asset correlations R are entirely determined by the PDs. In the ASRF model, asset correlations measure the sensitivity of loans to a macroeconomic risk factor and they should vary from one portfolio to another one, depending on the composition of the portfolio. But, in practice, Basel II provides banks with the formulas to compute R, instead to leave them computing this risk parameter using internal information. Consequently, RWAs depend strongly upon the value of the asset correlations and the main difference between regulatory capital requirements and economic capital requirements computed by using banks' internal data comes from the value of assets correlations. So, one issue arises to know if the "regulatory" asset correlations computed by using regulatory formulas – and the related regulatory capital requirements - are different from the "economic" asset correlations computed by using banks' internal data.

Secondly, in the ASRF model, there is a single "general" undetermined credit risk factor which represents the "state of the economy". However, borrowers are not equally sensitive to common systematic risk factors. In addition, borrowers' financial heath is linked to multiple sources of credit risk which are more or less specific to the risk segment to which they belong. Taking account for borrowers' heterogeneity obliges to expand the standard single risk factor model and to adopt a multifactor framework.

In a multifactor framework, groups of borrowers are exposed to additional systematic risk factors which are specific to their segment. It is important to emphasize that these additional risk factors could reinforce or attenuate the influence of the general systematic risk factor. Moreover, a multifactor model allows detecting potential concentration (diversification) effects coming from the strong (weak) dependence of borrowers to risk factors which are specific to their own risk segment. In case of realization of unfavorable value of one systematic risk factor, the number of defaults will increase and losses will climb to higher levels. In such a case, the contribution to the portfolio's segment which is exposed to this risk factor will raise, inducing an increase in total losses.

More generally, if the sensibility of exposures to the systematic risk factor which is specific to their segment is high, the relative contribution of this segment to the portfolio's total losses will be high, what corresponds to a situation of credit risk concentration in that segment. So, in a portfolio composed of several segments, using a multifactor model allows to compute the marginal contribution of each segment to total losses and observe either the impact of this segment on the concentration of losses or, on the contrary, the role the segment plays in the diversification of the portfolio credit risk.

In practice, this marginal contribution can be expressed under the form of a capital ratio by relating capital requirements needed to cover potential unexpected losses produced to this segment (computed at a given percentile - for instance 99.9% - of the probability distribution function of losses) to total exposures of the segment. In this way, we can assess portfolio's concentration and diversification in terms of capital ratio as a common metrics, showing how size and sector factors could contribute to increase or decrease the level of capital requirements relative to the level given by a single risk factor model.

4. The results: comparisons of regulatory and economic capital ratios

To conduct our analysis, we do not have access to complete detailed banking groups' internal information, and, in particular, to banks' internal rate of defaults. However, using BDF ratings histories of French firms as well as information about their debts provided by the French Credit Register gives us the opportunity to use an "as-if" approach and to compute very consistently internal asset correlations and economic capital requirements. A major advantage of this approach is to consider a single risk metric –the BDF ratings- across banks. Recall that data which are used to implement the econometric analysis and compute portfolios' credit risk parameters (among them the dependence structure shown by the covariance matrixes of risk factors) are: i) the time series of observed rates of default in the different segments for each banking group over the 2000-2011 period[8], and ii) the loans' amounts in the French Credit Register. The empirical rates of default were used as proxies for probabilities of default (PDs) in the Basel II capital requirements formulas. We use this information to compute capital requirements in each of the six French major banking groups. In each case, we compare three capital ratios:

- the regulatory ratio using the Basel II IRB Foundation formulas,
- the economic multifactor ratio computed by using a multifactor model which takes into account firm size and firm sector as additional risk sources,
- the economic single factor ratio computed by using the standard ASRF model in which the risk factor is a general undetermined factor e.g. not constrained by the regulatory formula.

It is interesting to compute also the single factor ratio, because the difference between capital requirements measures provided by the standard single factor model and the regulatory model relies directly on the value of asset correlation which is computed using portfolios default rates dynamic in the ASRF model while, as mentioned before, it is given by regulatory formulas in the regulatory model. On the other side, the difference between capital requirements provided by the single risk ASRF model and the multifactor model illustrates the role the additional risk factors play in the determination of portfolios' losses.

In the multifactor framework as well as in the single factor framework, total portfolio's required capital is computed by simulation of the risk factors given default thresholds and risk factor sensitivities, which are the outputs of an econometric model explaining the volatility of default rates over the 2000-2011 period. Given the credit risk parameters and a set of simulated risk factors, defaults in each sub-portfolio defined by crossing four size segments - or eight industries - with six rating grades are produced by

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8 As mentioned before, in the BDF ratings system, two ratings refer to default states: i) legal failure, which is bankruptcy, and ii) bank default, which corresponds in the BDF ratings system to severe banking problems –"incidents bancaires sérieux". We took the two forms of default to compute annual rates of default and to distinguish them by ratings grade.

drawing from a binomial probability with the number of firms in each sub-portfolio and the conditional default probability defined by econometric analysis result as parameters. Exposures are then defined as the average of firms' total loans amounts within classes crossing rating and sizes, i.e. we assume at this stage homogeneity in exposures within portfolios' segments.

In what follows, first, we will present results we obtained when decomposing the business loans portfolio of each major banking group in four size segment, taking firm size as additional systematic risk factor. Then, we present results obtained when taking firm sector as additional systematic risk factor.

4.1 Taking account for borrowers heterogeneity and potential credit risk concentration: the firm size as an additional source of systematic risk

Here, we consider firm size as a risk factor. The banking groups' portfolios were divided in the four size classes we defined previously, that is:

- a) very small firms, with turnover between M€ 0.75 and M€ 1.5,
- b) small firms, with turnover between M€ 1.5 and M€ 10,
- c) medium-sized firms, with turnover between M€ 10 and M€ 50,
- d) intermediate and large firms, with turnover over M€ 50.

Using this segmentation, we compute capital ratios associated to each segment considering three models: the single factor model, the multifactor model and the regulatory model. All information concerns the 2000-2011 period and is treated on an early basis.

It is likely that the portfolios under consideration are highly granular due to their size. Therefore, differences in capital requirements would come from credit risk concentration which corresponds to situations of strongly correlated defaults. If obligors were homogenous in terms of credit risk, capital ratios should not differ across obligors and/or portfolios' segments. On the contrary, if obligors are heterogeneous, a higher capital ratio in a given segment would indicate potential risk concentration. A straightforward source of heterogeneity is credit rating which is accounted for in the econometric analysis by the estimation of default thresholds. Size could be an additional source of credit risk heterogeneity in SME portfolios. If there is concentration risk, capital ratios should vary along this source of risk. Thus, the heterogeneity in capital charges will mainly come from the risk factors affecting the different portfolios' segments.

However, table 2 has shown that, apparently, there is not a considerable potential for credit risk concentration when considering the size of the portfolios and the distribution of exposures into size classes in the large banking groups operating in France. Indeed, there are patent differences in terms of portfolios' size and the composition of the portfolio varies from one group to another one. Notice that one exception could come from the higher share of intermediate and large firms' segment. But, all in all, these observations suggest that the evolution of potential credit losses in large portfolios might be sustainable when considering concentration risk. It's what our results tends to verify.

Table 4 shows the covariance matrixes of random effects in the size model at the aggregate level of a global portfolio composed of all exposures of the six banking groups. The covariance matrixes show average values of covariances over the time period, taking account for default rates volatility over time. Table 4 shows also the minimum and maximum values of covariances across banking groups[9]. There is a considerable systematic component driving the volatility of default rates. Indeed, the variance associated to the "general" factor, which is the random intercept in the GLMM model, has very high values. Secondly, the size class with the largest random effect is the very small businesses class, with an order of magnitude higher to the general factor. The random effects associated to the other size classes are generally small or equal to zero. Moreover, the general and the size specific risk factors are negatively and quite strongly correlated. This reflects lower risk levels, this negative correlation dampening the fluctuations of the general risk factor. Size factors and general factors tend to compensate to generate a lower level of credit risk. Thus, results suggest a very low potential for risk concentration on most of size segments. Finally, the estimated covariance matrices of random effects do not suggest a continuous and convex relationship between risk and size at the aggregate level.

[9] Results at the individual banking group level – not presented here - show that all covariance matrices share quite the same pattern when considering the size factors. However, differences across banks may exist, mainly in what concerns the medium-sized firms segment.Such differences mean either that banks are making different portfolio's choices or that they encounter different environmental conditions

Table 4: Covariance matrices results

A: portfolio composed of all exposures

	Very small	Small	Medium-sized	Intermediate & large	General
Very small	0.2125	0	0	0	-0.1564
Small	0	0.0379	0	0	-0.07336
Medium-sized	0	0	0	0	0
Intermediate & large	0	0	0	0.09849	-0.05119
General	-0.1564	-0.07336	0	-0.05119	0.2837

B: minimum values

	Very small	Small	Medium-sized	Intermediate & large	General
Very small	0.04381	0	0	0	-0.1468
Small	0	0	0	0	0
Medium-sized	0	0	0	0	0
Intermediate & large	0	0	0	0	-0.07054
General	-0.1468	0	0	-0.07054	0.1314

C: maximum values

	Very small	Small	Medium-sized	Intermediate & large	General
Very small	0.1913	0	0	0	-0.06112
Small	0	0.0379	0	0	0.05597
Medium-sized	0	0	0.0438	0	0.04892
Intermediate & large	0	0	0	0.1301	-0.02108
General	-0.06112	0.05597	0.04892	-0.02108	0.1314

Source: ACP-BDF, Directorate Research

Notes: for illustration : in the top panel A, 0.2125 corresponds to the correlation of borrowers belonging to the very small businesses portfolio to the systematic risk factor related to this sub portfolio. A high level of correlation corresponds to a high level of concentration within the segment. -0.1567 corresponds to the correlation between the general systemic factor and the size specific systemic factor. A large negative value captures a diversification effect mitigating the risk within the portfolio. Panel B and C reproduce the minimum and maximum values of the components of the correlation matrix across banking groups.

Table 5 shows the distribution of: (i) the ratio of regulatory capital requirements over capital requirements given by a multifactor model, and (ii) the ratio of regulatory capital requirements over capital requirements given by a single factor model across banking groups[10]. A ratio higher than 1 means that regulatory capital requirements are larger than economic requirements. Another view is to consider that a ratio higher than 1 in one given segment demonstrates that diversification effects coming

[10] Here, capital ratios represent average values of ratios over the period. They are computed using the average risk parameters (rate of default, covariances) values over the 2000-2011period. This period includes two downturn episodes, such that these average values could be considered as “through-the-cycle” values. These values and especially correlations, can change in case of realization of extreme events. However, two remarks could be made on this issue. Firstly, because our methodology uses importance sampling techniques, values of probability distribution of losses could be considered as stressed values. Secondly, it is possible to obtain stressed values of credit risk parameters by changing the observation window in order to measure the influence of “bad” years or “bad” realizations of the additional size or sector risk factors.

from the dependence structure in this segment are high. The result could come either from low value of covariance of random effect in this segment or from compensation between the risk factor specific to the size segment and the general factor. Notice that the regulatory capital ratios are computed using the IRB formulas of Basel II. The “other retail” formulas were used for the segments of micro-firms, very small firms and small firms, while the “corporate” formula was used for the medium-sized firms segments. Main results come as follows.

Table 5: Distribution of the ratios of regulatory to economic capital –distribution across banking groups, by size of firms

Size of firms in the portfolio	Multifactor model			Single factor model		
	min	mean	max	min	mean	Max
very small	2,0	3,4	6,5	1,5	1,8	2,0
small	1,2	1,5	2,7	1,3	1,6	1,9
medium-sized	0,9	2,1	4,6	2,2	2,7	3,5
intermediate & large	1,4	1,8	2,0	1,4	1,8	2,0

Notes 1. Left panel: ratio between, on the one hand, regulatory capital requirements, and, on the other hand, capital requirements derived from the multifactor model; right panel: ratio between, on the one hand, regulatory capital requirements, and, on the other hand, capital requirements derived from the single factor model model.

2. Regulatory capital ratios are computed accordingly to the “other retail” basel formula for the “very small” and “small” businesses and accordingly to the corporate Basel 2 formula for the other firm size classes.

Source: ACP-BDF, Directorate Research

Firstly, at the aggregate level for each group, the regulatory capital ratio does not underestimate credit risk. It remains true when looking at specific portfolio segments. Indeed, the level of capital requirements computed using the multifactor model are lower than the regulatory capital requirements computed using the Basel II formulas, except for the portfolio’s segment composed of loans to medium-sized firms in some banking groups where regulatory capital ratio is lower than economic capital ratios (see for instance the minimum value of 0.9 in that size segment). This result shows that, in practice, even if size concentration effects, which the regulatory approach of capital requirements does not take into account, could exist, the aggregate level of regulatory capital required to cover total portfolios’ unexpected losses do cover *de facto* potential concentration effects. However, in the (rare) cases where the level of the regulatory capital ratio is lower than the level provided by the size multifactor model, regulatory formulas could induce distortions in the capital allocation across size segments. The solution to this problem would be to complement the regulatory capital requirements to take into account results provided by a multifactor approach.

Secondly, the comparison of the ratios of regulatory requirements to economic capital requirements computed by using the single risk model shows that the regulatory formulas overestimate asset correlation. In other terms, regulatory approach overestimates the sensitivity of exposures to the business cycle approximated by the 'general' systematic risk factor.

Thirdly, the comparison of the ratios of regulatory capital requirements to economic capital requirements given by a multifactor factor and a single model shows that the latter is in most cases lower than the former. Thus, taking into account additional factors specific to size segments shows tends to lower in most case the economic capital requirements. Therefore, credit risk concentration due to a size effect appears to be limited. Moreover, the value of the ratios are close from one size to another one, what means that, on average, it seems better to manage portfolios composed of exposures of all sizes than to manage portfolios concentrated on one or a little number of sizes. This result which holds for quite all banking groups in all size segments varies in intensity across groups. Indeed, the dispersion of the ratios shows clearly the existence of divergences between banking groups when considering the level of economic capital. The potential for diversification given by the size risk factor varies across banking groups and banks are not equally efficient in managing the composition of their portfolios by firm size or, at least, they do not have the same opportunities to extract diversification benefits. Table 2 does show differences in the allocation of credit by firm size across banking groups. Therefore, this result is partly the consequence of different banking groups' policy in terms of supplied loans amount by firm. But, another important factor explaining the differences of capital ratio comes from the differences in the dependence structure between obligors by size.

3. 2 Taking account for borrowers heterogeneity and potential credit risk concentration: the firm sector as an additional source of systematic risk

The previous section tried to detect potential concentration linked to firm size in banking groups' business loans portfolios. Here, we present results considering potential concentration coming from sector systematic risk factors. To proceed, we segment portfolios' exposures by industry and use the same methodology we used to consider size effects. If concentration is strong within a sector, this sector will strongly contribute to the potential losses and higher level of capital will be required.

Table 6 shows the covariance matrices of random effects in the size model at the aggregate level of a global portfolio composed of all exposures of the six banking groups. Table 4 shows also the minimum and maximum values of covariances across banking groups11. Here, the variance associated to the "general" factor, which is the random intercept in the GLMM model, is not so high. The systematic component driving the volatility of default rates is not so high when one considers the sector segmentation. However, the values of the random effects associated to industry are often significant and very close from one industry to anoter one. In some case, such as in the services to business sector, the largest random effect is quite high. Results also show that industry random effects could either compensate or reinforce the general factor random effect. The general and the industry specific risk factors are negatively or positively correlated depending of the industry. For instance, in the construction and real estate sector, they reinforce each other, inducing larger capital requirements, while in the services to businesses they compensate. In total, results suggest a low potential for risk concentration on most industry segments.

[11] Results at the individual banking group level – not presented here - show that all covariance matrices share quite the same pattern when considering the size factors. However, differences across banks may exist, mainly in what concerns the medium-sized firms segment. Such differences mean either that banks are making different portfolio’s choices or that they encounter different environmental conditions

Table 6: Covariance matrices results

A: portfolio composed of all exposures

	Agriculture	Manufacturing industry	Construction & real estate	Retail	Wholesale	Transport	Services to businesses	Services to households	General
Agriculture	0.04065	0	0	0	0	0	0	0	-0.00425
Construction & real estate	0	0.05566	0	0	0	0	0	0	0.00867
Manufacturing	0	0	0.04411	0	0	0	0	0	0.03899
Retail	0	0	0	0.04467	0	0	0	0	-0.06182
Wholesale	0	0	0	0	0	0	0	0	0
Transport	0	0	0	0	0	0.005448	0	0	0.001139
Services to businesses	0	0	0	0	0	0	0.195	0	-0.1051
Services to households	0	0	0	0	0	0	0	0.007091	0.000161
General	-0.00425	0.00867	0.03899	-0.06182	0	0.001139	-0.1051	0.000161	0.1787

B: minimum values

	Agriculture	Manufacturing industry	Construction & real estate	Retail	Wholesale	Transport	Services to businesses	Services to households	General
Agriculture	0	0	0	0	0	0	0	0	-0.00425
Construction & real estate	0	0.03425	0	0	0	0	0	0	-0.01255
Manufacturing	0	0	0.04023	0	0	0	0	0	0.02974
Retail	0	0	0	0,0336	0	0	0	0	-0.05543
Wholesale	0	0	0	0	0,0049	0	0	0	-0.01418
Transport	0	0	0	0	0	0	0	0	0
Services to businesses	0	0	0	0	0	0	0.04973	0	-0,1321
Services to households	0	0	0	0	0	0	0	0	-0.00099
General	-0.00425	-0.01255	0.02974	-0.05543	-0.01418	0	-0,1321	-0.00099	0.05576

C: maximum values

	Agriculture	Manufacturing industry	Construction & real estate	Retail	Wholesale	Transport	Services to businesses	Services to households	General
Agriculture	0,0688	0	0	0	0	0	0	0	0.000269
Construction & real estate	0	0.1412	0	0	0	0	0	0	0.02186
Manufacturing	0	0	0.135	0	0	0	0	0	0.06105
Retail	0	0	0	0.04751	0	0	0	0	0
Wholesale	0	0	0	0	0.02999	0	0	0	0.000667
Transport	0	0	0	0	0	0.05264	0	0	0.02649
Services to businesses	0	0	0	0	0	0	0.2234	0	-0.01703
Services to households	0	0	0	0	0	0	0	0.03083	0.00783
General	0.000269	0.02186	0.06105	0	0.000667	0.02649	-0.01703	0.00783	0.1687

Table 7 shows ratios of regulatory capital to economic capital. Main results come as follows.

Table 7: Distribution of the ratios of regulatory capital to economic capital by industry across banking groups

	Regulatory capital requirements over capital requirements given by a multifactor model			Regulatory capital requirements over capital requirements given by a single factor model		
	min	mean	max	min	mean	max
agriculture	0,9	6,8	8,6	1,6	2,0	2,7
construction & real estate	2	11,4	14,3	1,3	1,7	2
manufacturing	0,9	4,8	6,5	1,3	1,7	1,9
retail	3,9	13,6	14,4	1,4	1,9	2
wholesale	2,3	8,7	13,9	1,3	1,7	2
transport	1,4	9,3	9,1	1,5	2,0	2,8
service to business	1,9	19,8	26	1,6	2,1	2,8
services to households	1,4	9,0	10,5	1,4	1,9	2,6

Source: ACP-BDF, Directorate Research

Note: regulatory capital ratios are computed as the weighted average of requirements computed using the "other retail" Basel 2 IRBF formula when the loan amount is lower than € 1 million and accordingly to the "corporate" formula when it is higher. Weights are the respective amounts of the two borrowers populations.

Firstly, when comparing regulatory capital ratios and multifactor or single factor capital ratios, we come again to the conclusion that regulatory capital ratio does not underestimate credit risk. The level of capital requirements computed using the multifactor framework are most of the time lower than the regulatory capital requirements computed using the Basel II formulas, except in the manufacturing sector (see minimum value in the table). This result shows that even if the regulatory approach of capital requirements does not take into account sector credit risk concentration, the aggregate level of regulatory capital cover total portfolios' unexpected losses. However, as mentioned above for size concentration, regulatory formulas could induce distortions in the capital allocation across industry.

Secondly, the comparison of the ratios of regulatory requirements to economic capital requirements computed by using the single risk model shows as in the size segmentation case that the regulatory formulas overestimate asset correlation. Thus, regulatory approach overestimates the sensitivity to the business cycle.

Thirdly, the comparison of the ratios of regulatory capital requirements to economic capital requirements given by a multifactor factor and a single model shows that the latter is in most cases significantly lower than the former. Multifactor capital ratio is higher than single factor capital ratio in a large number of sectors. Therefore, credit risk concentration due to an industry effect appears to be limited. However, the value of the ratios are different from one industry to another one, what could be the consequence of the industry composition in the real world but also means that, on average, it seems possible to manage portfolio's industry composition to modify the portfolio's credit risk level. It is more efficient to manage industry diversified portfolios than portfolios concentrated on one or a little number of sectors.

5. Conclusion

This paper provides results aiming to answer to the issue of the ability of regulatory capital requirements to hedge business loans portfolio credit risk. The paper considers the case of business loans portfolios held by the six major French banking groups. The paper uses a unique data combining information provided by the French Credit Register and the Banque de France firms' ratings system. One of the main benefits of this source of information is it allows treating real French banks' business loans portfolios. Another benefit comes from the use of objective measures of credit risk (rates of default) which are common to the portfolios of all major French banking groups.

The detection and measurement of credit risk in large portfolios needs the extension of the regulatory framework in order to introduce additional sources of systematic risk in the modeling of credit risk. In particular, to measure consistently credit risk in business loans portfolios, we have to take into account potential credit risk concentration. Therefore, this paper uses a multifactor portfolio's credit risk model, which is an extension of the standard asymptotic single risk factor model, to compute economic capital requirements taking account for such concentration phenomena. Here, additional risk factors are associated to size and industry portfolios' segmentation. Our findings demonstrate that the heterogeneity captured by credit ratings, the only source of heterogeneity in the asymptotic one factor framework, fails to describe the effective heterogeneity in default rates within large portfolios. Other factors might be at play. Indeed, size and industry risk factors appear to have significant effects on the heterogeneity of credit risk in business loans portfolios. Results show that additional industry factors tend to lower the capital requirements due to risk mitigation or diversification effects.

Our findings also demonstrate that regulatory capital ratio does not underestimate credit risk. The level of capital requirements computed using the multifactor framework are most of the time lower than the regulatory capital requirements computed by using the Basel II formulas. A possible caveat is that we use a quasi-bank default indicator, which is a mix of judiciary defaults and a restricted set of banking defaults instead of the Basel II definition of default, to assess the probability of default. However, the difference is significantly larger not to overturn our conclusion. Moreover, this risk metric is immune to the critique that the parameters of the banks ‘s internal models can be misspecified and be the source of an underestimation of the risk taken. This result shows that even if the regulatory approach of capital requirements does not take into account sector or size credit risk concentration, the aggregate level of regulatory capital cover total portfolios’ unexpected losses. However, in very rare cases where the level of the regulatory capital ratio is lower than the level provided by the size or sector multifactor models, regulatory formulas could induce distortions in the capital allocation across size or sector segments. The solution to this problem would be to complement the regulatory capital approach to take into account results provided by a multifactor approach. In that sense, the new supervisory guidelines calling for the use of economic capital models are welcome.

Adding new risk factors allows controlling for potential risk concentration which might affect the level of capital required to protect the banks’ solvency against unexpected losses. Even if business loans portfolios are highly granular, correlated defaults may generate credit concentration risk which pushes the level of economic capital above the level of regulatory capital. But, in fact, at least in the portfolios we have considered in this study, our findings show that additional risk factors do not contribute to increase economic capital requirements. Our results show that concentration effect is quite limited on average in most banking groups’ portfolios. On the contrary, strong diversification effects seem to be at play in French business loans portfolios.

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Appendix : Credit risk model specification[12]

The asymptotic multi-factor credit risk framework

Losses at the portfolio level can be defined as the sum of individual losses on defaulting loans in the portfolio, adjusted for the severity of individual losses; in other words, portfolio-level losses may be regarded as the sum of the losses given default for each individual loan in the portfolio that goes unpaid. Thus, if u_i is defined as the loss given default (LGD) of an obligor i and if $\mathbf{1}_{Di}$ is defined as the default indicator variable of obligor i, then the total portfolio losses L may be computed as follows:

$$L = \sum_{i=1}^{n} u_i \mathbf{1}_{D_i}$$

In structural credit risk models, such as the model devised by Merton (1974), default occurs if the value of an obligor's assets is smaller than the value of the obligor's debt that is due. Because asset and debt values may be difficult to observe, this framework has been extended by generalizing the modeling of default as the crossing of an unobservable threshold. Thus, the financial health of obligor i is represented by a latent (unobservable) variable U_i, and the level of U_i is determined by the realizations of risk factors that satisfy the following conditions:

$$U_i = \mathbf{w'_i}\, s + \sqrt{1 - \mathbf{w'_i}\, R\, \mathbf{w_i}}\ \varepsilon_i \quad (1)$$

where S is a vector of "systematic" risk factors with realization s, $\mathbf{w_i}$ is the vector of sensitivities (or factor loadings) of the i-th borrower to the systematic factors, and ε_i is a specific risk factor for borrower i. In the above equation, R is the correlation matrix of the risk factors. Assuming that the risk factors are multivariate Gaussian, the sensitivity to specific risk in equation (1) ensures that U_i is standard normal. Specific risk factors are assumed to be uncorrelated among obligors and also independent from the systematic factors. In this framework, default occurs if the latent variable U_i falls below a default threshold that is calibrated in accordance with the stationary (long term) default probability $\overline{p}_i$ of obligor i. In other words, if the standard normal cdf is denoted by Φ, then default occurs when the following conditions are satisfied:

$$1_{D_i} = 1 \Leftrightarrow U_i = \mathbf{w_i}' s + \sqrt{1 - \mathbf{w_i}' R\, \mathbf{w_i}}\ \varepsilon_i < \Phi^{-1}\left(\overline{p}_i\right)$$

12 This appendix relies on Dietsch, Fraisse, and Petey (2012).

Moreover, assuming specific risk can be entirely diversified away, then the realized losses can be approximated by their expected value conditional to s. Conditional portfolio losses can then be defined as follows:

$$L(s) \approx \sum_{i=1}^{n} u_i \Phi\left[\frac{\Phi^{-1}(\overline{p}_i) - \mathbf{w_i}' s}{\sqrt{1 - \mathbf{w_i}' R\, \mathbf{w_i}}}\right] \quad (2)$$

This framework is known as the asymptotic multi-factor framework of credit risk (e.g., Lucas et al., 2001). Equation (2) assumes that each obligor can be characterized by his individual default threshold and factor sensitivities. However, in retail loan portfolios, default rates are generally computed based on rating classes, and sensitivities to risk factors cannot be computed on an individual basis. Thus, assumptions are required to reduce the number of parameters of the loss variable. In particular, we assume that obligors who belong to the same rating notch r will share the same default threshold. We further assume that the vector of risk factor sensitivities will be the same for obligors who belong to the same segment of a portfolio. Hence, assuming the existence of a portfolio that is composed of K segments, losses can be rewritten as follows:

$$L(s) \approx \sum_{i=1}^{n} u_i \Phi\left[\frac{\Phi^{-1}(\overline{p}_r) - \mathbf{w_{ki}}' s}{\sqrt{1 - \mathbf{w_{ki}}' R\, \mathbf{w_{ki}}}}\right]$$

Thus, the adoption of a multi-factor structural approach of credit risk requires not only the specification of the dependence structure of risk factors but also the appropriate estimation of default thresholds and factor sensitivities.

The econometric estimation of the portfolio’s credit risk parameters

In this study, we extend the single factor model by considering new latent factors that can be linked to the observable characteristics of borrowers. To estimate default thresholds and factor sensitivities, we use an econometric model that belongs to the class of generalized linear mixed models (GLMMs) that combines fixed and random effects for observable and (latent) unobservable factors. Detailed presentations of GLMM models in credit risk modeling can be found in Frey and McNeil (2003) and McNeil and Wendin (2007).

If Y is defined as the $(N \times 1)$ vector of observed default data and if γ is defined as the $(K \times 1)$ vector of random effects, then the conditional expected default probability of obligor i may be expressed as follows:

$$E[Y_i = 1 | \gamma] = g(X\beta + Z\gamma)$$

where $g(\cdot)$ is a differentiable monotonic link function, Y_i is the default indicator variable for obligor i (Y_i takes a value of 1 if there is a default and equals 0 otherwise), X is a ($N \times P$) matrix that contains the (observed) fixed effects, and Z is the ($N \times K$) design matrix for the random effects. In the following applications, we will focus on the probit link function because the normal distribution is the underlying link function that is assumed by the Basel 2 framework of credit risk; thus, $g(x) = \Phi(x)$. The random effects are assumed to follow a multivariate standard normal distribution with covariance matrix G. In the equation above, β is the vector of parameters that is associated with fixed effects. Considering a portfolio of N obligors who are categorized into $r = 1, \ldots, R$ (non-default) rating classes and given a vector γ_t of random effects, the default probability of borrower i at time t may be expressed as follows:

$$P\left(Y_{ti} = 1 \middle| \gamma_t\right) = \Phi\left(x'_{ti}\,\mu_r + z'\gamma_t\right)$$

where μ_r denotes the vector of parameters from the fixed effect of the borrower's rating class. If the rating scale is properly built, we expect these thresholds to be ordered and increasing as credit quality decreases. In the above equation, $x'_{ti} = [0,\ldots,1,\ldots0]$ is a ($1 \times R$) vector of dummies that defines the rating of borrower i during time period t. Because we assume that borrowers within segments are interchangeable, the estimation of this vector does not involve individual borrowers but instead uses the quarterly default rates within segments. This approach leads to an assumption of borrower homogeneity for each credit rating that is examined.

Extending the one factor model also calls for a specification of the risk factors' dependence structure. By assuming that the general risk factor (the risk factor of the one factor model) represents the impact on default rates of variations in general economic conditions, it seems straightforward to consider that additional risk factors can reinforce or weaken the sensitivity of a given subset of firms in the portfolio to general economic conditions. This corresponds to the idea that a given sector or region can be either procyclical, cycle neutral or countercyclical. In order to capture these effects, we estimate the correlation between the general risk factor and a set of additional factors associated to a given segmentation of the portfolio. In order to keep the model tractable, we further assume that the additional factors, i.e. shocks that affect subgroups of the portfolio, are independent. This specification implies in particular that the inter-segment correlation is not directly attributable to the segments' risk factors but rather to the dependence between these latter factors and the general economic factor. The covariance structure we will focus on is of the form:

$$G = \begin{bmatrix} \sigma_1^2 & 0 & 0 & \sigma_{1,q+1} \\ 0 & \ddots & 0 & \sigma_{2,q+1} \\ 0 & 0 & \sigma_q^2 & \vdots \\ \sigma_{1,q+1} & \sigma_{2,q+1} & \cdots & \sigma_{q+1}^2 \end{bmatrix}$$

considering q latent segment factors and one systematic factor (denoted q+1, thus q+1= K). This last random effect defines a factor common to all obligors and reflects the heterogeneity in default rates related to time. This random effect corresponds to the heterogeneity in default rates attributable to time-heterogeneity, which is assumed to be related to general economic conditions. In this specification, the linear predictor in the logistic regression contains an intercept term that randomly varies at the year level, the highest level in the modelling, where all other effects are nested in. In other words, a random intercept is drawn separately and independently for each year. This structure implies that a given obligor is affected by two factors: the factor representative of general economic conditions and its industry risk factor or size risk factor.

Measuring potential concentration

To assess the credit risk of a given type of borrower within the portfolio, we compute the economic capital contribution of each borrower type. This calculation requires the portfolio-wide economic capital to be allocated to sub-portfolios or individual assets. From the findings of Tasche (1999) and Gouriéroux et al. (2000), the marginal contributions to a portfolio VaR can be expressed as the expected loss on a given exposure, conditional on losses reaching this VaR:

$$RCVAR_i = E\left[L_i \middle| L = VaR_\alpha(L)\right] = \frac{E\left[L_i \mathbf{1}_{VaR_\alpha(L)=L}\right]}{P\left[L = VaR_\alpha(L)\right]} \quad (3)$$

Equation (3) indicates that if there is a positive probability for losses to reach a porfolio's VaR, then the computation of marginal contributions will rely heavily on the ability to estimate individual losses as aggregate losses approach this VaR. Thus, in the context of a Monte Carlo simulation, the conditional mean may be based only on a limited number of simulations, producing unreliable estimates. To improve the estimation procedures, certain authors (Tasche, 2009, Glasserman and Li, 2005, Egloff and Leippold, 2010) have used importance sampling. Importance sampling consists of shifting the parameters of a distribution in ways that increase the likelihood of observing certain desired realizations of the risk factors. The main difficulty with respect to this approach relates to the choice of the alternative distribution F*. In this study, we follow the methodology of Tasche (2009) and shift only the risk factor (S) means in the following manner:

$$S_i^* = S_i - E_F\left[S_i\right] + \mu_i,\ \mu_i = E\left[S_i \middle| L = VaR_\alpha(L)\right]$$

The next step is the computation of conditional expectations (equation 3). Because the computation of VaR is accomplished through Monte Carlo simulations, one has both the realizations of the risk factors and the resulting credit losses. This information permits the utilization of the non-parametric Naradaya-Watson estimator for conditional expectations. If the standard normal density is used as the kernel and if h is used to denote the bandwidth of the kernel, then the estimator of the conditional expectation for risk factor k may be defined as follows:

$$\hat{E}\left[S_k \middle| L = VaR_\alpha(L)\right] = \frac{\sum_{t=1}^{T} S_k \phi\left(\frac{VaR - L_t}{h}\right)}{\sum_{t=1}^{T} \phi\left(\frac{VaR - L_t}{h}\right)}$$

$$h = 1.06\sigma_L T^{-1/5}$$

To limit the computational burden involved in the simulations of marginal contributions, we rely on the size of the portfolio under consideration and assume the complete diversification of idiosyncratic risk. This assumption allows for losses to be simulated using conditional probabilities instead of requiring the simulation of defaults (and their associated losses). Thus, given the assumed homogeneity of exposures within sub-portfolios, it is possible to compute a single marginal contribution based on the rating/modality of the segmentation variable rather than by proceeding at the asset level. Losses are then approximated by the following expression:

$$L \approx \sum_{j=1}^{N} u_{rk} \Phi\left(\frac{\Phi^{-1}(\bar{p}_r) - \mathbf{w}' s_k}{\sqrt{1 - \mathbf{w}'\Sigma\mathbf{w}}}\right)$$

Once the shifts in the means are computed for all of the risk factors, the next step in the analysis is to obtain realizations of the risk factors under the new distribution to once again compute the aggregate losses for the portfolio and the individual losses within each sub-segment and each rating grade. Tasche (2009, proposition 4.2), who refers to the work of Klebaner (2007), establishes that conditional on VaR, the expected losses under the natural distribution can be defined as follows:

$$E_F\left[L_i \middle| L = VaR_\alpha(L)\right] = \frac{E_{F*}\left[L_i R \middle| L = VaR_\alpha(L)\right]}{E_{F*}\left[R \middle| L = VaR_\alpha(L)\right]}$$

As discussed above, these conditional expectations can be computed with the Naradaya-Watson estimator, and simulations of risk factors and losses can be obtained under the shifted distribution. Finally, these expected losses can be aggregated across ratings for each modality of the segmentation variable to compute segment-wide economic capital requirements.

Débats Economiques et Financiers

1. M. Dietsch et H. Fraisse « De combien le capital réglementaire diffère t-il du capital économique: le cas des prêts aux enterprises par les grands groups français », Février 2013

Economic and Financial Discussion Notes

1. M. Dietsch and H. Fraisse, “How different is the regulatory capital from the economic capital: the case of business loans portfolios held by major banking groups in France”, February 2013

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