# Rules versus discretion in bank resolution*

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**Abstract**

We analyze the problem of a bank regulator who has the power to 'bail in' the debt of troubled banks, as is implied by newly designed bank resolution regimes. Allowing regulators to use their discretion in resolving banks permits them to act on the basis of their more precise, private information. However, regulators with discretion end up being excessively weak in order to avoid revealing adverse information and triggering bank runs. Optimally designed resolution regimes involve discretion whenever public news is favorable, but tie the regulator's hands with rules after bad news. The optimal regime can be implemented by supplementing discretionary bail-in powers with contingent capital instruments. We show that tighter capital and liquidity regulation, and having an effective lender of last resort, improve the efficacy of bail-in policies.

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**Keywords:** Bank resolution, rules, discretion, bankruptcy reform, bank runs, asymmetric information, contingent capital.

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# 1 Introduction

Current economic policy places strong emphasis on developing resolution regimes for failing banks. In the crisis of 2008, governments feared that the bankruptcy of major financial institutions would cause contagion and disrupt their provision of critical services to the real economy. Absent a workable alternative to bankruptcy, states recapitalized banks with public money. These bail-outs raised serious concerns about fairness and moral hazard, and in some cases they threatened fiscal stability. The G20 leaders have since vowed to end the 'too big to fail' problem, and the design of bank resolution regimes forms a key part of this agenda (G20 Leaders, 2013).

These regimes will complement bankruptcy law by giving regulators the discretion to take struggling important banks into resolution, and to re-capitalize them through 'bail-ins', i.e. by writing down debt at the expense of private creditors. One point of contention is whether discretionary bail-ins have sufficient *credibility* to provide investors with certainty, and to persuade them and large banks that the latter will no longer be deemed too big to fail. The Financial Stability Board (2011), for example, emphasizes that resolution plans are not credible if they create a risk of 'disruptions in domestic or international financial markets, for example, because of lack of confidence or uncertainty effects' (FSB 2011, p. 32). Similarly, Bulow and Klemperer (2015) argue that 'regulators are reluctant to actively force a recapitalization because doing so will send a negative signal about the bank's current financial status, possibly exacerbating a bad situation' (p. 6).

Such concerns about credibility suggest that some commitment to rules regarding bank resolution might be valuable. Indeed, bank regulators have shown that they are willing to give up some discretion: Contingent capital instruments (so-called CoCos), which write down or convert debt according to fixed contractual rules specified in advance, are being widely issued by banks and will count towards regulatory capital requirements in some jurisdictions (Avdjiev et al., 2013).

In this paper, we model the optimal design of bank resolution regimes, and consider the following four questions: (1) Why might full discretion be problematic for bank resolution authorities? (2) What are the central trade-offs in choosing between rules and discretion, and how should rules optimally be designed? (3) How do contingent capital instruments interact with resolution regimes? (4) How should the design of resolution policies interact with other financial policies, such as capital and liquidity regulation or liquidity support by a lender of last resort?

In our model, bank resolution policy is constrained by two frictions which are typical of the banking industry: First, banks are exposed to potential illiquidity and runs by uninsured short-term creditors, as in Diamond and Dybvig (1983). Second, bank regulators have access to private information about banks' financial health.[1]

[1]The regulator's private information may correspond, for example, to information gleaned in the course of conducting supervisory ratings excercises, such as CAMELS in the US, which are not publicly available. More recently, regulators have been conducting stress tests of bank balance sheets, the details of which remain largely secret. The

As a result of these frictions, regulators with full discretion have difficulty taking as strong and decisive an action as they would like. Discretion allows regulators to fine-tune resolution policies based on their private information. However, due to such fine-tuning, regulatory action provides news to the market. If the regulator 'bails in' a large portion of the bank's debt, market participants rationally infer that the bank must be under-capitalized. This revelation can trigger costly disruptions such as runs by short-term creditors.[2] Therefore, regulators with bad news and discretion have incentives to act as if they had better news. In equilibrium, they conduct excessively weak bail-in policies, leaving banks under-capitalized compared to the first best.[3]

Due to this *excessive weakness* problem, the optimal bank resolution regime generally involves some commitment to bail-in rules. We study rules which mandate pre-specified bail-in policies contingent on certain realizations of public news, but allow the regulator discretion after other realizations. When choosing between rules and discretion, the central trade-off facing regulators is between *accuracy and toughness*. Discretion allows fine-tuning, but it also opens the door to excessive weakness. By contrast, rules can create a commitment to tougher policies (with more bail-in), but they necessarily tie bail-in policies to noisy public news, thus sacrificing accuracy.

The optimal regime commits the regulator to a tough bail-in policy after *bad* public news. After good public news, regulators should be allowed discretion. This result can be understood in terms of the 'toughness vs. accuracy' tradeoff. Bad public news foreshadows bad private news, thereby increasing the expected value of tough policies. Thus toughness is a virtue and commitment is desirable. By contrast, good public news reduces the expected value of toughness, so regulators care relatively more about accuracy, and it is best to retain discretion. We also characterize the relationship between the optimal resolution regime and the quality of public information. In most relevant cases, commitment is more valuable, and should be used to a greater extent, when public signals are more informative. Even when discretion is problematic, it does not make sense to tie one's actions to a very noisy signal.

Contingent capital can exactly implement the optimal regime and substitute for explicit rule-writing. It automatically writes down debt whenever a publicly observable indicator, such as the bank's book or market value, falls below a pre-specified threshold. Thus, contingent capital en-

interaction between illiquidity and asymmetric information, which is emphasized by our theory, has been documented empirically by Iyer et al. (2013), who show that the announcement of regulatory action in an Indian bank, which signalled the outcome of a confidential review by the regulator, caused a substantial run by its uninsured creditors.

[2]In modern banking systems with deposit insurance, runs are prone to take place in wholesale credit markets, such as repo and commercial paper, rather than on retail deposits. Wholsesale runs are discussed in detail by Shin (2009), Gorton and Metrick (2012) and Krishnamurthy et al. (2014).

[3]For concreteness, in our model, we interpret the regulator's action as the fraction of bail-inable debt to write down. In reality, the set of regulatory actions is much broader, and our basic results can be applied more generally. The excessive weakness associated with discretion can also be interpreted as regulators acting *too late* in a crisis, or asking banks to raise an insufficient amount of capital in private markets. Our model suggests that pre-committing to a plan of action before the regulator has any private information about the state of the troubled bank would help to avoid these problems too.

forces tough write-downs whenever public news are sufficiently bad, without affecting discretion after good news. This replicates the optimal regime, which establishes a novel role for contingent capital in regulatory policy.

We show that other financial policies are complementary to effective resolution if they alleviate the threat of bank runs. First, we consider changes to banks' balance sheets which increase liquidity and decrease the probability of runs. Liquidity regulation, along the lines of Basel III's Liquidity Coverage Ratio, allows the regulator to target a measure of bank liquidity which is a *sufficient statistic* for the efficacy of resolution policy. Capital regulation can have a similar effect, but it is a blunter tool, since it does not directly target the relevant liquidity measures. Second, we study liquidity support by a lender of last resort. A lender of last resort can cover some of banks' liquidity shortfall if creditors run on the bank. Anticipating this response, creditors are less likely to run in the first place (Diamond and Dybvig (1983)). Thus the presence of an effective lender of last resort also lends credibility to bank resolution regimes.

Our paper thus suggests that liquidity regulation and last resort lending are natural complements to a successful bank resolution regime. In practice, there are limits to the coverage of regulation (e.g. fears of reducing the social value of intermediation, or the political influence of financial firms) and to the leniency of a lender of last resort (e.g. concerns about moral hazard). We argue that *at the margin,* liquidity regulation should be tougher, and lenders of last resort should lend against wider ranges of collateral, when efficient bank resolution is an important objective.

The trade-off between rules and discretion in bank resolution in our model looks quite different to that highlighted in macro-economic policy by Kydland and Prescott (1977) and Barro and Gordon (1983). In those models, the central bank (or government) moves last, and is tempted to create ex post inflation surprises to boost output. The central bank's action is anticipated in equilibrium and no information is revealed by its actions. In our setting, the move order is reversed - the regulator moves and then the public react - and the central concern motivating regulatory commitment is to avoid information revelation rather than sub-game perfection. In the monetary policy literature, our mechanism is closer to Cukierman and Meltzer (1986), where the central bank has private information about its objective function, and acts strategically in revealing information to the public.

Our paper also relates to a growing literature on the design of contingent capital instruments. Flannery (2005) was the first to propose 'reverse convertible debentures', which resemble today's CoCos. The subsequent literature has focused on the problem of multiple equilibria with market-based triggers (Hillion and Vermaelen 2004, Sundaresan and Wang 2014), alternative designs which overcome this problem (Pennacchi et al. 2013, Bulow and Klemperer 2015), and the impact of contingent capital on incentives and the value of the firm (Pennacchi 2010, Martynova and Perotti 2012, Albul et al. 2013). Flannery (2013) provides an excellent survey. Our paper is a complement to this literature. We take a step back from the details of contingent capital design and ask whether regulators would want to encourage *any* of these instruments, all of which commit the bank to debt write-downs or conversion into equity as a function of publicly-available information, rather than allowing the regulator to exercise discretion when the need for de-leveraging arises. Moreover, our analysis of contingent capital helps to motivate some of the key questions addressed by this literature. We show that the optimal resolution regime can *in principle* be implemented with contingent capital contracts, but only if these contracts can (i) avoid feedback effects and death spirals, and if (ii) their conversion is credibly beyond the regulator's control. Both issues have been central concerns in contingent capital design.

The paper is structured as follows: Section 2 describes our model of bank resolution. Section 3 describes equilibria when regulators have discretion, and Section 4 analyzes the optimal resolution regime when commitment is possible. Section 5 discusses the implementation of optimal regimes with contingent capital contracts. Section 6 analyzes complementarities between resolution regimes and other financial policies. Section 8 concludes and elaborates on policy implications. The Appendix contains all proofs not given in the text.

# 2 The model

The model has two dates, $t \in \{1,2\}$ and a single bank, which is subject to intervention by a regulator at date 1. At date 1, the bank has the following balance sheet.[4] Its liabilities are short-term, uninsured debt with face value $D$, and long-term 'bail-inable' bonds with face value $B$. Short-term debt are held by a unit mass of identical, risk-neutral creditors. We assume that short-term creditors have (absolute) priority over long-term creditors in case of insolvency. The bank's assets are long-term risky investments, which pay a random cash flow $V$ at date 2, with support $[\underline{v}, \overline{v}] \subset \mathbb{R}$.

The regulator observes the realization $V = v$ at date 1, and at the same time the public observes a signal $S$ with support $[\underline{s}, \overline{s}] \subset \mathbb{R}$. The distribution of $V$ given $S$ is $F(v|s)$, and a high $S$ is 'good news' about $V$ in the sense of first-order stochastic dominance:

$$\frac{\partial F(v|s)}{\partial s} < 0. \tag{1}$$

After observing $V$ the regulator may bail in $a \in [0, B]$ long-term bonds.[5] Bailing in means writing down the debt, so that the owners of bonds do not get paid, or converting it into equity,

[4] For now, we treat the bank's balance sheet as given at date 1. In Section 6 we will come back to this and discuss the issues that our analysis will raise for the regulation of the composition of the bank's assets and liabilities at a potential date 0, and in particular, point out the complementarity of balance sheet regulation with bail-in policies.

[5] Current policy proposals suggest that resolution authorities will first bail in bonds which were designated as 'bail-inable' at the point of issuance. If there are long-term bonds which cannot be bailed in, one could restrict the regulator's action to $a \in [0, B']$ for $B' < B$, without affecting our results.

so that they get paid in shares. After the bail-in, the face value of outstanding long-term bonds is $B-a$.

The public observes $a$, and uses it along with the signal $S$ to infer the regulator's information. Let $\beta(v|a,s)$ be the distribution of $v$ given public information, and let $E_\beta[V|a,s] = \int_{\underline{v}}^{\overline{v}} v d\beta(v|a,s)$ denote its conditional expectation.

After observing $a$, short-term creditors decide whether to 'withdraw' their debt, demanding an immediate repayment at date 1, or to roll over until date 2. We focus on the case where only news about the bank drives withdrawals, and short-term creditors withdraw late if they are indifferent. For this reason, we assume that short-term creditors incur a small non-pecuniary cost $\chi > 0$ if they withdraw early.[6]

Assets can be sold into a competitive market. There is a pool of risk-neutral outside buyers who can acquire assets and extract value $\lambda V$, where $0 < \lambda < 1$. The outside buyers observe only public information, i.e. the signal $S$ and the regulatory action $a$, and therefore the market value of assets at date 1 is $p = \lambda E_\beta[V|a,s]$. When a fraction $\phi$ of short-term creditors withdraws early (at date 1), the bank needs to sell a fraction $\sigma = \min\{1, \phi D/p\}$ to meet withdrawals. If $\sigma = 1$, then the bank runs out of assets and is insolvent at date 1.[7]

## 2.1 Welfare

The regulator seeks to maximize social welfare,[8] which is the sum of two components. First, asset sales to outside buyers cause a deadweight loss, since outside buyers extract less value from assets than banks can. This loss is given by $(1-\lambda)\sigma v$, where $\sigma$ is the fraction of assets sold to outside buyers, as discussed above.

Second, welfare depends on the bank's equity capital at date 2, which is $E(a,v) = v + a - (D + B)$. In particular, we assume that welfare (not including the deadweight cost of asset sales) is

$$U(E(a,v)),$$

where the function $U(E)$ is strictly concave and twice differentiable in $E$. In other words, we assume that social welfare first increases and then decreases in the level of bank capital. This is consistent with a large literature on bank capital structure, which we discuss in Subsection 2.3. The ideal level of bank capital is $E^\star$ , defined by $U'(E^\star) = 0$.

[6] Equivalently, one could assume that short-term debt earns a non-zero interest between dates 1 and 2.

[7] In Section 6, we will allow the bank to hold cash that could be used to meet some withdrawals. For simplicity, in this Section, we focus on the case with neither cash holdings nor withdrawals for liquidity reasons.

[8] Similar conclusions will follow if the regulator does not set equity to maximize social welfare, but nevertheless has a utility function which first increases and then decreases in the leverage of the bank. In particular, the regulator is likely to find that discretion leads to weaker action ex post than he would consider desirable ex ante.

Combining the two components, social welfare is

$$W = U(E(a,v)) - (1-\lambda)\sigma v.$$

Below, we consider how equilibrium withdrawals from the bank, and therefore asset sales $\sigma$, depend on the regulator's action $a$ and the information it reveals to the public. However, it is instructive to first consider what the regulator would do in the absence of any liquidity problems. In this case, his ideal action is

$$a^{\star}(v) = \begin{cases} 0 & \text{if } E^{\star} \leq v-(D+B), \\ E^{\star}+D+B-v & \text{if } v-D < E^{\star} < v-(D+B), \\ B & \text{if } v-D \leq E^{\star}. \end{cases}$$

We focus on the interesting case where regulators with different news $v$ have different ideal bail-in policies $a^{\star}(v)$. To this end, we assume that $a^{\star}(v) \neq a^{\star}(v')$ for some $v$ and $v'$, which is equivalent to

$$U'(\underline{v}-(D+B)) \geq 0 \geq U'(\bar{v}-D), \tag{2}$$

with at least one strict inequality.

## 2.2 Withdrawals and bank runs

We now describe the withdrawal game between the bank's short-term creditors. Each short-term creditor decides whether to withdraw early (at date 1) or late (at date 2). They will withdraw early if doing so reduces the chance of losing money when the bank defaults. We assume that

$$\underline{v} \geq D > \lambda \underline{v}. \tag{3}$$

This first part of this assumption implies that there is enough value in the bank to repay short-term creditors even with the worst possible realization of $V$ (although it may or may not be solvent overall, taking into account outstanding long term bonds). The second part of the assumption implies, however, there is not necessarily enough value to repay the short-term creditors at date 1 if a run occurs and the asset value realization is low. This part of the assumption makes the bank vulnerable to runs in the face of bad news. Assumption (3) ensures that bank runs are driven by self-fulfilling concerns about liquidity as in Diamond and Dybvig (1983), but not by concerns about solvency. The implications of solvency-driven bank runs are discussed in Subsection 2.3.

Equilibrium in the withdrawal game depends on whether the total liquidation value of assets $p$ exceeds the claims of short-term creditors $D$. When $p \geq D$, the unique equilibrium has no withdrawal with $\phi = 0$. The bank is never insolvent at $t = 1$, and the value of its remaining assets at $t = 2$ will be $v(1 - \phi D)/p$. Hence late withdrawers will be repaid in full if and only if $v(1-\phi D/p) \geq (1-\phi)D$. This is guaranteed because $p \geq D$ (by assumption) and $v \geq \underline{v} \geq D$ (by the lower bound in (3)). Therefore, short-term creditors get paid in full, no matter when they withdraw, and it is a dominant strategy to withdraw late to avoid the cost $\chi$.

When $p < D$, there are multiple equilibria. If everybody withdraws early ($\phi = 1$), the bank is insolvent at date 1. Early withdrawers get paid $p$ each, and late withdrawers get nothing. As long as $\chi$ is sufficiently small, nobody has an incentive to withdraw late, and the bank run scenario $\phi = 1$ is an equilibrium. If everybody withdraws late ($\phi = 0$), the bank remains solvent and can pay back all short-term creditors in full at because its assets will be worth at least $\underline{v} \geq D$. Then nobody has an incentive to withdraw early, and $\phi = 0$ is also an equilibrium. Finally, there is a third, unstable, equilibrium with $0 < \phi < 1$ and partial liquidation of the bank's assets.

For tractability, we assume that with multiple equilibria, one of the stable equilibria is picked based on the realization of independent sunspots. In particular, suppose that the bank run $\phi = 1$ is played with probability $\pi > 0$, and $\phi = 0$ is played with probability $1 - \pi$. The global games approach of Goldstein and Pauzner (2005) could, in principle, be used to endogenize $\pi$. We work with an exogenous $\pi$ in order to obtain a more tractable characterization of regulatory trade-offs.

A run is therefore possible, and occurs with probability $\pi$, if and only if $p = \lambda E_\beta[V|a,s] < D$. When this is satisfied, runs induce an expected social cost of $\kappa(v) = \pi(1-\lambda)v$. We assume that this cost is large, in the sense that a regulator prefers taking the 'wrong' bail-in action to triggering a run:

$$\kappa(v) > U(v + a - (D+B)) - U(v + a' - (D+B)) \text{ for all } a, a', v. \tag{4}$$

We further assume that runs cannot be triggered by the public signal alone,

$$\lambda E[V|\underline{s}] \geq D. \tag{5}$$

This restriction allows us to focus on the case of interest, which is where regulatory action itself might create runs by revealing information. The lowest asset value that can be revealed without triggering a potential run is $v_D = D/\lambda$. Finally, we assume that in the marginal state $v = v_D$, the regulator prefers a complete bail-in to inaction:

$$U(v_D - D) > U(v_D - (D+B)). \tag{6}$$

## 2.3 Remarks on the model setup

**Alternative interpretations of regulatory intervention.** While we present the case of bail-ins motivated by the desire to raise the level of bank equity, our qualitative results are much more general. In the Appendix, we consider a general function $U(a,v)$, which gives the regulator's utility as a function of his action $a$ and the bank's asset value $v$. In that setup, all our results hold as long as the regulator's utility is concave in $a$, and satisfies the condition $\partial^2 U/\partial a \partial v < 0$, so that the marginal benefit of intervention is lower when the regulator has good news.

The general specification $U(a,v)$ not only offers a robust analysis of bail-in policy, but also shows that our model is open to many alternative interpretations. For instance, one could also interpret $a$ as the *time* at which the regulator intervenes, in a model where regulators with bad private news prefer to intervene earlier than regulators with good private news. Alternatively, $a$ can also be interpreted not as a bail-in action, but as the quanitity and timing with which the regulator requires banks to raise new equity in the market.

Our analysis below shows that regulators with discretion and bad news undertake excessively weak bail-ins to avoid bank runs, and can therefore benefit from a commitment device. This intuition can also be applied to alternative interpretations: If $a$ is the timing of intervention, then regulators with discretion might act too late; if $a$ is a required equity injection, they might require a smaller or later equity injection than would be optimal.

**Alternative interactions between beliefs and welfare.** We present a model where adverse public beliefs are socially costly because they trigger illiquidity-driven bank runs. This case is particularly tractable, but we would expect our results on rules and discretion to hold in other settings. For example, instead of focusing on banks' legacy assets, one could imagine that adverse public beliefs increase the bank's cost of external finance at date 1, and therefore inhibit additional lending to the real economy.

Within our model, Assumption (3) allows us to focus on illiquidity-driven bank runs. The advantage of doing so is that we obtain a clean interaction between public beliefs and bank runs: Runs are possible if and only if $p = \lambda E_\beta[V|a,s] < D$, and this condition only depends on the conditional expectation of $V$. With solvency concerns, short-term creditors would worry about the date 2 value of bank assets, and the entire conditional distribution of $V$ would play a role in the withdrawal game. However, the interaction between beliefs and welfare would be qualitatively similar.

**Social welfare and bank equity.** We work with a general specification of social welfare as a function of bank equity, restricted only by Assumption (2). The first inequality in (2) implies that welfare is increasing in bank equity when equity is low. This seems uncontroversial: A large literature following Holmstrom and Tirole (1997) demonstrates that badly capitalized banks can cause credit rationing and macroeconomic distortions. Alternatively, banks' incentives to gamble for resurrection (Hellmann et al., 2000) or deadweight costs of bank default would also yield a social welfare function which suffers when banks are badly capitalized.

The second inequality in (2) implies that welfare can be reduced by excessive recapitalizations when banks are healthy. Dang et al. (2014) and Gorton and Winton (2014) argue that bank equity is more information-sensitive than debt, and therefore introduces asymmetric information costs when it is too high. Moreover, Calomiris and Kahn (1991) and Diamond and Rajan (2000) develop theories where debt, by acting as a hard claim, disciplines bank managers. These theories can be used to motivate our assumption: Beyond a certain point, more bank equity becomes socially costly.[9]

**Illiquidity.** The potential illiquidity of banks is central to our model. In particular, Assumption (3) states that short-term debt liabilities are large, relative to the amount cash the bank can raise at date 1 by liquidating its assets. This raises two potential concerns. First, if banks were required to hold sufficient equity capital ex ante, debt liabilities would be small relative to risky assets. Second, if there were a lender of last resort which could provide liquidity support, banks would not need to liquidate assets to raise cash. We are sympathetic to these arguments. Indeed, Section 7 formally shows that last resort lending and bank regulation are *complementary* to bail-in policy, precisely because they alleviate the threat of runs.

We effectively model a world in which these policies are *imperfect*, so that runs are still a potential outcome. This is a reasonable restriction: As discussed above, very high levels of bank equity may not be optimal. In practice, the toughness of capital regulation is further constrained by the political clout of the financial industry (Admati and Hellwig, 2014). Equally, perfect liquidity support by a lender of last resort is not desirable since it crowds out banks' private incentives to properly manage liquidity.

**Priority of short-term creditors.** We impose that short-term creditors have priority over long-term creditors (the holders of bail-inable debt $B$) when the bank is insolvent at date 2. This is an optimal arrangement. If short-term creditors were to rank pari passu with long term debt at date 2, then the bank would be more prone to runs and regulatory actions would be correspondingly weaker in equilibrium. This observation validates recent regulatory attempts to restructure banks to ensure that short-term debt is structurally senior to most long-term debt.

# 3 Equilibrium with discretion

If the regulator has discretion to freely choose $a$, then he is playing a signalling game with creditors. In particular, creditors use $a$ to infer information about $V$, since regulators with different values of $V$ prefer to choose different $a$. When $\lambda E_{\beta}[V|a,s] < D$, potential runs create a social loss $\kappa(v)$. Therefore, in choosing $a$, the regulator must consider the informational effect of $a$ and the cost of

[9] As the strongest proponents for higher levels of bank capital, Admati and Hellwig (2014), advocate that banks fund between 20% and 30% of investments with equity. Even this view is consistent with our assumptions in principle.

runs as well as its direct payoff. His effective objective function is

$$W(a,v,s,\beta) = U(v+a-(D+B)) - \kappa(v) \times \mathbf{1}(\lambda E_\beta[V|a,s] < D),$$

where $\mathbf{1}(.)$ denotes the indicator function. We now examine the equilibria of the regulatory signalling game, considering separately each realization of the public signal $s$.[10]

**Definition 1.** An *equilibrium with discretion* in state $s \in [\underline{s}, \overline{s}]$ is a bail-in rule $\alpha(v,s) \in [0,1]$ and beliefs $\beta(v|a,s)$ such that

- The bail-in rule $\alpha(v,s)$ solves $\max_{a\in[0,1]} W(a,v,s,\beta)$.
- Beliefs are consistent with Bayes' rule when possible.

## 3.1 Runs, minimal pooling and incentive compatibility

We can narrow down the properties of equilibria with discretion, which we collect in Lemma 1 below. First, runs do not occur on the equilibrium path. If they did, then the types of regulators which faced runs would deviate to an action which avoided a run (by equation (4)). This contradicts equilibrium, unless the regulator faces a run *regardless* of his action. But this last scenario requires overly pessimistic public beliefs. The proof of Lemma 1 shows that such beliefs are not consistent with Bayesian updating.

Second, equilibria feature minimal pooling. Regulators with very bad news ($v$ close to $\underline{v}$) must not reveal themselves to the public. If they did, they would open the door to runs, because the perceived liquidation value of assets would be too low to repay short-term creditors. Therefore they will pool with regulators with better signals to avoid runs. Runs are avoided only if regulators play the same action whenever $v$ is below a threshold $v_p(s)$, which is defined implicitly by9s

$$\lambda E[V|s, V \le v_p(s)] = D. \tag{7}$$

Intuitively, this condition states that if the public learns that $V$ is below$v_p(s)$, as it does when all regulators with $V \le v_p(s)$ play the same action, a run will just be avoided.

Finally, regulators' equilibrium actions are incentive compatible. To rule out profitable deviations, each type of regulator must prefer his equilibrium action to those of other types. Lemma 1 summarizes our results and further characterizes incentive compatibility.

**Lemma 1.** *In any equilibrium with discretion in state s, the bail-in rule $\alpha$ and beliefs $\beta$ satisfy the following conditions:*

[10] Definition 1 focuses on pure strategies. It is easy to see that the regulator is never indifferent between two actions, by the strict concavity of $U$ and Assumption (4).

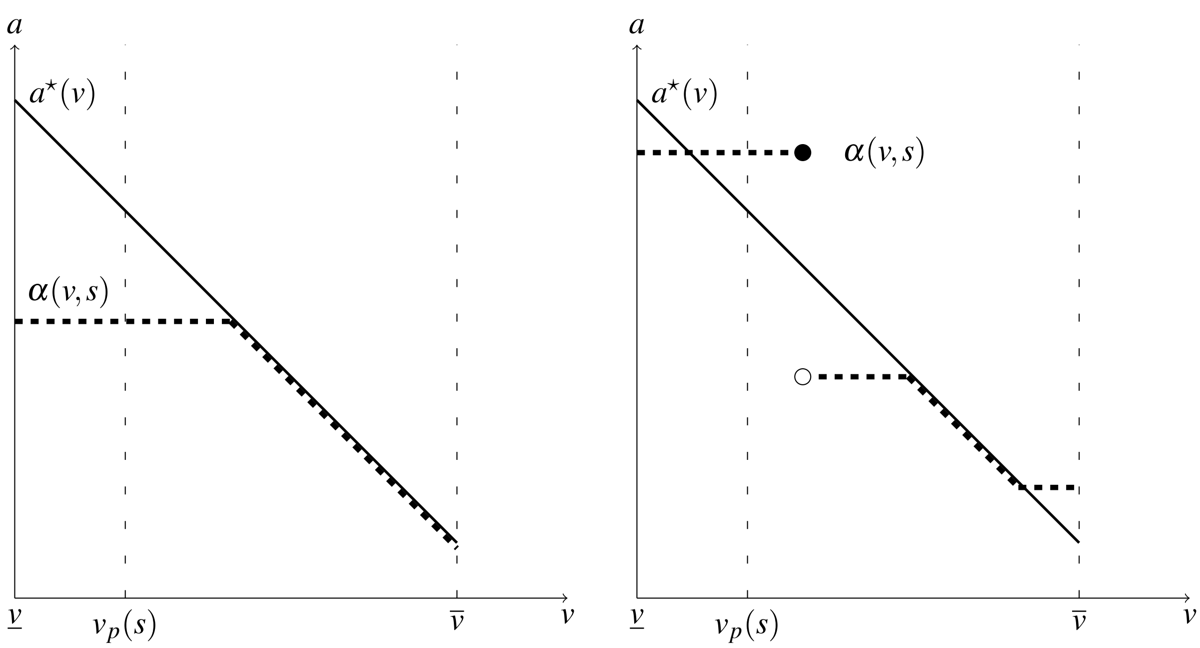
- No runs*:* $\lambda E_\beta[V|\alpha(v,s),s] \geq D$ for all $v$.

- Minimal pooling: $\alpha(v,s) = \alpha(\underline{v},s)$ *for all* $v \leq v_p(s)$.

- Incentive compatibility: $\alpha(v,s)$ *is weakly decreasing in* $v$ *and therefore differentiable in* $v$ *almost everywhere. Where it is differentiable, it is either flat,* $\frac{\partial\alpha(v,s)}{\partial v} = 0$*, or coincides with the ideal action,* $\alpha(v,s) = a^\star(v)$*. Moreover, if* $\alpha(v,s)$ *is discontinuous at* $v$*, then the regulator is indifferent between* $\alpha^+(v,s) = \lim_{t\downarrow v}\alpha(t,s)$ *and* $\alpha^-(v,s) = \lim_{t\uparrow v}\alpha(t,s)$*.*

Figure 1 illustrates the incentive compatibility conditions in Lemma 1. Panel (a) shows an equilibrium with continuous actions (fixing a public signal $s$). There is a pooling region where $\alpha(v,s)$ is flat and a separation region where it coincides with the ideal action $a^\star(v)$. Panel (b) shows a discontinuous case, where $\alpha(v,s)$ rises above $a^\star(v)$ and then jumps below it. Panel (b) also shows how separation and pooling regions can alternate. Moreover, both panels exhibit the minimal pooling result. In each case, the first pooling region, which starts at $\underline{v}$, must extend at least to the threshold $v_p(s)$.[11]

Figure 1: **Equilibrium bail-in actions**.




(a) Continuous case

(b) Discontinuous case

[11]This signalling game is different from standard models such as Spence (1973), since no cash transfers are made between the informed and uninformed players. Thus, our incentive compatibility conditions are as in Melumad and Shibano (1991) and Martimort and Semenov (2006), who analyze a screening problem without transfers.

## 3.2 Equilibrium selection

There are multiple equilibria with discretion. In fact, any action rule satisfying the conditions of Lemma 1 can be sustained by sufficiently severe off-equilibrium beliefs, which trigger a run whenever the regulator deviates from equilibrium play. This complicates the analysis of rules versus discretion.

We take a two-step approach. First, we rule out certain equilibria using an equilibrium selection criterion. Then, we give discretion the benefit of the doubt by comparing rules to the 'best' surviving equilibrium with discretion.

As a first step, we adapt the Cho and Kreps (1987) *intuitive criterion* to our context to discipline off-equilibrium beliefs. For an off-equilibrium action $a_0 \in \{a : \alpha(v,s) \neq a \forall v\}$, define

$$\sigma(a_0,s) = \{v : W(a_0,v,s,\beta) \geq W(\alpha(v,s),v,s,\beta)\}$$

as the set of signals for which the regulator would consider deviating. In the language of Cho and Kreps, this is the set of signals for which $a_0$ is not equilibrium-dominated.

Generally, Perfect Bayesian Equilibrium requires that the informed party has no incentive to deviate from their prescribed strategy for *given* beliefs of the uninformed players. The Cho-Kreps criterion additionally requires that she has no incentive to deviate for *any* beliefs which attach zero probability to equilibrium-dominated behavior.

In our context, the intuitive criterion rules out equilibria which are sustained by an unreasonable threat of runs. In particular, if there is a bail-in policy to which only regulators with good news would want to deviate, it does not seem credible that creditors should run when that policy materializes. It is easy to see that Cho and Kreps' original definition is equivalent to the following version in our model:

**Definition 2.** An equilibrium with discretion in state $s$ survives the *intuitive criterion* if there is no off-equilibrium action $a_0$ such that:

1. For some signal $v_0$, the regulator strictly prefers $a_0$ to his equilibrium action in the absence of a run:
$$U(v+a_0-(D+B)) > W(\alpha(v,s),v,s,\beta).$$

2. For all beliefs $\gamma$ with $Pr_\gamma[V \in \sigma(a_0,s)] = 1$, we have
$$\lambda E_\gamma[V|a_0, V \in \sigma(a_0,s)] \geq D,$$
so that a run would be avoided if creditors attached zero probability to types for which $a_0$ is equilibrium dominated.

The intuitive criterion does not select a unique equilibrium in our model. However, it does allow us to place meaningful restrictions on possible bail-in actions.

Recall that $v_D = D/\lambda$ is the lowest level of asset values which can be revealed without triggering a run. Suppose we can find an off-equilibrium action $a'$ such that some regulators would strictly prefer $a'$ to their equilibrium action, and only regulators with signals above $v_D$ would deviate to $a'$. Then, the equilibrium cannot survive the intuitive criterion, since beliefs which are confined to $V \geq v_D$ can never trigger a run. This reasoning allows us to narrow down equilibrium play considerably. Figure 2 illustrates the process of elimination.

First, we can rule out discontinuous bail-in actions, as in panel (a) of the figure. If there is a downward jump in equilibrium, then for the marginal signal $\hat{v}_1$, the regulator must be indifferent between his actions before and after the jump, $a_1^-$ and $a_1^+$. Only types close to $\hat{v}_1$ would want to deviate to $a' = a_1^+ + \varepsilon$ for small $\varepsilon > 0$. By the minimal pooling requirement of Lemma 1, we have $\hat{v}_1 \geq v_p(s) > v_D$. Thus, only types above $v_D$ would deviate to $a'$ for small $\varepsilon$, and according to our previous reasoning, the candidate equilibrium does not survive the intuitive criterion.

Second, we can put a restriction on bail-in when the regulator has the best possible signal. The idea is illustrated in panel (b) of Figure 2. Suppose that $\alpha(\bar{v}, s) > a^\star(\bar{s})$. Then there is either pooling for the highest signals followed by separation for slightly lower signals (dashed line), or complete pooling (solid line). In the dashed case, only regulators with high signals want to deviate to actions below $\alpha(\bar{v}, s)$. In the solid case, by our assumption in (6), a regulator with signal $v_D$ or below has no incentive to deviate to $a' \simeq 0$. In both cases, the candidate equilibrium fails to survive the intuitive criterion.

Combining the above arguments, surviving equilibria must be structured as in panel (c) of the figure. They either involve complete pooling on an action below $a^\star(\bar{s})$, or an initial pooling region followed by separation. In the second case, Lemma 1 implies that the initial pooling region must extend at least up to $v_p(s)$, and that in the separation region, actions must coincide with the ideal action $a^\star(v)$.

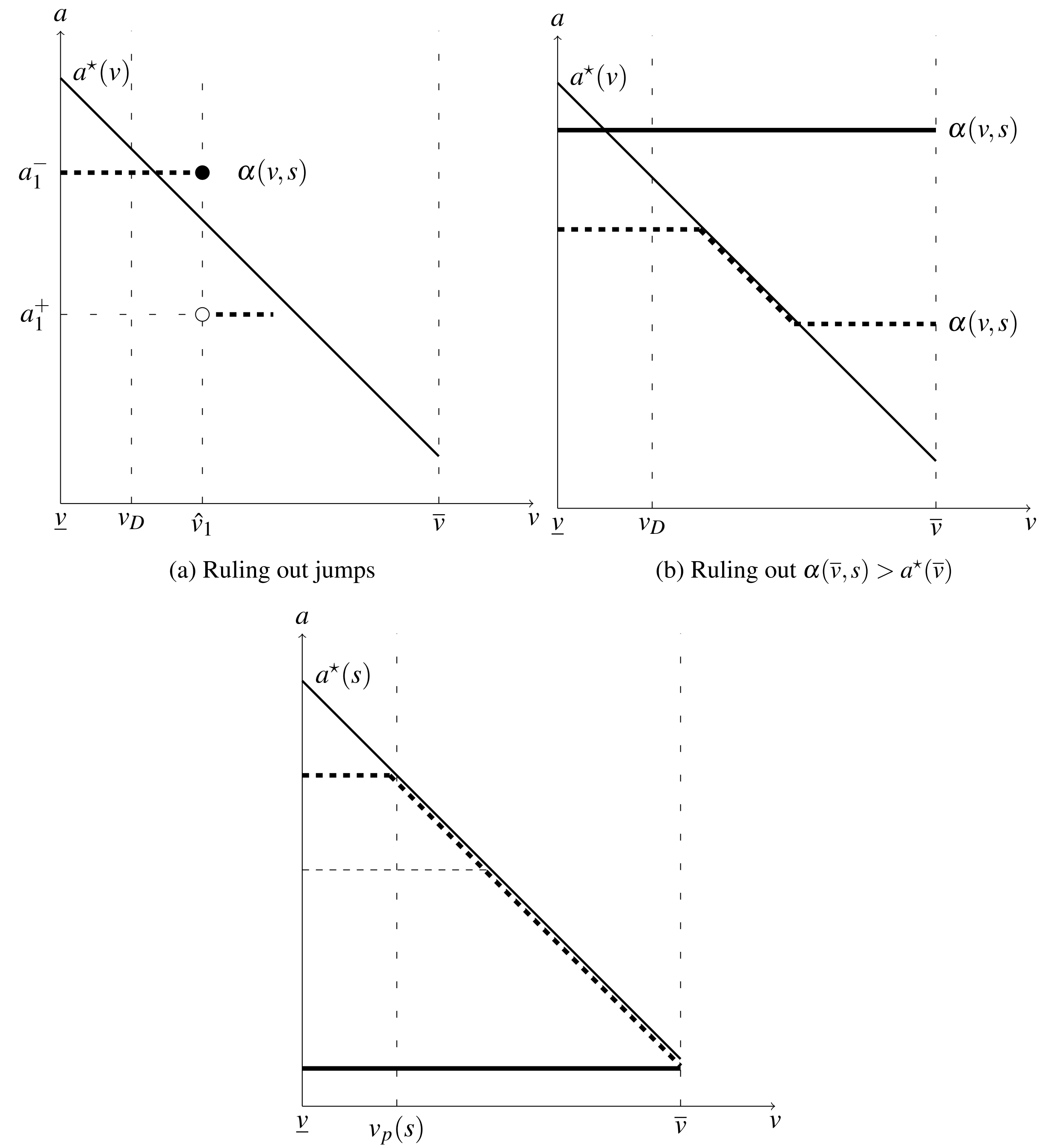
**Proposition 1.** *In any equilibrium surviving the intuitive criterion, the bail-in rule $\alpha$ satisfies*

$$\alpha(v, s) = \min\{a^\star(v), a'\}$$

*for some $a' \leq a^\star(v_p(s))$.*

This proposition demonstrates why discretion may be a problem. In surviving equilibria, no regulator takes an action above $a^\star(v_p(s))$, the ideal action of the 'minimal pooling type' $v_p(s)$. It is clear that regulators with worse news ($v < v_p(s)$) would like to take tougher actions. However, they cannot do so for fear of triggering runs. As a result, the equilibrium with discretion exhibits *excessive weakness* when the regulator has bad news. In this situation, the regulator needs to

Figure 2: **Applying the intuitive criterion**.




(a) Ruling out jumps

(b) Ruling out $\alpha(\overline{v},s) > a^\star(\overline{v})$

(c) Surviving equilibria

'pretend' to have better news, pool with higher types, and take weaker actions than he would like, in order to avoid a run.

As discussed in Section 2, the regulator's action $a$ can be given a broader interpretation. First, $a$ could stand for the timing of regulatory intervention in a failing bank. Then, Proposition 1 implies that regulators tend to react *too late* when banks are in trouble – another type of excessive weakness. Second, $a$ could stand for a regulatory requesting that the bank raise capital on the private market. Proposition 1 then reveals an incentive to ask for insufficient recapitalizations, driven by the desire to 'pool' with types who have better news.

What is the highest utility the regulator can achieve with discretion? By inspection of Figure 2, panel (c), it is clear that the best surviving equilibrium is the one with the smallest pooling region: Shrinking the pooling region increases the utility of regulators who now obtain separation, and leaves the utility of the 'poolers' unchanged.

**Corollary 1.** *For all realizations of the regulator's private information $v$, the highest payoff in a surviving equilibrium is achieved when $a' = a^\star(v_p(s))$. Hence, the highest expected payoff in a surviving equilibrium, conditional on public information $s$, is*

$$\bar{U}(s) = E[U(v + \min\{a^\star(v), a^\star(v_p(s))\} - (D+B))|s]. \tag{8}$$

Our graphical illustrations are drawn such that the ideal action $a^\star(v)$ is strictly decreasing. This is not the case in general. It is possible, for example, that regulators with bad news (low $v$) wish to conduct the maximal possible bail-in, $a^\star(v) = B$. In this case, discretion need not induce excessive weakness. If all regulators with news $v \leq v_p(s)$ wish to conduct the maximal policy $a^\star(v) = B$, then the ideal action already satisfies the minimal pooling requirement, and there is a surviving equilibrium in which the regulator plays $a^\star(v)$ for all realizations of $v$. In what follows, we focus on the more interesting case where discretion induces excessive weakness. Therefore, we impose the parametric restriction that $a^\star(v_p(s)) < B$, or equivalently

$$U'(v_p(s) - D) < 0. \tag{9}$$

In this case, the payoff in the best surviving equilibrium, as characterized in (8), simplifies to

$$\bar{U}(s) = E[U(E^\star - \max\{v_p(s) - v, 0\})|s], \tag{10}$$

where $E^\star$ is the preferred level of bank capital. This gives an intuitive characterization of the loss from discretion. The regulator achieves his preferred level of bank capital when he has good news ($v > v_p(s)$). However, when he has bad news, he conducts an excessively weak bail-in, and bank capital ends up below its preferred level at $E^\star - (v_p(s) - v)$.

To summarize, we have established that discretion leads to excessive weakness. We now turn to the implications for optimal policy, and in particular, for the trade-off between rules and discretion.

# 4 Optimal regimes and contingent capital

Suppose that the regulator is able to tie his hands before any information is revealed. He can credibly promise to take a certain action $A(s)$, *regardless of his private information,* when the public signal is $s$. An optimal rule lays out for which public signals the regulator will pre-commit to a particular action, and which action he will commit to in each case. The timing of events is now as follows:

- Date 0: The regulator announces a *commitment set* $\mathscr{C} \subset [\underline{s}, \bar{s}]$ of realizations of the public signal, and *commitment actions* $A(s) \in [0, B]$ for each $s \in \mathscr{C}$.[12]
- Date 1: The public signal $s$ is observed by everybody, and the private signal $v$ is simultaneously observed by the regulator.
- The regulator takes the bail-in action $a$. His choices depend on whether the public signal lies in the commitment set ($s \in \mathscr{C}$) or not:
  - If $s \in \mathscr{C}$, then the regulator is forced to take the commitment action $a = A(s)$.
  - If $s \notin \mathscr{C}$, then the regulator has discretion to choose $a \in [0, B]$, and plays the discretion game analyzed in Section 3. We give discretion the benefit of the doubt
- Date 2: Assets mature and social welfare is realized.

For states without commitment ($s \notin \mathscr{C}$), we give discretion the benefit of the doubt by considering the surviving equilibrium with discretion which yields the highest expected payoff $\bar{U}(s)$. By contrast, commitment to $A(s)$ gives the regulator an expected payoff of $E[U(v + A(s) - (D + B))|s]$ in state $s \in \mathscr{C}$. Importantly, a committed regulator need not worry about runs. His actions, being contingent only on public information, will not reveal any private information, and public information alone does not trigger runs by our assumption in (5).

## 4.1 Optimal rules

In writing an optimal rule, the regulator decides for which subsets of states $s$ to write a commitment, and which actions to commit to in those states, to maximize his expected utility.

[12] An alternative sequence is to have the regulator announce his commitment at date 1, either before or after he observes $s$. This will yield equivalent results. The only essential assumption is that a commitment is made *before* $v$ is observed by the regulator, since this allows him to avoid the signalling problem associated with discretion.

**Definition 3.** An *optimal rule* is a commitment set $\mathscr{C}^\star \subset [\underline{s}, \overline{s}]$ and a commitment action $A^\star(s)$ for each $s \in \mathscr{C}^\star$, which solve the problem

$$\max_{\mathscr{C} \in 2^{[\underline{w},\overline{w}]},(A(s))_{s\in C}} E[U(v+A(s)-(D+B)) \times 1(s \in C) + \bar{U}(s) \times 1(s \notin C)] \tag{11}$$

In choosing whether to make a commitment in state $s$, the regulator compares the value of playing the discretion game, $\bar{U}(s)$, to the expected payoff from playing a fixed action $A$, which is $E[U(v+a-(D+B))|s]$. The value of commitment to $A$ is therefore

$$VC(s) \quad = \max_A \quad E[U(v+A-(D+B))|s]. \tag{12}$$

The *optimal rule* is as follows: If $VC(s) < \bar{U}(s)$, then the regulator is better off playing the discretion game. Otherwise, then he chooses to commit to $A^\star(s) = \arg\max_A E[U(v+A-(D+B))|s]$.

The trade-off between commitment and discretion is illustrated in Figure 3. The regulator would never commit to a weak action $A_0 < a'$ (the lower bold line). This commitment takes the regulator further away from the ideal action $a^\star(v)$ than he would be in the discretion equilibrium (the bold dashed line), regardless of his private information $v$. Intuitively, since discretion induces excessive weakness, commitment to a weak action only makes things worse.

However, commitment to a tough action $A_1 > a'$ (the upper bold line) may be valuable. This commitment benefits the regulator by taking him closer to the ideal action whenever a low enough $v$ is realized, but hurts him when high a high $v$ is realized. When $v$ is low, the excessive weakness with discretion hurts the regulator, and he would like to commit to being tough.

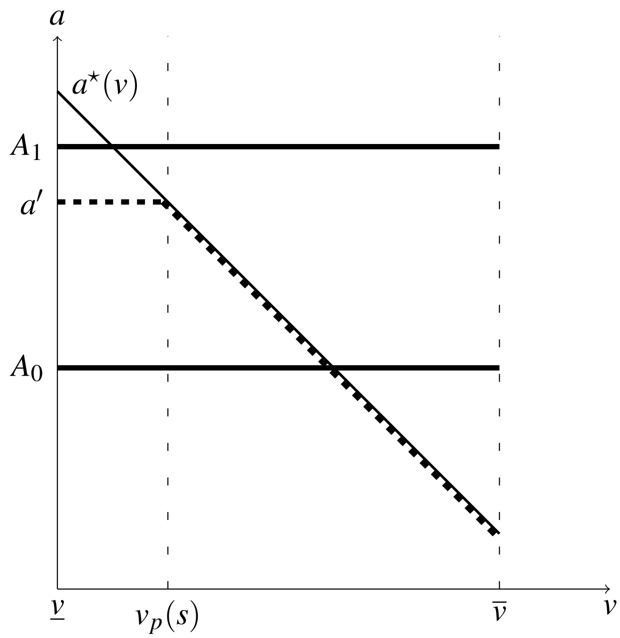
It follows that commitment to a high action like $A_1$ is valuable when public news $s$ is sufficiently bad. Adverse public news imply that low realizations of $v$ are likely, which is precisely when commitment benefits the regulator. The following proposition formalizes this intuition.

**Proposition 2.** *Suppose $VC(\underline{s}) > \bar{U}(\underline{s})$. Then, the optimal commitment set $\mathscr{C}^\star$ takes the shape $\mathscr{C}^\star = [\underline{s}, s^\star)$ or $\mathscr{C}^\star = [\underline{s}, s^\star]$ for some $s^\star \geq \underline{s}$. The optimal commitment actions solve $A^\star(s) = \arg\max_A VC(A,s)$, and are decreasing in s (and strictly decreasing whenever $A^\star(s) < 1$).*

This clearly illustrates the trade-off between commitment and discretion as a trade-off between toughness and accuracy. With discretion, the threat of runs leads to excessively weak bail-in policies, as regulators attempt to avoid revealing very bad news. The benefit of commitment is the ability to be tough when news are bad. The cost of commitment is that the regulator cannot adapt his action, even when he ends up with good news.

When the economic outlook based on public news is poor, regulators anticipate bad private news as well and therefore a greater need for tough bail-in policies. In this case, the benefits of

Figure 3: **Candidate commitment actions**.




commitment outweigh the costs. Commitment allows regulators to be tough without provoking runs. Conversely, when public news suggests that the economic outlook is good, regulators also anticipate good private news. In this case, the costs of commitment outweigh its benefits. The threat of runs is remote, and the excessive weakness induced by discretion is unlikely to affect the regulator. Therefore, the regulator prefers discretion. As a result, the regulator optimally writes rules which tie his hands whenever the economic outlook, as measured by public news, falls below a threshold $s^\star$.[13]

## 4.2 Optimal rules and the quality of public information

The value of commitment depends on the quality of public information. When the regulator gives up discretion, he is forced to ignore his private information and acts only on public signals. At first glance, a noisy public signal should therefore reduce the value of commitment by decreasing the accuracy of bail-in policies. For instance, regulators might worry that commitment based on market prices, e.g. through contingent capital, would subject policy to the whims of market sentiment. We show that this is true in a practically relevant region of the parameter space, but that the general effect is more nuanced.

We model a deterioration in the quality of public information as follows: Suppose that instead of $S$, the public observe a signal $\hat{S}$ with support $[\underline{s}, \overline{s}]$ which is less informative than $S$ in the sense

[13]Note that the threshold $s^\star$ is the state in which the regulator is indifferent between discretion and commitment. Therefore, it does not matter whether it is included in the commitment set or not.

of Blackwell (1953). Letting $h(s|v)$ and $\hat{h}(s|v)$ denote the conditional densities of $S$ and $\hat{S}$, the two signals are related by

$$\hat{h}(\hat{s}|v) = \int_{\underline{s}}^{\overline{s}} m(\hat{s},s)h(s|v)ds \tag{13}$$

where $m$ is a 'garbling function' satisfying $\int_{\underline{s}}^{\overline{s}} m(\hat{s},s)d\hat{s} = 1$.

Suppose that $\hat{S}$ is observed instead of $S$. The regulator continues to decide between discretion and commitment for each realization $s$. Is commitment less valuable given a noisy realization $\hat{S} = s$ than given a precise realization $S = s$? Perhaps surprisingly, the effect is ambiguous.

Recall that commitment to tough actions benefits the regulator when he has bad news (low $v$) and hurts him if he faces good news (high $v$). First, consider a low realization of the public signal, close to $\underline{s}$. Noisy bad news is less meaningful than precise bad news, so the distribution of $V$ given $\hat{S} = s$ is more *optimistic* than its distribution given $S = s$, in the sense of first-order stochastic dominance. Hence, the noise shifts probability mostly towards high $v$, where commitment is harmful, and the value of commitment falls. Second, consider a high realization of the public signal, close to $\overline{s}$. In this case, noise makes the conditional distribution more *pessimistic*, probability shifts mostly towards low $v$, and commitment becomes more valuable.

Since noise affects the value of commitment in an ambiguous way, its effect on the optimal rule is also ambiguous.

**Proposition 3.** *Suppose the public signal becomes $\hat{S}$ instead of $S$. Then the optimal commitment set $\mathscr{C}^\star = [\underline{s}, s^\star)$ shrinks if $s^\star$ is close to $\underline{s}$, and expands if $s^\star$ is close to $\overline{s}$. Moreover, the optimal commitment action $A^\star(s)$ falls if $s \in \mathscr{C}^\star$ is close to $\underline{s}$, and rises if $s \in \mathscr{C}^\star$ is close to $\overline{s}$.*

In reality, regulators would want to execute tough resolution policies only when banks are close to failure. Hence, the empirically relevant case is perhaps where $s^\star$ is close to $\underline{s}$. In this scenario, Proposition 3 shows that when the quality of public information deteriorates, a more cautious approach to rules-based resolution is warranted, in two dimensions. On the one hand, discretion becomes relatively more attractive, and the regulator should only commit for very bad news (the commitment set shrinks). On the other hand, in those states where he does commit, the rules should mandate a weaker response (the commitment actions fall).

When optimal rules are implemented with contingent capital, and if $s^\star$ is close to $\underline{s}$, noisier public information means that (i) less contingent debt should be issued, and (ii) the triggers on contingent capital should be set at a lower level. If the regulator worries about large deviations of market prices from fundamentals, for example, contingent capital with market-based triggers is naturally a less attractive option, even when it acts as a commitment device.

# 5 Contingent capital as a commitment device

Suppose that some long-term bonds are issued as contingent capital. The bank writes contracts with its investors which specify that long-term bonds with face value $\phi(s)$ will be written down or converted into equity when public news $s$ arrives.[14]

By specifying contingent capital in this way, we are making some implicit assumptions. First, we continue to treat the distribution of the public signal $S$, which plays the role of a trigger, as exogenous. In the case of a market-based trigger, this is a bad approximation in situations where conversion itself strongly affects prices. In other words, we assume that contingent debt is designed to prevent the strong feedback effects or 'death spirals' discussed by Sundaresan and Wang (2014). Second, we assume that the regulator has no direct influence over the realization of $S$. This is not guaranteed. For example, Bulow and Klemperer (2013) argue that the conversion of contingent convertibles with *regulatory capital* (i.e. book equity) triggers may not be credible, since regulators can affect measured regulatory capital by deciding when to require banks to write down non-performing assets.

Contracts used in practice trigger conversion whenever a publicly observable indicator falls below a certain threshold. If news get worse, the amount of debt converted always increases or stays the same in these structures. For the sake of realism, we therefore restrict $\phi(s)$ to be decreasing in $s$. The relevant conversions can then be implemented by ensuring that a fraction $\phi(s)$ of bonds have triggers greater than or equal to $s$.

The regulator's preferences are as before, but he cannot reverse the conversion of contingent capital. Hence, even when he acts with discretion, he faces the additional constraint $a \geq \phi(s)$. We show that this constraint creates sufficient commitment to implement the optimal rule. Given a contingent debt structure $\phi(s)$, an equilibrium with discretion and the intuitive criterion are defined as before, except that the regulator must choose $a \in [\phi(s), B]$.

**Definition 4.** A contingent debt structure is a decreasing function $\phi(s)$. A contingent debt structure $\phi$ implements the optimal rule if there exists equilibria with discretion for each $s$ which survive the intuitive criterion and in which the regulator's utility achieves the maximized value of problem (11).

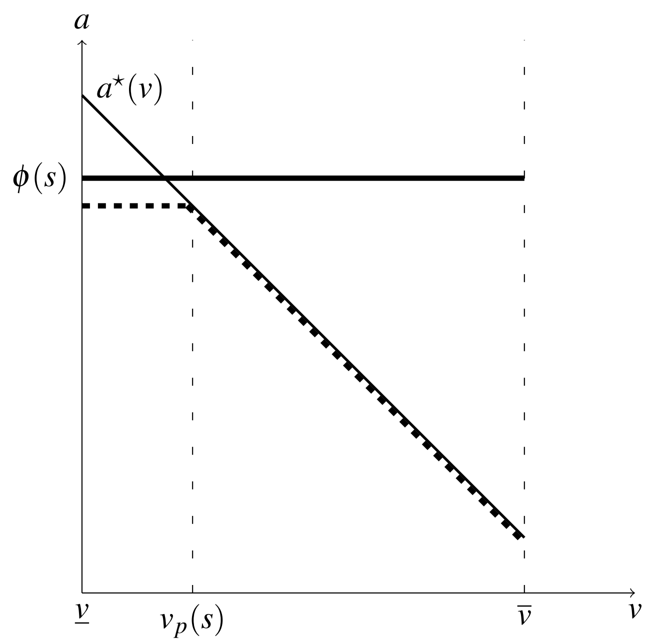
The natural candidate is a contingent debt structure which enforces the actions that the regulator would optimally commit to, i.e. $\phi(s) = A^\star(s)$ for states in the commitment set $s \in \mathscr{C}^\star$. Moreover, in states where commitment is not valuable, we would like contingent debt not to restrict her discretion, i.e. $\phi(s) = 0$ for $s \notin \mathscr{C}^\star$. The characterization of the optimal rule in Proposition 2 immediately

[14] Avdjiev et al. (2013) show that conversion-based contracts dominated the initial wave of issuance in 2009, but that more recently, the split between conversion and principal write-down CoCos has been roughly half-half.

shows that the candidate $\phi(s)$ is a decreasing function, so that it qualifies as a contingent debt structure.

We need to check that $\phi(s)$ induces a discretion equilibrium for each state $s$ which (i) gives the regulator the same utility as the optimal rule, and (ii) survives the intuitive criterion.

Figure 4: **Equilibrium with contingent debt**.




In states without commitment ($s \in \mathscr{C}^\star$), $\phi(s) = 0$, and the set of surviving equilibria is trivially the same as with the optimal rule. In states with commitment, the regulator would play $A^\star(s) = \phi(s)$ for sure under the optimal rule. We must verify that there is a surviving pooling equilibrium such that $\alpha(v,s) = \phi(s)$ for all $v$. These actions are sustained in equilibrium by having the public believe that $v$ is very low whenever they see an action other than $\phi(s)$. Figure 4 illustrates why such beliefs are reasonable in the sense of the intuitive criterion. Given that $\phi(s)$ worth of debt must convert, the only feasible off-equilibrium actions are $a' > \phi(s)$. If a run could be avoided, a regulator with the worst news $\underline{v}$ would always want to deviate to a higher action $a'$, since this would bring him closer to his ideal action $a^\star(\underline{v})$. Hence, it is not unreasonable for the public to believe that $v$ is low when a deviation is observed, and the equilibrium survives the intuitive criterion accordingly.

The optimal rule is implemented by the contingent debt structure

$$\phi(s) = A^\star(s) \times 1(s \in \mathscr{C}^\star).$$

This result illustrates a novel role for contingent capital in financial policy. Contingent capital hardwires the conversion of debt upon bad public news. This ties the regulator's hands in a helpful way.

When public news is bad, commitment to tough bail-in actions is valuable, since the threat of runs and the excessive weakness associated with discretion are imminent. In these states, the conversion of contingent debt provides quasi-commitment by mandating a tough bail-in policy beyond the regulator's control. When public news is good, discretion is preferable. Since contingent capital does not convert in these states, discretion is preserved exactly when it is most valuable.

In our implementation, the regulator has the option to take the bank into resolution and conduct further bail-ins ($a > \phi(s)$), even when some conversion of the contingent debt has been mandated. Contingent debt contracts in practice often have this feature: There is a trigger based on market or accounting information, but the regulator always has the option to intervene, even when the trigger has not been hit. It is interesting to note that in our implementation, this additional 'regulatory trigger' is not used. The regulator would only want to conduct an additional bail-in when he has very bad news. But doing so would reveal bad news to the public, triggering a run.

Hence, the regulator optimally refrains from pulling the additional regulatory trigger. Do regulatory triggers add any value at all? In a richer model, this may still be the case. For example, consider a setting where the public signal $S$ is observable, but only a noisier version $\hat{S}$ is privately contractible. Then commitment via contingent capital contracts is only possible based on $\hat{S}$. However, the regulator will wish to react by using additional regulatory triggers when the public news $S$ is worse than its contractible part $\hat{S}$. Moreover, he will not hesitate to do so, because $S$ is publicly known and acting upon it will not trigger runs.

Our results show that it can be helpful for banks to have a contingent capital structure designed according to the *regulator's* tastes. If there are externalities associated with bank distress, this structure will differ from banks' private preference. A step towards aligning the incentives of banks and regulators is to have contingent debt count towards regulatory capital requirements, as they currently do in Europe. However, there is no guarantee that this would align incentives exactly. Thus, a combination of contingent debt and hard-wired rules may be needed to achieve the right type of commitment.

We show that under our assumptions, the optimal rule can be exactly implemented by contingent capital structures. Thus, we suggest a novel role for contingent capital as a commitment device. Moreover, the assumptions we make to achieve implementation provide guidance on the design of contingent capital contracts and the choice of the trigger signal $S$. In particular, it should be the case that these contracts (i) do not lead to strong feedback effects and (ii) are based on triggers which are credibly beyond the regulator's control. Proposals for such designs are given by Hart and Zingales (2011), Bulow and Klemperer (2013), and Pennacchi et al. (2013), among others. Moreover, Proposition 3 in the last section implies that triggers should not be too noisy. This implies a potential trade-off between the use of market and book values in contingent capital design: Market values are more credibly beyond the control of the regulator than book values, but might also be considered noisier.

Note in addition, that one reason that practitioners give for banks' desire to issue CoCos is that they offers investors a way to avoid the risks associated with regulatory discretion over bail-ins. If investors are uncertain about the regulator's preferences, this can introduce additional risk for the buyers of bonds which may be deemed bail-inable. In our model, regulators' preferences are known and benevolent, but there will nevertheless be uncertainty associated with discretion because investors are not privy to the regulator's private information. Commitment has the virtue of avoiding not only excessive weakness, but also the risk associated with predicting the regulator's signal on the basis of public information.

# 6 The bank's balance sheet and ex ante regulation

So far, we have taken the bank's balance sheet as a primitive, determined exogenously at some prior date (date 0, say). This section examines how changes in the date 0 balance sheet affect the problems associated with discretion and the optimal rule. This exercise will point to ex ante policies, such as liquidity and capital regulation at date 0, which will allow bail-in policy to become more effective. Thus, we show that the ex ante liquidity and capital requirements of Basel III should be seen as *complementary* to the design of ex post resolution regimes.

The parameters of the bank's balance sheet in the baseline model are its short-term debt $D$, and its long-term bonds $B$. In this section, we also allow the bank to hold $C$ units of riskless cash. Moreover, we allow the exposure to risky assets to be scaleable. Assuming that one unit of risky investment, undertaken at date 0, yields exposure to the risky cash flow $V$ we have studied so far, let $X$ be the amount invested in risky assets at date 0, which yields exposure to a risky cash flow of $XV$ at date 1.

In order to determine the effect of balance sheet changes on the optimal rule, and eventually on the regulator's maximized utility, we consider the regulator's utility under discretion and rules.

When the regulator has discretion in state $s$, changes in the bank's balance sheet increase utility if they alleviate the regulator's incentives for excessive weakness. In terms of our graphical analysis, e.g. in panel (c) of Figure 2, such changes benefit the regulator if they reduce the threshold signal $v_p(s)$ and the size of the minimal pooling region. Recalling (10), the maximal utility in discretion is equal to $\bar{U}(s) = E[U(E^{\star} - \max\{v_p(s) - v, 0\})|s]$, so it depends on the bank's balance sheet only indirectly through the threshold $v_p(s)$.

In the model with cash holdings, we need to slightly modify the definition of $v_p(s)$: When the public learns that $V \leq v_p(s)$, it believes that the liquidation value of the bank's assets, i.e. of its cash and its risky assets, is just enough to cover its short-term debt liabilities. Thus, $v_p(s)$ in a model with cash holdings is defined by $C+\lambda E[XV|V \le v_p(s),s] = D$, or equivalently

$$E[V|V \le v_p(s),s] = \frac{D-C}{\lambda X}. \tag{14}$$

Since the left-hand side of this equation is strictly increasing in $v_p(s)$, it follows that a change in the balance sheet reduces $v_p(s)$ – and increases utility under discretion – if and only if it reduces the ratio

$$\Delta \equiv \frac{D-C}{\lambda X}. \tag{15}$$

The ratio $\Delta$ is a natural measure of *illiquidity*. The numerator measures the bank's liquidity shortfall, i.e. the difference between short-term liabilities and cash reserves. The denominator $\lambda X$ measures the number of risky assets available for liquidation, accounting for the discount at which they must be sold.

In states $s$ where the regulator chooses to commit, his utility is equal to

$$VC(s) = \max_a E[U(v+a-(D+B))|s], \tag{16}$$

which depends on capital structure only through the term $D+B$, the bank's total outstanding debt. By the envelope theorem, we have

$$\frac{dVC(s)}{d(D+B)} = -E[U'(v+A^\star(s)-(D+B))|s] = 0,$$

where $A^\star(s)$ is the maximizer in (16), and the second equality follows from the regulator's first-order condition. Thus, we find that changes in the bank's balance sheet affect the value of discretion only via the ratio $\Delta$, and leave the value of commitment unchanged.

**Proposition 4.** *The ratio $\Delta$ defined in (15) is a sufficient statistic for the response of the optimal rule to changes in the bank's balance sheet. If $\Delta$ increases, the optimal commitment set $\mathscr{C}^\star$expands, and the regulator's maximized expected utility decreases. If $\Delta$ decreases, the optimal commitment set $\mathscr{C}^\star$ shrinks, and the regulator's maximized expected utility increases.*

An increase in $\Delta$ makes the bank less liquid. This strengthens the threat of runs and increases the regulator's incentives to take weak actions. As a result, discretion becomes more problematic and he reacts by commiting over a larger range of states. Other things equal, this change makes the regulator worse off. The opposite happens when $\Delta$ decreases and the bank becomes more liquid. We now discuss how, in practice, liquidity and capital regulation can influence $\Delta$.

## 6.1 Liquidity regulation

Under the rules of Basel III, banks must ensure that their *liquidity coverage ratio* (LCR) does not fall below 1. The liquidity coverage ratio is calculated as the ratio of *high quality liquid assets* to *net cash outflows*, where the outflows are taken from a hypothetical 30-day stress scenario. High quality liquid assets are a weighted average of assets of the form $C + w_l X$, where $w_l < 1$ is a coefficient measuring the liquidity of non-cash assets.[15] Net cash outflows are a weighted average of liabilities. In particular, the Basel proposal puts a weight of 1 on short-term debt, i.e. it is assumed that short-term debt is withdrawn completely in the stress scenario. If long-term debt may not be withdrawn (as is the case for sufficiently long maturities), net cash outflows in our notation are simply equal to $D$, and the LCR requirement is $(C + w_l X)/D \geq 1$, which is equivalent to

$$\Delta = \frac{D-C}{\lambda X} \leq \frac{w_l}{\lambda}.$$

If this requirement is binding, a regulator controlling the LCR can directly influence the ratio $\Delta$ by adjusting the liquidity weight on risky assets $w_l$. By Proposition 4, this ratio is the only quantity that matters for the effectiveness of bail-in policy. It follows that Basel III's liquidity regulation is a very natural complement to bank resolution regimes. We do not model the determinants of optimal liquidity regulation, which is influenced by many factors, such as the cost of reduced maturity transformation by banks ex ante (Walther, 2015). However, our analysis demonstrates that *at the margin,* tough liquidity regulation becomes more desirable when illiquidity has an adverse impact on the efficacy of bank resolution.

## 6.2 Capital regulation

Capital regulation requires the ratio of bank equity to risk-weighted assets to be above a certain threshold $\kappa$. In our setting, a capital requirement at date 0 would constrain the bank's balance sheet to satisfy

$$\frac{X-(D+B)}{w_r X} \geq \kappa, \tag{17}$$

where $w_r > 0$ is a risk weight. The capital requirement can be rearranged to

$$\frac{D+B}{X} \leq 1 - \kappa w_r.$$

While liquidity regulation was able to directly target the ratio $\Delta$, which is important for bank

[15] For simplicity, we concentrate on the case with one non-cash asset. However, a similar argument would apply for any number of non-cash asset classes, in which case high quality liquid assets would be $C + \sum w_{li} X_i$ for some vector of weights $(w_{li})$.

resolution, capital regulation is a blunter tool. Tightening capital regulation, by raising $\kappa$ or $w_r$, forces the bank to reduce the ratio $(D+B)/X$. How this measure correlates with $\Delta$ is difficult to say without an explicit model of bank choices at date 0, which is beyond the scope of our paper.

However, for well-behaved preferences, a bank which is forced to reduce $(D+B)/X$ is likely to push on both margins, i.e. reduce both short-term debt $D/X$ and long-term debt $B/X$, relative to risky assets. This will lead to a reduction in $\Delta = (D-C)/\lambda X$, unless the decrease in $D/X$ is offset by a *bigger decrease* in the cash ratio $C/X$. It would be difficult to write a model in which a bank's cash holdings, which are not constrained by (17), respond more a tightening of the capital requirement than short-term debt, which *is* constrained by (17). Moreover, reserve requirements place a lower bound on $C/X$ in practice, so that the offsetting effect is unlikely to dominate.

Taking these arguments together, a tightening in the cpaital requirement is likely to reduce $\Delta$, increase the bank's liquidity, and increase the utility of the bail-in regulator we have studied. However, the transmission channel is less clear-cut than with liquidity requirements, which were able to target $\Delta$ directly. Liquidity regulation appears a more natural candidate for complementing bank resolution regimes in our setting, where regulator weakness in bailing-in is driven by fear of liquidity-draining bank runs.

# 7 The lender of last resort

We have worked under the assumption that liquidity support by lenders of last resort is insufficient to eliminate the threat of runs. The motivation for this assumption is as discussed in Section 2: Instituting a very lenient lender of last resort may not be optimal due to moral hazard concerns, and empirical reality features binding limits on liquidity support, such as collateral requirements. In this section, we show that partial liquidity support is complementary to bail-in policy.

Suppose that at date 1, a lender of last resort is willing to grant the bank a loan $L$ per unit of risky asset investment. For simplicity, we suppose that the maximal loan $L$ does not react to public or private information, and that the lender of last resort does not act strategically. These feedbacks would introduce an additional fixed point problem to the model, but as long as the coverage of liquidity support is imperfect, the qualitative effects with feedback will be similar. Moreover, the lender of last resort can only take on a fraction $\eta$ of the bank's assets as collateral. This might reflect pre-determined rules about acceptable collateral.

If the bank sells a fraction $z$ of its risky assets, it obtains $\lambda E[V|a,s]z$ in the market, and it can borrow $\max\{1-z,\eta\}L$ from the lender of last resort. Thus, the maximum amount of liquidity the bank can obtain at date 1 is

$$\max_z \left[\lambda E[V|a,s]z + \max\{1-z,\eta\}L\right]$$
$$= \lambda E[V|a,s] + \eta \max\{L - \lambda E[V|a,s], 0\} \tag{18}$$

The expression is easy to interpret. Selling assets on the market brings $\lambda E[V|a,s]$. Suppose that per risky asset, the loan available from the lender of last resort exceeds the amount that it can be sold for, i.e. $L > \lambda E[V|a,s]$. Then the bank raises the most liquidity by using the lender of last resort, up to the maximum possible extent $\eta$. This further increases liquidity by $\eta \max\{L - \lambda E[V|a,s], 0\}$.

Repeating the analysis of the withdrawal game in Section 2 reveals that bank runs become a possibility when the expression in (18) is strictly less than outstanding short-term debt $D$. Suppose that this is the case when the public learn the worst possible news, i.e.

$$\lambda \underline{v} + \eta \max\{L - \lambda \underline{v}, 0\} < D. \tag{19}$$

Then, by to the argument of Section 3, discretionary bail-in policies will induce excessive weakness for regulators who observe $V \leq v_p(s)$, where the pooling threshold $v_p(s)$ is defined by

$$E\left[V|V \leq v_p(s), s\right] = \begin{cases} \frac{D-\eta L}{\lambda(1-\eta)}, & \text{if } L \geq D \\ \frac{D}{\lambda}, & \text{if } L < D. \end{cases}$$

The first case, $L \geq D$, corresponds to one where the lender of last resort would be able to rescue any bank if $\eta = 1$, i.e. if it accepted all projects as collateral. Under the restriction (19), the threshold $v_p(s)$ is decreasing in $\eta$. Therefore, in the region where the lender of last resort is strong in principle but constrained by collateral requirements, loosening the collateral requirement will reduce the regulator's incentives to be excessively weak in his bail-in policy. Intuitively, a more lenient lender of last resort reduces the threat of bank runs, and as a result the regulator will worry less about revealing bad news when conducting bail-in policies.

By a parallel argument to Proposition 4, it follows that in this parametric region, loosening the lender of last resort's collateral requirement is complementary to bank resolution policy. We do not model the other trade-offs involved in setting collateral requirements, such as concerns about moral hazard. But *at the margin*, liquidity support by lenders of last resort should be more generous when effective bank resolution is considered an important policy objective.

# 8 Conclusion

We have built a signaling model in which bank regulators have information about the financial condition of banks which investors do not have. We have shown that in the presence of such information, effective bank resolution can be inhibited by regulatory authorities' incentives to be excessively weak. In our model, regulators may prefer to leave banks under-capitalized, because undertaking a tough re-capitalization before it is expected would reveal regulators' pessimistic outlook, and so could spook investors and lead to adverse consequences such as bank runs. More generally, whenever regulators are believed to have private information, signaling concerns will limit the regulator's willingness to step in early or take other tough action to resolve banks.

The implication of our result is that it is always desirable for the regulator to use some amount of commitment to limit discretion ex post in a crisis, even if this means tying regulation to a more noisy, publicly observable, source of information. Commitment is most valuable after bad public news, since such events foreshadow the need for tough resolution policy. We show that optimally designed contingent capital issues by banks (e.g. CoCos) can help implement such a commitment.

Our theory has three broad implications for resolution policy, and for financial policy in general. First, allowing regulators discretion in resolution regimes is not always a virtue. It is widely agreed that resolution authorities should put in place plans for bail-ins in systemically important banks. But our paper goes further in pointing out that it is very important that these plans are made *binding* after adverse public news, for example after severe declines in banks' market or book values, in order to avoid the excessive weakness problem that arises from signaling. Second, regulators should welcome the use of properly designed contingent capital contracts, which recapitalize failing banks automatically. Finally, bank resolution and 'going concern' policies such as the regulation of banks' balance sheets and last resort lending, should not be considered separate activities. Because they ease the 'run constraint' on the regulatory release of information, going concern policies help improve the credibility of plans to write-down or convert bank debt in a crisis. Thus, there is a natural and important complementarity between going-concern policies and the effective design of resolution regimes.

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# APPENDIX

Unless otherwise stated, proofs are given for the general case where the regulator's utility (in the absence of a run) is $U(a,v)$, where $U(a,v)$ is twice differentiable, strictly concave in $a$ and satisfies the submodularity condition

$$\frac{\partial^2 U(a,v)}{\partial a \partial v} < 0 \tag{20}$$

The exposition in the text corresponds to the special case where (abusing notation slightly),

$$U(a,v) = U(v + a - (D+B)).$$

## Proof of Lemma 1

1. *No runs.* Let $\rho(a,s) = \mathbf{1}(\lambda E_\beta[V|a,s] < D)$. Suppose some type $v'$ faces a run in equilibrium, $\rho(\alpha(v',s),s) = 1$. Then a run must occur regardless of the regulator's action, $\rho(a,s) = 1$ for all $a$, because otherwise, assumption (4) implies that type $v'$ would deviate to an action that does not trigger a run. Then the regulator's action maximize $U(a,v) - \kappa(v)$, implying $\alpha(v,s) = a^\star(v)$ for all $v$. Let $v'' = \inf\{v : a^\star(v) = a^\star(\overline{v})\}$. Bayes' rule and assumption (3) give

$$E_\beta[V|\alpha(\overline{v},s),s] = E[V|V \geq v'',s] \geq E[V|s] \geq D.$$

Thus type $\overline{v}$ cannot face a run, a contradiction.

2. *Minimal pooling.* Suppose not. Assume that equilibrium actions are weakly decreasing in $v$, as will be established in part 3. Then $\alpha(\underline{v},s) > \alpha(v_p(s),s)$. Let $v' = \sup\{v : \alpha(v,s) = \alpha(\underline{v},s)\}$. Then Bayes' rule and assumption (3) give $E_\beta[V|\alpha(\underline{v},s),s] = E[V|V \in [\underline{v},v']] < D$. Thus type $\underline{v}$ must face a run, contradicting the result in part 1.

3. *Incentive compatibility.* In equilibrium, each type must prefer his action to anybody else's:

$$U(\alpha(v,s),v) \geq U(\alpha(v',s)v) \text{ for all } v' \neq v \tag{21}$$

Suppose $\alpha(v,s)$ is not weakly decreasing in $v$ so that for some $v' > v$ we have $\alpha(v',s) > \alpha(v,s)$. Let $a = \alpha(v,s)$ and $a' = \alpha(v',s)$. Condition (21) implies $U(a',v') - U(a,v') \geq 0 \geq U(a',v) - U(a,v)$. But by the submodularity condition (20) and $a' > a$, we have

$$\frac{\partial}{\partial t} U(a',t) - U(a,t) < 0$$

implying $U(a',v') - U(a,v') < U(a',v) - U(a,v)$, a contradiction.

When $\alpha(v,s)$ is differentiable at $v$, the function $U(\alpha(v',s),s)$ differentiable in $v'$ at $v = v'$. The incentive condition (21) then implies the first-order condition

$$\frac{\partial U(\alpha(v,s),v)}{\partial a}\frac{\partial \alpha(v,s)}{\partial v} = 0.$$

Thus either $\frac{\partial \alpha(v,s)}{\partial v} = 0$ or $\frac{\partial U(\alpha(v,s),v)}{\partial a} = 0$. The latter case gives $\alpha(v,s) = a^\star(v)$ as required.

When $\alpha(v,s)$ is discontinuous at $v$, then suppose $U(\alpha^+(v,s),s) > U(\alpha^-(v,s),s)$. By continuity of $U$, for small enough $\varepsilon$, there exists a type $v-\varepsilon$ who prefers action $\alpha^+(v,s)$ to his own, contradicting equilibrium. If $U(\alpha^+(v,s),s) < U(\alpha^-(v,s),s)$ the contradiction follows for some type $v+\varepsilon$. Thus $U(\alpha^+(v,s),s) = U(\alpha^-(v,s),s)$ as required.

## Proof of Proposition 1

We prove the proposition in two steps. First, we show that $\alpha(v,s)$ is continuous in $v$ in any surviving equilibrium. Second, we show that the highest type never bails in more than his preferred action: $\alpha(\bar{v},s) \leq a^\star(\bar{v})$.

In both steps, we show that a candidate equilibrium violating the proposed condition does not survive the intuitive criterion. To do this, we find an off-equilibrium action $a_0$ such that types below$v_D = D/\lambda$ would never to $a_0$, and that some type above $v_D$ strictly prefers $a_0$ to his equilibrium action. For such an action, and for all beliefs $\gamma$ with $Pr_\gamma[S \in \sigma(a_0,s)] = 1$, we have $\lambda E_\gamma[V|a_0, V \in \sigma(a_0,s)] \geq D$, so that the equilibrium does not survive the intuitive criterion.

1. *Continuity of* $\alpha(v,s)$. Take any equilibrium where $\alpha(v,s)$ has a discontinuity at some $v$. By Lemma 1, we have $v \geq v_p(s)$ (minimal pooling), and type $v$ is indifferent between the actions before and after the jump (incentive compatibility). Consider the off-equilibrium action $a_0 = \alpha^-(v,s)+\varepsilon$. Only types above $v-\delta(\varepsilon)$ would deviate to $a_0$, where $\delta(\varepsilon) \to 0$ as $\varepsilon \to 0$. Since $v \geq v_p(s) > v_D$, only types above $v_D$ would deviate to $a_0$ when $\varepsilon$ is small. Finally, note that for all $\varepsilon > 0$, there exists a type $v+\hat{\delta}(\varepsilon)$ who strictly prefers $a_0$ to his equilibrium action.

2. *Highest type's action satisfies* $\alpha(\bar{v},s) \leq a^\star(\bar{v})$. Take any equilibrium with $\alpha(\bar{v},s) > a^\star(\bar{v})$, and let $v' = \min\{v : \alpha(v,s) = \alpha(\bar{v},s)\}$ be the lowest type who takes the same action as $\bar{v}$. There are two possible cases: complete pooling with $v' = \underline{v}$, or partial separation with $v' \geq v_p(s)$.

Case (i): $v' = \underline{v}$. All types take action the same pooling action $a'$. When this action is lower than the ideal action of type $v_D$, i.e. $a' \leq a^\star(D)$, then it is also lower than the ideal action of any type below. Thus only types above $v_D$ would deviate to the off-equilibrium action $a_0 = a^\star(\bar{v})$, and type $\bar{v}$ strictly prefers $a_0$ to his equilibrium action.

When $a' > a^\star(D)$, then we have $U(\alpha(D,s),D) > U(1,D) > U(0,D)$ by assumption (6). It follows from the submodularity condition (20) that no types below $v_D$ would deviate to the off-equilibrium action $a_0 = 0$. If type $\bar{v}$ wants to deviate to this action, i.e. $U(0,\bar{v}) > U(a',\bar{v})$, then we are done. Otherwise, define $a''$ as the action that makes her indifferent: $U(a'',\overline{v}) = U(a',\overline{v})$. Take an off-equilibrium action $a_0 = a'' - \varepsilon$, where $\varepsilon > 0$, which type $\overline{v}$ strictly prefers to his equilibrium action. Only types above $\overline{v} - \delta(\varepsilon)$ would deviate to $a_0$, where $\delta(\varepsilon) \to 0$ as $\varepsilon \to 0$. Thus only types above $D$ would deviate to $a_0$ when $\varepsilon$ is small.

Case (ii): $v' \geq v_p(s)$. By Lemma 1 (incentive compatibility) and continuity, every type below $v \leq v'$ takes an equilibrium action which is weakly below his preferred action $a^\star(v)$. Thus only types above $v'$ would deviate to an off-equilibrium action $a_0 \in (a^\star(\overline{v}), \alpha(\overline{v}, s))$, and type $\overline{v}$ strictly prefers such an $a_0$ to his equilibrium action, which completes the proof.

## Proof of Proposition 2

We show that it the essentially unique optimal commitment set is either empty or an interval $\mathscr{C}^\star = [\underline{s}, s^\star]$. The characterization of the optimal commitment action in the second case follow immediately, and the proof of Proposition 5 demonstrates that $A^\star(s)$ is decreasing in $s$.

In an optimal rule, the regulator must commit in almost all states $s$ for which the value of commitment $VC(s)$ is greater than $\bar{U}(s)$, and retain discretion in almost all others. Using the definition of $VC(s)$ in (12) and the characterization of $\bar{U}(s)$ in (8), and defining $J(a,v) = U(a,v) - U(\min\{a^\star(v), a^\star(v_p(s))\}, v)$, we have $VC(s) - \bar{U}(s) = \max_a E[J(a,v)|s]$. It is easy to see that $J(a,v) < 0$ for all $a < a^\star(v_p(s))$. Hence, the regulator must commit whenever the effective value of commitment, defined as

$$EVC(s) = \max_{a \geq a^\star(v_p(s))} E[J(a,v)|s] \tag{22}$$

is positive. For all $a \geq a^\star(v_p(s))$, $J(a,v)$ is strictly decreasing in $v$. By the assumption of first-order stochastic dominance in (1) and the argument of Rothschild and Stiglitz (1974), it follows that $EVC(s)$ is strictly decreasing in $s$.

If $VC(\underline{s}) > U(\underline{s})$, then $EVC(\underline{s}) > 0$, and by continuity of $U$, there exists a $s^\star$ such that $EVC(s) \geq 0$ for all $s \leq s^\star$and $EVC(s) < 0$ for all $s > s^\star$ (if $s^\star < \overline{s}$). Thus, the regulator must commit for almost all $s \in [\underline{s}, s^\star]$, and retain discretion for almost all other $s$, as required.

## Proof of Proposition 3

Denote the effective value of commitment, as defined in (22), by $EVC(s)$ if $S$ is observed and by $E\hat{V}C(s)$ if $\hat{S}$ is observed. First, we show that $E\hat{V}C(\underline{s}) < EVC(\underline{s})$ and $E\hat{V}C(\overline{s}) > EVC(\underline{s})$. Second, we show that there exists a $s_{low} > \underline{s}$ such that (i) the commitment threshold $s^\star$ decreases if $s^\star \leq s_{low}$ and (ii) the optimal commitment action $A^\star(s)$ falls for all $s \leq s_{low}$. Conversely, there exists a $s_{high} < \overline{s}$ such that (i) the commitment threshold $s^\star$ falls if $s^\star \geq s_{high}$ and (ii) the optimal commitment action $A^\star(s)$ falls for all $s_{high} \leq s \leq s^\star$ if $s^\star > s_{high}$.

1. $E\hat{V}C(\underline{s}) < EVC(\underline{s})$ *and* $E\hat{V}C(\bar{s}) > EVC(\underline{s})$. Let $h_V(v)$, $h_S(s)$ and $h_{\hat{S}}(\hat{s})$ denote the marginal densities of $V$, $S$ and $\hat{S}$. By Bayes' rule and the garbling condition (13), we have

$$\begin{aligned} \hat{g}(v|\hat{s}) &= \frac{\hat{f}(s|v)h_V(v)}{h_{\hat{S}}(\hat{s})} = \frac{h_V(v)}{h_{\hat{S}}(\hat{s})}\int_{\underline{s}}^{\bar{s}} m(\hat{s},s)f(s|v)ds \\ &= \int_{\underline{s}}^{\bar{s}} k(\hat{s},s)g(v|s)ds \end{aligned}$$

for all $\hat{s} \in [\underline{s},\bar{s}]$, where $k(\hat{s},s) = \frac{h_S(s)}{h_{\hat{S}}(\hat{s})}m(\hat{s},s)$. Integrating both sides over $v \in [\underline{v},\bar{v}]$, we have $\int_{\underline{s}}^{\bar{s}} k(\hat{s},s)ds = 1$. Now integrating over $v \in [\underline{v},v']$, we have $\hat{G}(v'|\hat{s}) = \int_{\underline{s}}^{\bar{s}} k(\hat{s},s)G(v'|s)ds$. It follows from (1) that $G(v'|\bar{s}) < \hat{G}(v'|\hat{s}) < G(v'|\underline{s})$ for all $\hat{s}$. Thus the distribution of $V$ given $\hat{S} = \underline{s}$ first-order stochastically dominates its distribution given $S = \underline{s}$, and the distribution of $V$ given $\hat{S} = \bar{s}$ is dominated by its distribution given $S = \bar{s}$. Repeating the argument of Proposition 2 implies the proposed inequalities.

2. *Existence of* $s_{low}$ *and* $s_{high}$. We show the existence of $s_{low}$. The proof for $s_{high}$ is analogous.

From the analysis above, and by continuity, there is a $s' > \underline{s}$ such that $E\hat{V}C(s) < EVC(s)$ for all $s \le s'$. Note that the original optimal commitment threshold solves $EVC(s^\star) = 0$. If $s^\star \le s'$, then $E\hat{V}C(s^\star) < 0$. By the argument of Proposition 2, $E\hat{V}C(s)$ is increasing in $s$, and therefore, the new optimal commitment threshold, which solves $E\hat{V}C(\hat{s}^\star) = 0$, must lie below $s^\star$.

The original optimal commitment action $A^\star(s)$ solves $E[\frac{\partial}{\partial a}U(c^\star(s),v)|S=s] = 0$. By the submodularity condition (20), $\frac{\partial}{\partial a}U(a,v)$ is decreasing in $v$. Using the result on first-order stochastic dominance above, it follows that $E[\frac{\partial}{\partial a}U(a,v)|S=\underline{s}] > E[\frac{\partial}{\partial a}U(a,v)|\hat{S}=\underline{s}]$, implying

$$E[\frac{\partial}{\partial a}U(A^\star(s),v)|\hat{S}=\underline{s}] < 0.$$

Thus the optimal commitment action given the lowest signal, which solves

$$E[\frac{\partial}{\partial a}U(\hat{A}^\star(s),v)|\hat{S}=\underline{s}] = 0,$$

must lie below $c^\star(\underline{s})$. By continuity, there exists a $s'' > \underline{s}$ such that $\hat{A}^\star(s) < A^\star(s)$ for all $s \le s''$ for which an optimal commitment action is defined.

Combining the arguments above, we can set $s_{low} = \min\{s', s''\}$, which has the desired properties.

## Proof of Proposition 5

First, we verify that the proposed function $\phi(s) = A^\star(s) \times 1(s \in \mathscr{C}^\star)$ is non-increasing in $s$ and qualifies as a contingent debt structure. Second, we show that it implements the optimal rule.

1. $\phi(s)$ *is non-increasing.* By Proposition 2, we have $\phi(s) = 0$ for all $s$ or

$$\phi(s) = \begin{cases} A^\star(s), & s \leq s^\star \\ 0, & s > s^\star \end{cases}$$

for some $s^\star$, where $c^\star(s)$ satisfies the first-order condition $\frac{\partial}{\partial a} E[U(c^\star(s), v)|s] = 0$. By the submodularity condition (20), $\frac{\partial}{\partial a} U(a, v)$ is decreasing in $v$. Using the assumption of first-order stochastic dominance in (1) and the argument of Rothschild and Stiglitz (1974), it follows that $\frac{\partial}{\partial a} E[U(a, v)|s] = 0$ is decreasing in $s$ for all $a$. By concavity of $U$, it follows that $c^\star(s)$ is decreasing in $s$. Hence $\phi(s)$ is non-increasing everywhere (but with a potential jump discontinuity at $s^\star$).

2. $\phi(s)$ *implements the optimal rule.* For $s \notin \mathscr{C}^\star$, the game is identical to that without contingent debt, and it is immediate from Proposition 1 that an equilibrium with the desired properties exists. For $s \in C$, we consider the pooling equilibrium with $\alpha(v, s) = \phi(s)$, and show that it survives the intuitive criterion. The only feasible off-equilibrium actions have $a_0 > \phi(s)$. The gain from deviating from $\phi(s)$ to $a_0$ for type $v$ is $U(a_0, v) - U(\phi(s), v)$. By the submodularity condition, this gain is highest for type $\underline{v}$. Thus whenever any type prefers $a_0$ to his equilibrium action, type $\underline{v}$ prefers it too. Thus whenever $\sigma(a_0, s)$ is non-empty, there is a belief $\gamma$, which places all mass on $\underline{v} \in \sigma(a_0, s)$, such that $\lambda E_\gamma[V|a_0, V \in \sigma(a_0, s)] = \underline{v} < D$, and the equilibrium survives the intuitive criterion.