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The Effects of a Low Interest Rate Environment on Life Insurers

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Abstract

Low interest rates are becoming a threat to the stability of the life insurance industry, especially in countries such as Germany, where products with relatively high guaranteed returns sold in the past still represent a prominent share of the total portfolio. This contribution aims to assess and quantify the effects of the current low interest rate phase on the balance sheet of a representative German life insurer, given the current asset allocation and the outstanding liabilities. To do so, we generate a stochastic term structure of interest rates as well as stock market returns to simulate investment returns of a stylized life insurance business portfolio in a multi-period setting. Based on empirically calibrated parameters, we can observe the evolution of the life insurers' balance sheet over time with a special focus on their solvency situation. To account for different scenarios and in order to check the robustness of our findings, we calibrate different capital market settings and different initial situations of capital endowment. Our results suggest that a prolonged period of low interest rates would markedly affect the solvency situation of life insurers, leading to a relatively high cumulative probability of default, especially for less capitalized companies. In addition, the new reform of the German life insurance regulation has a beneficial effect on the cumulative probability of default, as a direct consequence of the reduction of the payouts to policyholders.

Keywords: Life Insurers; Minimum Return Guarantees; Low Interest Rates; Risk Assessment; Solvency II.

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1 Introduction

According to the European Insurance and Occupational Pension Authority (EIOPA)'s recent Financial Stability Report (EIOPA, 2013), the current low interest yield environment represents the most prominent risk for life insurers, which are currently struggling to pay guaranteed rates of return and to maintain strong profitability in the long term. Moreover, a study conducted by Swiss Re (2012) highlights that in some jurisdictions, such as Germany, the U.S. and Italy, the exposure to interest rate risks of savings products with guaranteed minimum return appears to be particularly high.[1] In Germany, life insurers typically offer products with minimum return guarantees and minimum profit participation, where the maximally allowed minimum return is set by the regulator based on the presently achievable interest rates. The minimum return set at the inception cannot be changed during the lifetime of the contract. The natural implication of this product feature is the simultaneous presence of products with different minimum returns in the insurers' portfolio. In addition, the Financial Stability Review 2013 released by the Deutsche Bundesbank (2013) reports the results of stress scenarios conducted on German life insurers. According to the report, persistently low interest rates would be deleterious for the solvency situation of a subset of insurers. Particularly under the most severe scenario, more than one-third of all life insurers operating in Germany would not be able to meet the regulatory capital requirements by 2023.[2] Finally, the report indicates that high guaranteed returns are the main threat for the solvency of German life insurers.

In the literature, products with minimum guaranteed return and their exposure to the interest rate risk are extensively analyzed. A paper by Holsboer (2000) describes qualitatively the potential impact that the worldwide downward trend in interest rates could have on the life insurance industry in the presence of products with minimum guarantees. The paper highlights how the duration mismatch between the asset and the liability side plays a major role. As the Japanese case showed, when prevailing interest rates are substantially lower than they were at the time of the inception of the contracts, the existing stock of liabilities becomes more expensive to fund, as assets that come

[1] The study analyzes mainstream products for all key insurance markets worldwide: in Germany endowment insurance policies (*Kapitallebensversicherung*) (that in 2011 accounted for a 57% share of total premiums), in the U.S. *deferred fixed annuities*, *universal life* and *participating whole life* (together accounting for 70% of US life insurers' general account reserves in 2011) and in Italy with-profits and whole life policies (*polizze rivalutabili*).

[2] The analysis was conducted under the currently in force capital requirements, i.e. Solvency I.

due would be reinvested at a lower rate of return. The duration (and convexity) mismatch was also analyzed in Lee and Stock (2000): their theoretical analysis, although not specific to life insurers but rather concerning financial institutions in general, indicates that interest rate risk can be eliminated via a duration-match strategy, whereas in the presence of embedded options, the same strategy can induce significant duration and convexity mismatches, the latter being the most detrimental for the value of the equity. More specific to the life insurance industry, Briys and De Varenne (1997), in a theoretical setting, provide more consistent interest rate elasticity and duration measurements and argue that due to embedded options, the actual duration of liabilities is significantly different from the traditional Macaulay method. Minimum profit participation mechanisms are the focus of Grosen and Løchte Jørgensen (2000). They present a theoretical analysis of two different types of profit participation contracts which also include a surrender option. In their contract specification, the guaranteed return and the profit participation mechanism play a fundamental role. The authors highlight how a deterioration of the earning possibilities can be deleterious in the presence of high bonus returns (e.g. a minimum return guarantee). Kling et al. (2007a) conduct a numerical analysis for three different profit distribution policies and suggest that allowing the management to accumulate resources during years of high asset returns in order to distribute them in the case of an underperformance in bad years substantially reduces the shortfall risk of the insurer while allowing for higher guaranteed interest rates. Kling et al. (2007b) analyze the interaction between the profit distribution policy of a life insurer under German regulatory rules, the guaranteed return on the contracts and the asset allocation. Their findings show that the reserve quota[3] strongly influences the shortfall probability and that high guaranteed interest rates have a major influence on the solvency of the insurer when the capital buffer is at a low level. Gatzert (2008) conduct numerical analyses by including additional sources of risks (such as surrender and mortality risk) and find out that the main product features, such as early death or early surrender of the policyholder, are the significant drivers of the default risk of the insurer. The surrender option value embedded in many life insurance contracts was also analyzed by Albizzati and Geman (1994), who stress how changes in prevailing interest rates are a major challenge for insurers that provide products featuring such an option: as policyholders in times of highly volatile financial market returns may opt for more attractive investments elsewhere, life insurers have the incentive to offer higher guaranteed

[3] The ratio between the capital buffer and the technical reserves.

returns with a consequent increase in their interest rate risk exposure. Schmeiser and Wagner (2012) investigate the relation between interest rate guarantees, solvency requirements and asset allocation for life insurers. Their findings suggest that if the risk-free interest rate (i.e. the return on the bond portfolio) approaches the guaranteed interest rate, the equity capital of the insurers drifts towards zero, while the investment strategy tends to become less risky.

An empirical contribution that attempts to quantitatively estimate the impact of distribution policies and guaranteed interest rates for the German life insurers in the presence of low capital market returns was proposed by Wedow and Kablau (2011). The authors forecast different adverse capital market scenarios and observe the development of the Bonus and Rebate Provisions reserve (BPR), an important balance sheet item in the German life insurance industry because products very often embed profit participation.[4] Their findings suggest that given the outstanding stock of products with guaranteed minimum return, only in the case of very adverse market conditions would the buffer quickly run out of funds, thus leading to a direct reduction in equity capital with severe consequences for the solvency situation.

Thus in our contribution, we attempt to assess and quantify the effects of a prolonged period of low interest rates on the solvency situation of a representative (average) German life insurer by proposing a balance sheet model of a life insurer that attempts to incorporate many of the features that are usually analyzed separately in the literature, namely *i*) a minimum profit participation, *ii*) an additional return distribution mechanism, *iii*) a dynamic asset and liability portfolio where the former follows a multiple asset allocation and the latter features an existing stock of savings products with different minimum guaranteed returns, and finally *iv*) a distinction between book values and market values.[5] The last feature is an especially distinctive trait of our analysis, since existing studies such as the Deutsche Bundesbank Financial Stability Review (Deutsche Bundesbank, 2013), are based on the current Solvency I capital requirements and therefore rely on book values. The focus on Germany is justified for two reasons: on the one hand, as highlighted by Swiss Re (2012), Germany is one of the most exposed countries with respect to interest rate risk, mainly because of the popularity of minimum guaranteed products in the German life insurance market, and because

[4] A part of the BPR serves as a capital buffer during times of lower return on assets, therefore its role during a prolonged period of low market return is crucial.

[5] The distinction between book value and market value becomes necessary due to the fact that the profit distribution mechanism is based on German GAAP, i.e. book value accounting, whereas Solvency II regulation is based on a market-consistent valuation.

of the minimum participation mechanism imposed by the regulator. On the other hand, the present monetary policy conducted by the European Central Bank (ECB) results in extraordinarily low interest rates, especially for German sovereign bonds. Hence, German life insurers are not only facing an expensive stock of products sold with a high minimum guaranteed return, but they are also facing poor investment results, since German sovereign bonds account for a large portion of the aggregated asset portfolio of life insurers. A similar situation was observed in Japan in the 1990s: Then, the Japanese life insurance industry faced a long period of very low interest rates since the mid-1990s coupled with relatively high return guarantees on the outstanding stock of life insurance contracts.[6] As a result, profitability and the solvency level of the Japanese life insurance industry decreased dramatically, with seven mid-size Japanese life insurers declaring insolvency between 1997 and 2001 (The Life Insurance Association of Japan, 2013). Thus, the current situation represents the biggest challenge the German life insurance industry has faced in the last few decades, and in our view, a comprehensive analysis of the problem is still missing in the literature. In addition, our contribution allows us to assess the impact of the newly introduced reform of the German life insurance regulation (*"Lebensversicherungsreformgesetz"*) on default probabilities and to compare it with the previous regulation.

Our results indeed suggest that a prolonged period of low interest rates progressively reduces the solvency situation of life insurers featuring a liability portfolio with a guaranteed minimum return. Depending on the initial capital endowment and on the capital market development, life insurers might experience severe financial distress over the medium term, with a subset of companies struggling to maintain past promises. However, the newly introduced reform of the German life insurance regulation allows for reductions of the payouts to policyholders which substantially improves the solvency situation of the less capitalized insurers and thereby reduces the cumulative probability of default under the more adverse capital market scenarios.

The paper is organized as follows: In section 2, we introduce the model and its characteristics, where we describe a life insurer's asset side and the liability side, as well as the regulatory framework. In section 3, we describe the data and the calibration adopted. Section 4 discusses the main findings. Section 5 concludes the analysis.

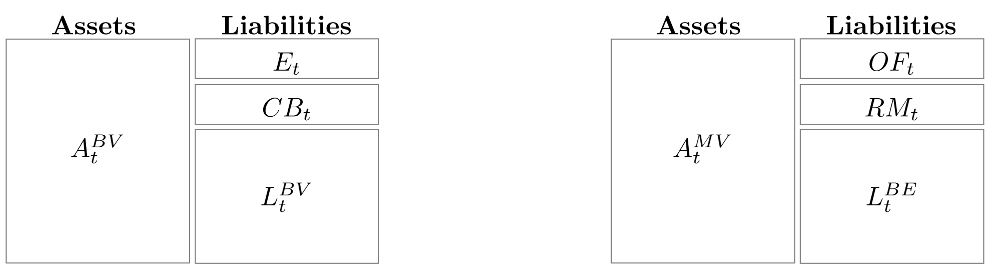
[6] A more comprehensive description of the Japanese case is provided, among others, by Holsboer (2000) and Hoshi and Kashyap (2004).

2 The Model

2.1 Book Value and Market Value Balance Sheets

We introduce both the book value and market value balance sheet of a representative life insurer with limited liability by calibrating both the asset side and the liability side to be as realistic as possible given several data availability constraints. Figure 1 depicts the book value balance sheet (left side) and the market value balance sheet (right side) of the insurer at time t: A_t^{BV} and L_t^{BV} correspond to the book value of assets and liabilities respectively. E_t is the equity capital endowment and CB_t is a capital buffer. The latter is a residual item that clears the balance sheet in every period and thereby serves as a smoothing account that allows the insurer to distribute more return during years of low asset returns and store funds during years of higher assets return.[7]

Figure 1: Book Value and Market Value Balance Sheet at time t



OF_t represents the market value of the own funds, which is the difference stemming from the market valuation of assets and liabilities. The latter corresponds to the sum of the *best estimate* of liabilities (L^{BE}) and a *risk margin* (RM).[8] The reason for the distinction between book value and market value balance sheets is twofold: on the one hand, book values allow us to determine *i*) the yearly earnings of the insurer and *ii*) the final payout of the cohort maturing at time t; on the other hand, the presence of a market value balance sheet is justified by the fact that the future Solvency II regulation will require both assets and liabilities to be *marked-to-market* in order to provide a market-consistent evaluation of the solvency situation.

[7] A thorough description of the item and its function follows in section 2.3.1

[8] In the fifth Quantitative Impact Study (QIS5) (2010), the Risk Margin is defined as "a part of technical provisions" which ensures "that the value of technical provisions is equivalent to the amount that insurance and reinsurance undertakings would be expected to require in order to take over and meet the insurance and reinsurance obligations" in case of insolvency. This is equivalent to the expected cost of capital accruing to the undertaking in the case of portfolio transfer.

2.2 The Asset Side

2.2.1 The Portfolio Structure

According to the German Insurers Association (GDV), the aggregated asset portfolio at book values of life insurers operating in Germany in 2012 amounted to 768.9 bn €.[9] The share of bonds and debentures was 92.5% of total assets, the amount of stocks was 5.3%[10] and real-estate-related assets amounted to 3.8%.[11] For our analysis, we consider the most recent outstanding structure of the aggregated asset portfolio to be the representative asset allocation. We aggregate similar items into 6 distinct classes, namely Sovereign Debt, Mortgage Pfandbriefe (Covered Bonds), Bank Bonds, Corporate Bonds,[12] Stocks and Real Estate. Figures for 2012 are reported in Table 1. Due to the long-term nature of the liabilities, life insurers tend to favor long-term investments in fixed-income securities both because of the need for a stable source of income and because of the need to minimize the duration gap between the asset and the liability side. In order to model the duration of the asset side, we refer to data provided by GDV, which estimates the average asset portfolio modified duration to be within the range 7.5 - 9 years.[13] Thus, for the sake of simplicity we assume that the insurer only invests in Sovereign Bonds with 25 years to maturity,[14] Mortgage Pfandbriefe with 15 years to maturity (Covered Bonds), and in Credit Institutions Bonds and Corporate Bonds with 10 years to maturity and that it holds the securities until redemption.[15] Due to the lack of available data on the actual maturity breakdown of the asset side, we set the initial weight for every Sovereign bucket term to maturity to 4%, for Mortgage Pfandbriefe to 6.6% and for the remaining classes to 10%[16] which results in a portfolio with an average of 9.25 years to maturity and an initial average modified duration across all 4 bond asset classes of roughly 7.36 years (see figure 2). For simplicity reasons we abstract from credit risk. Thus we assume that Mortgage Pfandbriefe, Credit

[9] Gesamtverband der Deutschen Versicherungswirtschaft e.V.: Statistical Yearbook of German Insurance 2013.

[10] The figure includes Shares and Participating Interests, see the Statistical Yearbook of German Insurance 2013 (GDV).

[11] The remaining 1.7% is classified as other investments.

[12] GDV reports loans to credit institutions and loans to corporations: due to data availability, we use banks bonds and corporate bonds data as proxies.

[13] We wish to thank GDV for providing us with this data.

[14] Although instruments with a time to maturity higher than 10 years represent roughly 15% of the outstanding Sovereign Debt market in Germany and their nominal value is roughly 1/3 of the nominal value of bonds with 10 years maturity, we assume that the insurer concentrate its purchases on the long end of the yield curve. For a more detailed overview of the maturity structure of the German Sovereign Debt, see the secondary market data provided by the Federal Government Debt Management Agency.

[15] The model allows for exceptions to this rule if liquidity is insufficient.

[16] Wedow and Kablau (2011) use a similar assumption.

Institutions Bonds and Corporate Bonds pay a constant spread different for every maturity (e.g. similar to a liquidity premium) above the Sovereign Debt Securities. We use the monthly statistics provided by the Deutsche Bundesbank and calculate the observed spreads in different time windows (see Table 2).[17] Finally it is worth noting that the absence of credit risk on the bond portfolio has a profound impact on the solvency situation: in fact, bonds that pay a higher yield are subject to a higher probability of default. This would, of course, negatively impact the solvency situation of the insurer and therefore our results.

2.2.2 The Asset Side Dynamics

We simulate an underlying capital market development by means of stochastic processes. In order to simulate the term structure of interest rates, and therefore the driving force of our asset side, we employ the model presented by Cox et al. (1985) (CIR model).[18] The model introduces the following interest rate dynamics under the risk neutral measure $\mathcal{Q}$

$$dr(t) = k(\theta - r(t))dt + \sigma_r \sqrt{r(t)} dW_r^{\mathcal{Q}}(t) \tag{1}$$

where $W_r^{\mathcal{Q}}(t)$ is a standard Brownian motion under $\mathcal{Q}$, $r(t)$ is the instantaneous interest rate, $k > 0$ is the speed of adjustment, $\theta > 0$ is the mean reversion level and $\sigma_r > 0$ is the volatility of the short rate dynamics. In addition, assuming the absence of arbitrage and a market price of risk $\lambda(t, r)$ of the special form $\lambda(t, r) = \lambda_0 \sqrt{r(t)}$, the short interest rate dynamics under the objective measure $\mathcal{P}$ can be written as follows

$$dr(t) = [k\theta - (k + \lambda\sigma_r)r(t)]dt + \sigma_r \sqrt{r(t)} dW_r^{\mathcal{P}}(t). \tag{2}$$

[17] Data are available on the Macroeconomic Time Series Database, Deutsche Bundesbank

[18] The CIR model is a wide-spread interest rate model: although its ability to reproduce observed term structure of interest rates has been challenged over the years (see for instance Chan et al. (1992)), evidence of its reliability is mixed, with some authors finding major drawbacks (see for instance Lamoureux and Witte (2002)) and others finding positive features (see for instance Brown and Schaefer (1994)). However, despite its shortcomings, many studies still rely on it (see for instance Maurer et al. (2013) and Gerstner et al. (2008)). For the purpose of the present paper, the model (and its characteristics) has the desired property of being widely known among researchers and of being relatively easy to calibrate and to adapt to specific needs (i.e. calibration of different interest rate scenarios). Moreover, as pointed out by an anonymous referee, the calibrations that we propose should mitigate some of the undesired properties of the model in the context of a low interest rate environment.

where $W_r^{\mathcal{P}}(t)$ is a standard Brownian motion under $\mathcal{P}$. Moreover, the model allows the pricing of a zero coupon bond B according to

$$B(t,T) = A(t,T)e^{-H(t,T)r(t)} \tag{3}$$

where t is the time spot and T is the maturity time of the bond. $A(t,T)$ is defined as

$$A(t,T) = \left[\frac{2\gamma e^{(k+\lambda+\gamma)(T-t)/2}}{(\gamma+k+\lambda)(e^{\gamma(T-t)}-1)+2\gamma}\right]^{\dfrac{2k\theta}{\sigma_r^2}}, \tag{4}$$

the discount factor $H(t,T)$ is defined as

$$H(t,T) = \left[\frac{2(e^{\gamma(T-t)}-1)}{(\gamma+k+\lambda)(e^{\gamma(T-t)}-1)+2\gamma}\right] \tag{5}$$

and finally

$$\gamma = \sqrt{(k+\lambda)^2 + 2\sigma_r^2)}. \tag{6}$$

Thus the CIR enables us to generate a term structure of interest rates which we employ both to estimate future bonds' coupons and to determine the market value of assets and liabilities.[19]

Stock and real estate returns evolve over time following a Geometric Brownian Motion (GBM)[20] which is specified as follows:

$$\partial S(t) = \mu\, S(t)\, dt + \sigma_s\, S(t)\, dW_s^{\mathcal{P}}(t) \tag{7}$$

where μ is the drift rate and and σ_s is the volatility of the return. The closed form solution to equation (7) is given by

$$S(t) = S(0)e^{\left(\mu - \dfrac{\sigma^2}{2}\right)t + \sigma_s W_s^{\mathcal{P}}(t)}. \tag{8}$$

Finally, the three resulting processes, namely the GBM for stocks, the GBM for real estate and the instantaneous interest rate process of the CIR model, are correlated by means of the Cholesky

[19] For further details refer to Brigo and Mercurio (2006)

[20] Recent literature has proposed other distributional assumptions that are more robust to market dynamics: for a good overview, see for instance Jiménez and Arunachalam (2011).

decomposition technique.[21]

2.2.3 The Book and Market Value of Assets

We assume that all bonds are bought at par and that stocks and real estate are bought for their respective market value at time t. The book value of the b^{th} bond-like asset class $B^{b,BV}$ at the purchasing time t equals its face value $B^{b,FV}$ and its market value $B^{b,MV}$

$$B^{b,BV}_{(t,T)} = B^{b,FV}_{T} = B^{b,MV}_{(t,T)} \tag{9}$$

where T is its maturity date. Analogously, the book value of the k^{th} stock-like asset class $S^{k,BV}$ at the purchasing time t equals its purchasing cost $S^{k,FV}$ and its market value $S^{k,MV}$

$$S^{k,BV}_{t} = S^{k,FV_t} = S^{k,MV}_{t}. \tag{10}$$

Following the prudential approach of the German GAAP, if during the holding period the market value of an asset drops below its book value, the latter must be adjusted downwards.[22] Therefore, to account for such a rule and thus determine the book value in every period, we introduce the market valuation of assets. For the b^{th} bond-like asset class, the market value is given by the following equation:

$$B^{b,MV}_{(t,T-\tau)} = \sum_{j=\tau+1}^{T} \left(\frac{B^{b,FV}_{(T-\tau)} \cdot i^{b}_{c,(T-\tau)}}{1 + i_{d,(t,j-\tau)}} \right) + \frac{B^{b,FV}_{(T-\tau)}}{1 + i_{d,(t,T-\tau)}} \tag{11}$$

where τ is the life time of the bond that has already passed by at time t, i_c is the coupon and i_d is the discount rate applied to that particular asset class.[23] We use the term structure of interest rates to evaluate bonds, which implies that every future coupon payment and the face value are discounted using the interest rate with equivalent maturity at time t.[24] Regarding stock-like asset

[21] The discrete versions of both the CIR and the GBM are described in Appendix (A.1 and A.2). For further mathematical details, see, for instance, Hull (2010) or Björk (2004).

[22] We apply a simplified version of the "Minimum Value Accounting Principle", in German *Niederstwertprinzip*.

[23] As mentioned in section 2.2.1, we reproduce an outstanding asset allocation where bonds were bought at different points in time and therefore differ in their time to maturity and coupons. We thus index each bond-like asset class b with a different time to maturity $T - \tau$ (see figure 2).

[24] Government Bonds will be discounted using the term structure of interest rates, whereas for the other 3 asset classes, we will use a discount factor that incorporates the corresponding spread.

classes, the market value at time t follows the GBM evolution over time. At time $t = 0$, we assume that market values and book values coincide[25], but as time progresses, market values are adjusted according to the underlying GBM simulation. The market value of the k^{th} asset class at time t is defined as follows

$$S_t^{k,MV} = \begin{cases} S_{t-1}^{k,MV} + (1-\vartheta)\cdot(S_t^{k,*} - S_{t-1}^{k,MV}), & \text{if } S_t^{k,*} > S_{t-1}^{k,MV} \\ S_t^{k,*}, & \text{otherwise} \end{cases} \tag{12}$$

where $S_t^{k,*}$ represents the evolution of assets values as given by the GBM, and where a fraction ($\vartheta \leq 1$) of the increase in $S_t^{k,*}$ is subtracted from the value, as it is considered a cashed-in dividend payment.[26] Book values of bonds are given by the following condition:

$$B_{(t,T-\tau)}^{b,BV} = \begin{cases} B_{(t,T-\tau)}^{b,MV}, & \text{if } B_{(t,T-\tau)}^{b,MV} < B_{(t-1,T-\tau)}^{b,BV} \\ B_{(t,T-\tau)}^{b,MV}, & \text{if } B_{(t-1,T-\tau)}^{b,BV} \leq B_{(t,T-\tau)}^{b,MV} \leq B_{(T-\tau)}^{b,FV} \\ B_{(T-\tau)}^{b,FV}, & \text{if } B_{(t,T-\tau)}^{b,MV} > B_{(T-\tau)}^{b,FV} \end{cases} \tag{13}$$

where the book value of the b^{th} asset class with $T-\tau$ time to maturity adjusts to the market value in time t if the latter falls below its face value. Moreover, after a depreciation has occurred in t, the book value can still recoup its face value as long as the market value rises and exceeds the corresponding face value. Analogously, for stocks and real estate, the book value dynamics is given by the following conditions

$$S_t^{k,BV} = \begin{cases} S_t^{k,MV}, & \text{if } S_t^{k,MV} < S_{t-1}^{k,BV} \\ S_t^{k,MV}, & \text{if } S_{t-1}^{k,BV} \leq S_t^{k,MV} \leq S^{k,FV_{t-n}} \\ S_{t-1}^{k,FV}, & \text{if } S_t^{k,MV} > S^{k,FV_{t-n}} \end{cases} \tag{14}$$

[25] This is a necessary assumption, since we do not have any information on the time of the purchase of both stocks and real estate.

[26] We model real estate assets similarly to stocks. Due to the lack of data, we were not able to estimate the cash flow provided by yearly rents, therefore we use a simplified approach and treat real estate as stocks.

where $S^{k,FV_{t-n}}$ is the purchasing price of the asset at time $t-n$ with $n = 1, 2, 3....$ Finally, we can write the aggregate book value of assets at time t

$$A_t^{BV} = A_t^{b,BV} + A_t^{k,BV} \tag{15}$$

where

$$A_t^{b,BV} = \sum_{b=1}^{N^b} \sum_{\tau=0}^{T} B_{(t,T-\tau)}^{b,BV} \tag{16}$$

$$A_t^{k,BV} = \sum_{k=1}^{N^k} S_t^{k,BV}, \tag{17}$$

and its analogous aggregate market value

$$A_t^{MV} = A_t^{b,MV} + A_t^{k,MV} \tag{18}$$

where

$$A_t^{b,MV} = \sum_{b=1}^{N^b} \sum_{\tau=0}^{T} B_{(t,T-\tau)}^{b,MV} \tag{19}$$

$$A_t^{k,MV} = \sum_{k=1}^{N^k} S_t^{k,MV} \tag{20}$$

with N^b and N^k being the number of bond-like asset classes and stock-like asset classes respectively.

2.3 The Liability Side

The liability side of a representative life insurer is modeled with the intention of reproducing $i)$ an existing stock of contracts sold in the past that will mature as time progresses and will be constantly replaced by new contracts and $ii)$ a dynamic structure dependent on the underlying capital market development. Each cohort of contracts carries the guaranteed return in force at the inception. Given the limited amount of data on the contract portfolio structure, we infer a plausible liability structure that attempts to reproduce empirically observed dynamics, such as the evolution over time of the average outstanding guaranteed interest rate and the return distribution policy.

2.3.1 The Regulatory Framework

The regulatory framework currently in force in Germany allows a maximum technical interest rate for discounting policy reserves. This implies that once the insurer decides on the rate to use, at least this technical rate must be credited to policyholders' accounts every year.[27] The maximum return an insurer can guarantee is 60% of the 10-year moving average of the interest rate of 10-year-maturity Government Federal security (*reference interest rate*).[28] However, if the reference interest rate at some point in time falls below the guaranteed return, insurers must build an additional reserve by substituting the guaranteed rate as a discount factor with the reference interest rate, thus increasing the amount of reserves at that particular point in time.[29] In figure 3, we plot both the interest rate on the 10-year maturity Government Federal security and its 10-year moving average. The 60% figure of the latter drives the dynamics of the maximal guaranteed interest rate allowed by the regulator: by observing the reaction of the regulator to the changes in the reference interest rate over time, we can infer that adjustments in the maximum technical rate occur as soon as the reference interest rate approaches the rate in force.[30] Formally, there is no obligation for life insurers either to provide a minimum guaranteed return or to provide it at the maximum allowed. However, it has become common practice within the industry to provide long-term insurance products that include yearly guaranteed return set at the maximum allowed (once the contract has been sold with a certain minimum return, it cannot be changed until maturity), with an additional return on top of that. The latter is a variable that the insurer has at its disposal to improve the attractiveness of its product. In addition, profit participation has to comply with a minimum distribution of returns. According to German regulation, at least 90% of profit generated by tied assets (i.e. assets backing

[27] §2, paragraph (1) of the German Directive for the calculation of policy reserves (*Deckungsrückstellungsverordnung* - DeckRV).

[28] §65, paragraph (1) of the German Insurance Supervision Code (*Versicherungsaufsichtsgesetz* - VAG)

[29] In German *Zinszusatzreserve*: this is regulated by the §341, paragraph 2 of the German GAAP (*Handelsgesetzbuch* - HGB) and §2, paragraph (5) of the German Directive for the calculation of policy reserves (*Deckungsrückstellungsverordnung* - DeckRV). According to the German Supervisory Authority (BaFin), this constitutes "an additional provision to the premium reserve introduced in response to the lower interest rate environment" which aims at offsetting "their lower investment income in the future" (BaFin 2012 Report). However, the calculation of the additional reserve is based on a 15-year horizon, thus implying that expected residual time to maturity exceeding 15 years will be calculated using the original discount factor.

[30] In order to simulate the behavior of the regulatory authority and therefore the guaranteed return of the new incoming cohort of contracts, we extract the regulator reaction function as described in the Appendix (A.3).

the single insurance contract), 75% of mortality profits (i.e. profits stemming from the difference between first-order actuarial assumptions and actual mortality developments) and at least 50% of other profit sources (i.e. mainly administrative costs) must be transferred to policyholders.[31] Moreover, the insurer can decide on a distribution of additional returns on top of the minimum profit participation.[32] In addition, the insurer has to decide on the way in which the additional return gets assigned to policyholders. For this purpose, life insurance companies under the German legislation have three different accounts at their disposal: funds to be immediately paid out to policyholders are conveyed to a *direct deposit*, funds to be paid out within a certain time horizon are conveyed into the so-called *committed Provision for Premium Refunds* (committed PPR), while funds that can be withdrawn during years of bad return or funds that can be accumulated during years of good returns are stored in the *uncommitted Provision for Premium Refunds* (uncommitted PPR).[33] The uncommitted PPR and part of the committed PPR can be considered as the capital buffer CB_t depicted in figure 1 that allows returns to be smoothed over time.[34] These items are not considered tied reserves, which is indeed the case for the direct deposit and the remaining part of the committed PPR, and therefore are eligible as part of the own funds.[35] BaFin data for 2012 report that the entire PPR accounted for ca. 7.7% of technical reserves, making it the second biggest item in the aggregated balance sheet of life insurers operating in Germany. An additional change in the regulation of profit participation in Germany has introduced the obligation for life insurers to share a minimum amount of the *hidden reserves* (i.e. difference between market and book values) with policyholders.[36] Every year, life insurers must transfer at least 50% of the hidden reserves stemming from the assets backing those policies that come due during the year. This implies that a relatively small proportion of customers profit each year from the existence of hidden reserves.[37]

[31] §4, paragraphs 3-5 of the German Directive on minimum profit participation (*Mindestzuführungsverordnung* - MindZV).

[32] Assekurata data show how the industry was able to distribute a rather stable amount of additional returns over time.

[33] In German *gebundene Rückstellung für Beitragsrückerstattung* and *ungebundene Rückstellung für Beitragsrückerstattung* respectively.

[34] Part of the committed PPR (in German *Schlussüberschussanteilfonds*) is considered tier 1 capital, therefore it counts as capital endowment of the insurer. See GDV's official position on eligible own funds for German Life Insurers (2007).

[35] For a more extensive discussion on the German Profit Distribution regulatory framework, see Maurer et al. (2013).

[36] §153, German Insurance Contract Law (*Versicherungsvertragsgesetz* - VVG).

[37] GDV reports that on average every year, 5% of total customers profit from hidden reserves. In our model, the share of customers that receive a part of the hidden reserves in every year is 4%.

In the case of negative hidden reserves, policyholders do not participate in losses.

Reform of the Regulatory Framework

On August 1, 2014, the German Parliament has introduced a reform on the regulatory framework of life insurance companies.[38] The main changes can be summarized as follows[39]:

- the minimum share of yearly mortality profits to be transferred to policyholders has been increased from 75% to 90%. In addition, negative results on the minimum profit participation of asset returns can be compensated with mortality profits and other profits (*cross-subsidization among profit sources*); [40]
- the 50% minimum participation on hidden reserves that accrues to policyholders when terminating the contract must only be distributed to policyholders if the hidden reserves of the insurer's bond portfolio exceed the positive difference between the market-consistent value of the underwriting portfolio and the technical provisions (calculated at book values).[41] For stock-like asset classes the old regulation remains unchanged.

2.3.2 The Liability Structure

In order to model the liability side and to fit it with the available data, we allow the insurer to sell only a homogeneous product with a fixed 25 years to maturity, for which the policyholder commits to make one premium payment every year during the entire period and receive a lump-sum benefit payment at the end of the contractual period.[42] We assume that as older cohorts of contracts mature, they are constantly replaced by new cohorts of contracts carrying the maximum allowed guarantee in force at the time of inception. In addition, neither mortality nor surrender risk

[38] The Life Insurance Reform Law (in German *Gesetz zur Absicherung stabiler und fairer Leistungen für Lebensversicherte - Lebensversicherungsreformgesetz*) was published on August 1, 2014 and entered into force on August 6, 2014.

[39] The Reform introduces other substantial modification on the existing regulation but these changes do not affect our model.

[40] The regulatory framework on minimum profit participation entails the possibility for the insurer to reduce the payouts to policyholders if there are unforeseen investment losses or if it is necessary to ensure an insurer's solvency, conditional on the approval of the regulatory authority: this feature has been strengthened by the reform, however such possibility is close in spirit to a default, therefore we do not take it into consideration in our analysis.

[41] In German *Sicherungsbedarf*.

[42] We model a product with the main characteristics of an endowment policy in which the premium is normalized to unit.

are considered, with only financial risk thus being taken into consideration. Moreover, the premium does not comprise costs (loading factors), which is justified by the fact that administrative costs of the insurer are, in turn, also not mapped.[43] These assumptions have a strong impact on the final results, in particular in the absence of policyholders' reactions. By assuming no surrender risk and a constant inflow of new contracts, the insurer benefits from a stable source of funding which is crucial either when interest rates stay at a low level or when interest rates start to rise again. In the first case the cost of the (high) guarantees contained in the existing portfolio increases. Due to the fear of cross-subsidization from new to old policyholders the attractiveness of new insurance policies with lower guarantees would decrease. This reduces the funding of the insurer. By contrast, as interest rates start to rise again, policyholders with lower guarantees might cancel their contracts, leading again to a reduction in funding. Thus policyholders' behavior might significantly worsen the solvency situation of the insurer and substantially increase the probability of default.[44]

The contract time to maturity is justified by *i*) the observed average duration of life insurance contracts in the German market, *ii*) by the need of approximating the observed dynamics that data on the average outstanding guaranteed interest rate show and by *iii*) reproducing a plausible duration mismatch with the asset side. GDV reports an average modified duration of the total liability portfolio in Germany in the range of 11 - 13 years. However, typical endowment policies considerably differ in duration compared to typical annuity products. In fact, according to GDV (2005), the duration of a typical endowment policy is 12 years, whereas a typical annuity product ranges from 17 to 24 years depending on the characteristics of the product.[45] By assuming the simultaneous presence in the portfolio of 25 cohorts of contracts, each with different terms to maturity, we can reproduce an outstanding average of 13 years to maturity with a modified duration of ca. 11.11 years (see figure 2).[46] Moreover, we assume that in the last 25 years, the insurer sold 1 cohort of contracts in every year and that each cohort carries the maximum guaranteed return allowed at the inception, as reported in figure 3. This simplification allows us to approximate

[43] A similar product is modeled in Kling et al. (2007b) and Bauer et al. (2006), while a more complex payoff structure is modeled, for example, by Gerstner et al. (2008).

[44] A thorough discussion on some of these aspects can be found in Babbel (2001). We would like to thank an anonymous referee for pointing out this relevant aspect.

[45] GDV (2005) reports the following estimations for typical life insurance contracts in Germany: endowment policies (*Kapitallebensversicherungen*) 12 years, annuity with lump-sum benefits option (*Rentenversicherungen mit Kapitalwahlrecht*) 17 years and annuity without lump-sum benefit option (*Rentenversicherungen ohne Kapitalwahlrecht*) 24 years.

[46] We employ the Fisher-Weil method and use as discount rates the average German yield curve in 2013.

the 3.12% average guaranteed interest rate for 2013 as reported by Assekurata market surveys (Assekurata, 2012).[47]

We report the initial composition of the underwriting portfolio in Table 3. As a result, the portfolio displays an average guaranteed rate of 3.11% at the end of 2013 and contracts with 4% guaranteed return account for 24% of the total portfolio.[48] In addition, Assekurata estimates an average total return on life insurance policies, i.e. the return including profit distribution on top of the guaranteed rate, across all survey participants and across all product types,[49] which allows us to determine the current book value of each policyholder's account as per end 2013.[50] Finally, the average duration gap estimated by GDV ranges between 3.5 and 4 years. In our model, we are able to reproduce an initial duration gap of ca. 3.75 years.

2.3.3 Development of the Policyholder Accounts over Time

Cliquet-style guarantees are a typical product feature in Germany (see, for instance, Kling et al. (2007b), Bauer et al. (2006) and Gerstner et al. (2008)). The policy provides at least the guaranteed interest rate every year, and due to the regulation on the *minimum participation rate*, the policyholder might receive an additional return on top of the guarantee, depending on the total return of the asset side. The product is similar to the European Participation Contract introduced by Grosen and Løchte Jørgensen (2000). In every period, the insurer can decide how much additional return can be distributed based on regulatory constraints and the financial information available up to the decision moment.[51] Each policyholder's account develops over time as follows

$$l_t^{i,p,BV} = l_{t-1}^{i,p,BV} \cdot \left[1 + max\left(r_t^{i,g}, r_t^p\right)\right] + \pi_t^i \tag{21}$$

[47] The Market Surveys conducted by Assekurata (Table 5) report average data on the guaranteed interest rates and the profit distribution since 2004. In 2012, the study covered 77 Life Insurers operating in Germany, accounting for more than 96% of the market in terms of premiums.

[48] The robustness check shows that applying the same method (i.e. a weighted average of a 25-year rolling window) to older data, the estimation on the portfolio structure remains roughly in line with Assekurata data. However, the simple inference of our model tends to slightly underestimate the true value, but deviations always remain below 0.09%.

[49] Cf. Table 5.

[50] Data available only from 2004. We assume that older cohorts of contracts received the same total return (i.e. the oldest figure available, 2004) since inception. Compounding the total return and all premium payments, we can thus derive the book value of each cohort of contracts.

[51] We denote with the superscript p values including additional returns and with superscript g values including only the minimum regulatory return distribution.

where i is the tariff generation indicating the minimum guaranteed return fixed at contract inception, $l_{t-1}^{i,p,BV}$ is the book value of the account at time $t-1$, $r_t^{i,g}$ is the regulatory minimum rate of return, r_t^p is a rate of return that incorporates the additional return [52] and π_t is the annual premium which the policyholder commits to pay upfront for the entire duration of the contract. By aggregating all the cohorts in the portfolio (N^l) at time t, we obtain the book value of liabilities which we can express as

$$L_t^{p,BV} = \sum_{i=1}^{N^l} l_t^{i,p,BV}. \tag{22}$$

In order to determine how much return will be distributed in every year, we adopt a simple rule that aims to approximate the behavior of German life insurers with respect to additional returns distribution.[53] German life insurers in the past tended to credit a rather stable rate of return to policyholders' accounts, accumulating funds during years of high return on assets and using these funds during years of low return on assets.[54] However, while in Kling et al. (2007b) and Bauer et al. (2006) the rate of return including additional profit distribution r^p was exogenously given (and kept constant along the simulation), here r^p is fully endogenized and determined as a result of a set of regulatory and financial constraints. We thus make sure that policyholders receive a fair amount of additional returns based on their respective regulatory minimum guaranteed return. This implies that the lower the guaranteed rate of return is, the (relatively) higher the additional return allocation will be.

The return on assets R^a is the key variable that the insurer considers when deciding on the profit distribution. It is defined as follows:

$$R_t^a = \sum_{b=1}^{N^b} \sum_{j=\tau+1}^{T} \left(B_{(t,j-\tau)}^{b,BV} \cdot i_{c,(j-t)}^b \right) + max \left\{ \sum_{k=1}^{N^k} \vartheta \cdot \left(S_t^{k,*} - S_{t-1}^{k,MV} \right), 0 \right\} - \left(\sum_{b=1}^{N^b} d_{(t,T-\tau)}^b + \sum_{k=1}^{N^k} d_t^k \right). \tag{23}$$

The first 2 terms of the right-hand side represent the amount of coupons and dividends respectively that are cashed in at time t, while the last term represents the sum of the *depreciations* or

[52] For further details see Appendix (A.4).

[53] Since the profit distribution mechanism in the German life insurance market is rather complex, we present a simplified version. We assume that the insurer credits in every period at least the minimum regulatory return to the policy holders' accounts directly. This allows us to approximate without loss of generality the financial dynamics typical of life insurance savings products for the German market.

[54] The same underlying assumption is presented in Kling et al. (2007b), Bauer et al. (2006) and more recently in Maurer et al. (2013).

appreciations for both bonds and stocks during the same period. If market values drop below book values, the insurer must register a depreciation in the value of its assets, which in turn lowers its return. If the value increases, the opposite applies. Since policyholders under German regulation are entitled to receive at least 90% of R_t^a exceeding the guaranteed return, 75% of returns from mortality development and 50% from other sources of income, the insurer needs first to compare the amount of funds that must be credited to policyholders in order to comply with regulation. Although we only account for the financial performance of the insurer and neglect underlying mortality developments, we need to account for returns stemming from mortality assumptions, as they represent an important source of income for life insurers that can potentially alleviate the impact of bad financial performances.[55] Moreover, data in table 4 show that such returns were rather stable (in percentage of total liabilities) over time in the German life insurance industry, and we can therefore model returns from mortality assumptions to be a fixed (deterministic) share (r^q) of total liabilities at book values. [56] Hence, the insurer needs to account for the marginal growth of L that complies with minimum regulatory profit participation and thereby determine the terminal value of all outstanding cohorts of contracts. The minimum value for each cohort at time t is given by

$$l_t^{i,g,BV} = l_{t-1}^{i,p,BV} \cdot (1 + r_t^{i,g}) \tag{24}$$

where $r_t^{i,g}$ is determined as follows

$$r_t^{i,g} = \frac{l_{t-1}^{i,p,BV} \cdot (1 + r^i) + max\Big\{0, 0.9 \cdot R_t^{i,a} - l_{t-1}^{i,p,BV} \cdot r^i\Big\} + 0.75 \cdot R_t^{i,q} - l_{t-1}^{i,p,BV}}{l_{t-1}^{i,p,BV}} \tag{25}$$

where r^i is the minimum guaranteed return assigned at the inception, $R_t^{i,a}$ is the return on assets belonging to cohort i (i.e. tied assets) given by

$$R_t^{i,a} = R_t^a \cdot \frac{l_{t-1}^{i,p,BV}}{A_{t-1}^{BV}}, \tag{26}$$

[55] We neglect profits stemming from costs reduction and other sources, since values are persistently negative over time (see table 4).

[56] We use the typical actuarial notation q to identify mortality returns. For further details on the profit distribution in the German Life Insurance industry, see Führer and Grimmer (2010).

and $R_t^{i,q}$ is the return from mortality belonging to cohort i given by

$$R_t^{i,q} = r^q \cdot l_{t-1}^{i,p,BV} \tag{27}$$

being the share of mortality returns belonging (proportionally) to each cohort. Finally, the aggregate value of all cohort of contracts growing at the minimum regulatory rate is given by

$$L_t^{g,BV} = \sum_{i=1}^{N^l} l_t^{i,g,BV} \tag{28}$$

In addition to this, the insurer must increase the technical reserves as soon as the guaranteed return of each cohort of contracts (excluding the cohort that gets liquidated in time t) exceeds the reference interest rate (r_t^{ref}).[57] Therefore, when $r^i > r_t^{ref}$, the following accounting criterion applies

$$IR_t^i = \frac{l_t^{i,g,BV} \cdot (1+r^i)^{(T-\tau)}}{(1+r_t^{ref})^{(T-\tau)}} - l_t^{i,g,BV} \tag{29}$$

where IR_t^i is the additional *interest rate reserve* that must be set aside on top of the otherwise i^{th} technical provision as if only the guaranteed return were granted. In contrast, if $r^i \leq r_t^{ref}$, the discount factor applied to equation (29) would be r^i, thus implying $IR_t^i = 0$.[58] Moreover, at least 50% of the hidden reserves must be transferred every year to the cohort of policyholders whose contracts have matured during the period. The insurer must thus take into account the additional cash outflow in case hidden reserves are positive. This is calculated as follows

$$HR_t^{ph} = \begin{cases} \dfrac{l_t^{1,g,BV}}{L_t^{g,BV}} \cdot 0.5 \cdot \left(A_t^{MV} - A_t^{BV}\right), & \text{if } A_t^{MV} > A_t^{BV} \\ 0, & otherwise \end{cases} \tag{30}$$

where $l_t^{1,g,BV}$ is the cohort of contracts that mature and $\left(A_t^{MV} - A_t^{BV}\right)$ are the amount of reserves available at time t. Therefore, equation (24) can be rewritten including the additional interest rate

[57] The 10-year moving average of the 10-year Government Federal security. See section 2.3.1.

[58] However, the calculation of the interest rate reserve is based on a 15-year horizon, thus implying that for those contracts with expected residual time to maturity greater than 15 years, the part exceeding 15 years will be discounted using the original discount factor, i.e. r^i.

reserves and the additional cash outflow stemming from hidden reserves

$$L_t^{g,BV} = \sum_{i=1}^{N^l} \left(l_t^{i,g} + IR_t^i \right) + HR_t^{ph}. \tag{31}$$

Additional return can only be distributed to policyholders if

$$(R_t^a + R_t^q) > R_t^{ph,g} \tag{32}$$

with

$$R_t^{ph,g} = L_t^{g,BV} - L_{t-1}^{p,BV} \tag{33}$$

being the regulatory minimum amount of funds that must be accredited to the policyholders' accounts in every period. If at time t equation (32) is fulfilled, the insurer can distribute returns to shareholders (dividends) and additional returns to policyholders. Dividends are determined as follows

$$R_t^{sh} = \begin{cases} R_t^a + R_t^q - R_t^{ph,g}, & \text{if} \quad (R_t^a + R_t^q - R_t^{ph,g}) \leq \delta \cdot (R_t^a + R_t^q) \\ \delta \cdot (R_t^a + R_t^q), & \text{otherwise} \end{cases} \tag{34}$$

whereas dividends are calculated as residual funds and cannot exceed the threshold $\delta \cdot R_t^a$ with $\delta << 1$. After dividends are determined, the funds that are left over can be additionally distributed to policyholders so that

$$\widehat{R}_t^{ph,p} = R_t^a + R_t^q - R_t^{ph,g} - R_t^{sh}. \tag{35}$$

According to the following conditions, the insurer decides how much of $\widehat{R}_t^{ph,p}$ will be actually distributed as additional return to policyholders in time t:

$$R_t^{ph,p} = \begin{cases} \widehat{R}_t^{ph,p}, & \text{if} \quad v \cdot R_{t-1}^{ph,p} \leq \widehat{R}_t^{ph,p} \leq u \cdot R_{t-1}^{ph,p} \\ v \cdot R_{t-1}^{ph,p}, & \text{if} \quad \widehat{R}_t^{ph,p} < v \cdot R_{t-1}^{ph,p} \\ u \cdot R_{t-1}^{ph,p}, & \text{if} \quad \widehat{R}_t^{ph,p} > u \cdot R_{t-1}^{ph,p} \end{cases} \tag{36}$$

where $v < 1 < u$. [59] In contrast, if constraint (32) is not fulfilled, the insurer decides to set both R_t^{sh}

[59] If $R_{t-1}^{ph,p} = 0$, the insurer would choose $R_{t-i}^{ph,p}$ where $i = 2, 3...n$ until a positive value is found. Moreover $R_t^{ph,p}$ cannot be negative, as policyholders do not participate in downside risk.

and $R_t^{ph,p}$ to zero. The mechanism for the additional return distribution that we propose has three desirable characteristics: *i*) it makes sure that the distribution of funds complies with regulation, *ii*) it ensures that additional returns are distributed smoothly and fairly (among policyholders)[60] over time and *iii*) it withdraws or stores funds in CB to offset years with relatively low and high returns respectively. Once the decision on dividends (R_t^{sh}) and additional return to policyholders ($R_t^{ph,p}$) has been made, the insurer can determine the book value of L which is given by

$$L_t^{p,BV} = L_t^{g,BV} + R_t^{ph,p} \tag{37}$$

and finally liquidate the cohort that is due at time t

$$l_t^{1,p,BV} = l_{t-1}^{1,p,BV} \cdot \Big[1 + max\Big(r^1, r_t^p\Big)\Big] + HR_t^{ph} \tag{38}$$

where l^1 refers to the oldest cohort held in the portfolio.

2.3.4 The Market Value of Liabilities

In order to obtain a market-consistent valuation of liabilities, we first define the best estimate of liabilities as the discounted minimum final payment which the insurer has to make at the end of the contract.[61] This is equivalent to the *present value* of future cash outflows, net of future inflows: the purpose is to estimate a market-consistent level of the total indebtedness of the insurer at time t. [62] The present value of each cohort of contracts is given by

$$l_t^{i,BE} = \frac{l_t^{i,p,BV} \cdot (1 + r^i)^{(T-\tau)}}{(1 + i_{d,(t,T-\tau)})^{(T-\tau)}} \tag{39}$$

[60] For further details on how r_p is determined, see Appendix (A.4).

[61] Our approach does not lead to a *fair valuation* as presented among others by Grosen and Løchte Jørgensen (2000) or later by Bauer et al. (2006) and Gatzert (2008). This is due to the fact that our valuation does not include potential additional returns but only considers the final payment as if the policyholder's account grew only at the guaranteed return from t onward. This is justified by the fact that we are mainly interested in assessing the solvency situation of insurers and therefore, since there is no obligation for the insurer to pay additional returns on top of the guaranteed return, we only account for the guaranteed return.

[62] The 2009 Solvency II Directive explicitly recalls the principle of *best estimation* for the valuation of liabilities. We thus interpret the present value as being the best estimation of the value of the lowest final payment at that particular point in time, recalling the absence of surrender options and mortality risk.

where T is the maturity date, τ is the time already matured at time t, and consequently $T - \tau$ is the remaining time to maturity of cohort i. Finally $i_{d,(t,T-\tau)}$ is the discount factor applied to the final payoff, where $(t, T-t)$ indicates the point in time t and the corresponding $T - \tau$ maturity. We use the term structure of interest rates generated by the CIR model as discount factors. By aggregating all the outstanding contracts at time t, we obtain the best estimate of the technical reserves

$$L_t^{BE} = \sum_{i=1}^{N^l} l_t^{i,BE}. \tag{40}$$

Finally, we can write the market value of liabilities as follows

$$L_t^{MV} = L_t^{BE} + RM_t \tag{41}$$

where RM_t is the risk margin estimated at time t. Since RM_t is determined as a function of the solvency capital requirement (SCR), a description follows in section 2.5.

Reform of the Regulatory Framework

Under the new regulatory framework as of August 2014, equation (25) has been substituted by the following equation[63]

$$r_t^{i,g} = \frac{l_{t-1}^{i,p,BV} \cdot (1 + r^i) + \left(max\left\{ \left(R_t^{i,a} - l_{t-1}^{i,p,BV} \cdot r^i \right)^-, 0.9 \cdot R_t^{i,a} - l_{t-1}^{i,p,BV} \cdot r^i \right\} + 0.9 \cdot R_t^{i,q} \right)^+ - l_{t-1}^{i,p,BV}}{l_{t-1}^{i,p,BV}}. \tag{42}$$

in which $(\cdot)^+$ and $(\cdot)^-$ take only positive and negative values respectively, and 0 otherwise. Moreover, equation (30) has been substituted by the following equation[64]

$$HR_t^{ph} = 0.5 \cdot \left(max\left\{ A_t^{b,MV} - A_t^{b,BV}, 0 \right\} - max\left\{ L_t^S - L_t^{BV}, 0 \right\} \right) \cdot \frac{l_t^{1,g,BV}}{L_t^{g,BV}} \tag{43}$$

[63] See Article 6, §4 of the Life Insurance Reform Law (in German *Lebensversicherungsreformgesetz*).
[64] See Article 1, §3 of the Life Insurance Reform Law (in German *Lebensversicherungsreformgesetz*).

where

$$L_t^S = \sum_{i=1}^{N^l} l_t^{S,i}, \tag{44}$$

$$l_t^{S,i} = \frac{l_t^{i,g,BV} \cdot (1 + r^i)^{(T-\tau)}}{(1 + min\{r^i, i_{d(t,10)}\})^{(T-\tau)}} \tag{45}$$

where $i_{d,(t,10)}$ is the discount factor applied to the final payoff, in which $(t, 10)$ indicates the point in time t and 10 represents the years to maturity.[65] As for the market value of liabilities, we use the term structure of interest rates generated by the CIR model to determine the discount factors.

2.4 The Free Cash Flow

The free cash flow dynamics determines the amount of funds that are reinvested in each year. This is given by the following equation

$$FCF_t = R_t^a + R_t^q + \sum_{i=1}^{N^l} \pi_t^i + \sum_{a=1}^{N^b} B_{(t,0)}^{b,FV} - l_t^{1,p,BV} - R_t^{sh} + \Big(\sum_{b=1}^{N^b} d_{(t,T-\tau)}^b + \sum_{k=1}^{N^k} d_t^k \Big) \tag{46}$$

where inflows come from return on assets (R_t^a), return from mortality developments (R_t^q), premiums and the payoff from matured bonds ($\sum_{b=1}^{N^b} B_{(t,0)}^{b,BV}$).[66] Outflows include the cohort of contracts that is liquidated at time t ($l_t^{1,p,BV}$) and possibly dividends (R_t^{sh}). Moreover, the depreciations or appreciations we subtracted from the R_t^a must be added back as they do not reflect any change in the actual cash flow. An additional constraint which the insurer faces in every period concerns the reinvestment of funds. The amount of funds that needs to be reinvested every year must be at least equal to the available free cash flow. Hence, we can impose the following constraint

$$FCF_t \geq \sum_{i=1}^{N^l} \pi_t^i + \sum_{i=1}^{N^l-1} \Big[l_{t-1}^{i,p,BV} \cdot max\Big(r_t^{i,g}, r_t^p\Big) \Big] \tag{47}$$

which is given by the sum of the marginal growth of the outstanding contracts ($N - 1$, as 1 gets liquidated in every period and the new one receives its first return after 1 year), their relative premiums and the premium received from the newly issued cohort of contracts. If, on the one hand,

[65] The calculation of L^S and $l^{S,i}$ is based on a 15-year horizon.

[66] Premiums include those coming from ongoing contracts and from the newly issued cohort of contracts.

constraint (47) is fulfilled, the insurer reinvests an amount of funds equivalent to FCF_t and thereby possibly increases reserves stored in CB. On the other hand, if constraint (47) is not fulfilled, the insurer can sell assets which have a market value higher than the book value and thereby cash in the positive difference.[67] In other words, it is possible to obtain the required additional liquidity by selling assets that are trading above their book value. Therefore, in the case of an insufficient level of funds at hand (FCF_t), the additional liquidity needed is given by

$$HR_t^* = \sum_{i=1}^{N^l} \pi_t^i + \sum_{i=1}^{N^l-1} \left[l_{t-1}^{i,p,BV} \cdot max\left(r_t^{i,g}, r_t^p \right) \right] - FCF_t \tag{48}$$

where HR_t^* corresponds to the *funding gap*. For the sake of simplicity, we let the insurer adopt a straightforward trading strategy where asset classes are sold sequentially from the most to the least liquid and from the closest to the more distant to maturity, until the value HR_t^* is reached or at least minimized. The sequence follows the following rule: bonds are traded from the most liquid and with the shortest time to maturity (i.e. the oldest cohorts in a portfolio) to the least liquid and longest time to maturity. The degree of liquidity is given by the premium that every bond-like asset class pays over the corresponding sovereign interest rate as explained in section 2.2.1. In general, relatively higher coupons are substituted with lower ones with exactly the same face value and time to maturity.[68] If hidden reserves of the bond portfolio were not sufficiently high, the insurer would proceed in selling stocks and real estate (conditioned on their market value being greater than the book value). The insurer can replace as many assets as needed in order to comply with constraint (47) as long as assets with a market value higher than their book value are available. Otherwise, the management would reinvest only the available funds.[69] Finally, the reinvestment strategy follows a simple rule. As introduced in section 2.2.1, we assume that the insurer adopts a static investment strategy and chooses the allocation of funds according to the relative portfolio weights as reported in table 1. Since the aim is to maintain the asset allocation (and the level of risk) unchanged over time, we must condition the allocation of funds to both the FCF_t (amount

[67] We assume that the insurer sells high valuable assets and immediately reinvests an equivalent amount necessary to keep the book value unchanged.

[68] We do not assume any transaction cost and aim to maintain a relatively constant average duration in the asset portfolio.

[69] Please note that under these circumstances, the residual item B would become smaller as the balance clears the new amount of assets with the liabilities.

of funds available in t) and the amount of bonds to be replaced (bonds that came due in t).[70]

2.5 The Solvency Situation

The interplay between the asset and the liability side determines the solvency situation of the insurer at the end of every year. We calculate in every year the (total balance sheet) one year *Value-at-Risk* with a 99.5% confidence interval (VaR_α) as required by Solvency II regulation and compare it with the market value of the available capital (OF). Formally, we can express the solvency condition required by Solvency II as follows [71]

$$SCR_t := \underset{x}{\operatorname{argmin}} \Big\{ \mathcal{P}\Big(OF_t - \frac{OF_{t+1}}{1 + r_{f(t,1)}} > x \Big) \leq 1 - \alpha \Big\} \tag{49}$$

where the *Solvency Capital Requirement* (SCR) is defined as the smallest amount x satisfying (49), $r_{f(t,1)}$ is the 1-year maturity risk-free rate and α represents the confidence interval (here 99.5%). Condition (49) ensures that the probability that the loss over one year exceeds the SCR is less or equal to $1 - \alpha$. Moreover, as introduced in section 2.1, we need to calculate the RM as required by Solvency II regulation in order to estimate OF. The general method for calculating the RM is expressed as follows

$$RM_t = \frac{\sum_{t \geq 1}^{T} SCR_t}{(1 + r_{f(t,t+1)})^{t+1}} \cdot CoC \tag{50}$$

where $\sum_{t \geq 1}^{T} SCR_t$ is the projection of the solvency capital necessary to cover the entire life (T) of the liability portfolio discounted to the present time t using the risk-free term structure. Finally, CoC is the Cost-of-Capital rate that yields the RM_t of the entire portfolio at time t.[72] However, due to the complexity of the calculations required in order to assess the expected amount of SCR in every period until the portfolio redemption, we rely on average figures for European Life Insurers presented by EIOPA in the Final Report of the QIS 5 (2010). The report provides average RM calculations as a percentage of L^{BE} for with profit life insurance liabilities: we denote with ρ the markup on top of L^{BE} which we assume to remain constant over time. In the Appendix (A.6), we provide a detailed overview of the calculations to obtain the SCR in every period and thereby assess

[70] A detailed description of the reinvestment strategy follows in the Appendix (A.5).
[71] See, for instance, Bauer et al. (2012) and Christiansen and Niemeyer (2012)
[72] Under Solvency II regulation, the CoC is assumed to be fixed at 6%.

the solvency situation of the insurer in every period. To conclude and summarize the dynamics of the balance sheet adjustment over time, we report in figure 4 the timeline of the entire decision process that the insurer undertakes in every period.

3 Data and Calibrations

We calibrate different initial situations for the capital endowment and subsequently observe the solvency situation under different capital market scenarios. In table 6, we report the calibration for the CIR model and for the GBM. The CIR parameters were estimated from empirically observed data. We follow the method proposed in Brigo et al. (2009) and calibrate the parameters k, θ and σ_r based on the overnight interbanking interest rate prevailing in Germany[73] by means of a maximum likelihood estimation, in which λ has been set to 10%.[74] The parameters for the GBM were estimated on the DAX[75] and DREITS[76] indices for stocks and real estate respectively. Table 6 reports 3 different calibrations which we use to simulate 3 different capital market scenarios. By changing the long-term equilibrium interest rate in the CIR model, we are able to reproduce structurally different interest rate levels, as the model features a mean reverting behavior. Calibration 1 represents the situation in which interest rates stay at a similar level as observed in 2013 and is therefore our baseline scenario ($\theta = 0.02$). Calibration 2 ($\theta = 0.01$) reproduces an interest rate level similar to the level observed in Japan from the end of 1990s up to date (i.e. Japanese-like scenario), and 3 ($\theta = 0.03$) reproduces a scenario in which interest rates gradually recover towards a higher level.[77] The term structure of interest rates for Mortgage Pfandbriefe, Bank Bonds and Corporate Bonds is determined by adding the average spread observed between 1999 and 2013 on top of the CIR simulated term structure.[78] Moreover, we correlate the 3 stochastic processes by

[73] Until 1999, we used the overnight rate FIBOR, and subsequently EONIA (end of month data).

[74] As explained in Brigo and Mercurio (2006) a constant market price of risk is common practice for the Vasicek model, which can be similarly applied to the CIR model. Moreover, it is possible to derive an observed market price of risk by calculating the mean square error between the theoretical and the observed term structure (Fischer et al., 2003)). However, taking into consideration the time window 1973-2013, which includes big shocks such as the introduction of the euro and the great recession, the estimated coefficient was $\lambda = -0.39$. Therefore, we chose an arbitrary calibration of $\lambda = -0.1$, which allows for a more realistic (moderately concave) shape of the yield curve.

[75] Deutscher Aktien IndeX - Main German Stock Index.

[76] Diversified Real Estate Investment Trusts (REITS).

[77] It is worth noticing that calibration number 3 tries to simulate an interest rate environment similar to the one observed after the introduction of the euro, therefore still below market returns observed in the 1990s but above the returns observed after the 2008 crisis.

[78] See data in Table 2

means of deterministic correlation coefficients that we report in table 6, which in turn allows us to influence stock and real estate markets through the short-term interest rate process.

In table 7, we report the different initial capital endowment situations: we specify 5 different balance sheets (BS) with increasing amount of own funds at book values. We keep the equity capital level in percentage of assets fixed, which according to BaFin data for 2012 amounted to ca. 1.7% of the aggregated liability side of German life insurers.[79] Moreover, we calibrate different initial amounts of funds stored in CB: we consider a set of German Life Insurers that represents ca. 90% of the market in terms of annual premiums and extract the distribution of available capital at book value ($E + CB$) at the end of 2012. We then select the 10^{th}, 30^{th}, 50^{th}, 70^{th} and 90^{th} percentile respectively and derive the amount of funds stored in the CB by keeping E constant. By doing so, we aim to estimate the resilience of the balance sheet to adverse capital market conditions given different initial indebtedness levels (i.e. leverage ratios). Moreover, we introduce additional constraints such as *i*) the amount of market value increments cashed in as dividends ($\vartheta = 50\%$), *ii*) the maximum allowed dividends paid to shareholders ($\delta = 5\%$), *iii)* the corridor for the additional return distribution (a, b) and *iv*) the fixed amount in percentage of total premiums of returns from mortality assumptions (r_q) and *iv)* the fixed markup (ρ) used to calculate the RM. We project each balance sheet 10 years forward under 10000 CIR and GBM iterations (i.e. the underlying capital markets simulation). At the end of every year, we simulate 10000 1-year developments of the asset portfolio and liability portfolio in order to assess the solvency situation.

4 Results

We first discuss the results from the book value perspective under the 3 different capital market calibrations and then the solvency situation (i.e. the market value perspective) for the 5 different initial capital endowments (BS $1-5$). Figure 5 reports the results of the simulation using the capital markets calibration 1, i.e. our baseline scenario (see table 6). Figure 6 reports the results for calibration 2, i.e. the Japanese-like scenario, and 7 reports the result for calibration 3, i.e. the most favorable scenario. Each figure depicts growth rates of A and L and the levels of hidden reserves in the upper left and right corner respectively, whereas the solvency situation is depicted

[79] BaFin Statistics on primary insurers, 2012. Data are expressed at book values.

in the bottom left and right corner. We compare the 5 different initial capital endowments (see table 7) by using the same underlying capital markets simulation.

4.1 Book Value Perspective

The first dynamics we examine are reported in figure 5.1 (fig. 5): we compare the *total rate of return* (including returns from mortality) with the *actual growth rate of the policyholders' accounts* (excluding interest rates reserves and the participation in hidden reserves). All initial capital endowments display a similar development regarding both the total rate return and the return to policyholders. This is due to *i*) the lower return (i.e. lower cash flow) that the insurer is able to achieve given the chosen asset allocation and *ii*) the fact that on average, no additional return is distributed and therefore policyholders' accounts grow in most cases at their guaranteed return.[80] Moreover, the insurer is forced to increase the level of technical reserves due to the decrease in the discount factor (i.e. the reference interest rate) and to distribute part of the hidden reserves. As interest rates remain persistently low, the reference interest rate adjusts downward and the value of the asset portfolio increases, implying the creation of additional technical reserves and the increase in payout benefits to policyholders. The substitution of the discount factor is a mere accounting adjustment, since no change occurs in the final payout to policyholders. The main effect of such additional technical reserve is the increased (mandatory) claim that policyholders have on the cash flow generated by the asset side, which entails that fewer funds are left over for dividends payments and additional returns. In order to reduce the *funding gap* and therefore to avoid financial distress, the insurer would always cash in hidden reserves as long as they are available.[81] As figure 5.2 shows, in 2014 the aggregate market value of assets is relatively high compared to its book value as a consequence of low interest rates (i.e. discount factors). A part of those reserves must be paid out to policyholders and used to offset the lower return on assets. Under all initial capital endowment calibrations, the pace of the decrease in hidden reserves follows a similar development. During the first 3 years, the outstanding liability structure is very expensive to fund compared to the generated cash flow, both because of the guaranteed returns and because of the hidden reserves sharing regulation. If lower capitalized insurers suffer a higher funding gap, more

[80] Recalling condition 32, the insurer would avoid additional return distribution, since the return on assets is below the return of the policyholders.

[81] Recall constraint 48.

capitalized insures would need to share a proportionally higher amount of hidden reserves. This partially explains why under all initial capital endowments, hidden reserves decrease at a similar pace. Furthermore, part of the decrease in hidden reserves is the result of the substitutions of (old) bonds with relatively higher coupons that come due with bonds with lower coupons. Finally, since the stochastic process underlying the term structure features a mean reverting behavior, interest rates eventually rise toward their long-term equilibrium and therefore further contribute to the average decline in disposable hidden reserves. Therefore, high value assets need to be replaced in order to increase the level of the cash flow and thereby keep up with the funds that need to be transferred to policyholders.[82] As time passes and old expensive contracts are liquidated, the balance sheet adjusts towards the new lower interest rate level.

Figure 6 reports the results of the most adverse scenario. As interest rates stay at an even lower level, the effect on the cash flow dynamics is threefold: *i*) the total rate of return is lower than it was under calibration 1, *ii*) the need for additional technical reserves increases and *iii*) the amount of hidden reserves to be shared with policyholders is higher. The 3 effects together result in a funding gap represented in figure 6.1: in fact, the total rate of return on average becomes insufficient to keep up with the increase in policyholders' accounts. The dynamics we observe in the reduction of hidden reserves is similar to the baseline scenario: under all initial capital endowments, hidden reserves decrease quickly during the first 3 years, due to bond portfolio adjustment to new market interest rates, hidden reserves participation and the funding gap.

Finally, figure 7 reports the results of the most favorable situation where interest rates are at a higher level. The insurer under this capital market calibration enjoys both a higher rate of return and a lower amount of funds that need to be transferred to policyholders as required by regulation. Figure 7.2 reports the development of hidden reserves. The reason why hidden reserves still fade out over the medium term is twofold: on the one hand, as interest rates slowly converge to a higher level, the insurer might still need to dissolve hidden reserves during the first years; on the other hand, as old bonds come due, the market value of the asset portfolio adjusts to the new interest rate level.[83]

[82] Recall that our trading strategy replaces assets from the most liquid and the closest to maturity to the least liquid and with the longest time to maturity.

[83] The CIR model features a mean reverting process: this implies that given the initial calibration, interest rates tend to be lower at the beginning of the simulation and gradually converge toward the 3% level.

4.2 The Market Value Perspective

We now consider the solvency situation under the capital markets calibration 1 (figure 5). In figure 5.3, we report the *solvency ratio* (defined as $\frac{OF_t}{SCR_t}$). The increase in the market value observed on the asset side is more than offset by the increase in the market value of liabilities. This is due to the duration mismatch of the balance sheet: the higher average duration of the liability side results in a higher sensitivity to interest rate changes. Since we use the simulated term structure of interest rates to discount both assets and liabilities, a generalized decrease in discount factors affects the longer maturities more heavily than the shorter ones.[84] This results in a higher increase in the value of liabilities compared to assets with strong implications on the solvency situation of the insurer: the higher present value of liabilities becomes detrimental for the solvency situation of almost all initial capital endowments. In 2014, both BS 1 and BS 2 already have, on average, an amount of own funds that is below the regulatory requirement. As the interest rate level remains low, the solvency situation deteriorates: figure 5.4 reports the *cumulative probability of default* (calculated as the number of iterations with negative OF) which peaks by 2022 and reaches 6.5% for the least capitalized insurer. Moreover, we can also calculate an expected cumulative probability of default across all initial capital endowments: each initial capital endowment roughly represents 20% of life insurers operating in Germany, therefore the weighted average of the cumulated default probability at the end of 2023 is 3.95%.

Under capital markets calibration 2 (figure 6), the solvency situation becomes worse across all initial capital endowments. In figure 6.3, we can see how low interest rates increase the present value of liabilities to a point where even the median balance sheet (BS 3) would not be able to meet the required solvency capital. In figure 6.4, the cumulative probabilities of default have similar developments as under calibration 1, but here defaults occur earlier. Moreover, the expected probability of default across all balance sheets reaches 4.17% by 2023. Under this capital market calibration, the average solvency situation deteriorates earlier: we already observe defaults in 2016, compared to calibration 1, where we observe defaults starting in 2017. Moreover, the median solvency ratio for the least capitalized insurer reaches 100% a year later (2019) compared to calibration 1 (2018).

Finally, under capital markets calibration 3 (figure 7), solvency concerns are moderate. The

[84] Discount factors for Mortgage Pfandbriefe, Bank Bonds and Corporate Bonds are obtained by adding the liquidity premium on top of the simulated term structure of interest rates.

reason lies in the fact that the decrease in the present value of liabilities is stronger than the decrease in the value of assets. This is once more the effect of the higher sensitivity of the liability side, that through its longer duration reacts to changes in discount factors more strongly than the asset side. The net effect turns out to be beneficial for the solvency situation of the insurers.

Reform of the Regulatory Framework

Figure 8 depicts the results of the simulations under the new regulatory framework as of August 2014. Figures 8.1, 8.2 and 8.3 report the cumulative probability of default under calibration 1, calibration 2 and calibration 3 respectively. Compared to the results reported in figures 5.4, 6.4 and 7.4, we see that the reform, at first sight, has a generalized beneficial impact on the cumulative probability of default. The lower hidden reserves payout to policyholders together with the possibility of subsidizing negative results on asset returns with other profit sources, decrease both the cash outflows and the present value of liabilities thereby reducing the probability of default of the insurer. Moreover, the effect seems to be stronger under calibration 2 rather than calibration 1: this is no surprise, since the reduction of both hidden reserves payouts and the minimum profit participation under calibration 2 is stronger than under calibration 1 due to (on average) lower interest rates. In fact, as interest rates decline sharply, L^S increases thereby reducing the share of hidden reserves to be distributed, i.e. HR^{ph}; the same holds true for the profit participation, i.e. $r^{i,g}$, in which mortality profits can offset lower asset returns.

Finally, it is worth mentioning that the possibility of reducing the minimum profit participation through a *cross-subsidization* between profit sources is close in spirit to a haircut on the value of the policyholders' accounts.

5 Discussion and Conclusion

The present analysis attempts to assess and quantify the effects of a prolonged period of low interest rates on life insurance companies in the presence of a stock of old policies with expensive guarantees. The model we develop allows us to assess the resilience of a stylized German life insurer to a protracted period of low interest rates. Our results suggest that a protracted period of low interest rates would markedly affect the solvency situation of life insurance companies, in particular of the less capitalized companies. In line with the Deutsche Bundesbank Financial Stability Review (Deutsche Bundesbank, 2013), a subset of companies might default should interest rates remain at the present low level (calibration 1). Under a more severe interest rate scenario, (a Japanese-like situation - calibration 2) the solvency situation would deteriorate faster compared to a scenario similar to the present interest rate level. In contrast, a gradual increase in interest rates would be beneficial with a more than proportional improvement of the median solvency situation and decrease of expected default probabilities. This is due to the asset and liabilities duration mismatch typical of the life insurance business (i.e. the higher sensitivities of longer maturities compared to shorter ones), where a protracted low interest rate environment would cause more damages than a sudden increase in interest rates would. Moreover, we are able to show that the recent change in the German life insurance regulation substantially improves the situation, especially for less capitalized companies which would not be able to bear the losses stemming from their liabilities. Yet, this improvement comes at the expense of lower benefit payments to policyholders who would experience a reduction on the minimum profit participation and therefore a haircut on their claims.

However, the results of our analysis are strongly dependent on both the calibration of the model and on the necessary simplifications adopted. The solvency situation depends heavily on the duration mismatch between assets and liabilities: as we do not possess detailed company-level information, we were forced to imply a representative level of duration mismatch. Thus, a slight increase (or decrease) in the duration mismatch could lead to different results. In addition, the model relies on simplifying assumptions that considerably influence the final result. In general, the model is a reduced version of a life insurer's balance sheet without product lines diversification, group diversification or reinsurance activities. Moreover, more general equilibrium implications such as an underlying economic development or policyholders' reactions are missing. The last point is particularly relevant in the light of the newly allowed reduction in the payouts to policyholders, which could cause substantial reputational costs for the insurer. Finally, limited data availability remains a major impediment for a more precise analysis. For instance, a more precise estimation of the liability structure and the asset portfolio allocation could significantly improve our analysis and add robustness to our results.

In conclusion, the present analysis aims at introducing an analytic tool which can be easily adapted to other countries and other regulatory frameworks in order to assess the effects that the current financial market condition is having on life insurers, or, in general, on providers of long-term financial promises such as pension funds. Since the current situation is relatively new for the industry, more analyses should be done in this area, especially with a focus on possible reactions of life insurers to such an environment. In addition, in light of the new requirements for the Forward Looking Assessment of Own Risk (FLAOR) underlying the preparatory phase for S II, the analytic tool of this article could be of particular interest for both regulators and the industry.

Moreover, the analysis might be relevant for the conduct of monetary policy. In fact, a loose monetary policy with the aim of low bond yields has two major side effects: *i*) the solvency situation of financial institutions deteriorates as interest rates remain low, and *ii*) low capital market returns can create incentives for investors towards more risky investments. The latter is commonly referred to in the literature as the risk of *gambling for redemption*.[85] It could have a strong impact on liability driven businesses such as life insurers and pension funds: as yields in the bond market are kept artificially low, the bonds' default risk might be underpriced, thereby creating incentives for managers to assume a higher firm risk level than would be acceptable in "normal" times. Moreover, a consequence of increased risk taking could be a further reduction of the average duration of the bond portfolio, which, in turn, would widen the duration gap. Adding such features to our model could change the results substantially: On the one hand, a riskier investment allocation would *i)* yield higher expected return on assets (and higher expected cash flows) and thereby possibly decrease the funding gap and the associated probability of default, and *ii)* increase the default probability of the asset portfolio, which, in turn, would increase the probability of default of the insurer. On the other hand, the internal model which we developed to calculate the SCR, would point to higher capital holdings due to an increase in tail outcomes in the asset portfolio and an increase in the duration mismatch. Thus, the net effect of such a trade-off is not clear and should be analyzed further.

[85] See, for instance Rajan (2005) and Antolin et al. (2011).

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A Appendix

A.1 The Cox-Ingersoll-Ross Model

The simulation of the continuous time first-order auto regressive process (2) can be obtained by means of the following recursive equation with discretization time t_i:

$$r_{t_i} = k\theta\Delta t + (1 - k\Delta t)r_{t_{i-1}} + \sigma\sqrt{r_{t_{i-1}}\Delta t}\varepsilon_{t_i} \tag{51}$$

with $\varepsilon \sim \mathcal{N}(0,1)$ and $\Delta t = t_i - t_{i-1}$ (Brigo et al., 2009).

A.2 The Geometric Brownian Motion

The simulation of the process follows the recursive version of equation (8) with discretization time t_i:

$$S_{t_i} = S_{t_{i-1}} e^{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}\varepsilon_{t_i}} \tag{52}$$

with $\varepsilon \sim \mathcal{N}(0,1)$ and $\Delta t = t_i - t_{i-1}$ (Brigo et al., 2009).

A.3 The Regulator's Reaction Function

The Regulator in the model reacts to the changes in the reference interest rate (r_t^{ref}) according to the following rule:

$$\begin{cases} r_{t+1}^g = r_t^g - \omega, & \text{if} \quad r_t^{ref} \leq r_t^g \\ r_{t+1}^g = r_t^g + \omega, & \text{if} \quad r_t^{ref} \geq r_t^g + \omega \\ r_{t+1}^g = r_t^g, & \text{otherwise} \end{cases} \tag{53}$$

where r_t^g is the maximum allowed guaranteed return at time t and ω is the marginal change decided by the regulator. Consistent with the observed changes in the technical interest rate in recent years, we assume $\omega = 50$ basis points. Please note that time steps are monthly, therefore adjustments do not necessarily happen at the end of the year.

A.4 Additional Return to Policyholders

By determining the final value of L at time t which is given by

$$L_t^{p,BV} = L_t^{g,BV} + R_t^{ph,p} \tag{54}$$

we can indirectly determine r_t^p by solving the following equation:

$$L_t^{p,BV} = \sum_{i=1}^{N} l_t^{i,BV} \cdot \Big[1 + max\Big(r_t^{i,g}, x\Big)\Big] \tag{55}$$

where x is the total offered rate across all cohorts of contracts. We solve the equation by means of the solver routine in Matlab©. It is worth noting that determining r_t^p starting from the total amount of funds to be distributed has the desirable property of maintaining *fairness* among policyholders with respect to additional return distribution. Indeed, choosing a unique r^p across all tariff generations combined with a max operator ensures that the higher the guaranteed return of the cohort, the (relatively) lower the amount of additional return it gets assigned.

A.5 The Reinvestment Strategy

If the amount of funds coming from bonds due in time t is smaller or equal to the FCF_t, the amount of cash allocated to each asset classes follows the following rule for bond-like asset classes

$$B_{(t,T)}^{b,BV} = B_{(t,0)}^{b,FV} + \frac{\sum_{\tau=1}^{T} B_{(0,T-\tau)}^{b,BV}}{A_0^{BV}} \cdot \Big(FCF_t + HR_t^* - \sum_{b=1}^{N^b} B_{(t,0)}^{b,FV}\Big) \tag{56}$$

and for stock-like asset classes

$$S_t^{k,BV} = \frac{S_0^{k,BV}}{A_0^{BV}} \cdot \Big(FCF_t + HR_t^* - \sum_{b=1}^{N^b} B_{(t,0)}^{b,FV}\Big) + S_{t-1}^{k,BV} \tag{57}$$

where $B_{(t,0)}^{b,FV}$ is the amount of funds returned from the bonds that came due, $\frac{\sum_{\tau=1}^{T} B_{(0,T-\tau)}^{b,BV}}{A_0^{BV}}$ and $\frac{S_0^{k,BV}}{A_0^{BV}}$ are the relative weights of the asset class (bonds and stocks respectively) in the portfolio (fixed from $t = 0$). This implies that the insurer first replaces the book value of bonds that came due and the residual funds are distributed among all asset classes according to their relative weight in the portfolio. If the amount of funds coming from bonds due in time t is greater than the available (for reinvestment) funds, then the portfolio is re-balanced as follows for bond-like asset classes

$$\frac{\sum_{\tau=1}^{T} B_{(t,T-\tau)}^{b,FV} - B_{(t,0)}^{b,FV} + B_{(t,T)}^{b,BV}}{A_{t-1}^{FV} - \sum_{b=1}^{N^b} B_{(t,0)}^{b,FV} + FCF_t + HR_t^*} \stackrel{!}{=} \frac{\sum_{\tau=1}^{T} B_{(0,T-\tau)}^{b,BV}}{A_0^{BV}} \tag{58}$$

and for stock-like asset classes

$$\frac{S_t^{k,BV}}{A_{t-1}^{FV} - \sum_{b=1}^{N^b} B_{(t,0)}^{a,FV} + FCF_t + HR_t^*} \stackrel{!}{=} \frac{S_0^{k,BV}}{A_0^{BV}} \tag{59}$$

where $B_{(t,T)}^{b,BV}$ and $S_t^{k,BV}$ represents the allocation of funds for bonds and stocks respectively. Regarding the stock-like investments, it might happen that their book value must be reduced compared to the previous period, i.e. $S_t^{k,BV} < S_{t-1}^{k,BV}$, which in turn also implies a proportional reduction in its market value according to the ratio $\frac{S_t^{k,MV}}{S_t^{k,BV}}$. However, given that we are in a situation where the amount of bonds that came due is bigger than the available cash, the ratio is equal to 1, as our trading strategy would cash-in all reserves available.

A.6 The Solvency Requirements

We define the SCR_t as the minimum amount of capital to be held in t in order to ensure solvency in $t+1$ with at least a probability of $1-\alpha$, which can be expressed as follows

$$\mathcal{P}\{OF_{t+1} \geq 0\} \geq 1-\alpha. \tag{60}$$

or equivalently as follows

$$\mathcal{P}\Big\{\widetilde{A_{t+1}^{MV}} - \big(\widetilde{L_{t+1}^{BE}} + \widetilde{RM_{t+1}}\big) \geq 0\Big\} \geq 1-\alpha. \tag{61}$$

Since $\widetilde{A_{t+1}^{MV}} = \big(SCR_t + RM_t + L_t^{BE}\big) \cdot \big(1 + \tilde{r}_{t+1}\big)$ with $\tilde{r}_{t+1}$ being the 1-year stochastic growth rate of assets, we can rewrite equation 61 as follows

$$\mathcal{P}\Big\{\big(SCR_t + RM_t + L_t^{BE}\big) \cdot \big(1 + \tilde{r}_{t+1}\big) - \big(\widetilde{L_{t+1}^{BE}} + \widetilde{RM_{t+1}}\big) \geq 0\Big\} \geq 1-\alpha. \tag{62}$$

Since RM_t is a function of L_t^{BE}, we can substitute it with $RM_t = \rho \cdot L_t^{BE}$ and $RM_{t+1} = \rho \cdot \widetilde{L_{t+1}^{BE}}$ respectively and obtain the following expression

$$\mathcal{P}\Big\{ \Big(\widetilde{SCR_t} + (1+\rho) \cdot L_t^{BE}\Big) \cdot (1 + \tilde{r}_{t+1}) - (1+\rho) \cdot \widetilde{L_{t+1}^{BE}} \geq 0 \Big\} \geq 1 - \alpha \tag{63}$$

where SCR_t is the minimum amount of capital the insurer must hold in t in order to comply with a default probability of α. In order to determine the distribution of $\widetilde{OF}_{t+1}$ in every period and thereby determine SCR_t, we need to determine $\tilde{r}_{t+1}$ and $\widetilde{L}_{t+1}^{BE}$. To obtain $\tilde{r}_{t+1}$, we first determine $\widetilde{A}_{t+1}^{MV}$: we project 1 year ahead the bond portfolio at book values and value it at time $t+1$ according to the following equation

$$\widetilde{B}_{(t+1,T-\tau-1)}^{b,MV} = \sum_{j=\tau+1}^{T-1} \left(\frac{B_{(T-\tau-1)}^{b,FV} \cdot i_{c,(T-\tau-1)}^{b}}{1 + \tilde{i}_{d,(t+1,j-\tau)}} \right) + \frac{B_{(T-\tau-1)}^{b,FV}}{1 + \tilde{i}_{d,(t+1,T-\tau-1)}} \tag{64}$$

where coupons cashed in and bonds due at $t+1$ are added on top of the market value of the remaining bonds. The discount factor ($\tilde{i}_d$) is the risk-free rate simulated with a 1-year ahead CIR model, where the starting point of the interest rate process (r_0) is the yearly average of the instantaneous rate of the underlying capital market CIR simulation at time t. All other parameters (i.e. α, θ, σ and λ)) remain unchanged. Furthermore, depending on the asset class, it includes a liquidity spread as explained in section 2.2.3. Stocks and real estate are simulated using GBM, in which the starting point is the market value of the assets at time t ($S_t^{t,MV}$) and parameters μ and σ remain unchanged.[86] Finally, we sum up all asset classes and obtain the value of the asset side at time $t+1$, which is given by

$$\widetilde{A}_{t+1}^{MV} = \sum_{b=1}^{N^b} \sum_{\tau=1}^{T} \widetilde{B}_{(t+1,T-\tau-1)}^{b,MV} + \sum_{k=1}^{N^k} \widetilde{S}_{t+1}^{k,MV}. \tag{65}$$

The stochastic growth rate of assets is then given by

$$\tilde{r}_{t+1} = \frac{\widetilde{A}_{t+1}^{MV} - A_t^{MV}}{A_t^{MV}}. \tag{66}$$

[86] We assume the interest rate process and the GBMs to be correlated, as we did for the underlying capital market simulation.

For the liability side, we follow the same approach: we determine the sum of all ongoing cohorts of contracts valued at time $t+1$ which we define as follows

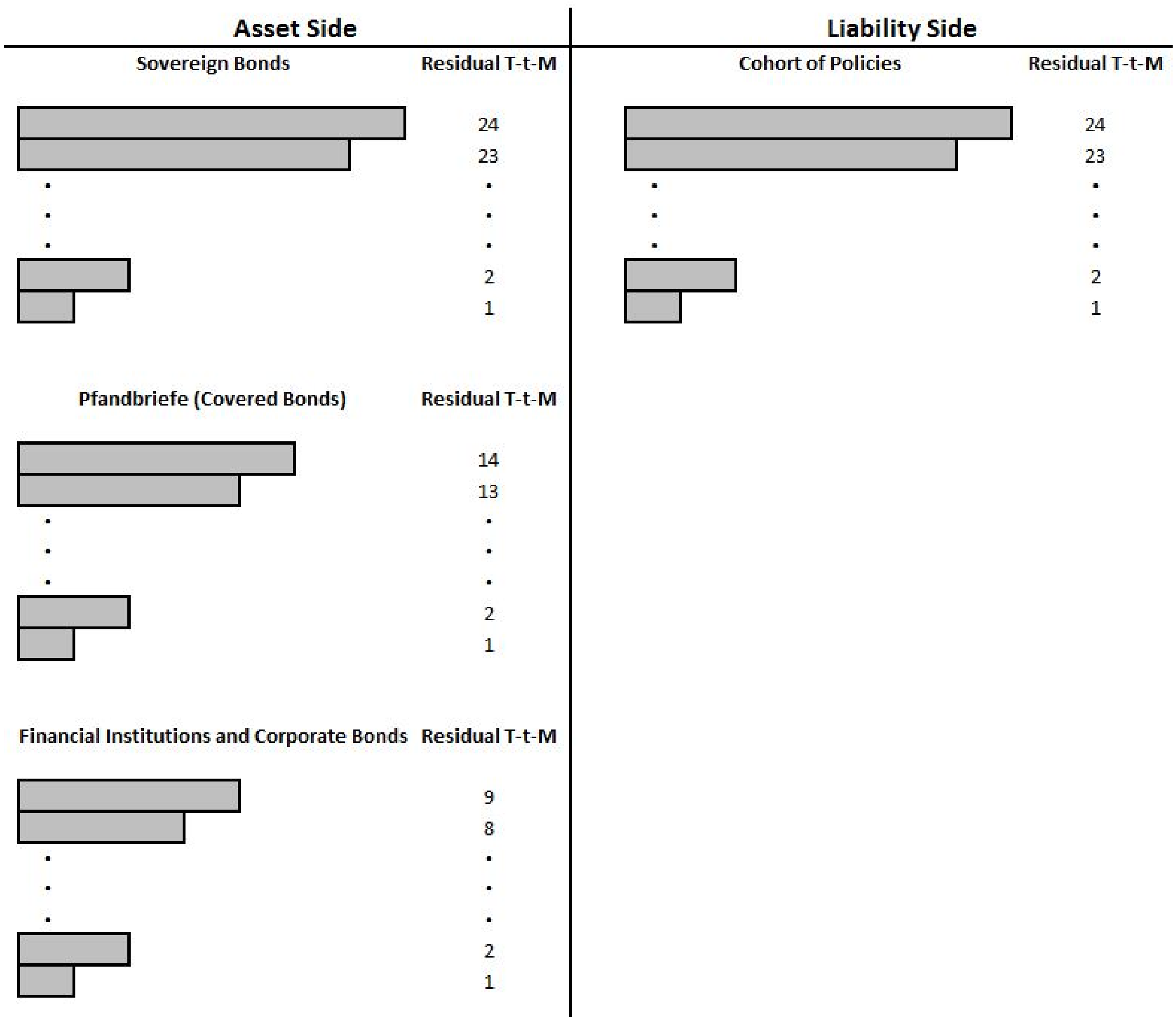
$$\widetilde{L}_{t+1}^{BE} = \sum_{i=1}^{N^l} \tilde{l}_{t+1}^{i,BE} = \sum_{i=1}^{N^l} \frac{l_{(t+1,T-\tau-1)}^{i,g,BV} \cdot \left(1 + max\left(0.9 \cdot \tilde{r}_{t+1}, r^i\right)\right)^{(T-\tau-1)}}{\left(1 + \tilde{i}_{d,(t+1,T-\tau-1)}\right)^{(T-\tau-1)}} \tag{67}$$

where the $l^{i,BV}$ is calculated 1 period ahead taking into account the minimum profit participation occurring in $t+1$. Consequently, $\tilde{l}^{i,BE}$ is determined on the $t+1$ level of the policyholder's account, where the discount factor $\tilde{i}_d$ is the risk-free rate simulated with a 1-year ahead CIR model. Finally, $(t+1, T-\tau-1)$ indicates the point in time $t+1$ and the corresponding maturity of $T-\tau-1$ years.[87] Once we obtain $\tilde{r}_{t+1}$ and $\widetilde{L}_{t+1}^{BE}$, we can estimate SCR_t and assess the solvency situation.

[87] The oldest cohort will not be discounted, as the final payment occurs at time $t+1$

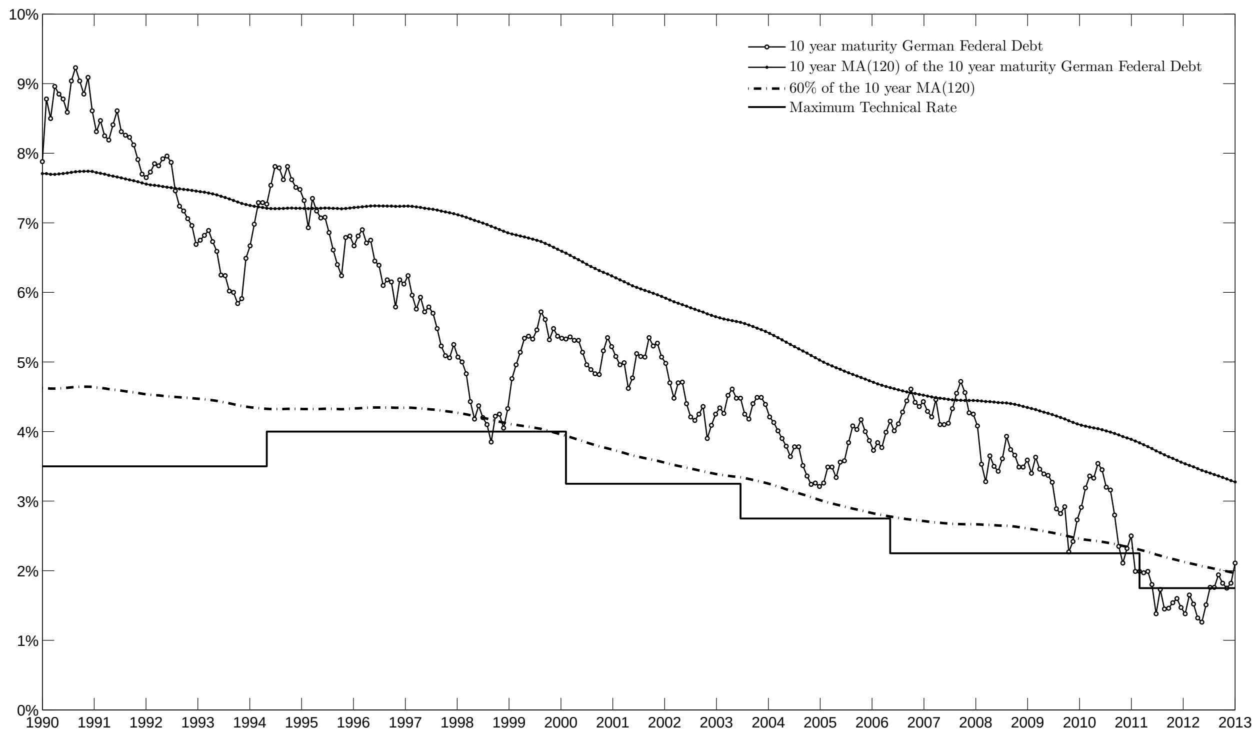
B Figures

Figure 2: Time to Maturity (T-t-M) Structure of Assets and Liabilities.



* Residual Time to maturity expressed in years. Please note that the Time to Maturity remains fixed over time, whereas the duration might vary according to the interest rate level.

Figure 3: Maximum Allowed Guaranteed Return



source: Deutsche Bundesbank and BaFin

Figure 4: Time line of the Balance Sheet in time t

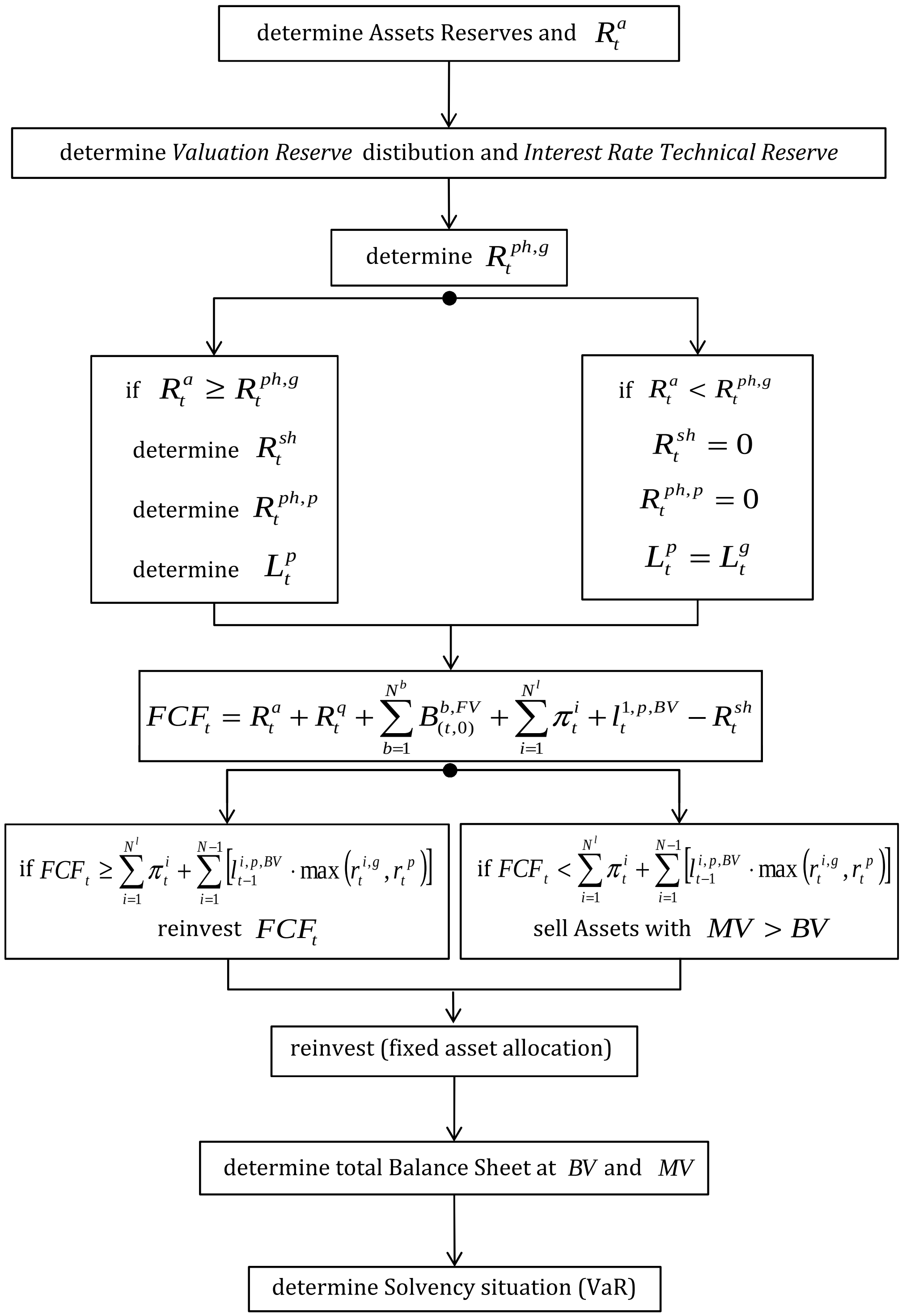


Figure 5: Results Capital Markets calibration 1 (median values)

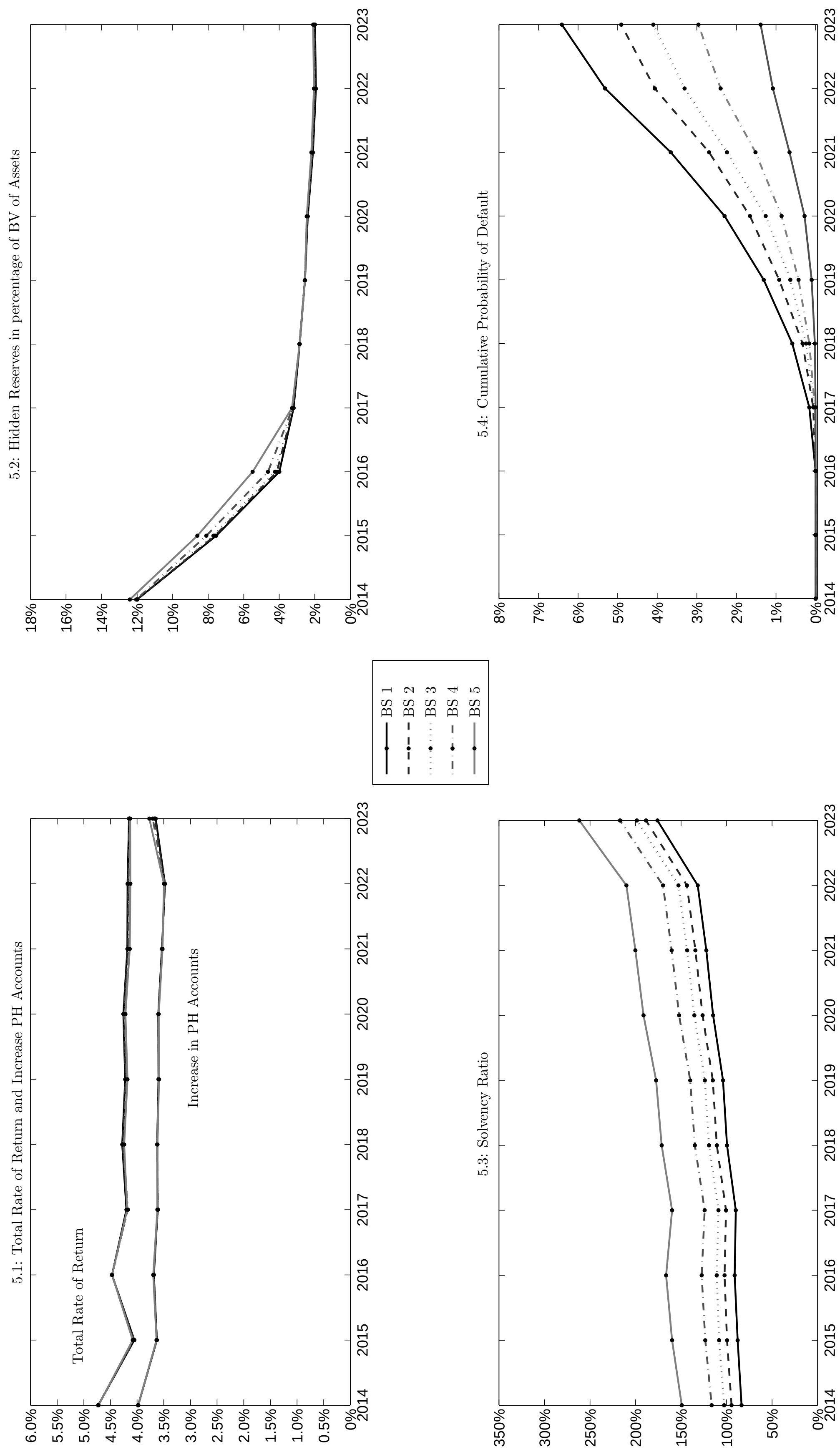


Figure 6: Results Capital Markets calibration 2 (median values)

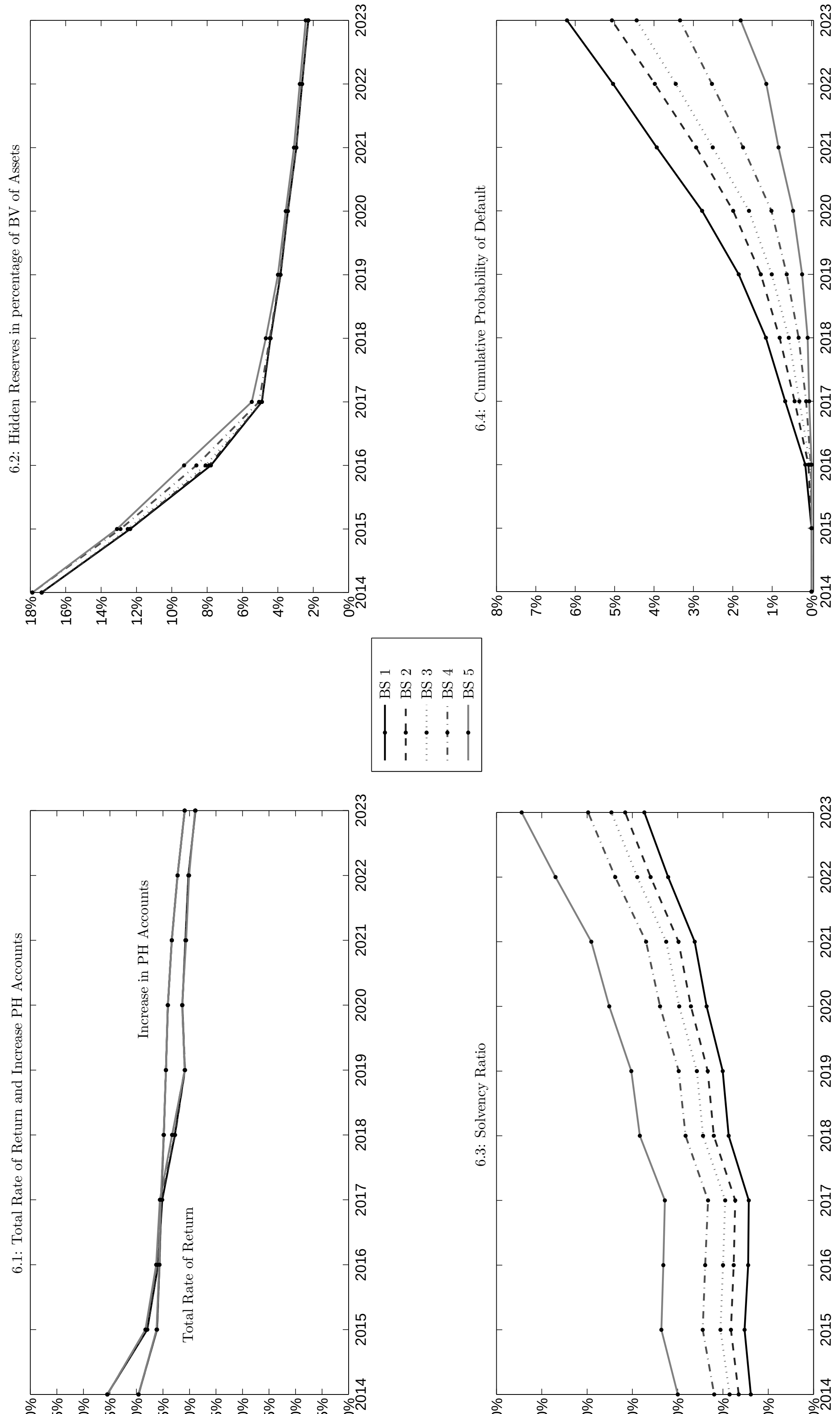


Figure 7: Results Capital Markets calibration 3 (median values)

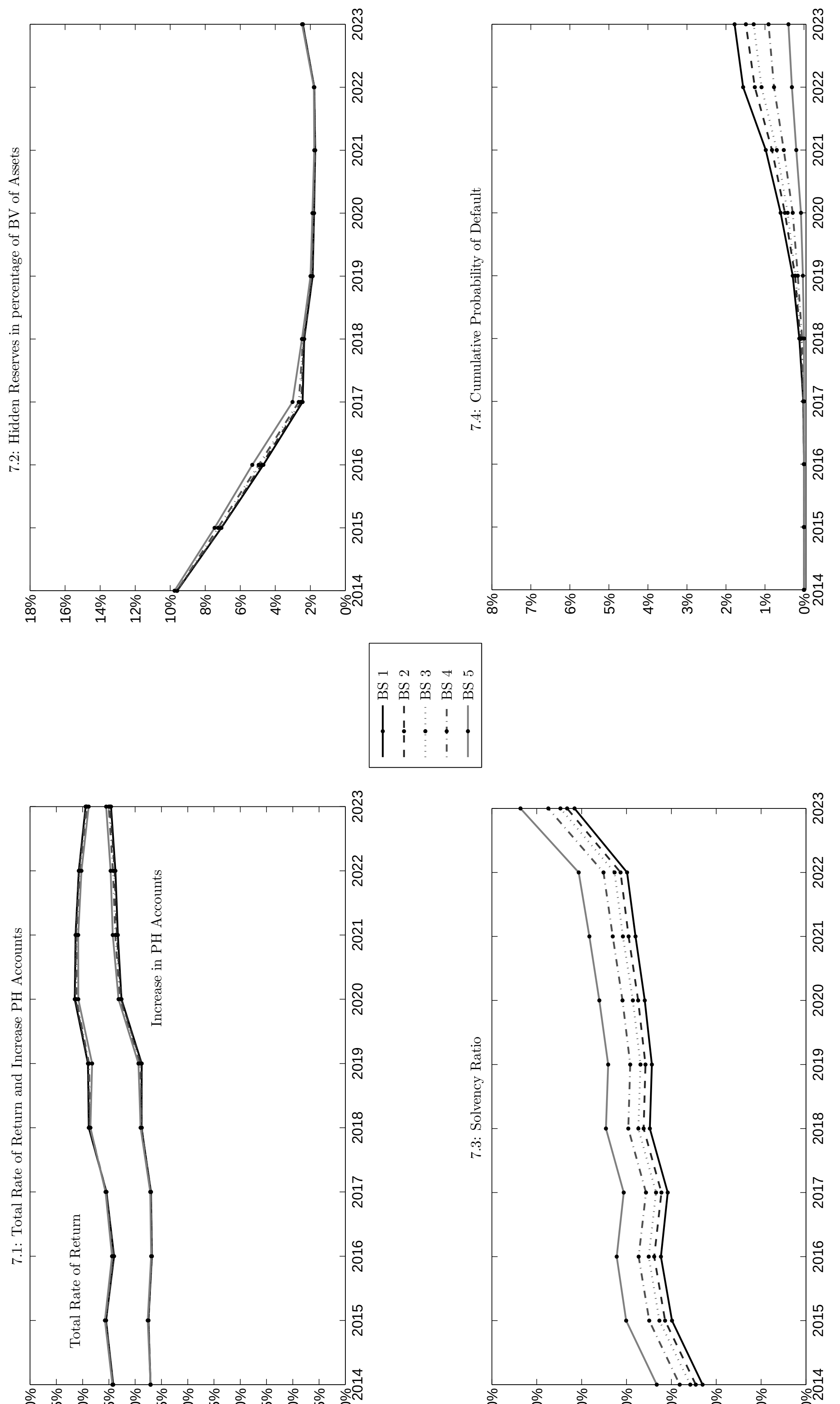
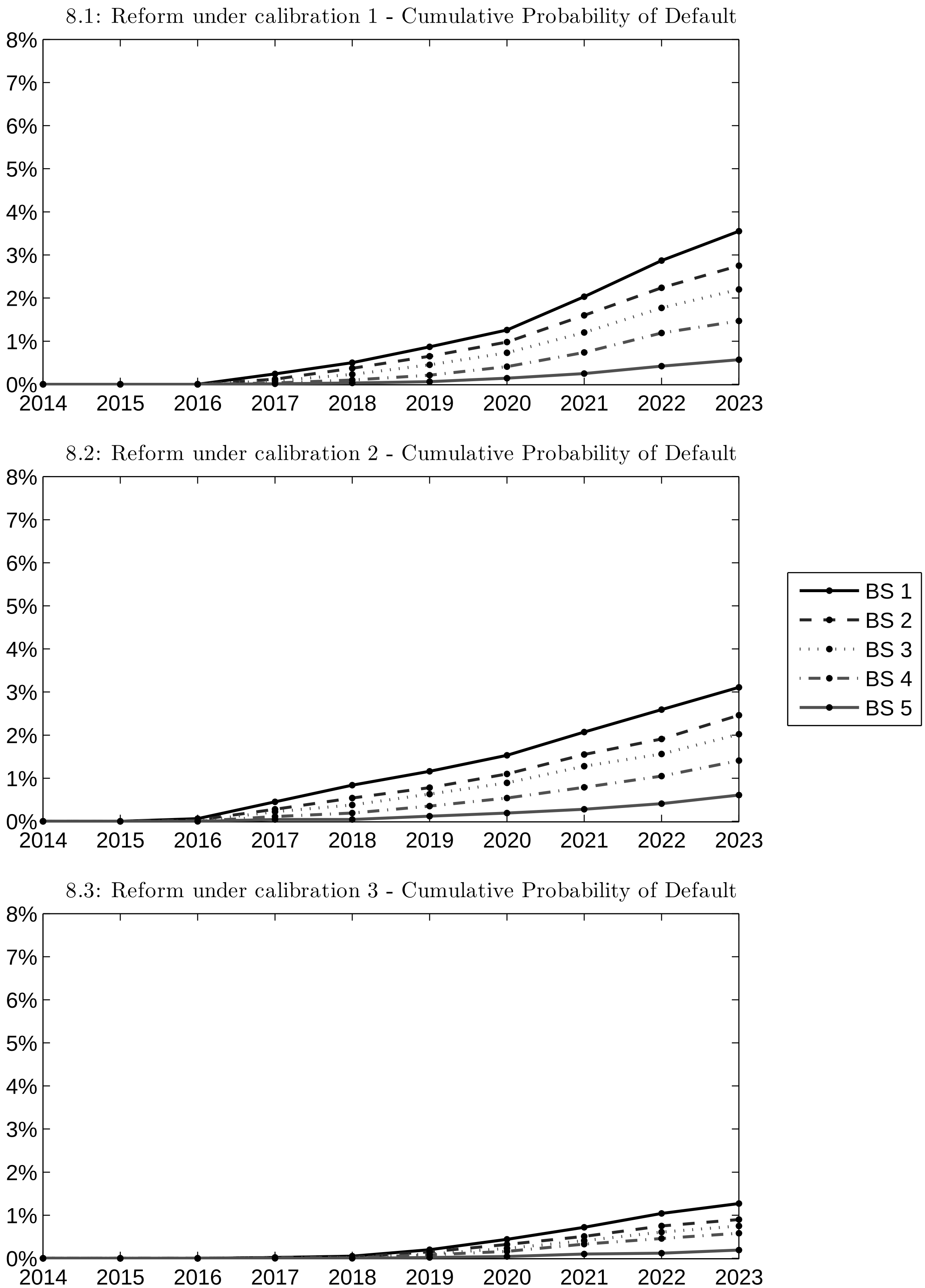


Figure 8: Cumulative Probability of Default after the Reform of the Regulatory Framework



C Tables

Table 1: Breakdown of the Asset Side by Asset Class

Asset Classes	**share***
Sovereign Debt	34.4%
Mortgage Pfandbriefe	34.0%
Credit Institutions Bonds	13.4%
Corporate Bonds	9.2%
Stocks	5.3%
Real Estate	3.8%

Source: Statistical Yearbook of German Insurance 2013 (GDV) and authors' calculations.
* Bonds held through funds were split 80% Sovereign and 20% Pfandbriefe. Assets classified under "Other Investments" (1.7%) were proportionally redistributed among all other asset classes.

Table 2: Interest Rate Differentials versus Term Structure of Listed German Federal Securities

Maturity	1999 – 2013*	1999 – 2008**	2008 – 2013***
Mortgage Pfandbriefe†			
1	0.55%	0.31%	0.94%
2	0.51%	0.30%	0.86%
3	0.51%	0.30%	0.84%
4	0.50%	0.30%	0.84%
5	0.50%	0.30%	0.82%
6	0.50%	0.31%	0.81%
7	0.50%	0.32%	0.79%
8	0.50%	0.32%	0.78%
9	0.50%	0.33%	0.77%
10	0.51%	0.34%	0.77%
11	0.52%	0.36%	0.78%
12	0.53%	0.37%	0.80%
13	0.55%	0.38%	0.83%
14	0.57%	0.39%	0.87%
15	0.60%	0.40%	0.91%
Credit Institutions Bonds			
1	0.49%	0.24%	0.93%
2	0.48%	0.24%	0.92%
3	0.47%	0.24%	0.90%
4	0.46%	0.23%	0.86%
5	0.46%	0.22%	0.88%
6	0.42%	0.23%	0.75%
7	0.41%	0.23%	0.75%
8	0.39%	0.23%	0.68%
9	0.33%	0.21%	0.57%
10	0.33%	0.21%	0.57%
Corporate Bonds‡			
1 – 10	1.63%	1.06%	2.67%

*) Monthly data from January 1999 to December 2013.

**) Monthly data from January 1999 to August 2008.

***) Monthly data from September 2008 to December 2012.

† Data for the term structure of Mortgage and Public Pfandbriefe available from January 2000.

‡ Due to the lack of data, differentials were calculated on the average maturity of the sample and assumed to be constant for all maturities (1 – 10).

Table 3: Liability Portfolio Composition at $t = 2013$

inception period	guaranteed return	relative weight in portfolio
1/1989 – 6/1994	3.50%	22%
7/1994 – 6/2000	4.00%	24%
7/2000 – 12/2003	3.25%	14%
1/2004 – 12/2006	2.75%	12%
1/2007 – 12/2011	2.25%	20%
1/2012 – 12/2013	1.75%	8%
weighted average	3.11%	

Note: weights are determined by using the number of months within the inception period.

Table 4: Breakdown per source of Return net of the guaranteed Rate of Return (tsd. €)

year	liabilities	assets	mortality	costs & others	total	$\frac{mortality}{liabilities}$
2002	512,935	1,339	4,590	(697)	5,231	0.9%
2003	521,670	5,886	4,697	(1,502)	9,081	0.9%
2004	545,310	7,878	4,478	(2,218)	10,139	0.8%
2005	562,009	10,668	5,569	(1,796)	14,441	1.0%
2006	578,381	9,337	6,363	(1,477)	14,223	1.1%
2007	595,236	8,533	6,381	(1,161)	13,754	1.1%
2008	607,796	892	6,498	(575)	6,815	1.1%
2009	627,966	5,485	6,463	(130)	11,819	1.0%
2010	654,133	6,569	6,460	(871)	12,158	1.0%
2011	666,677	4,481	6,518	(717)	10,282	1.0%
2012	693,484	4,545	5,729	(600)	9,675	0.8%
					average	1.0%

Source: BaFin, Primary Life Insurer, 2002-2012.

Table 5: Assekurata Market Surveys on Life Insurance Contracts in Germany

year	total weighted avg.	min. weighted avg.	max. weighted avg.	avg. total return	share of 4% g.r.
2004	3.51%	2.75%	3.80%	4.43%	28.56%
2005	3.50%	2.75%	3.69%	4.28%	29.15%
2006	3.49%	2.75%	3.77%	4.24%	29.55%
2007	3.45%	2.63%	3.76%	4.23%	30.17%
2008	3.42%	2.57%	3.67%	4.34%	26.69%
2009	3.39%	2.52%	3.62%	4.26%	29.44%
2010	3.33%	2.52%	3.56%	4.19%	27.17%
2011	3.26%	2.44%	3.52%	4.08%	26.00%*
2012	3.19%	2.41%	3.47%	3.94%	22.15%
2013	3.12%	2.38%	3.45%	3.68%	21.50%

Note: the total weighted average indicates the average of the guaranteed return among all the survey's participants. The total average return indicates the return including the profit distribution and the share of 4% g.r. indicates the relative weight of highest guaranteed return in the underwriting portfolio.
* No precise value reported.

Table 6: Capital Markets Calibrations

Geometric Brownian Motion

Stocks	$\mu = 0.072$	$\sigma = 0.22$	$S_0 = 9552.16^*$
Real Estate	$\mu = 0.052$	$\sigma = 0.19$	$S_0 = 838.26^*$

CIR model

parameters	**calibration 1**	**calibration 2**	**calibration 3**
θ	0.02	0.01	0.03
k	0.201	0.201	0.201
σ	0.114	0.114	0.114
λ	-0.10	-0.10	-0.10
r_0	0.0045*	0.0045*	0.0045*

Correlation Matrix

	DAX	DREITS	FIBOR/EONIA
DAX	1	0.93	-0.65
DREITS	-	1	-0.61
FIBOR/EONIA	-	-	1

Time series from January 1973 until December 2013 (end of month)
Source: Datastream, Bundesbank
$^{*)}$ values as per December 2013 for DAX, DREITS and EONIA respectively

Table 7: Model's Parameters

Balance Sheet's Capital Endowment

	BS 1	**BS 2**	**BS 3**	**BS 4**	**BS 5**
$\frac{CB_0}{L_0^{p,BV}}$	0.036	0.044	0.05	0.06	0.084
$\frac{E_0}{L_0^{p,BV}+B_0}$			0.017		
$\frac{E_0+CB_0}{A_0^{BV}}$	$\cong 0.051$	$\cong 0.058$	$\cong 0.063$	$\cong 0.072$	$\cong 0.093$

Other Parameters

ω	0.005	marginal change in technical rate
ϑ	0.5	share of market value increase cashed in as dividends and rents
δ	0.05	shareholder dividends constraint
π_t^i	1	premium paid by cohort i
N^l	25	cohorts of contracts simultaneously held in portfolio
N^b	4	bond-like asset classes
N^k	2	stock-like asset classes
r^q	0.01	mortality return as share of book value of liabilities
v	0.8	lower bound for additional return distributions
u	1.2	higher bound for additional return distributions
α	0.995	confidence interval for the VaR
ρ	0.0183	fixed markup to calculate the RM