# Stress Testing and Bank Lending*

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**Abstract**

Stress tests can affect banks' lending behavior. Since regulators care about lending, banks' reactions affect the test's design and create a feedback loop. We demonstrate that there may be multiple equilibria due to strategic complementarity, leading to fragility in the form of excess default or insufficient lending to the real economy. The stress tests may be too soft or too tough. Regulators may strategically delay stress tests. We also analyze bottom-up stress tests and banking supervision exams.

Keywords: Bank regulation, stress tests, bank lending, feedback

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# DISCLOSURE STATEMENT

I have nothing to disclose.

– Joel Shapiro

I have nothing to disclose.

– Jing Zeng

# 1 Introduction

Stress tests, a new policy tool for bank regulators, were first used in the recent financial crisis and have become regular exercises since the crisis. They assess a bank's ability to withstand adverse shocks and are generally accompanied by requirements intended to boost the capital of banks that are found to be at risk.

Naturally, bank behavior changes in response to stress testing exercises. Acharya, Berger, and Roman (2018) find that all banks that underwent the U.S. SCAP and CCAR tests reduced their risk by raising loan spreads and decreasing their commercial real estate credit and credit card loan activity.[1]

Regulators must take banks' reactions into account when conducting the tests. One might posit that if regulators want to boost lending, they might make stress tests softer. Indeed, in the case of bank ratings, Agarwal et al. (2014) show that state-level banking regulators give banks higher ratings than federal regulators (due to concerns over the local economy), which leads to more bank failures.

In this paper, we study the *feedback* effect between stress testing and bank lending. Banks may take too much risk or not lend enough. Regulators anticipate this by designing a stress test that is either tough or soft. Nevertheless, the regulator may fail to maximize surplus (its objective) because the interaction between the bank and the regulator may be self-fulfilling and result in coordination failures, leading to either excess default or inefficiently low levels of lending to the real economy.

In the model, there are two sequential stress testing exercises. For simplicity, there is one bank that is tested in both exercises. Each period, the bank decides whether to make a risky loan or to invest in a risk-free asset. The regulator can observe the quality of the risky loan and may require the bank to raise capital (which we refer to as "failing" the stress test) or may not. Therefore, stress tests in the model are about gathering information and taking actions based on that information (while considering the reaction of banks), rather than optimally choosing how to reveal information to the market.[2] This is in line with the annual exercises during non-crisis times, when runs are an unlikely response to stress test results.

The regulator may be one of two types: lenient or strategic. The regulator knows its type, and all other agents are uncertain about it. A lenient regulator is behavioral and conducts uninformative stress test exercises.[3] A strategic regulator maximizes surplus. Its decision

[1]Cortés et al. (forthcoming), Connolly (2017) and Calem, Correa, and Lee (2017) have similar findings.

[2]The theoretical literature mostly focuses on this latter point; we discuss the literature in the next section.

[3]In the text, we demonstrate that the results can be qualitatively similar if the possible types of the

to fail a bank depends on the trade-off between the cost of forgone credit and the benefit of reducing costly default. After the first stress test result, the bank updates its beliefs about the regulator's preferences, decides whether to make a risky loan, and undergoes a second stress test. Thus, the regulator's first stress test serve two purposes: to possibly boost capital for the bank in the first period *and* to signal the regulator's willingness to force the bank to raise capital in the second period.

Banks may take too much or too little risk from the regulator's point of view. On the one hand, the bank may take too much risk due to its limited downside. On the other hand, the bank's owners may take too little risk to avoid being diluted by a capital raising requirement.[4] The bank's anticipated choice affects the toughness of the stress test, and the toughness of the stress test affects the bank's choice.

The regulator faces a natural trade-off in conducting the first stress test:

First, the strategic regulator may want to build a reputation for being lenient, to try to increase the bank's lending in the second period. Since the lenient regulator does not require banks to raise capital, there is a "soft" equilibrium in which the strategic regulator builds the perception that it is lenient by passing a bank that should fail. This is reminiscent of the EU's 2016 stress test, which eliminated the pass/fail grading scheme and found only one bank to be undercapitalized.[5]

Second, the strategic regulator may want to build a reputation for not being lenient, which can prevent future excess risk-taking. This leads to a "tough" equilibrium in which the regulator builds the reputation that it is not lenient by failing a bank that should pass. The U.S. has routinely been criticized for being too tough: imposing very adverse scenarios, not providing the model to banks, accompanying the test with asset quality reviews, and conducting qualitative reviews all combine to create a stringent test.[6]

Finally, there is one more type of equilibrium - one in which the regulator doesn't engage in reputation building and rates the bank in accordance with the bank's quality.

There may be multiple equilibria that co-exist, leading to a natural coordination failure. This occurs due to a subtle strategic complementarity between the strategic regulator's choice of toughness in the first stress test and the bank's second-period risk choice. The less likely

regulator are "strict" (i.e., it always fails banks) and strategic (defined as above). We also discuss microfoundations for the lenient regulator's preferences.

[4]Thakor (1996) provides evidence that the adoption of risk-based capital requirements under Basel I and the passage of FDICIA in 1991 led to banks substituting risky lending with Treasury investments, potentially prolonging the economic downturn.

[5]That bank, Monte dei Paschi di Siena, had already failed the 2014 stress test and was well known by the market to be in distress.

[6]A discussion of this and the very recent tilt towards leniency is in "Banks rest hopes for lighter regulatory burden on Fed's Quarles," by Pete Schroeder and Michelle Price, *Reuters*, October 25, 2017.

the strategic regulator is to pass the bank in the first period, the more risk the bank takes in the second period when it observes a pass (as it believes the regulator is more likely to be lenient). This prompts the strategic regulator to be even tougher, and leads to a self-fulfilling equilibrium. This implies that the presence of stress tests may introduce fragility in the form of too many defaults or suboptimal levels of lending to the real economy.

We also demonstrate that a regulator may conduct an uninformative stress test or strategically delay the test. This is an extreme version of the "soft" equilibrium described above. The irregular timing of stress testing in Europe is in line with this result.

We show that when recapitalization becomes more difficult, stress tests are less informative. Recapitalization may become more difficult because of either the scarcity of capital or lucrative alternative uses for capital. In this situation, passing a bad bank or failing a good bank is less costly since the possibility of recapitalization is low in any case.

When the bank is more systemic, stress tests are more informative, thus indicating that regulators tailor stress tests to bank size and linkages. Regulators frequently debate and revise criteria for deciding which banks should be included in stress tests.

In stress testing exercises, by examining the banking system, a regulator may uncover information about liquidity and systemic linkages that individual banks may be unaware of. In our model, this is the motivation for why the regulator has private information. However, we also analyze the case in which the regulator uncovers only the information about asset quality that the bank already knows. This is similar to banking supervision exams or "bottom-up" stress tests in which the regulator allows the bank to perform the test (as in Europe). We find that results may be more or less informative depending on the weight that the regulator places on lending vs. costly defaults.

The multiplicity of equilibria naturally raises the issue of how a particular equilibrium may be chosen. One way might be if regulators could commit, ex-ante, to a way to use the information that they collect (as in games of Bayesian Persuasion). In practice, this might mean announcing stress test scenarios in advance or allowing banks to develop their own scenarios. The regulator might also take costly actions to commit by auditing bank data (e.g., asset quality reviews).

In the model, uncertainty about the regulator's type plays a key role. Given that (i) increased lending may come with risk to the economy, and (ii) bank distress may have systemic consequences, there is ample motivation to keep this information/intention private. This uncertainty may also arise from the political process. Decision making may be opaque, bureaucratic, or tied up in legislative bargaining.[7] Meanwhile, governments with a mandate

[7] Shapiro and Skeie (2015) provide examples of related uncertainty around bailouts during the financial crisis.

to stimulate the economy may respond to lobbying by various interest groups or upcoming elections.[8]

There is little direct evidence, but much indirect evidence, of regulators behaving strategically during disclosure exercises. The variance in stress test results to date seem to support the idea of regulatory discretion.[9] Beyond Agarwal et al. (2014), cited above, Bird et al. (2015) show that U.S. stress tests were soft towards large banks and tough with poorly capitalized banks, affecting bank equity issuance and payout policy. The recent Libor scandal revealed that Paul Tucker, deputy governor of the Bank of England, made a statement to Barclays' CEO that was interpreted as a suggestion that the bank lower its Libor submissions.[10] Hoshi and Kashyap (2010) and Skinner (2008) discuss accounting rule changes that the government of Japan used to improve the appearance of its financial institutions during the country's crisis.[11]

**Theoretical Literature**

Our paper identifies the regulator's reputation concern as a source of feedback effects (and hence fragility) in the banking sector. In a different context, Ordoñez (2013, 2017) show that banks' reputation concerns, which provide discipline to keep banks from taking excessive risk, can lead to fragility and a crisis of confidence in the market. Other theories have predicted self-fulfilling banking lending freezes due to interdependence of banks' lending opportunities (Bebchuk and Goldstein, 2011) and fear of future fire sales (Diamond and Rajan, 2011).

There are a few papers on reputation management by a regulator. Morrison and White (2013) argue that a regulator may choose to forbear when it knows that a bank is in distress because liquidating the bank may give the regulator the reputation of being unable to screen and trigger contagion in the banking system. Boot and Thakor (1993) also find that bank closure policy may be inefficient due to reputation management by the regulator, but this is due to the regulator being self-interested rather than being worried about social welfare consequences, as in Morrison and White (2013). Shapiro and Skeie (2015) show that a regulator may use bailouts to stave off depositor runs and forbearance to stave off excess risk-taking by banks. Our paper uses the reputation management modelling framework to

[8] Thakor (2014) discusses the political economy of banking.

[9] The 2009 U.S. SCAP was widely perceived as a success (Goldstein and Sapra, 2014), with subsequent U.S. tests retaining credibility. European stress tests have varied in perceived quality (Schuermann, 2014), with the early versions so unsuccessful that Ireland and Spain hired independent private firms to conduct stress tests on their banks.

[10] The CEO of Barclays wrote notes at the time on his conversation with Tucker, who reportedly said, "It did not always need to be the case that [Barclays] appeared as high as [Barclays has] recently." This quote and a report on what happened appeared in the *Financial Times* (B. Masters, G. Parker, and K. Burgess, Diamond Lets Loose Over Libor, *Financial Times*, July 3, 2012).

[11] Nevertheless, stress tests do contain significant information that is valued by markets (Flannery, Hirtle, and Kovner (2017) demonstrate this and survey recent evidence).

illustrate a feedback effect not present in these papers; the strategic complementarity between the regulator's stress test and the bank's lending decision leads to multiple equilibria and fragility.

There are several recent theoretical papers on stress tests.[12] Quigley and Walther (2018) show that more disclosure by a bank regulator decreases the amount of information that banks provide to the public, and that the regulator may take advantage of this to stop runs. Bouvard, Chaigneau, and de Motta (2015) show that transparency is better in bad times and opacity is better in good times. Goldstein and Leitner (2018) find a similar result in a very different model, in which the regulator is concerned about risk sharing (the Hirshleifer effect) between banks. Williams (2017) looks at bank portfolio choice and liquidity in this context. Orlov, Zryumov, and Skrzypacz (2018) show that the optimal stress test will test banks sequentially. Faria-e-Castro, Martinez, and Philippon (2016) demonstrate that stress tests will be more informative when the regulator has a strong fiscal position (to stop runs). In contrast to these papers, in our model, *reputational incentives* drive the regulator's choices, not commitment to a disclosure rule.[13] In addition, we focus on capital requirements and banks' endogenous choice of risk as key elements of stress testing; the papers listed above focus on information revelation to prevent bank runs.[14]

# 2 The model

We consider a model with three risk-neutral agents: the regulator, the bank and a capital provider. The model has two periods $t \in \{1, 2\}$ and the regulator conducts a stress test for the bank in each period. We assume that the regulator has a discount factor $\delta \geq 0$ for the payoffs from the second period, where $\delta$ may be larger than 1 (as, e.g., in Laffont and Tirole, 1993). The discount factor captures the relative importance of the future of the banking sector for the regulator. For simplicity, we do not allow for discounting within a period.

We now provide a very basic timeline of each period. In each period $t$, where $t = \{1, 2\}$, there are three stages:

1. Bank investment choice;

[12] There are a few papers on regulatory disclosure. Goldstein and Sapra (2014) survey the disclosure literature to describe the costs and benefits of information provision for stress testing. Prescott (2008) argues that more information disclosure by a bank regulator decreases the amount of information that banks provide to the regulator. Bond, Goldstein, and Prescott(2010) and Bond and Goldstein (2015) analyze government interventions that rely on and endogenously determine market information.

[13] Quigley and Walther (2018) and Bouvard, Chaigneau, and de Motta (2015) do not have commitment or reputation.

[14] In Dogra and Rhee (2018), the regulator commits to a disclosure rule but banks may choose their risk profile to satisfy the stress testing regime, leading to 'model monoculture'.

2. Stress test and (possible) recapitalization;

3. Payoffs realize.

We proceed in the following subsections to discuss each aspect in detail: the bank, the stress test, recapitalization, the preferences of the regulator, and reputation.

## 2.1 The Bank

At stage 1, the bank raises one unit of fully insured deposits, which mature at stage 3.[15] The bank can choose between two possible investments. The first is a safe asset that returns $R_0 > 1$ at stage 3. The second is a risky loan, whose quality $q_t$ can be good ($g$) or bad ($b$). The prior probability that the loan is good is denoted by $\alpha$. A good loan ($q_t = g$) repays $R$ with probability 1 at stage 3, whereas a bad loan ($q_t = b$) repays $R$ with probability $1 - d$ and 0 otherwise at stage 3. We assume that the expected return of the risky loan is higher than that of the safe investment, representing the risk-return trade-off:

**Assumption 1.** $[\alpha + (1 - \alpha)(1 - d)]\, R > R_0$.

At stage 3, the bank uses the payoff of its investment to repay the deposits, and pays out the residual profit (if there is any) to its owners as dividends.

In order to focus on the regulator's reputation building incentives when conducting the stress test in the first period, we make the simplifying assumption that in the first period, the bank has extended the risky loan.[16]

## 2.2 Stress testing

We assume that only the regulator learns the credit quality of the risky loan (through the stress test). In Section 6, we demonstrate that the main results do not change if the bank also knows this information. The regulator could have generated private information from having done a stress test on many banks. In this case, it may have gathered more information on asset values and liquidity. Given this, the regulator may understand more about systemic risk and tail risk (not modeled here). This is an element of the macroprudential role of stress tests.

[15] In an earlier version of this paper, we remove the assumption that deposits are fully insured and allow the bank's liabilities to be priced by the market. All of our qualitative results remain. These results are available upon request.

[16] Allowing endogenous loan origination effort in the first period does not alter the reputation-building incentives we demonstrate in Section 4.

At stage 2, the regulator conducts the stress test. It first observes the quality $q_t$ of the bank's risky loan and then decides whether to require the bank to raise capital. We will henceforth refer to the regulatory action of requiring the bank to raise capital as "failing", and not requiring the bank to raise capital as "passing."[17]

The stress test in the model, therefore, is not about conveying information to the market about the health of the bank. The test provides the regulator with information on the bank's health, which the regulator uses by requiring a recapitalization. Nevertheless, the stress test accompanied by the recapitalization does convey information.[18] This information is about the type of the regulator, which is private information (this is defined below). In the second period, the bank reacts to this information inferred from the first-period stress test, forming the basis of the reputation mechanism.

## 2.3 Recapitalization

If a bank fails the stress test, we assume that the bank is required to raise one unit of capital, kept in costless storage with zero net return, so that the bank with the risky loan will not default at stage 3 even if its borrower does not repay.[19] There is a capital provider who can fund the bank. We assume that the capital provider's outside option for its funding is an alternative investment that produces a total return of $\rho > 1$, which we call the opportunity cost of capital. The opportunity cost of capital is high ($\rho = \rho_H$) with probability $\gamma$, and low ($\rho = \rho_L$) with probability $1 - \gamma$. We assume that the $\rho$ is realized after the stress test, when the bank approaches the capital provider for funds, and is publicly observable.

We make the following assumption about the expected return on the risky loan:

**Assumption 2.** *$R < \rho_H$ and $(1 - d)R \geq \rho_L$.*

This assumption implies that recapitalization is feasible only with probability $1 - \gamma$, when the opportunity cost of capital is low, regardless of the risky loan's quality. First, if the opportunity cost of capital is high, then the expected value of a good loan is lower than the capital provider's outside option. This also implies that recapitalization is infeasible for the bad loan. Second, if the opportunity cost of capital is low, then the expected value of a bad loan is higher than the capital provider's outside option. This also implies that recapitalization is feasible for the good loan.

[17]To be precise, a "fail" is an announced requirement for the bank to recapitalize. We will allow for recapitalizations to be attempted but not to work out, which we still consider a fail.

[18]The stress test results themselves are cheap talk in the model, but the recapitalizations incur costs (and benefits) for the regulator, making signaling possible.

[19]For simplicity, we assume that capital earns zero net return. The results do not change if capital is reinvested in the safe investment with a return $R_0 > 1$.

We assume that the capital provider has some bargaining power $\beta$ due to the scarcity of capital, enabling it to capture a fraction of the expected surplus of the bank. Thus, raising capital results in a (private) dilution cost for the bank's owners. The banking literature generally views raising equity capital as costly for banks (for a discussion, see Diamond (2017)). We model this cost as dilution due to the bargaining power of a capital provider, which fits our scenario of a public requirement by a regulator, though other mechanisms that impose a cost on the bank when trying to shore up capital would also work.[20] We make the following assumption on the effect of recapitalization:

**Assumption 3.** $[\alpha + \gamma(1-\alpha)(1-d)]\,(R-1) < R_0 - 1.$

This assumption looks at the decision of the bank at stage 1 of whether to choose the risky loan or the safe asset. The bank considers its expected payoff given its priors and expectations about the regulator's actions. The assumption implies that if the expected dilution from recapitalization was sufficiently large, the bank's owners will not find it worthwhile to originate the risky loan. More specifically, the left-hand side of the above expression takes into account that, if the bank originates a risky loan, and the loan is good (with probability $\alpha$), the bank's owners receive, at most, the residual payoff of $R-1$ after repaying the debtholders; if the loan is bad, however, the bank may be required to raise capital, in which case it receives the value of the loan when recapitalization is infeasible (with probability $\gamma$); when recapitalization is feasible, the worst thing that could happen to the bank's owners is that they surrender all of their equity and have a payoff of zero.

## 2.4 Regulatory preferences

The regulator's objective function is to maximize social welfare. This includes the net payoff of the asset chosen by the bank and externalities from the bank's risky lending. We now detail these externalities.

There are two social costs of risky lending. The first is the cost to society of a bank default at stage 3. Specifically, if a bank operates without being recapitalized and the borrower repays 0 at stage 3, the bank defaults and a social cost to society $D$ is incurred. The cost of bank default may represent the cost of financing the deposit insurance payout,[21] the loss of value from future intermediation that the bank may perform, the cost to resolve the bank, or the cost of contagion.

[20] For example, the bank may be forced to sell assets at fire-sale prices. This is a loss in value for the bank. And those who are purchasing the assets are distorting their investment decisions, as in our model. Hanson, Kashyap, and Stein (2011) discuss this effect and review the literature on fire sales.

[21] The deposit insurance payout would be costly if (i) deposit insurance weren't fairly priced, or (ii) there were a cost (e.g., political) of using the deposit insurance fund.

The second social cost of risky lending is the capital provider's opportunity cost: the alternative investment that goes unfunded when the capital provider recapitalizes the bank. This is incurred only if $\rho = \rho_L$.

We make the following assumption about the social costs of risky lending.

**Assumption 4.** $dD > \rho_L - 1 > 0$.

This assumption states that a strategic regulator (a strategic regulator, defined formally in the next section, maximizes social welfare) finds it beneficial to recapitalize a bank whose risky loan is known to be bad, but not a bank whose risky loan is known to be good.

Finally, we add one more potential externality, which we call the social benefit of risky lending: if the bank originates a risky loan at stage 1, it generates a positive externality equal to $B$. Broadly, increased credit is positively associated with economic growth and income for the poor (across both countries and U.S. states; see Demirgüç-Kunt and Levine (2018)).[22] Nevertheless, despite the broad evidence that U.S. stress tests reduced risky lending cited in the introduction, Cortés et al. (forthcoming) indicate stress tests do not change aggregate lending. This may imply that the benefit $B$ is low.

## 2.5 Regulator reputation

The regulator can be one of two types: strategic or lenient. The strategic type trades off the social benefits and costs associated with recapitalization when deciding whether to fail a bank. The lenient type is behavioral and always passes the bank. A lenient type can also be considered an uninformative (or uninformed) type, as its test does not screen banks. Agents may view this type as not conducting "serious" stress test exercises. In subsection 7.2, we demonstrate that our qualitative results still hold if we replace the behavioral lenient type with a behavioral strict type who always fails banks and recapitalizes them.[23]

The regulator knows its own type, but during the stress test in period $t$ (where $t = \{1, 2\}$), the owners of the bank and the capital provider are uncertain about the regulator's type. These agents believe that, with probability $1 - z_t$, the regulator is strategic; with probability $z_t$, they believe the regulator to be a lenient type. In our model, $z_1$ is the probability that nature chooses the regulator to be a lenient type. The term $z_2$ is the updated belief that the regulator is a lenient type after the first-period stress test.

[22] Moskowitz & Garmaise (2006) provide causal evidence of the social effects of credit allocation, such as reduced crime.

[23] The behavior of the lenient type regulator can be microfounded in a model in which it has a high social net benefit of risky lending and the strategic type has a low social net benefit of risky lending. Specifically, if the lenient type regulator has a low cost of bank default $D'$ or a large benefit from risky lending $B'$, then passing the bank with certainty is, indeed, the unique equilibrium strategy.

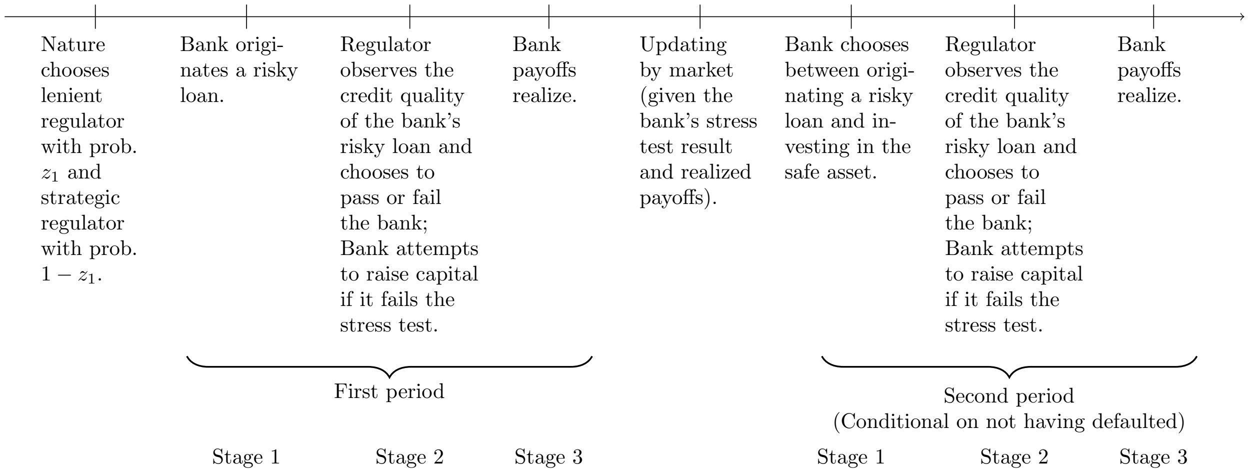


Figure 1: Timeline of events

## 2.6 Summary of timing

The regulator conducts stress testing of the bank in first period, and again in the second period if the bank has not defaulted in the first period. If the bank defaults in the first period, the bank is closed down and does not continue into the second period. At the beginning of the second period, the beliefs about the regulator's type are updated depending on the result of the bank's stress test and the realized payoff of the bank in the first period. The timing is illustrated in Figure 1.

We assume that the probability that the risky loan opportunity is good in the second period is independent of whether the risky loan opportunity is good in the first period, and that the type of the regulator is independent from the quality of the bank's risky loans. Furthermore, the regulator's type remains the same in both periods.

We use the equilibrium concept of Perfect Bayesian equilibrium.

# 3 Stress testing in the second period

We begin the analysis of the model by using backward induction, and characterize the equilibrium in the second period. We first characterize the strategic regulator's stress test strategy at stage 2, taking as given the bank's investment decision at stage 1.

If the bank invests in the safe asset at stage 1, it is clear that the bank will not default and, therefore, requires no capital at stage 2. We will focus on describing the equilibrium stress test outcome given that the bank extends a risky loan at stage 1.

Since the game does not continue after the second period, the regulator has no reputational incentives. The stress test strategy of the strategic regulator at stage 2 depends on the

| | Strategic regulator | Lenient regulator |
|---|---|---|
| $q_2 = g$ | Pass | Pass |
| $q_2 = b$ | Fail | Pass |

Table 1: Equilibrium stress testing in the second period.

quality of the bank's risky loan $q_2 \in \{g, b\}$. Specifically, the strategic regulator passes the bank if and only if the loan is good, as Assumption 4 implies. Table 1 depicts the regulator's equilibrium stress testing strategy.

At stage 2, the bank attempts to raise one unit of capital if it fails the stress test. Since failing the stress test reveals that the bank's loan is of bad quality, the total value of the bank's equity (including the capital provider's equity) post-recapitalization is $(1-d)R$, because the one unit of capital raised will all be paid out to the depositors at maturity. The capital provider's outside option is equal to the expected return on the forgone alternative investment, $\rho$. Assumption 2 implies that the total surplus is positive if and only if the opportunity cost of capital is low ($\rho = \rho_L$).

If recapitalization is feasible, we define $1-\phi$ as the fraction of equity that the bank's owners retain. In order to determine this fraction, we now examine how the surplus is split between the capital provider and the bank's owners. When recapitalization is feasible, the capital provider's outside option is $\rho_L$. We assume that the bank's outside option is 0, as the regulator compels the bank to be recapitalized. The total surplus is, therefore, $(1-d)R - \rho_L$. We use the Nash bargaining solution to define the split of the surplus, where the capital provider gets a fraction of the surplus determined by its bargaining power $\beta$. The transfer from the bank's owners to the capital provider is given by the right-hand side of Eq. 1 below, which is equal to the capital provider's outside option plus the fraction of surplus it obtains through bargaining power. Therefore, the equity given to the capital providers is a fraction $\phi$, determined by:

$$\phi(1-d)R = \rho_L + \beta\left[(1-d)R - \rho_L\right]. \tag{1}$$

We can now analyze the bank's investment decision at stage 1. At stage 1, the bank anticipates the fraction of equity $\phi$ it will need to sell to capital providers in exchange for capital. The bank originates a risky loan if and only if:

$$\underbrace{\left[\alpha + (1-\alpha)\left[z_2 + (1-z_2)\gamma\right](1-d)\right](R-1)}_{\text{pass, or fail but recapitalization infeasible}} + \underbrace{(1-\alpha)(1-z_2)(1-\gamma)(1-\phi)(1-d)R}_{\text{fail and recapitalized}} \geq R_0 - 1. \tag{2}$$

The bank originates a risky loan if and only if the expected payoff to the bank's owners is higher when it originates a risky loan (represented by the left-hand side of Eq. 2) than when it invests in the safe investment (represented by the right-hand side of Eq. 2). Notice that the expected payoff to the bank's owners when it originates a risky loan consists of two terms. First, if the bank does not raise capital and there is no default, it receives the net payoff $R-1$ at stage 4. This is the case if the loan is (i) good, (ii) bad and the regulator is lenient so that the bank passes the stress test (and the loan does not default), or (iii) bad and the bank fails the stress test but recapitalization is infeasible (and the loan does not default). Second, if the bank fails the stress test and is recapitalized, which is the case if the loan is bad and the regulator is strategic, the bank's owners face dilution during recapitalization, and, thus, their payoff is only the retained share $1-\phi$ of the bank's equity. The equity is priced after the stress test and reflects the equilibrium choice of the regulator in the second period.

**Proposition 1.** *In the second period, there exists a unique equilibrium, in which the bank originates a risky loan if and only if the probability that the regulator is lenient* $z_2 \geq z_2^*$, *where* $z_2^* < 1$ *is defined by* $\Delta(z_2^*) = 0$, *where:*

$$\Delta(z_2) \equiv \underbrace{[\alpha + (1-\alpha)(1-d)](R-1) - (R_0 - 1)}_{\textit{profit differential without recapitalization}} - \underbrace{(1-\alpha)(1-z_2)(1-\gamma)\left(\rho_L + \beta\left[(1-d)R - \rho_L\right] - (1-d)\right)}_{\textit{dilution cost of recapitalization}}. \tag{3}$$

*If the bank extends a risky loan at stage 2, the lenient regulator passes the bank with certainty, and the strategic regulator passes the bank with certainty if and only if the bank's loan is good.*

*Moreover, there exists* $\bar{\beta} < 1$, *such that* $z_2^* > 0$ *if and only if* $\beta > \bar{\beta}$.

This proposition states that the bank's incentive to originate a risky loan takes two factors into account. On the one hand, the bank benefits from originating the risky loan because it produces a higher expected profit than the safe investment (the profit differential term in $\Delta(z_2)$). On the other hand, the bank faces a dilution cost whenever it is required to raise capital, because the capital provider extracts rents (the dilution cost in $\Delta(z_2)$). When recapitalized, which occurs with probability $(1-\alpha)(1-z_2)(1-\gamma)$, the bank's cost of funding increases from $1-d$ to $\rho_L + \beta\left[(1-d)R - \rho_L\right]$. Here, $1-d$ represents the bank's cost of repaying depositors, taking into account the deposit insurance, and $\rho_L + \beta\left[(1-d)R - \rho_L\right]$ is how much the bank must pay the capital provider. Since the bank faces the possibility of failing the stress test and, thus, having to raise capital only if it extends a risky loan, the bank originates the risky loan only if the gains from higher NPV outweigh the potential

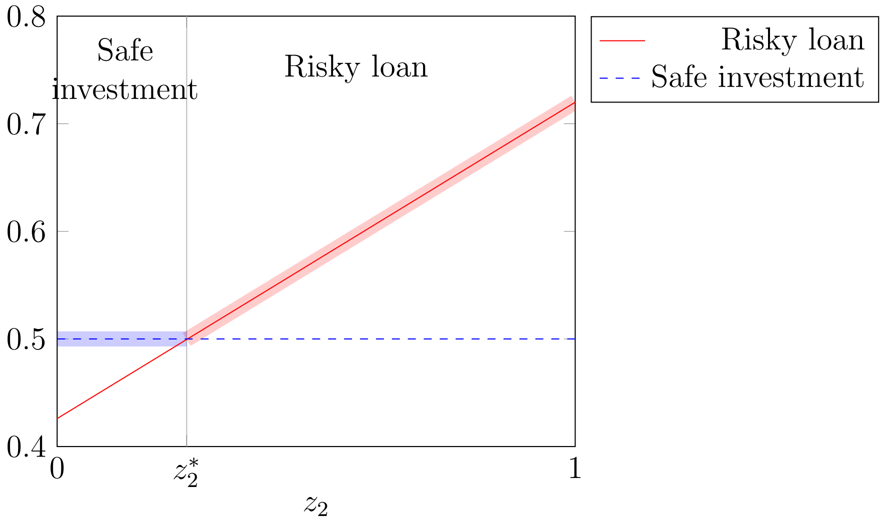


Figure 2: The expected payoff to the bank's owners given that it makes a risky loan (red solid line) and a safe investment (blue dashed line), respectively. The shaded portion of the lines represents the bank's equilibrium project choice. The parameters used in this plot are: $R_0 = 1.5$, $R = 2$, $\alpha = 0.3$, $d = 0.4$, $\rho_L = 1.1$, $\gamma = 0.2$, $\beta = 0.25$ and $D = 0.5$. These parameters imply that $z_2^* = 0.25$.

dilution cost of recapitalization.

Importantly, the bank originates a risky loan only if the regulator's reputation of being the lenient type is sufficiently high– i.e. $z_2 \geq z_2^*$– as illustrated in Figure 2. This is the case because the lenient type regulator does not require the bank to raise capital even if the bank's risky loan is bad (Table 1), thus reducing the expected dilution cost to the bank. For the remainder of the baseline model analysis, we impose the following additional assumption to restrict attention to the interesting set of the parameter space in which the bank's project choice indeed varies depending on the regulator's reputation, i.e., $z_2^* \in (0, 1)$.

**Assumption 5.** *$\beta > \bar{\beta}$, where $\bar{\beta}$ is defined in Proposition 1.*

Since the regulator's reputation $z_2$ determines the bank's investment decision in equilibrium, we now consider how the regulator's reputation affects surplus. Let $U_R$ and $U_0$ denote the strategic regulator's expected surplus from the bank in the second period when the bank originates a risky loan and invests in the safe asset, respectively. We can express the expected surplus as follows:

$$U_R = [\alpha + (1 - \alpha)(1 - d)]\, R - 1 + X, \tag{4}$$

$$U_0 = R_0 - 1, \tag{5}$$

where $X$ represents the net social benefits of risky lending, given by:

$$X \equiv B - (1-\alpha)\left[\gamma dD + (1-\gamma)(\rho_L - 1)\right]. \tag{6}$$

When the bank extends a risky loan, the strategic regulator internalizes the net social benefits of risky lending, consisting of the positive externality of bank lending $B$, as well as the social costs of a potential bank default. Conditional on a bad loan (with probability $1-\alpha$), the expected social costs of a potential bank default include the expected cost of bank default $dD$ if recapitalization is infeasible (with probability $\gamma$) and the forgone net return from the capital providers' alternative investment $\rho_L - 1$ when the bank is recapitalized (with probability $1-\gamma$). Notice that $X$ encapsulates all of the externalities from risky lending; the following analysis will use only $X$ rather than the individual components.

It then follows from Proposition 1 that the strategic regulator's expected surplus, for a given reputation $z_2$, denoted by $U(z_2)$, is given by

$$U(z_2) = \begin{cases} U_R, & \text{if } z_2 > z_2^*, \\ U_0, & \text{if } z_2 < z_2^*, \\ \lambda U_R + (1-\lambda)U_0 \text{ for some } \lambda \in [0,1], & \text{if } z_2 = z_2^*, \end{cases} \tag{7}$$

where we have taken into account that, if $z_2 = z_2^*$, the bank is indifferent between originating a risky loan and investing in the safe asset. Thus, the bank may employ a mixed strategy and randomize between the two investment choices with some probability $\lambda$.

The regulator internalizes the social benefits and costs of risky lending, whereas the bank cares only about the private cost of recapitalization. Therefore, the bank's investment choice characterized in Proposition 1 generally differs from the socially optimal choice. The more the bank expects the regulator to be the lenient type, the more the bank is willing to originate a risky loan. However, originating a risky loan is socially preferred only if the net social benefits of risky lending X are sufficiently high. This is illustrated in Figure 3. In the following analysis, we demonstrate that the divergence in preferences between the strategic regulator and the bank leads to reputation-building incentives for the strategic regulator that depend on the net social benefits of risky lending $X$.

# 4 Stress testing in the first period

In this section, we analyze the regulator's equilibrium stress testing strategy for the bank in the first period, given the equilibrium in the second period. In particular, we consider the

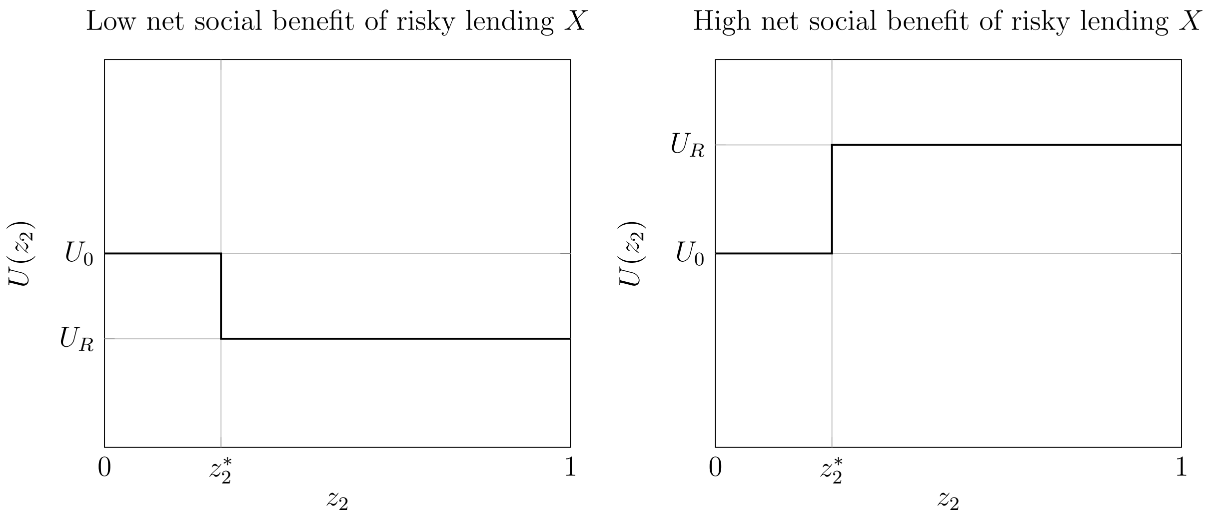


Figure 3: The strategic regulator's expected surplus for a given reputation $z_2$. The parameters used in this plot are the same as those in Figure 2, implying that $U_0 = 0.5$. In addition, in the left panel, $B = 0.1$, implying that $X = 0.016$ and $U_R = 0.456 < U_0$, whereas in the right panel, $B = 0.2$, implying that $X = 0.116$ and $U_R = 0.556 > U_0$.

strategic regulator's incentives to pass the bank in the first period at stage 2. These incentives are driven by concerns for the bank in the first period and the reputational consequences of the regulator's observable decision to pass or fail the bank.

At stage 2, the regulator takes the posterior beliefs held by the bank in the second period about the regulator's type as given. These posterior beliefs depend on the stress test result and the bank's realized payoff in the first period. Let $z_2^R$ denote the posterior belief about the probability that the regulator is the lenient type, given that the bank passes the stress test in the first period and realizes a payoff of $R$. If the bank passes the stress test in the first period and realizes a payoff of 0 instead, the bank defaults and does not continue to the second period. Let $z_2^f$ denote the posterior belief about the probability that the regulator is the lenient type, given that the bank fails the stress test in the first period. As will become clear, this posterior belief does not depend on the bank's realized payoff in the first period.

The regulator's incentive to pass the bank, given the quality of the bank's risky loan $q_1 \in \{g, b\}$, can be described by the net gain of passing the bank relative to failing the bank, which we denote by $G_{q_1}(z_2^R, z_2^f)$:

$$\begin{aligned} G_g(z_2^R, z_2^f) &= \underbrace{(1-\gamma)(\rho_L - 1)}_{\text{first period bank surplus effect}} + \underbrace{\delta \left[U(z_2^R) - U(z_2^f)\right]}_{\text{reputation effect}}, \\ G_b(z_2^R, z_2^f) &= \underbrace{(1-\gamma)\left[(\rho_L - 1) - dD\right]}_{\text{first period bank surplus effect}} + \underbrace{\delta \left[(1-d)U(z_2^R) - \left[(1-d) + d(1-\gamma)\right] U(z_2^f)\right]}_{\text{reputation effect}}. \end{aligned} \quad (8)$$

In both expressions, the first term represents the net gain in terms of the expected surplus in the first period, and the second term represents the reputation concern in terms of the expected surplus in the second period. The first term takes into account that, when the bank fails, recapitalization is feasible only with probability $1-\gamma$. In that case, the first-period bank surplus effect of passing the bank relative to failing it is equal to the capital provider's alternative investment (which can now be realized with a pass) less the expected cost of a bank default (which may also be realized with a pass) if the quality of the investment is bad.

The first-period bank surplus effect is positive if the first bank's risky loan is good, as there is no risk of default. This effect is negative if the risky loan is bad, given Assumption 4.

The reputation effect depends on the regulator's posterior reputation after it grades the first-period bank and the payoffs are realized. Given that the lenient type regulator passes the bank, if the strategic regulator fails the bank in the first stress test, it is revealed to be strategic ($z_2^f = 0$); the bank will then realize that it will be recapitalized in the second period if its risky investment is of bad quality. In contrast, if the strategic regulator passes the bank in the first stress test, it is pooled with the lenient type regulator, who also passes the bank. In equilibrium, the posterior probability that the regulator is the lenient type, given that the bank passes the first stress test and then realizes a payoff of $R$, is given by

$$z_2^R(\pi_g, \pi_b) = \frac{[\alpha + (1-\alpha)(1-d)]z_1}{[\alpha + (1-\alpha)(1-d)]z_1 + [\alpha\pi_g + (1-\alpha)(1-d)\pi_b](1-z_1)}, \tag{9}$$

where $\pi_g$ and $\pi_b$ denote the strategic regulator's probability of passing the bank in the first stress test, given that the bank's risky loan is good or bad, respectively. As a result, passing or failing the bank in the first period affect the reputation of the regulator and may lead to different investment decisions by the bank in the second period.

The following lemma establishes the set of possible equilibrium stress testing strategies, which narrows down our analysis.

**Lemma 1.** *In any equilibrium, the stress testing strategy of the strategic regulator is one of the following:*

***Informative:*** *it passes the bank if and only if the bank's risky loan is good;*

***Soft:*** *it passes the bank with certainty if the bank's risky loan is good and passes the bank with positive probability $\pi_b^* > 0$ if the loan is bad; or*

***Tough:*** *it passes the bank with probability $\pi_g^* < 1$ if the bank's risky loan is good and fails the bank with certainty if the loan is bad.*

This lemma follows from the fact that, in any equilibrium, the strategic regulator faces strictly greater incentives to pass a bank with a good risky loan than a bank with a bad risky loan. This can be seen in Eq. 8. Passing a bank with a bad risky loan results in a possible costly default, while passing a bank with a good risky loan does not have this possibility. A bank default generates two costs. First, it generates a social cost of default $D$ in the first period. Second, it leads to a loss of expected surplus $U(z_2)$ in the second period. This implies that the probability with which the strategic regulator passes a bank with a good risky loan ($\pi_g$) is weakly larger than the probability with which it passes a bank with a bad risky loan ($\pi_b$). Thus, Lemma 1 represents all possible equilibrium strategies of the strategic regulator in the first period.

We now show that for intermediate levels of the net social benefits of risky lending $X$, there is a unique equilibrium in which the strategic regulator's stress testing strategy in the first period is identical to its strategy in the second period (and is informative).

**Proposition 2.** *There exist cutoffs for the net social benefits of risky lending $\overline{X}$ and $\underline{X}$, with $\underline{X} < \overline{X}$, such that for $X \in [\underline{X}, \overline{X}]$, there exists a unique equilibrium in the first period in which the stress testing strategy of the strategic regulator in the first period is identical to that in the second period, described in Proposition 1, and is informative.*

For intermediate levels of the net social benefit of risky lending ($X \in [\underline{X}, \overline{X}]$), the expected surplus for the strategic regulator is not too sensitive to the bank's investment decision in the second period. That is, for intermediate $X$, the social values of the risky project and the safe asset are close, so the bank's investment choice in the second period has little effect on the regulator's surplus, and the regulator can choose its static optimum stress testing strategy in the first period. Proposition 2 shows that, in this case, the equilibrium is unique and is informative.

In the following sections, we will show that the other two types of equilibria described in Lemma 1 can arise if the net social benefits of risky lending $X$ is either low or high, and depend on the regulator's reputation-building incentives.

## 4.1 Low net social benefits of risky lending $X < \underline{X}$

When the net social benefit of risky lending $X$ is low (e.g., due to a large cost of bank default $D$), the expected surplus to the strategic regulator from the bank in the second period is higher when the bank invests in the safe asset. If the concerns about excessive risk-taking by the bank in the second period are sufficiently large, in order to reveal its willingness to fail a bank during the second stress test, the strategic regulator may want to fail the bank with a risky loan in the first period even when the loan is good.

In the following proposition, we demonstrate that the strategic regulator's reputation-building incentives to reduce excessive risk-taking by the bank leads to another equilibrium,

| | Strategic regulator | Lenient regulator |
|---|---|---|
| $q_1 = g$ | Pass with probability $\pi_g^* < 1$ | Pass |
| $q_1 = b$ | Fail | Pass |

Table 2: Equilibrium stress testing in the first period when the strategic regulator wants to build reputation to reduce excessive risk-taking by the second bank in the second period.

in addition to the equilibrium in which the strategic regulator's stress testing strategy in the first period is identical to its strategy in the second period.

**Proposition 3.** *For low net social benefits of risky lending $X < \underline{X}$, there exists an equilibrium in the bank's first period stress test that is either informative or tough (as described in Lemma 1). Specifically, there exist $\bar{z}_1$ and $\delta_g$ such that:*

- *If $z_1 \leq \bar{z}_1$,*
  - *for $\delta < \delta_g$, the unique equilibrium is informative,*
  - *for $\delta \geq \delta_g$, the informative equilibrium coexists with tough equilibria.*
- *If $z_1 > \bar{z}_1$,*
  - *for $\delta < \delta_g$, the unique equilibrium is informative,*
  - *for $\delta > \delta_g$, the unique equilibrium is tough,*
  - *for $\delta = \delta_g$, the informative equilibrium coexists with tough equilibria.*

Proposition 3 shows that, for certain parameters, the regulator's equilibrium stress testing strategy in the first period is the same as its strategy in the second period. This is illustrated in Table 1. However, this proposition also shows that the strategic regulator's reputation-building incentives to reduce excessive risk-taking by the bank in the second period can lead to an equilibrium with a tough stress test in the first period. Table 2 depicts the stress testing in the tough equilibrium in the first period described in Proposition 3.

In the informative equilibrium, the strategic regulator passes the bank with a good risky loan in the first period, which maximizes the expected surplus from the bank. Failing the bank in this case could result in a costly recapitalization of the bank with no benefit, since the good loan will not default. However, in the tough equilibrium, by failing this bank, the strategic regulator is able to reveal its type and, thus, reduce the bank's incentive to engage in excessive risk-taking in the second period. Building this reputation to reduce excessive risk-taking is worthwhile when the regulator's reputation concern ($\delta$) is sufficiently high, so that the regulator's reputational benefit outweighs the short-term efficiency loss.

Proposition 3 also indicates that the informative and tough equilibria coexist when $z_1 \leq \bar{z}_1$ and $\delta \geq \delta_g$. This is due to a strategic complementarity between the regulator's first-period stress test and the bank's belief updating process in the second period.

Specifically, the strategic regulator's stress testing strategy and the bank's belief updating process are strategic complements when the net social benefits of risky lending are low ($X < \underline{X}$). Here, the bank realizes that the strategic regulator's surplus is lower when the bank makes the risky loan or, equivalently, when the strategic regulator is perceived to be lenient (with high likelihood). If the bank conjectures that the strategic regulator adopts a tougher stress test strategy (lower $\pi_g$), the bank infers that the regulator who passes the bank in the first period is more likely to be lenient (higher $z_2^R$). Consequently, the bank increases its risk taking in the second period after a pass result in the first period, resulting in *even lower* expected surplus for the strategic regulator. In turn, this further decreases the net gain for the strategic regulator from passing the bank in the first period, justifying a tougher testing strategy. It is, indeed, this strategic complementarity that leads to equilibrium multiplicity.[24]

U.S. stress tests have generally been regarded as much tougher than European ones. First, the Federal Reserve performs the stress test itself on data provided by the banks (and does not provide the model to the banks), whereas in Europe, it has been the case that the banks themselves perform the test. Second, the U.S. stress tests have regularly been accompanied by Asset Quality Reviews, whereas this has been infrequent for European stress tests. Third, one of the most feared elements of the U.S. stress tests has been the fact that there is a qualitative element that can be (and has been) used to fail banks.[25] A possible reason for this is that European authorities prioritized stimulating lending more, given the slow recovery after the crisis.

[24] Note that Proposition 3 implies that there also exists multiplicity in the knife-edge case in which $\delta = \delta_g$ for $z_1 > \bar{z}_1$. This is for a different reason than the strategic complementarity discussed above. In this case, the bank's belief updating process in the second period does not feedback to influence the regulator's first-period stress test, because in both types of equilibria (informative and tough), the bank employs the same investment strategy in the second period, and invests in the risky loan if and only if it passes the stress test in the first period. Instead, such multiplicity stems from the fact that the bank's investment decision in the second period follows a threshold strategy. Therefore, a range of stress testing strategies (in terms of the mixing probability $\pi_g$) leads to posterior beliefs held by the bank that are consistent with the same investment strategy in the second period, implying the same reputation effect on the regulator's stress testing incentives that justifies the mixed strategies.

[25] The qualitative element for domestic banks was removed in March 2019 (see "US financial regulators relax Obama-era rules," by Kiran Stacey and Sam Fleming, Financial Times, March 7, 2019).

| | Strategic regulator | Lenient regulator |
|---|---|---|
| $q_1 = g$ | Pass | Pass |
| $q_1 = b$ | Pass with probability $\pi_b^* > 0$ | Pass |

Table 3: Equilibrium stress testing in the first period when the strategic regulator wants to build reputation to incentivize lending by the bank in the second period.

## 4.2 High net social benefits of risky lending $X > \overline{X}$

If the strategic regulator fails the first-period bank and recapitalizes it, the regulator reveals that it is strategic. The bank then faces a strong incentive to invest in the safe investment in the second period, in order to avoid failing the stress test. If the strategic regulator passes the bank in the first period, however, it pools with the lenient regulator, increasing the incentive for the bank to engage in risky lending in the second period. If the benefit of lending by the bank in the second period is sufficiently large, the regulator may want to pass the bank in the first period, even when its risky loan is bad, in order to gain a reputation for leniency.

In the following proposition, we demonstrate that for high $X$, there is still an equilibrium in which the strategic regulator's stress testing strategy in the first period is identical to its strategy in the second period. However, reputation-building incentives to encourage lending by the second-period bank can lead to a soft equilibrium for the bank's stress test.

**Proposition 4.** *For high net social benefits of risky lending $X > \overline{X}$, there exists an equilibrium in the first bank's stress test that is either informative or soft (as described in Lemma 1). Specifically, there exist $\bar{z}_1$ (defined in Proposition 3) and $\delta_b$, such that:*

- *If $z_1 \leq \bar{z}_1$, the unique equilibrium is informative.*
- *If $z_1 > \bar{z}_1$:*
  - *for $\delta < \delta_b$, the unique equilibrium is informative,*
  - *for $\delta > \delta_b$, the unique equilibrium is soft, and*
  - *for $\delta = \delta_b$, the informative equilibrium coexists with soft equilibria.*

Proposition 4 shows that, for certain parameters, the regulator's equilibrium stress testing strategy in the first period is the same as its strategy in the second period (illustrated in Table 1). However, this proposition also shows that the strategic regulator's reputation-building incentives to encourage lending by the bank in the second period can lead to an equilibrium with a soft stress test in the first period. Table 3 depicts the stress testing in the soft equilibrium in the first period, described in Proposition 4.

Passing the bank with a bad risky loan is costly, as it incurs an expected cost of default that is higher than the social cost of capital. However, in the soft equilibrium, by passing the bank, the strategic regulator is able to increase the perception that it is the lenient type, as the lenient type regulator always passes the bank. This is useful to the strategic regulator when this induces the bank to originate a risky loan in the second period. In other words, the regulator enjoys a positive reputation effect from passing the bank with a bad risky loan in the first period.

Proposition 4 identifies the necessary and sufficient conditions for the existence of a soft equilibrium with reputation building to incentivize lending to exist. First, the regulator's prior reputation of being lenient ($z_1$) must be sufficiently high, so that the posterior reputation of the regulator is sufficiently lenient after passing the bank in the first period to induce the bank to originate a risky loan in the second period. Second, the reputation concern ($\delta$) of the strategic regulator must be sufficiently high, so that the regulator's reputational benefits outweigh the short-term efficiency loss when passing the bank with a bad risky loan.

In contrast to the result in the previous subsection about the existence of multiple equilibria, when the net social benefit of risky lending is high ($X > \overline{X}$) there is a unique equilibrium.[26] This is because here the regulator's stress testing strategy and the bank's belief updating process are strategic substitutes (whereas in the previous subsection they were strategic complements). Here, the bank realizes that the strategic regulator's surplus is larger when the bank extends the risky loan or, equivalently, when the strategic regulator is perceived as lenient (with sufficiently high probability). If the bank conjectures that the strategic regulator adopts a soft stress testing strategy (higher $\pi_b$), the bank infers that the regulator who passed the bank in the first period is more likely to be the strategic type (lower $z_2^R$). Consequently, the bank may refrain from originating a risky loan in the second period after a pass result in the first period, resulting in *lower* expected surplus for the strategic regulator. In turn, this reduces the net gain for the strategic regulator from passing the bank in the first period. Because of this strategic substitutability, the same type of equilibrium multiplicity does not arise.[27]

While the initial European stress tests performed poorly (e.g., passing Irish banks and Dexia), one might argue that during crisis times, the main focus was preventing runs - and

[26]Note that Proposition 4 implies that there exists multiplicity in the knife-edge case in which $\delta = \delta_b$ for $z_1 > \bar{z}_1$. As in the case with low net social benefits of risky lending $X < \underline{X}$, this is again due to the threshold nature of the bank's investment decision in the second period rather than due to the feedback between the regulator's first-period stress test and the bank's belief updating process,

[27]While, formally, strategic complementarity arises only when $X$ is low, the driver for this is the assumption that the behavioral type is lenient. When we change the behavioral type to a "strict" type who always fails the bank, strategic complementarity arises only when $X$ is high and leads to a co-existence of informative and *soft* equilibria. We describe this in Section 7.2.

without a fiscal backstop it was hard to maintain credibility (Faria-e-Castro, Martinez, and Philippon, 2016). We argue that in normal times, a stress test may be soft to incentivize banks to lend to the real economy. This may explain the 2016 EU stress test, which eliminated the pass/fail criteria, reduced the number of banks stress tested by about half, used less-adverse scenarios than did the U.S. and UK, and singled out only one bank as undercapitalized - Monti dei Paschi di Siena, which had failed the previous (2014) stress test and was well known to be in distress.

# 5 Discussion

Having characterized the equilibria of the model, we discuss the implications of the model in this section. First, we point to the possibility that stress tests may be strategically delayed. Second, we consider how stress tests may vary with the availability of capital. Third, we explore the implications of the model for stress test design when banks are systemic. Finally, we discuss the meaning of multiple equilibria in the model.

## 5.1 Strategic delay of stress tests

There may exist an equilibrium in which both types of regulator pass the bank in the first period with certainty. This is equivalent to an economy in which the regulator does not conduct stress tests for the bank in the first period.

**Corollary 1.** *For high net social externalities of risky lending ($X > \overline{X}$), the regulator passes the bank in the first period in equilibrium with certainty if $\delta \geq \delta_b$ and $z_1 \geq z_2^*$, where $z_2^* > \bar{z}_1$ is defined in Proposition 1, and $\bar{z}_1$ is defined in Proposition 3.*

The timing of European stress tests has been quite irregular compared with the twice yearly U.S. exercises (European tests were conducted in 2010, 2011, 2014, 2016, and 2018). Delay in this situation may be a way of choosing to be soft.

## 5.2 Availability of capital

Let $\gamma_1$ denote the probability that recapitalization is infeasible in the first period. The following corollary assesses how the availability of capital in the first period affects the regulator's stress testing strategy.

**Corollary 2.** *When net social benefits of lending are:*

- *Low ($X < \underline{X}$): an increase in the probability that recapitalization is infeasible ($\gamma_1$) strictly shrinks the parameter space ($\beta, z_1, \delta$) for which an informative equilibrium exists, and strictly enlarges that for which a tough equilibrium exists.*

- *High ($X > \overline{X}$): an increase in the probability that recapitalization is infeasible ($\gamma_1$) strictly shrinks the parameter space ($\beta, z_1, \delta$) for which an informative equilibrium exists, and strictly enlarges that for which a soft equilibrium exists.*

The corollary demonstrates that when there is a higher probability that recapitalization is infeasible in the first period, the strategic regulator's reputation-building incentives are exacerbated, and the stress test becomes less informative. This is because the strategic regulator trades off the cost/benefit of recapitalizing the bank in the first period against the regulator's cost/benefit of affecting the bank's investment decision in the second period. While the reputation effect depends only on the bank's updated belief about the regulator's type, the cost of passing a bad bank or failing a good bank in the first period is smaller if recapitalization is infeasible in the first period with a high probability.

This result is related to that of Faria-e-Castro, Martinez, and Philippon (2016) in that, as recapitalizing the bank becomes more difficult, the test becomes less informative. Nevertheless, we demonstrate this link through a dynamic reputation model, whereas they have the regulator committing upfront to the informativeness of the stress test.

The U.S.'s swifter recovery from the crisis means that capital raising for banks was likely to be easier. Our model implies that stress tests will be more informative in this situation, which appears consistent with reality.

## 5.3 Stress tests of systemic banks

Let $D_1$ denote the social cost of a potential bank default in the first period. The following corollary assesses how the the social cost of a potential bank default in the first period affects the regulator's stress testing strategy.

**Corollary 3.** *When net social benefits of lending are:*

- *Low ($X < \underline{X}$): an increase in $D_1$ does not affect the parameter space ($\beta, z_1, \delta$) for which either an informative equilibrium or a tough equilibrium exist.*

- *High ($X > \overline{X}$): an increase in $D_1$ strictly enlarges the parameter space ($\beta, z_1, \delta$) for which an informative equilibrium exists, and strictly shrinks that for which a soft equilibrium exists.*

The corollary demonstrates that when the social cost of a potential bank failure in the first period is higher, the strategic regulator becomes less soft/more informative when facing reputation-building incentives to incentivize lending ($X > \overline{X}$). This occurs because, when considering whether to pass a bad bank in the first period, the strategic regulator trades off the cost/benefit of recapitalizing the bank in the first period against the cost/benefit of affecting the bank's investment decision in the second period. While the reputation effect depends only on the bank's updated belief about the regulator's type, the cost of passing a bad bank in the first period is larger if the cost of a potential bank failure in the first period is larger. The strategic regulator's stress testing strategy when facing reputation-building incentives to curb excessive risk-taking ($X < \underline{X}$) is unaffected since, in this case, the main focus is whether to pass or fail a good bank, which does not run the risk of default.

In both the U.S. and Europe, since the inception of stress tests, there have been ongoing debates about how large/systemic a bank must be in order to be included in the stress testing exercise. To the extent that larger and more systemic banks have a higher expected cost of default, our model predicts that they should be subject to (weakly) more informative tests.

### 5.4 Multiplicity

The fact that we have potentially coexisting equilibria raises the issue of how a particular equilibrium may be chosen. Commitment by the regulator in an ex ante stage would facilitate this. Of course, in a crisis, committing to future actions may not be feasible. The regulator has access to several policy variables that might prove useful as commitment devices. Committing to how signals from banks are used is standard in the Bayesian Persuasion literature, but it requires substantial independence from political pressure and processes that are well-defined. A more practical alternative is committing to stress test scenarios, which can be more soft or less soft, given the effect desired. Asset quality reviews also commit more resources and reveal more information about bank positions.

## 6 The bank and the regulator both learn the asset quality

In this section, we consider a stress test in which the signal about the quality $q_t$ of the bank's risky loan observed by the regulator during the test is also observed by the bank. This could be the case because:

- the stress test uncovers only the private information that the bank already has about its loan quality.[28] This is, indeed, the case for banking supervision examinations. These exams are conducted on a regular basis by collecting information and assessing the health of banks on multiple dimensions, and they have real effects.[29] They do not use information from the entire banking system to assess the position of each bank (which can be the source of the regulator's private information in the baseline model). In the United States, this has historically been conducted using the CAMEL rating system, though, in recent years, variations on this rating system have been implemented;[30] or

- the stress test produces/uncovers new information, but regulators share that information with the bank. This second case resembles the European stress test exercises, which use an approach whereby the regulator provides the model and basic parameters to banks, which then perform the test themselves.[31] In contrast, the U.S. uses an approach whereby the regulators perform the test and do not provide all of the information about the model or the results.[32]

As in the baseline model, the equilibrium in the second period for a given belief $z_2$ held by the bank is as described in Lemma 1.

Unlike in the baseline model, here, in the second period, the bank forms posterior beliefs $z_2 = z^R_{2,q_1}$ ($z_2 = z^f_{2,q_1}$) about the probability that the regulator is lenient given that the bank passes (fails) the stress test in the first period *and* the loan quality in the first period is $q_1$. Therefore, in the first period, taking the bank's posterior beliefs described above as given, the strategic regulator's incentives to pass the bank are characterized by $G_{q_1}(z^R_{2,q_1}, z^f_{2,q_1})$, where $G_{q_1}(\cdot)$ is defined by Eq. 8.

In equilibrium, as in the baseline model, the posterior belief that the regulator is lenient, given that the bank fails the stress test in the first period, is $z^f_{2,q_1} = 0$ since only a strategic regulator fails a bank. In addition, the posterior belief of the bank that the regulator is

[28]For example, in Walther and White (2018), the regulator and the bank both observe the bank's asset value, while creditors do not. They consider the effectiveness of bail-ins in this scenario.

[29]Agarwal et al. (2014) demonstrate real effects of exams: leniency leads to more bank failures; a higher proportion of banks unable to repay TARP money in the crisis; and a larger discount on assets of banks liquidated by the FDIC. Hirtle, Kovner, and Plosser (2018) demonstrate real effects of more banking supervision effort (measured by hours).

[30]The RFI/C(D) system was recently supplanted by the LFI system for large financial institutions. See (https://www.davispolk.com/files/2018-11-06_federal_reserve_finalizes_new_supervisory_ratings_system_for_large_financial_institutions.pdf).

[31]Note that we do not model the inherent moral hazard problem when a bank is permitted to do its own stress test.

[32]These are sometimes referred to as "bottom-up" (banks do the test) and "top-down" (regulator does the test) approaches, but definitions of these terms vary. See Baudino et al. (2018) and Niepmann and Stebunovs (2018) for a discussion of top-down vs. bottom up approaches. The U.S. recently made more information available about its test after complaints about opacity by banks (https://uk.reuters.com/article/uk-usa-fed-stresstests/fed-gives-u-s-banks-more-stress-test-information-unveils-2019-scenarios-idUKKCN1PU2GE).

lenient, given that it had a loan of quality $q_1$ and passed the first stress test is given by:

$$z^R_{2,q_1}(\pi_{q_1}) = \frac{z_1}{z_1 + (1 - z_1)\pi_{q_1}}. \tag{10}$$

We can now compare the results in this case to those in the baseline model and examine the effect of bank information on the regulator's equilibrium stress testing strategy.

**Proposition 5.** *The equilibria when the bank has information about the risky loan's quality $q_t$ are characterized as follows.*

- *For intermediate levels of net social benefits of risky lending $X \in [\underline{X}, \overline{X}]$, there exists a unique informative equilibrium (as described in Proposition 2).*
- *For low net social benefits of risky lending $X < \underline{X}$, there exists an equilibrium for the first bank's stress test, which is either informative or tough. The parameter space in terms of $(\beta, z_1, \delta)$ for which an informative equilibrium exists is strictly larger than in the baseline model, and that for which a tough equilibrium exists is identical to that in the baseline model.*
- *For high net social benefits of risky lending $X > \overline{X}$, there exists an equilibrium for the first bank's stress test, which is either informative or soft. The parameter space in terms of $(\beta, z_1, \delta)$ for which an informative equilibrium exists is strictly smaller than in the baseline model, and that for which a soft equilibrium exists is strictly larger than in the baseline model.*

Proposition 5 provides two key insights. First, when the net social benefits of risky lending are low ($X < \underline{X}$), the bank having knowledge of the risky loan's quality reduces the strategic regulator's reputation concerns, resulting in a more informative stress test. The reason is as follows. Since the strategic regulator is more likely to pass a bank with a good loan than a bank with a bad loan, after a pass on the first stress test, the bank's posterior belief about the likelihood that the regulator's type is lenient is lower if the loan was good quality than if it was bad quality. That is, $z^R_{2,g} \leq z^R_2 \leq z^R_{2,b}$. As a result, passing the good bank in the first period is less likely to induce the bank to originate a risky loan in the second period. This reduces the strategic regulator's concern about excessive risk-taking, resulting in a more informative stress test in the first period.

Second, when the net social benefits of risky lending are high ($X > \overline{X}$), the bank having knowledge of the risky loan's quality exacerbates the strategic regulator's reputation concerns, resulting in a less informative stress test. The reason is as follows. Since $z^R_2 \leq z^R_{2,b}$ as argued above, passing the bank with a bad loan in the first period is more likely to induce the bank to originate a risky loan in the second period, exacerbating the strategic regulator's incentives to pass the bank to incentivize lending.

Therefore, banking supervision exams (or stress tests conducted by the banks themselves) will be less informative than stress tests conducted by the regulator when regulators are concerned about lending. This is in line with the evidence in Agarwal et al. (2014). On the other hand, when bank defaults are more of a concern, banking supervision exams (or stress tests conducted by the banks themselves) will be more informative than stress tests conducted by the regulator.

# 7 Different types for the behavioral regulator

In the baseline model, we assumed that one type of regulator was behavioral and always passed the bank in its stress test - the lenient type. In this section, we demonstrate the robustness of our main results by considering two alternative models: one in which the behavioral regulator fails the bank and recapitalizes it if and only if the bank has a bad-quality loan (we call this the *truthful type*), and one in which the behavioral regulator always fails the bank and recapitalizes it regardless of the loan's quality (we call this the *strict type*). These results show that reputation-building incentives for the strategic regulator are present and can lead to multiple equilibria similar to those in our baseline model whenever the behavioral type's stress testing strategy differs from (i.e., is either softer or tougher than) that of the strategic regulator when there are no reputation concerns (as in the second period).

## 7.1 A truthful type regulator

In this subsection, we consider an alternative model in which the behavioral regulator (the truthful type) fails the bank and recapitalizes it if and only if the bank has a loan that is bad.[33] In this case, both the strategic and the truthful regulators use an identical stress testing strategy in the second period. Anticipating the regulator's stress testing strategy, the bank's lending behavior in the second period does not depend on its belief about the regulator's type. As a result, the strategic regulator faces no reputation-building incentives, since it cannot influence the bank's lending strategy through its reputation. The following proposition states that, in this case, the equilibrium is always informative, analogous to the informative equilibrium of the baseline model.

[33] Piccolo and Shapiro (2018) consider a reputation-based model of a credit rating agency where the behavioral type is truthful. However, in that paper, the strategic type's objective is not welfare maximization.

**Proposition 6.** *Consider the model with a truthful type regulator. There exists a unique equilibrium in which, in each period, if the bank originates a risky loan, the strategic regulator passes the bank with certainty if and only if the bank's loan is good.*

## 7.2 A strict type regulator

In this subsection, we consider an alternative model in which the behavioral regulator always fails the bank and recapitalizes it. The following proposition characterizes the equilibria with the strict regulator and demonstrates that the types of equilibria when the behavioral regulator is lenient are also the only types of equilibria when the behavioral regulator is strict.

**Proposition 7.** *Consider the model with a strict type regulator. In the second period, there exists a unique equilibrium in which, if the bank extends a risky loan, the strategic regulator passes the bank with certainty if and only if the bank's loan is good.*

*An equilibrium exists in the first period, and the possible equilibria are as described in Lemma 1.*

- *For $X \in [\underline{X}, \overline{X}]$, there exists a unique equilibrium in which the stress testing strategy of the strategic regulator in the first period is identical to that in the second period (informative).*
- *For $X < \underline{X}$, the equilibrium is either informative or tough. There can exist a unique equilibrium, or an informative equilibrium can coexist with a tough equilibrium. In particular, the equilibrium is unique unless $\delta = \delta_g$, where $\delta_g$ is defined in Proposition 3.*
- *For $X > \overline{X}$, the equilibrium is either informative or soft. There can exist a unique equilibrium, or an informative equilibrium can coexist with a soft equilibrium.*

Notice that, in contrast to the baseline model, when the behavioral regulator is strict, equilibrium multiplicity arising from the regulator's self-fulfilling reputation concern exists only with a soft equilibrium and not with a tough equilibrium. This is because, in this case, the strategic regulator's stress testing strategy and the bank's belief updating process are strategic complements only when the net social benefits of risky lending are high ($X > \overline{X}$). Here, the bank realizes that the strategic regulator's surplus is larger when the bank originates the risky loan and the regulator is perceived to be strategic. If the bank conjectures that the strategic regulator adopts a softer stress test strategy (higher $\pi_b$), the bank infers that the regulator who passes the bank in the first period is more likely to be the strategic regulator (who would be relatively soft). Consequently, the bank is more likely to originate a risky loan in the second period after a pass result in the first period, resulting in higher expected surplus for the strategic regulator. This further increases the net gain for the strategic regulator from passing the bank in the first period, justifying a softer testing strategy.[34]

# 8 Conclusion

A recent addition to the regulatory toolkit, stress tests provide assessments of bank risk in adverse scenarios. Regulators respond to negative information by requiring banks to raise capital. However, regulators have incentives to be tough, by asking even some safe banks to raise capital, or to be soft, by allowing some risky banks to get by without raising capital. These incentives are driven by the weight that the regulator places on lending in the economy versus stability. Banks respond to the softness of the stress test by altering their lending policies. We demonstrate that in equilibrium, regulators may act tough and discourage lending or act soft and encourage lending. These equilibria can be self-fulfilling, and the regulator may get trapped in one of them, leading to a loss of surplus. Banking supervision exams will lead to similar results but may be less informative.

It would be interesting to extend the model to study stress testing with multiple banks in a macroprudential setting. Examining further the relationship between reputation management and commitment would be worthwhile.

[34]As in the baseline model, there exists multiplicity in the knife-edge case in which $\delta = \delta_g$ when the net social benefits of risky lending are low ($X < \underline{X}$), due to the threshold nature of the bank's investment decision in the second period.

# A Proofs

## A.1 Proof of Proposition 1 (Second-period bank choice)

$\Delta(z_2)$ given by Eq. 3 is obtained by substituting Eq. 1 into Eq. 2 to eliminate $\phi$ and rearranging.

Notice that $\Delta(z_2)$ is strictly increasing in $z_2$. Moreover, $\Delta(1) > 0$ is implied by Assumption 1, and $\Delta(z_2) \to -\infty$ as $z_2 \to -\infty$. Therefore, a unique $z_2^*$ as defined by $\Delta(z_2^*) = 0$ exists, where $z_2^* < 1$.

We now derive a condition for $z_2^* > 0$. This is the case if and only if:

$$\begin{aligned}\Delta(0) = {} & [\alpha + \gamma(1-\alpha)(1-d)]\,(R-1) - (R_0 - 1) \\ & + (1-\alpha)(1-\gamma)(1-\beta)\,[(1-d)R - \rho_L] < 0. \end{aligned} \tag{11}$$

Notice that the above expression is strictly decreasing in $\beta$. Moreover, Assumption 3 implies that $\Delta(0) < 0$ for $\beta = 1$. Therefore, there exists a unique $\bar{\beta} < 1$, such that $z_2^* > 0$ if and only if $\beta > \bar{\beta}$, where $\bar{\beta}$ is defined by:

$$[\alpha + \gamma(1-\alpha)(1-d)]\,(R-1) - (R_0 - 1) + (1-\alpha)(1-\gamma)(1-\bar{\beta})\,[(1-d)R - \rho_L] = 0. \tag{12}$$

∎

## A.2 Proof of Proposition 2 (First-period equilibrium for $X \in [\underline{X}, \overline{X}]$ is informative)

Let $\underline{X}$ be defined such that $U_R = U_0$, i.e.,

$$[\alpha + (1-\alpha)(1-d)]\,R - 1 + \underline{X} = R_0 - 1. \tag{13}$$

Let $\overline{X}$ be defined such that $(1-d)U_R = [(1-d) + d(1-\gamma)]\,U_0$, i.e.,

$$(1-d)\left([\alpha + (1-\alpha)(1-d)]\,R - 1 + \overline{X}\right) = [(1-d) + d(1-\gamma)]\,(R_0 - 1). \tag{14}$$

It is straightforward to show that $\overline{X} > \underline{X}$.

We now consider the case in which $X \in [\underline{X}, \overline{X}]$. Notice that $X \geq \underline{X}$ and the fact that $z_2^R(\pi_g, \pi_b) \geq z_2^f = 0$ for all $(\pi_g, \pi_b)$ imply that $U_R \geq U(z_2^R(\pi_g, \pi_b)) \geq U(z_2^f) \geq U_0$. It follows that:

$$
\begin{aligned}
G_g(z_2^R(\pi_g,\pi_b),0) &= (1-\gamma)(\rho_L-1)+\delta\left[U(z_2^R(\pi_g,\pi_b))-U(0)\right] \\
&\geq (1-\gamma)(\rho_L-1)>0.
\end{aligned}
\tag{15}
$$

Moreover, they also imply that:

$$
\begin{aligned}
G_b(z_2^R(\pi_g,\pi_b),0) &= (1-\gamma)\left[(\rho_L-1)-dD\right] \\
&\qquad + \delta\left[(1-d)U(z_2^R(\pi_g,\pi_b))-\left[(1-d)+d(1-\gamma)\right]U(0)\right] \\
&\leq (1-\gamma)\left[(\rho_L-1)-dD\right] \\
&\qquad + \delta\left[(1-d)U_R-\left[(1-d)+d(1-\gamma)\right]U_0\right]<0,
\end{aligned}
\tag{16}
$$

where the second inequality follows because of Assumption 4 and the fact that $X \leq \overline{X}$.

Since $G_g(z_2^R(\pi_g,\pi_b),0)>0>G_b(z_2^R(\pi_g,\pi_b),0)$ for all $(\pi_g,\pi_b)$, in this case, there exists a unique equilibrium in which the strategic regulator passes the bank in the first period if and only if the risky loan is good. ∎

## A.3 Proof of Proposition 3 (First-period equilibrium for $X < \underline{X}$ can be informative or tough)

Recall that $\underline{X}$ is defined such that $U_R = U_0$ (and is given by Eq. 13). Therefore, for all $X < \underline{X}$, we have $U_R < U_0$. $z_2^R(\pi_g,\pi_b) \geq z_2^f = 0$ thus implies that $U(z_2^R(\pi_g,\pi_b)) \leq U(z_2^f) = U(0)$. It follows that:

$$
\begin{aligned}
G_b(z_2^R(\pi_g,\pi_b),0) &= (1-\gamma)\left[(\rho_L-1)-dD\right] \\
&\qquad + \delta\left[(1-d)U(z_2^R(\pi_g,\pi_b))-\left[(1-d)+d(1-\gamma)\right]U(0)\right] \\
&\leq (1-\gamma)\left[(\rho_L-1)-dD\right] \\
&\qquad + \delta\left[(1-d)U(0)-\left[(1-d)+d(1-\gamma)\right]U(0)\right]<0.
\end{aligned}
\tag{17}
$$

This implies that $\pi_b = 0$. Before we proceed to prove this proposition, let us define $\bar{z}_1 < z_2^*$ as the level of prior reputation such that $z_2^R(1,0) = z_2^*$, i.e.,

$$
\frac{\left[\alpha+(1-\alpha)(1-d)\right]\bar{z}_1}{\left[\alpha+(1-\alpha)(1-d)\right]\bar{z}_1+\alpha(1-\bar{z}_1)} = z_2^*.
\tag{18}
$$

We now characterize the two possible types of equilibria: an informative equilibrium (i.e, one with $\pi_g = 1$) exists if and only if $G_g(z_2^R(1,0),0) \geq 0$, and a tough equilibrium (i.e., one with

$\pi_g < 1$) exists if and only if $G_g(z_2^R(\pi_g, 0), 0) \leq 0$ for some $\pi_g < 1$. More specifically, a tough equilibrium with $\pi_g \in (0,1)$ exists if and only if $G_g(z_2^R(\pi_g, 0), 0) = 0$ for some $\pi_g \in (0,1)$, and a tough equilibrium with $\pi_g = 0$ exists if and only if $G_g(z_2^R(0,0), 0) \leq 0$.

- If $z_1 \leq \bar{z}_1$, we have $z_2^R(1,0) \leq z_2^*$. Since $z_2^R(\pi_g, 0)$ is decreasing in $\pi_g$ and $z_2^R(0,0) = 1$, there exists $\hat{\pi}_g \in [0,1)$, such that $z_2^R(\hat{\pi}_g, 0) = z_2^*$ and $z_2^R(\pi_g, 0) \geq z_2^*$ if and only if $\pi_g \leq \hat{\pi}_g$. Using Eq. 7, we have

$$G_g(z_2^R(\pi_g, 0), 0) = (1-\gamma)(\rho_L - 1) + \begin{cases} 0, & \text{if } \pi_g > \hat{\pi}_g, \\ \delta\left[U_R - U_0\right)], & \text{if } \pi_g < \hat{\pi}_g, \\ (1-\lambda)\delta\left[U_R - U_0\right)] \text{ for some } \lambda \in [0,1], & \text{if } \pi_g = \hat{\pi}_g. \end{cases} \tag{19}$$

  Note that the first term of the above expression is positive by Assumption 4. From this expression, we have that, first, the informative equilibrium with $\pi_g = 1$ exists for all $\delta$. Second, consider the tough equilibria. Recall that the definition of $\underline{X}$, which is given by Eq. 13, implies that $U_R < U_0$ for all $X < \underline{X}$. The tough equilibrium with $\pi_g = 0$ thus exists if and only if $\delta \geq \delta_g$, where $\delta_g$ is defined by

$$(1-\gamma)(\rho_L - 1) + \delta_g\left[U_R - U_0\right] = 0. \tag{20}$$

  In addition, there may exist tough equilibria with $\pi_g \in (0,1)$. Specifically, if $\delta > \delta_g$ and $z_1 < \bar{z}_1$, which implies that $\hat{\pi}_g < 1$, then a tough equilibrium with $\pi_g = \hat{\pi}_g$ exists; if $\delta = \delta_g$, then a continuum of equilibria with $\pi_g \in [0, \hat{\pi}_g]$ exist. To summarize, a tough equilibrium with $\pi_g < 1$ exists if and only if $\delta \geq \delta_g$.

- If $z_1 > \bar{z}_1$, we have $z_2^R(\pi_g, 0) \geq z_2^R(1,0) > z_2^*$ for all $\pi_g \in [0,1]$, since $z_2^R(\pi_g, 0)$ is decreasing in $\pi_g$. This implies that, for all $\pi_g$, $U(z_2^R(\pi_g, 0)) = U_R < U(0) = U_0$ and thus $G_g(z_2^R(1,0), 0) \leq (<) 0$ for $\pi_g \in [0,1]$ if and only if $\delta \geq (>) \delta_g$. Therefore, the unique equilibrium is informative with $\pi_g = 1$ if $\delta < \delta_g$, the unique equilibrium is tough with $\pi_g = 0$ if $\delta > \delta_g$, and a continuum of equilibria with $\pi_g \in [0,1]$ exist if $\delta = \delta_g$.

∎

## A.4 Proof of Proposition 4 (First-period equilibrium for $X > \overline{X}$ can be informative or soft)

Since $X > \overline{X} > \underline{X}$, we have $G_g(z_2^R(\pi_g, \pi_b), 0) > 0$ for all $(\pi_g, \pi_b)$, as shown in the proof of Proposition 2. Therefore, $\pi_g = 1$ in any equilibrium.

We can now characterize the two possible types of equilibria: the informative equilibrium (i.e., one with $\pi_b = 0$) exists if and only if $G_b(z_2^R(1, 0), 0) \leq 0$, and a soft equilibrium (i.e., one with $\pi_b > 0$) exists if and only if $G_b(z_2^R(1, \pi_b), 0) \geq 0$ for some $\pi_b \in (0, 1]$. More specifically, a soft equilibrium with $\pi_b \in (0, 1)$ exists if and only if $G_b(z_2^R(1, \pi_b), 0) = 0$ for some $\pi_b \in (0, 1)$, and a soft equilibrium with $\pi_b = 1$ exists if and only if $G_b(z_2^R(1, 1), 0) \geq 0$.

- If $z_1 \leq \bar{z}_1$, we have $z_2^R(1, \pi_b) \leq z_2^R(1, 0) \leq z_2^*$ for all $\pi_b \in [0, 1]$, since $z_2^R(1, \pi_b)$ is decreasing in $\pi_b$. This implies that, for all $\pi_b$, $U(z_2^R(1, \pi_b)) = U(0) = U_0$ and thus $G_b(z_2^R(1, \pi_b), \pi_b) < 0$ for all $\pi_b$. Therefore the unique equilibrium is informative.

- If $z_1 > \bar{z}_1$, we have that $z_2^R(1, 0) > z_2^*$. Recall that Eq. 18 implies that $\bar{z}_1 < z_2^*$. Let us distinguish between the two cases: $z_1 \geq z_2^*$ and $z_1 \in (\bar{z}_1, z_2^*)$.

  - For $z_1 \geq z_2^*$, we have that $z_2^R(1, \pi_b) \geq z_2^R(1, 1) = z_1 \geq z_2^*$ for all $\pi_b \in [0, 1]$, where the first inequality follows because $z_2^R(1, \pi_b)$ is decreasing in $\pi_b$. It follows that $U(z_2^R(1, \pi_b)) = U_R > U_0 = U(0)$ for all $\pi_b \in [0, 1]$, where $U_R > U_0$ is implied by $X > \overline{X} > \underline{X}$. Therefore, the unique equilibrium is informative with $\pi_b = 0$ if $\delta < \delta_b$, the unique equilibrium is soft with $\pi_b = 1$ if $\delta > \delta_b$, and a continuum of equilibria with $\pi_b \in [0, 1]$ exist if $\delta = \delta_b$, where $\delta_b$ is defined by:

    $$(1 - \gamma)\left[(\rho_L - 1) - dD\right] + \delta_b\left[(1 - d)U_R - \left[(1 - d) + d(1 - \gamma)\right]U_0\right] = 0. \quad (21)$$

    Note that the first term of the left-hand side of the above expression is negative by Assumption 4, and the second term is positive for all $X > \overline{X}$, as implied by the definition of $\overline{X}$, which is given by Eq. 14.

  - For $z_1 \in (\bar{z}_1, z_2^*)$, we have that $z_2^R(1, \pi_b)$ is decreasing in $\pi_b$, and $z_2^R(1, 0) > z_2^* > z_1 = z_2^R(1, 1)$. Therefore there exists $\hat{\pi}_b \in (0, 1)$, such that $z_2^R(1, \hat{\pi}_b) = z_2^*$ and $z_2^R(1, \pi_b) \geq z_2^*$ if and only if $\pi_b \leq \hat{\pi}_b$. We thus have:

    $$G_b(z_2^R(1, \pi_b), 0) = (1 - \gamma)\left[(\rho_L - 1) - dD\right]$$

$$+\begin{cases} \delta\left[(1-d)U_R-\left[(1-d)+d(1-\gamma)\right]U_0\right], & \text{if } \pi_b<\hat{\pi}_b, \\ \delta\left[-d(1-\gamma)U_0\right], & \text{if } \pi_b>\hat{\pi}_b. \\ \lambda\delta\left[(1-d)U_R-\left[(1-d)+d(1-\gamma)\right]U_0\right] & \\ \quad +(1-\lambda)\delta\left[-d(1-\gamma)U_0\right] \text{ for some } \lambda\in[0,1], & \text{if } \pi_b=\hat{\pi}_b. \end{cases} \tag{22}$$

From the above expression, we have that, for $\pi_b > \hat{\pi}_b$, $G_b(z_2^R(1,\pi_b),0) < 0$; for $\pi_b < \hat{\pi}_b$, $G_b(z_2^R(1,\pi_b),0) \geq 0$ if and only if $\delta \geq \delta_b$, Therefore, the unique equilibrium is informative with $\pi_b = 0$ if $\delta < \delta_b$, the unique equilibrium is soft with $\pi_b = \hat{\pi}_b$ if $\delta > \delta_b$, and a continuum of equilibria with $\pi_b \in [0,\hat{\pi}_b]$ exist if $\delta = \delta_b$.

To summarize, for $z_1 > \bar{z}_1$, the unique equilibrium is informative with $\pi_b = 0$ if $\delta < \delta_b$, the unique equilibrium is soft with $\pi_b > 0$ if $\delta > \delta_b$, and the informative equilibrium coexists with soft equilibria if $\delta = \delta_b$.

∎

## A.5 Proof of Corollary 1 (Strategic delay of stress tests)

This corollary follows immediately from the proof of Proposition 4. ∎

## A.6 Proof of Corollary 2 (Availability of capital)

This corollary follows immediately from Propositions 3–4, the implicit function theorem, and the observation that $G_g(\cdot)$ is decreasing in $\gamma_1$, and $G_b(\cdot)$ is increasing in $\gamma_1$. ∎

## A.7 Proof of Corollary 3 (Stress test of systemic banks)

This corollary follows immediately from Propositions 3–4, the implicit function theorem, and the observation that $G_g(\cdot)$ is independent of $D_1$, and $G_b(\cdot)$ is decreasing in $D_1$. ∎

## A.8 Proof of Proposition 5 (Bank and regulator both learn asset quality)

We prove this proposition by analyzing the three regions of $X$ separately.

- $X \in [\underline{X}, \overline{X}]$. The proof is identical to the proof of Proposition 2.

- $X < \underline{X}$. The characterization of the equilibrium follows the logic of the proof of Proposition 3. We have that $G_b(z_{2,b}^R(\pi_b), z_{2,b}^f) < 0$ for all $\pi_b$, and, therefore, $\pi_b = 0$ in any equilibrium. The equilibrium is, thus, either informative or tough. The informative equilibrium (i.e., one with $\pi_g = 1$) exists if and only if $G_g(z_{2,g}^R(1), 0) \geq 0$, whereas a tough equilibrium (i.e., one with $\pi_g < 1$) exists if and only if $G_g(z_{2,g}^R(\pi_g), 0) \leq 0$ for some $\pi_g < 1$.

  We first show that the parameter space for which the informative equilibrium exists is (weakly) larger than in the baseline model, and that for which the tough equilibrium exists is (weakly) smaller than in the baseline model. This follows because, for any $\pi_g \in [0,1]$, $G_g(z_{2,g}^R(\pi_g), 0) \geq G_g(z_2^R(\pi_g, 0), 0)$, since $G_g(z_2^R, 0)$ is decreasing in $z_2^R$ and $z_{2,g}^R(\pi_g) < z_2^R(\pi_g, 0)$.

  We now show that the parameter space for which the informative equilibrium exists is strictly larger than in the baseline model, by demonstrating that there exist parameter values for which the unique equilibrium in the baseline model is tough, whereas the informative equilibrium exists in the model in which the bank observes $q_t$. Specifically, suppose that $z_1 = \bar{z}_1 + \epsilon$, where $\epsilon > 0$, and $\delta > \delta_g$, where $\delta_g$ is defined by Eq. 20. In this case, $z_1 > \bar{z}_1$ implies that the unique equilibrium is tough, by Proposition 3. Recall that $z_1 > \bar{z}_1$ implies that $\Delta(z_2^R(1,0)) > 0$. Since $z_{2,g}^R(1) < z_2^R(1,0)$ and $\Delta(z_2)$ is increasing in $z_2$, we have that, there exists $\epsilon$ sufficiently small such that $\Delta(z_2^R(1,0)) > 0 > \Delta(z_{2,g}^R(1))$. This implies that $G_g(z_{2,g}^R(1), 0) > 1$ and therefore the informative equilibrium indeed exists when the bank observes $q_t$.

  Finally, we show that the parameter space for which a tough equilibrium exists is identical to the baseline model. Since $G_g(z_2^R, 0)$ is decreasing in $z_2^R$, and $z_{2,g}^R(\pi_g)$ is decreasing in $\pi_g$, $G_g(z_{2,g}^R(\pi_g), 0)$ is increasing in $\pi_g$, and, therefore, a tough equilibrium exists if and only if $G_g(z_{2,g}^R(0), 0) = G_g(z_1, 0) \leq 0$. Similarly, we have that $G_g(z_2^R(\pi_g, 0), 0)$ is increasing in $\pi_g$, and, therefore a tough equilibrium in the baseline model exists if and only if $G_g(z_2^R(0,0), 0) = G_g(z_1, 0) \leq 0$. It follows that the condition and, thus, the parameter space for which a tough equilibrium exists is identical to the baseline model.

- $X > \overline{X}$. The characterization of the equilibrium follows the logic of the proof of Proposition 4. Recall that $z_{2,g}^R(\pi_g) > z_{2,g}^f = 0$ for all $\pi_g \in [0,1]$. We then have that $G_g(z_{2,g}^R(\pi_g), z_{2,g}^f) > 0$ for all $\pi_g$, and, therefore, $\pi_g = 1$ in any equilibrium. The equilibrium is, thus, either informative or soft. The informative equilibrium (i.e., one with $\pi_b = 0$) exists if and only if $G_b(z_{2,b}^R(0), 0) \leq 0$, whereas a soft equilibrium (i.e., one with $\pi_b > 0$) exists if and only if $G_b(z_{2,b}^R(\pi_b), 0) \geq 0$ for some $\pi_b > 0$.

We first show that the parameter space for which the informative equilibrium exists is (weakly) smaller than in the baseline model, and that for which the soft equilibrium exists is (weakly) larger than in the baseline model. This follows because, for any $\pi_b \in [0,1]$, $G_b(z^R_{2,b}(\pi_b), 0) \geq G_b(z^R_2(1, \pi_b), 0)$, since $G_b(z^R_2, 0)$ is increasing in $z^R_2$ and $z^R_{2,b}(\pi_b) \geq z^R_2(1, \pi_b)$.

We now show that the above statement holds with strict inequality by demonstrating that there exist parameter values for which the informative equilibrium is the unique equilibrium in the baseline model but only soft equilibria exist in the model in which the bank observes $q_t$. Specifically, suppose that $\beta > \bar{\beta}$, $z_1 = \bar{z}_1 - \epsilon$, where $\epsilon > 0$, and $\delta > \delta_b$, where $\delta_b$ is defined by Eq. 21. In this case, $z_1 < \bar{z}_1$ implies that the unique equilibrium in the baseline model is informative, by Proposition 4. Recall that $z_1 < \bar{z}_1$ implies that $\Delta(z^R_2(1,0)) < 0$. Since $z^R_{2,b}(0) > z^R_2(1,0)$ and $\Delta(z_2)$ is increasing in $z_2$, we have that, there exists $\epsilon$ sufficiently small, such that $\Delta(z^R_2(1,0)) < 0 < \Delta(z^R_{2,b}(0))$. This and $\delta > \delta_b$ imply that $G_b(z^R_{2,b}(0), 0) > 0$. Therefore, an informative equilibrium does not exist when the bank observes $q_t$, whereas by continuity of $G_b(z^R_{2,b}(\pi_b), 0)$ in $\pi_b$, a soft equilibrium does exist.

∎

## A.9 Proof of Proposition 6 (Behavioral regulator is truthful)

Let $z_t$ denote the bank's ex ante belief in period $t \in \{1, 2\}$ that the regulator is the truthful type. Following backward induction, we first solve for the equilibrium in the second period. Analogous to Proposition 1, the equilibrium in the second period is characterized in the following lemma.

**Lemma 2.** *In the second period, there exists a unique equilibrium, in which the bank originates a risky loan if and only if $\beta \leq \bar{\beta}$, where $\bar{\beta}$ is defined in Proposition 1.*

This lemma follows similar logic as in the proof of Proposition 1. Given that the behavioral regulator passes the second bank at stage 3 if and only if the loan is good, the bank originates a risky loan at stage 1 if and only if $\Delta(0) \geq 0$, which is the case if and only if $\beta \leq \bar{\beta}$.

Since this lemma implies that the second bank's investment choice does not depend on the regulator's reputation, this proposition then follows. ∎

## A.10 Proof of Proposition 7 (Behavioral regulator is strict)

Let $\upsilon_t$ denote the bank's ex ante belief in period $t \in \{1, 2\}$ that the regulator is the strategic type. That is, a higher $\upsilon_t$ reflects a belief by the bank that the regulator is likely to be softer.

Following backward induction, we start by considering the second period. As in the baseline model, we focus on describing the equilibrium stress test outcome given that the bank originates a risky loan at stage 1. The strategic regulator at stage 2 passes the bank if and only if the loan is good, as implied by Assumption 4.

Conditional on failing the stress test, the probability that the bank is good is given by:

$$\alpha_2^S = \frac{\alpha(1-\upsilon_2)}{\alpha(1-\upsilon_2)+(1-\alpha)}. \tag{23}$$

Recapitalization is feasible if and only if the opportunity cost of capital is low ($\rho = \rho_L$), in which case the equity given to the capital providers is a fraction $\phi^S$, determined by:

$$\phi^S \left[\alpha_2^S + (1-\alpha_2^S)(1-d)\right] R = \rho_L + \beta\left(\left[\alpha_2^S + (1-\alpha_2^S)(1-d)\right] R - \rho_L\right). \tag{24}$$

At stage 1, the bank originates a risky loan if and only if:

$$\begin{aligned}&\left[\alpha\left[1-(1-\upsilon_2)(1-\gamma)\right] + (1-\alpha)\gamma(1-d)\right](R-1)\\ &\qquad + (1-\gamma)(1-\phi^S)\left[\alpha(1-\upsilon_2)+(1-\alpha)(1-d)\right] R \geq R_0 - 1.\end{aligned} \tag{25}$$

Analogous to Proposition 1, the equilibrium in the second period is characterized in the following lemma.

**Lemma 3.** *In the second period, there exists a unique equilibrium. There exists a unique threshold $\upsilon_2^*$, such that, in equilibrium, the bank originates a risky loan if and only if the probability that the regulator is strategic $\upsilon_2 \geq \upsilon_2^*$, given by $\Delta^S(\upsilon_2^*) = 0$, where*

$$\begin{aligned}\Delta^S(\upsilon_2) \equiv &\left[\alpha + (1-\alpha)(1-d)\right](R-1) - (R_0 - 1)\\ &- \alpha(1-\upsilon_2)(1-\gamma)\left(\rho_L - 1 + \beta\left[R-\rho_L\right]\right)\\ &- (1-\alpha)(1-\gamma)\left(\rho_L - (1-d) + \beta\left[(1-d)R - \rho_L\right]\right).\end{aligned} \tag{26}$$

*Moreover, $\upsilon_2^* < 1$ if and only if $\beta < \bar{\beta}$, where $\bar{\beta}$ is defined in Proposition 1. Finally, there exists $\underline{\beta} < \bar{\beta}$, such that $\upsilon_2^* > 0$ if and only if $\beta > \underline{\beta}$.*

*Proof.* Eq. 26 is obtained by substituting Eq. 24 into Eq. 25 to eliminate $\phi^S$. Moreover, notice that $\Delta^S(\upsilon_2)$ is increasing in $\upsilon_2$, and $\Delta^S(1) = \Delta(0)$. Therefore, $\upsilon_2^* < 1$ if and only if $\beta < \bar{\beta}$, since $\bar{\beta}$ is defined such that $\Delta(0) = 0$ at $\beta = \bar{\beta}$. Finally, $\upsilon_2^* > 0$ if and only if $\Delta^S(0) < 0$. This is the case if and only if $\beta > \underline{\beta}$, where $\underline{\beta}$ is defined by

$$\begin{aligned}[\alpha + (1-\alpha)(1-d)]\,(R-1) - (R_0 - 1) \\ &- \alpha(1-\gamma)\left(\rho_L - 1 + \underline{\beta}\,[R - \rho_L]\right) \\ &- (1-\alpha)(1-\gamma)\left(\rho_L - (1-d) + \underline{\beta}\,[(1-d)R - \rho_L]\right) = 0. \end{aligned} \tag{27}$$

We have that $\underline{\beta} < \bar{\beta}$ because $\Delta^S(\upsilon_2)$ is increasing in $\upsilon_2$ and decreasing in $\beta$. ∎

Note that the term $\Delta^S(\upsilon_2)$ looks different than the analogous expression in the baseline. The dilution term is different because, first, even a good bank is recapitalized if the regulator is strict (the second line of Eq. 26), and second, a bad bank is always recapitalized and not just when the regulator is strategic (the third line of Eq. 26).

As in the baseline model, we impose the following additional assumption (instead of Assumption 5 of the baseline model) to restrict attention to the interesting set of the parameter space in which the bank's project choice indeed varies depending on the regulator's reputation, i.e., $\upsilon_2^* \in (0, 1)$.

**Assumption $5^S$.** *$\beta \in (\underline{\beta}, \bar{\beta})$, where $\underline{\beta}$ and $\bar{\beta}$ are defined in Lemma 3 and Proposition 1, respectively.*

It then follows that the strategic regulator's expected surplus from the bank in the second period, for a given reputation $\upsilon_2$, denoted by $U^S(\upsilon_2)$, is given by:

$$U^S(\upsilon_2) = \begin{cases} U_R, & \text{if } \upsilon_2 > \upsilon_2^*, \\ U_0, & \text{if } \upsilon_2 < \upsilon_2^*, \\ \lambda U_R + (1-\lambda)U_0 \text{ for some } \lambda \in [0,1], & \text{if } \upsilon_2 = \upsilon_2^*. \end{cases} \tag{28}$$

We now move to analyze the equilibrium stress test of the regulator for the bank in the first period, given the equilibrium in the second period. The incentives of the strategic regulator to pass the first-period bank are given by:

$$\begin{aligned} G_g^S(\upsilon_2^p, \upsilon_2^R) &= (1-\gamma)(\rho_L - 1) + \delta\left[U^S(\upsilon_2^p) - U^S(\upsilon_2^R)\right], \\ G_b^S(\upsilon_2^p, \upsilon_2^R, \upsilon_2^0) &= (1-\gamma)\left[(\rho_L - 1) - dD\right] \\ &\quad + \delta\left[(1-d)U^S(\upsilon_2^p) - (1-d)U^S(\upsilon_2^R) - d(1-\gamma)U^S(\upsilon_2^0)\right]. \end{aligned} \tag{29}$$

Analogous to Eq. 8, the first term in both equations in Eq. 29 represents the net gain in terms of the expected surplus from the bank in the first period, and the second term represents the reputation concern in terms of the expected surplus from the bank in the second period. In contrast to the baseline setup, passing the first-period bank reveals that the regulator is strategic- i.e., $\upsilon_2^p = 1$. Subsequently, the bank continues to the second period with probability 1 if its risky loan is good, or with probability $1-d$ if its risky loan is bad. The term $\upsilon_2^R$ ($\upsilon_2^0$) is the posterior belief held by the bank about the probability that the regulator is strategic, given that the first-period bank *fails* the stress test and that the realized payoff is $R$ (0).[35] Since the bank recapitalizes with probability $1-\gamma$ after failing the stress test, it continues to the second period with probability 1 if its payoff is $R$ and with probability $1-\gamma$ if its payoff is 0. In equilibrium, the posterior probabilities are given by:

$$\begin{aligned}\upsilon_2^R(\pi_g,\pi_b) &= \frac{\left[\alpha(1-\pi_g)+(1-\alpha)(1-d)(1-\pi_b)\right]\upsilon_1}{\left[\alpha(1-\pi_g)+(1-\alpha)(1-d)(1-\pi_b)\right]\upsilon_1+\left[\alpha+(1-\alpha)(1-d)\right](1-\upsilon_1)},\\ \upsilon_2^0(\pi_b) &= \frac{(1-\pi_b)\upsilon_1}{(1-\pi_b)\upsilon_1+(1-\upsilon_1)}.\end{aligned} \tag{30}$$

In particular, if the first bank fails the stress test, the bank updates its belief to $\upsilon_2^R \le \upsilon_1$ or $\upsilon_2^0 \le \upsilon_1$ if the bank realizes a payoff of $R$ or 0, respectively, reflecting the fact that the strategic regulator is less likely than the strict regulator to fail a bank. Moreover, we have $\upsilon_2^R < \upsilon_2^0$, since the strategic regulator is more likely to fail a bad bank than a good bank.

We can now prove this proposition by considering the three regions of $X$ once again.

- $X \in [\underline{X}, \overline{X}]$. Recall that $\upsilon_2^p = 1 \ge \upsilon_2^0(\pi_b) \ge \upsilon_2^R(\pi_g,\pi_b)$ for all $(\pi_g,\pi_b)$. Therefore, $X \ge \underline{X}$ implies that $U_R \ge U^S(\upsilon_2^p) \ge U^S(\upsilon_2^0(\pi_b)) \ge U^S(\upsilon_2^R(\pi_g,\pi_b)) \ge U_0$ for all $(\pi_g,\pi_b)$. It follows that:

$$\begin{aligned}G_g^S(\upsilon_2^p,\upsilon_2^R(\pi_g,\pi_b)) &= (1-\gamma)(\rho_L-1)+\delta\left[U^S(\upsilon_2^p)-U^S(\upsilon_2^R(\pi_g,\pi_b))\right]\\ &\ge (1-\gamma)(\rho_L-1) > 0.\end{aligned} \tag{31}$$

  Moreover, we have:

$$\begin{aligned}&G_b^S(\upsilon_2^p,\upsilon_2^R(\pi_g,\pi_b),\upsilon_2^0(\pi_b))\\ &= (1-\gamma)\left[(\rho_L-1)-dD\right]\\ &\qquad + \delta\left[(1-d)U^S(\upsilon_2^p)-(1-d)U^S(\upsilon_2^R(\pi_g,\pi_b))-d(1-\gamma)U^S(\upsilon_2^0(\pi_b))\right]\\ &\le (1-\gamma)\left[(\rho_L-1)-dD\right]+\delta\left[(1-d)U_R-\left[(1-d)+d(1-\gamma)\right]U_0\right] < 0,\end{aligned} \tag{32}$$

  where the last inequality follows from $X \le \overline{X}$.

[35] We abuse notation slightly in that in the baseline model, the superscript $R$ refers to a realization of $R$ after a pass, while here, $R$ refers to a realization of $R$ after a fail.

Since $G^S_g(\upsilon^p_2, \upsilon^R_2(\pi_g, \pi_b)) > 0 > G^S_b(\upsilon^p_2, \upsilon^R_2(\pi_g, \pi_b), \upsilon^0_2(\pi_b))$ for all $(\pi_g, \pi_b)$, there exists a unique (informative) equilibrium in which the regulator passes the bank in the first period if and only if the risky loan is good.

- $X < \underline{X}$. The fact that $\upsilon^p_2 = 1$ is greater than or equal to both $\upsilon^R_2(\pi_g, \pi_b)$ and $\upsilon^0_2(\pi_b)$ implies that $U^S(\upsilon^p_2)$ is less than or equal to both $U^S(\upsilon^R_2(\pi_g, \pi_b))$ and $U^S(\upsilon^0_2(\pi_b))$ for all $(\pi_g, \pi_b)$. It follows that:

$$\begin{aligned}
&G^S_b(\upsilon^p_2, \upsilon^R_2(\pi_g, \pi_b), \upsilon^0_2(\pi_b)) \\
&= (1-\gamma)\left[(\rho_L - 1) - dD\right] \\
&\qquad + \delta\left[(1-d)U^S(\upsilon^p_2) - (1-d)U^S(\upsilon^R_2(\pi_g, \pi_b)) - d(1-\gamma)U^S(\upsilon^0_2(\pi_b))\right] \\
&\le (1-\gamma)\left[(\rho_L - 1) - dD\right] + \delta\left[(1-d)U^S(\upsilon^p_2) - \left[(1-d) + d(1-\gamma)\right]U^S(\upsilon^p_2)\right] < 0.
\end{aligned} \tag{33}$$

Therefore, $\pi_b = 0$ in any equilibrium. Let us define $\bar{\upsilon}_1 > \upsilon^*_2$ as the level of prior reputation $\upsilon_1$ such that $\upsilon^R_2(1, 0) = \upsilon^*_2$, where $\bar{\upsilon}_1 \in (\upsilon^*_2, 1)$, i.e.,

$$\frac{(1-\alpha)(1-d)\bar{\upsilon}_1}{(1-\alpha)(1-d)\bar{\upsilon}_1 + \left[\alpha + (1-\alpha)(1-d)\right](1-\bar{\upsilon}_1)} = \upsilon^*_2. \tag{34}$$

We can now characterize the two possible types of equilibria: the informative equilibrium with $\pi_g = 1$ exists if and only if $G^S_g(1, \upsilon^R_2(1, 0)) \ge 0$, and a tough equilibrium with $\pi_g < 1$ exists if and only if $G^S_g(1, \upsilon^R_2(\pi_g, 0)) \le 0$ for some $\pi_g < 1$.

  - If $\upsilon_1 \ge \bar{\upsilon}_1$, we have that $\upsilon^R_2(\pi_g, 0) \ge \upsilon^R_2(1, 0) \ge \upsilon^*_2$ for all $\pi_g \in [0, 1]$, since $\upsilon^R_2(\pi_g, 0)$ is decreasing in $\pi_g$. This implies that, for all $\pi_g$, $U^S(\upsilon^R_2(\pi_g, 0)) = U(1)$ and thus $G^S_g(1, \upsilon^R_2(\pi_g, 0)) > 0$ for all $\pi_g$. Therefore the unique equilibrium is informative.
  - If $\upsilon_1 < \bar{\upsilon}_1$, we have that $\upsilon^R_2(1, 0) < \upsilon^*_2$. Recall that Eq. 34 implies that $\bar{\upsilon}_1 > \upsilon^*_2$. Let us distinguish between the two cases: $\upsilon_1 \le \upsilon^*_2$ and $\upsilon_1 \in (\upsilon^*_2, \bar{\upsilon}_1)$.
    * For $\upsilon_1 \le \upsilon^*_2$, we have that $\upsilon^R_2(\pi_g, 0) \le \upsilon^R_2(0, 0) = \upsilon_1 \le \upsilon^*_2$, where the first inequality follows because $\upsilon^R_2(\pi_g, 0)$ is decreasing in $\pi_g$. It follows that $U^S(\upsilon^R_2(\pi_g, 0)) = U_0 < U^S(1) = U_R$ for all $\pi_g \in [0, 1]$. Therefore, the unique equilibrium is informative with $\pi_g = 1$ if $\delta < \delta_g$, the unique equilibrium is tough with $\pi_g = 0$ if $\delta > \delta_g$, and a continuum of equilibria with $\pi_g \in [0, 1]$ exist if $\delta = \delta_g$, where $\delta_g$ is defined by Eq. 20.
    * For $\upsilon_1 \in (\upsilon^*_2, \bar{\upsilon}_1)$, we have that $\upsilon^R_2(\pi_g, 0)$ is decreasing in $\pi_g$, and $\upsilon^R_2(0, 0) = \upsilon_1 > \upsilon^*_2 > \upsilon^R_2(1, 0)$. Therefore there exists $\hat{\pi}^S_g \in (0, 1)$, such that $\upsilon^R_2(\hat{\pi}^T_g, 0) =$

$\upsilon_2^*$ and $\upsilon_2^R(\pi_g, 0) \leq \upsilon_2^*$ if and only if $\pi_g \geq \hat{\pi}_g^S$. We thus have

$$G_g^S(1, \upsilon_2^R(\pi_g, 0)) = (1-\gamma)(\rho_L - 1) \\ + \begin{cases} 0, & \text{if } \pi_g < \hat{\pi}_g^S, \\ \delta\left[U_R - U_0\right], & \text{if } \pi_g > \hat{\pi}_g^S, \\ (1-\lambda)\delta\left[U_R - U_0\right] \text{ for some } \lambda \in [0,1], & \text{if } \pi_g = \hat{\pi}_g^S. \end{cases} \tag{35}$$

From the above expression, we have that, for $\pi_g < \hat{\pi}_g$, $G_g^S(1, \upsilon_2^R(\pi_g, 0)) > 0$; for $\pi_g > \hat{\pi}_g$, $G_g^S(1, \upsilon_2^R(\pi_g, 0)) \leq 0$ if and only if $\delta \geq \delta_g$, where $\delta_g$ is defined by Eq. 20. It follows that, the unique equilibrium is informative with $\pi_g = 1$ if $\delta < \delta_g$, the unique equilibrium is tough with $\pi_g = \hat{\pi}_g$ if $\delta > \delta_g$, and a continuum of equilibria with $\pi_g \in [\hat{\pi}_g, 1]$ exist if $\delta = \delta_g$.

To summarize, for $\upsilon_1 < \bar{\upsilon}_1$, the unique equilibrium is informative with $\pi_g = 1$ if $\delta < \delta_g$, the unique equilibrium is tough with $\pi_g < 1$ if $\delta > \delta_g$, and the informative equilibrium coexists with tough equilibria if $\delta = \delta_g$.

- $X > \overline{X}$. This implies that $X > \underline{X}$, which in turn implies that $G_g^S(\upsilon_2^p, \upsilon_2^R(\pi_g, \pi_b)) > 0$ for all $(\pi_g, \pi_b)$, as shown for the first case in which $X \in [\underline{X}, \overline{X}]$. Therefore, $\pi_g = 1$ in any equilibrium.

  We can now characterize the two possible types of equilibria: the informative equilibrium with $\pi_b = 0$ exists if and only if $G_b^S(1, \upsilon_2^R(1, 0), \upsilon_2^0(0)) \leq 0$, and a soft equilibrium with $\pi_b > 0$ exists and only if $G_b^S(1, \upsilon_2^R(1, \pi_b), \upsilon_2^0(\pi_b)) \geq 0$ for some $\pi_b > 0$.

  - If $\upsilon_1 \geq \bar{\upsilon}_1$, we have $\upsilon_2^R(1, 0) \geq \upsilon_2^*$. First, the informative equilibrium exists for all $\delta$. To see this, we have $\upsilon_2^0(0) \geq \upsilon_2^R(1, 0) \geq \upsilon_2^*$, implying that $U^S(1) = U^S(\upsilon_2^R(1, 0)) = U_R$ and therefore

$$G_b^S(1, \upsilon_2^R(1, 0), \upsilon_1(0)) = (1-\gamma)\left[(\rho_L - 1) - dD\right] - \delta d(1-\gamma)U^T(s_1) < 0,$$

    for all $\delta$. Second, consider the tough equilibrium. Notice that $G_b^S(1, \upsilon_2^R(1, \pi_b), \upsilon_2^0(\pi_b))$ is increasing in $\pi_b$, which follows because $G_b^S(1, \upsilon_2^R, \upsilon_2^0)$ is decreasing in $\upsilon_0^R$ and $\upsilon_2^0$, while $\upsilon_0^R(1, \pi_b)$ and $\upsilon_2^0(\pi_b)$ given by Eq. 30 are decreasing in $\pi_b$. That $G_b^S(1, \upsilon_2^R(1, \pi_b), \upsilon_2^0(\pi_b))$ is increasing in $\pi_b$ then implies that

$$\inf_{\pi_b \in (0,1]} G_b^S(1, \upsilon_2^R(1, \pi_b), \upsilon_2^0(\pi_b)) = G_b^S(1, \upsilon_2^R(1, 1), \upsilon_2^0(1)). \tag{36}$$

Therefore an equilibrium with $\pi_b > 0$ exists if and only if $G_b^S(1, \upsilon_2^R(1,1), \upsilon_2^0(1)) = G_b^S(1,0,0) \geq 0$. Using Eq. 29 and the definition of $\delta_b$ in Eq. 21, this is the case if and only if $\delta \geq \delta_b$.

– If $\upsilon_1 \in [\upsilon_2^*, \bar{\upsilon}_1)$, we have $\upsilon_2^R(1,0) < \upsilon_2^* \leq \upsilon_2^0(0) = s_1$. First, consider the informative equilibrium. This equilibrium exists for all $\delta$ if

$$(1-d)\left[U_R - U_0\right] - d(1-\gamma)U_R \leq 0.$$

Otherwise, the informative equilibrium exists if and only if $\delta \leq \delta_b^S$, where $\delta_b^S > \delta_b$ is defined by

$$(1-\gamma)\left[(\rho_L - 1) - dD\right] + \delta_b^S\left[(1-d)\left[U_R - U_0\right] - d(1-\gamma)U_R\right] = 0.$$

Second, following the same logic as for previous case in which $\upsilon_1 \geq \bar{\upsilon}_1$, we have that, a soft equilibrium with $\pi_b > 0$ if and only if $G_b^S(1,0,0) \geq 0$ or, equivalently, $\delta \geq \delta_b$.

– If $\upsilon_1 < \upsilon_2^*$, we have $\upsilon_2^R(1,\pi_b) \leq \upsilon_2^R(1,0) \leq \upsilon_2^0(0) = \upsilon_1 < \upsilon_2^*$, where the first inequality follows because $\upsilon_2^R(1,\pi_b)$ is decreasing in $\pi_b$. This implies that, for all $\pi_b$, $U^S(\upsilon_2^R(1,\pi_b)) = U^S(\upsilon_2^0(\pi_b)) = U_0$. Therefore, the unique equilibrium is informative with $\pi_b = 0$ if $\delta < \delta_b$, the unique equilibrium is soft with $\pi_b = 1$ if $\delta > \delta_b$, and a continuum of equilibria with $\pi_b \in [0,1]$ exist if $\delta = \delta_b$.

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