Survival of Hedge Funds: Frailty versus Contagion*

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Abstract

In this paper we develop a new methodology to analyse the dynamics of liquidation risk dependence in the hedge funds industry. This dependence results either from a common exogenous factor (shared frailty), or from contagion phenomena, which occur when an endogenous behaviour of a fund manager impacts the probability of liquidation of other funds. Our empirical analysis shows that the common factor, the sensitivities to this factor and the contagion scheme admit interpretations in terms of liquidity risks. The factor is related in a nonlinear way to standard proxies of rollover and margin funding liquidity risks. The sensitivities to the factor, that are funding liquidity risk exposures, depend on the redemption and leverage policies adopted by the funds managers. Finally, the causal scheme captures a part of the reinforcing spiral between funding and market liquidity.

Keywords: Hedge Fund, Contagion, Dynamic Count Model, Systemic Risk, Stress-Tests, Funding Liquidity.

JEL classification: G12, C23.

1 Introduction

The rather short lifetimes [1] of a majority of hedge funds (HF) and the dependence between their liquidations motivate the interest of investors, academics and regulators in HF survival analysis. Several economic reasons explain why a HF manager decides to liquidate a fund. Loosely speaking, this decision can be taken since either the income of the fund manager is not sufficient, or the market conditions are not appropriate for managing the fund in an efficient way. Let us discuss these two aspects.

i) Large cash withdrawals

The income of a fund manager is coming from management fees which are indexed in a complicated way on the performance of the fund, but also on the total Asset Under Management (AUM). In particular, when outflows are important it may become uninteresting to continue to manage the fund. Moreover, this effect is amplified by the specific high water mark fee structure implemented by the HF manager [see e.g. Brown, Goetzmann, Liang (2004), Darolles, Gourieroux (2013) for the description of fees]. This effect impacts the liability component of the balance sheet of the fund. It can be a source of liquidation dependence, for instance during a funding liquidity crisis when the fund managers experience jointly large cash withdrawals, while having difficulty in borrowing and being obliged to diminish their use of leverage. It also arises with the withdrawal of some prime brokers, and it is amplified by the use of debt to create the needed leverage. If the prime brokers simultaneously quit several funds, we get a frailty phenomenon, that is, a common risk factor. This frailty effect is exogenous, even when there is a herding behaviour of prime brokers, as long as their decision is not triggered by past liquidation events.

[1] The global annual liquidation rate for hedge funds between 1994 and 2003 has been around 8%-9%, which corresponds to a median lifetime of 6-7 years. However, the liquidation rate is considerably varying according to the year and management style, with values between about 4% and 30% for that period [see e.g. Getmanski, Lo, Mei (2004), Chan, Getmanski, Haas, Lo (2007), Table 6.14]. Moreover, the liquidation rate depends significantly on the definition of liquidation and on the database.

ii) Market liquidity

If HF portfolios are invested in illiquid assets, it can be difficult and risky to continue to manage funds during a market liquidity crisis. Indeed, the fire sales of a given fund manager will consume the market liquidity of a given class of illiquid assets. The first consequence of such fire sales is a price pressure on these assets, which implies a decrease of the market value of all funds holding these assets in their portfolio. This effect concerns the asset component of the balance sheet, and is often called contagion in the HF literature. A well-known example is the default of the Russian sovereign debt in August 1998, when Long Term Capital Management (LTCM) and many other fixed-income HF suffered catastrophic losses over the course of a few weeks. Then, the failure of one of these funds increases the probability of liquidation of other funds. Another example is the increase of margin calls for the hedge funds with large exposure in subprime-related fixed income securities, which forced them to sell securities held in their portfolios in the recent financial crisis.

In this paper, two causes of liquidation risk dependencies arise.

i) There exist underlying exogenous stochastic factors, which have a common influence on the liquidation intensities of the individual HF. In the credit risk literature, these factors are called systematic risk factors or frailties [Duffie et al. (2009)]. Such common exogenous shocks also exist in the joint analysis of HF lifetimes and likely capture the funding liquidity risk.

ii) Risk dependency can also arise when a shock to one fund has an impact on other funds. This is the so-called contagion effect. In the case of credit risk, contagion is generally due to the debt structure, when some banks or funds invest in other banks or funds [see e.g. Upper, Worms (2004) and Gourieroux, Heam, Monfort (2013)]. For HFs it corresponds mainly to the market liquidity risk.

To our knowledge this paper is the first one to introduce both frailty and contagion effects in HF survival models, and to measure the magnitudes of these effects. The liquidation intensity of an individual HF is assumed to depend on the lagged observations of liquidation counts in the same and in the other management styles, as well as on a common unobservable dynamic factor. The former explanatory variables represent contagion, whereas the latter represents the shared frailty. The specification allows to disentangle the two types of liquidation risk dependence by exploiting the time lag that contagion necessitates to produce its effects. The model is applied at the semi-aggregate level of the management styles. This allows us to focus on the underlying systematic dynamic factors as well as on the contagion within and between management styles, since the hedge fund specific aspects of the liquidation become negligible after the aggregation. The liquidation counts per management style are assumed to follow an autoregressive Poisson model with both frailty and contagion effects. We develop the appropriate methods to estimate this dynamic model and to reconstitute the values of the underlying unobservable factor. The common factor, the sensitivities of the liquidation counts to this factor and the contagion scheme have all interpretations in terms of liquidity. The underlying factor provides a measure of rollover and margin funding liquidity risks with two regimes. The sensitivities of the factor provide the liquidity exposures of the different management styles; they are linked to the redemption frequency and management of gates by the HF managers. Finally, the estimated causal scheme captures a part of the spiral effect between funding and market liquidity risks highlighted in Brunnermeier, Pedersen (2009). Such a Poisson dynamic model is especially appropriate to analyze in a dynamic framework the consequences of stress on either funding or market liquidity, and then to design the possible policies to diminish some of these consequences. The analysis of underlying common risk factors and contagion is the first step before studying the impact of HF on systemic risk for the global financial markets, and the possible cascade into a global financial crisis.

We focus on the liquidation times and do not consider the losses on Net Asset Value during the liquidation process. [2] The analysis of HF lifetimes is generally based on a duration model, which can

[2]It would be much more difficult to measure the loss given liquidation of hedge funds than the loss given default of large corporations for the following three reasons: *i)* The exposure at liquidation are self-reported and have to be carefully checked, *ii)* The hedge funds cannot issue bonds for refinancing and thus there exists no market value of their liquidation

be either parametric, semi-parametric, or nonparametric. Typically, in a first step analysis researchers study how the liquidation intensity depends on the age of the HF, and/or on calendar time. This is done by appropriately averaging the observed liquidation rates [see e.g. the Kaplan-Meier non parametric estimation of the hazard function in Baba, Goko (2006), Figure 1, for the description of age dependence, or the log-normal hazard function used in Malkiel, Saha (2005)]. In a second step, a parametric specification of the (discrete-time) liquidation intensity can be selected, such as a logit, or a probit model to analyze the possible determinants of liquidation. The explanatory variables can be time independent HF characteristics, such as the management style, the domicile country (off-shore vs domestic), the minimum investment, variables summarizing the governance structure, such as the existence of incentive management fees, and their design (high water mark, hurdle rate), the announced cancellation policy (redemption frequency, lockup period), the experience and education level of the manager [Boyson (2010)]. Regressors can also include time dependent HF individual characteristics, such as lagged individual HF return, realized return volatility and skewness, asset under management (AUM) and the recent fund inflows [see e.g. Baquero, Horst, Verbeek (2005), Malkiel, Saha (2005), Chan, Getmansky, Haas, Lo (2007), Section 6.5.1, Boyson (2010)], as well as time dependent market characteristics [Chan, Getmansky, Haas, Lo (2007), Section 6.6.1, Carlson, Steinman (2008)] and the competitive pressure, measured by the total number of HFs [Getmansky (2010)]. Finally, the parametric and nonparametric approaches can be combined in the proportional hazard model introduced by Cox (1972), as in Brown, Goetzmann, Park (2001), Baba, Goko (2006), Gregoriou, Lhabitant, Rouah (2010).

While the above survival models are useful for a descriptive analysis of liquidation intensity, these models are not always appropriate for liquidation risk prediction, for evaluation of systematic risk, or for capturing the observed liquidation clustering, as required in a stress testing analysis. For instance,

risk, *iii)* The hedge fund portfolios can include a significant proportion of illiquid assets, which require time to be sold at a reasonable price during the liquidation process.

for such purposes it is not suitable to introduce time dependent explanatory variables in survival models. Indeed, the future liquidation risk can only be analyzed after predicting the future values of the time dependent explanatory variables. This is a difficult task as it requires a joint dynamic model for these variables and the liquidation indicators. Furthermore, since the duration models considered in the literature assume the independence of individual liquidation risks given the selected observed explanatory variables, liquidation correlation has not yet been included in the HF survival models.

From the investor's point of view, liquidation is a risk, which concerns both the timing of the payoffs and the value of the fund at liquidation time. It is partially at the discretion of the fund manager, who can slow down or accelerate liquidation by an appropriate management of gates, for instance. The liquidation risk is similar to default risk, prepayment risk, or lapse risk encountered on corporate bonds, credit derivatives, or life insurance contracts, respectively. Liquidation risk is often neglected in standard portfolio management practices. It is a hidden risk, in contrast to the visible risk directly measured by the volatility of returns, i.e. the market risk. A fund with a small visible market risk can have a large hidden liquidation risk. Liquidation risk is especially important for the following three types of market participants:

i) Most of the money invested in HF in the yearly 2000's comes from institutional investors, including endowments, foundations, corporate and public plans, and insurance (if they have a minimal capital) [Casey, Quirk, Acito (2004)]. They invest on a long term basis, are interested in the low correlation of HF returns with traditional assets classes, like equity and bonds, and want to avoid the consequences of a short term liquidity crisis.

ii) The regulators have to monitor both the funding and the market liquidity risks. They may modify and control the funding liquidity exposure by means of restrictions on the use of leverage, of the redemption frequency and of the minimal requirements for investing in a HF. Regulators may limit the market liquidity exposure by applying the Basel III approach, for instance by introducing additional reserves based on liquidity stress scenarios.

iii) The funds of funds can be very sensitive to liquidation risk dependencies between individual funds. An appropriate survival analysis can help to detect the funds of funds, which are too sensitive to such systematic risk.

The paper is organized as follows. In Section 2 we describe the dataset on hedge fund liquidation used in our empirical study. We aggregate the liquidation counts per management style. In Section 3 we introduce the Poisson contagion model with dynamic frailty. The model with autoregressive gamma frailty is especially convenient, since it provides a joint affine dynamics for the frailty and the liquidation counts. This facilitates the prediction of future liquidation risk as well as the estimation of parameters in such a nonlinear setting with unobservable factors. Dynamic models with contagion and frailty are estimated in Section 4. We assess the relative magnitude of contagion and shared frailty phenomena when we study liquidation risks dependence across different management styles. We carefully distinguish between the direct frailty effect and the amplification of the exogenous systematic shocks through the contagion network. We also discuss the interpretations of the causal scheme and of the factor sensitivities in terms of liquidity risks. We derive in Section 5 the filtered values of the underlying unobservable factor. We show that this underlying factor is related in a nonlinear way with standard proxies of the rollover and margin funding liquidity risks and discuss the observed nonlinearity in terms of endogenous switching regimes. In Section 6 we illustrate our methodology by an application to dynamic stress tests of HF portfolios, that evaluates the stress effects on the term structure of liquidation risk. We perform different stress analyses by means of either the systematic factor, its dynamic, or the magnitude of the contagion in the spirit of the new regulations for financial stability. Section 7 concludes. Technical proofs are gathered in appendices and supplementary materials.

2 Hedge funds data on liquidation

2.1 The database and data filtering

The Lipper TASS database [3] consists of monthly returns, Asset Under Management (AUM) and other HF characteristics for individual funds from February 1977 to June 2009. The relevant information for our study concerns the HF status. The database categorizes HF into ”Live” and ”Graveyard” funds. The ”Live” funds are presented as still active. There are several reasons [4] for a fund to be included in the Graveyard database. For instance, these funds *i)* no longer report their performance to TASS, *ii)* are liquidated, *iii)* are merged or restructured, *iv)* are closed to new investors. A HF can be listed in the Graveyard database only after being listed in the Live database. The TASS dataset includes 6097 funds in the ”Live” database and 6767 funds in ”Graveyard”. In our analysis, we consider only the HF, which are reported as ”Live”, or ”Liquidated” (status code 1) [5]. The latter are 2533 funds. Moreover, in order to account for the time needed to pass from ”Live” to ”Graveyard” in TASS, we have transferred to the ”Graveyard” database the 273 funds of the ”Live” database with missing data at least for April, May, June 2009. Among these funds, 23 are considered as liquidated according to this criterion.

We apply a series of filters to the data. First, we have selected only funds with Net Asset Value (NAV) written in USD. This currency filter avoids double counting, since the same fund can have shares written in USD and EUR for example. After applying the currency code filter, we have 3183 funds in the ”Live” base, and 1881 liquidated funds. Second, we have selected only funds with monthly reporting frequency. Nevertheless, we have also included the funds with quarterly reporting frequency,

[3] Tremont Advisory Shareholders Services. Further information about this database is provided on the website http://www.lipperweb.com/products/LipperTASS.aspx.

[4] Graveyard status code: 1=fund liquidated; 2=fund no longer reporting to TASS; 3=TASS unable to contact the manager for updated information; 4=fund closed to new investment; 5=fund has merged into another entity; 7=dormant fund; 9=unknown.

[5] Chan, Getmansky, Haas, Lo (2007) have regarded as liquidated all Graveyard funds in status code 1, 2 or 3.

when the intermediate monthly estimated returns were available. Third, to keep the interpretation in terms of individual funds, we eliminate the funds of funds and, for funds with multiple share classes, we eliminate duplicate share classes from the sample. Finally, we select the nine management styles with a sufficiently large size. These are Long/Short Equity (LSE), Event Driven (ED), Managed Futures (MF), Equity Market Neutral (EMN), Fixed Income Arbitrage (FI), Global Macro (GM), Emerging Markets (EM), Multi Strategy (MS), and Convertible Arbitrage (CONV). This allows us to use the Poisson approximation for the analysis of the liquidation counts per management style (see Section 3). After applying all these filters, we get 2279 funds in the ”Live” database and 1520 liquidated funds. The distribution by style of alive and liquidated funds in the database is reported in Table 1.

[Insert Table 1: The database]

The largest management style in the database of alive and liquidated funds is Long/Short Equity Hedge (about 40%), followed by Managed Futures, Multi-Strategy and Event Driven (each about 10%).

The age of an individual HF is measured since the inception date reported in TASS. Thus, this age is the official age and does not take into account the incubation period.

2.2 Summary statistics

Figures 1 and 2 display the subpopulation sizes and the liquidation rates over time for different management styles, without distinguishing the age of the HF.

[Insert Figure 1: Subpopulation sizes of HF]

[Insert Figure 2: Liquidation rates of HF]

We observe in Figure 1 the HF market growth between 2000 and 2007, and the sharp decrease due to the 2008 financial crisis. However, the effect of the crisis is less pronounced for HF following a Global Macro strategy. Figure 2 shows liquidation clustering both with respect to time and among categories.

One liquidation clustering due to the Long Term Capital Management (LTCM) debacle is observed in Summer 1998 and is especially visible for the Emerging Markets and Global Macro categories. Another liquidation clustering is observed in the 2008 crisis, but did not include the Global Macro strategy. In fact, for several management styles, the increase of the liquidation rates started before the beginning of the crisis. This finding is likely due to a bubble phenomenon. Indeed, we observe in Figure 1 the increase in the number of funds and also the number of fund managers. It has likely be accompanied by a decrease of the average skill of the new entering fund managers, which has implied the larger observed liquidation rates.

Let us now focus on the age effect. We provide in Figure 3 the smoothed nonparametric estimates of the liquidation intensity by management style. The estimates are obtained from the Kaplan-Meier estimators of the survival functions.

[Insert Figure 3: Smoothed estimates of liquidation intensity]

These estimates feature similar patterns, with a maximum at age of about 4 years. Table 2 provides the estimated liquidation intensities at the maximum, and at ages 0 and 100 months. We observe that the intensity functions of different management styles are not proportional. This suggests that the proportional hazard model should not be used for these data.

[Insert Table 2: Maximum and boundary values of the liquidation rates]

There can also exist cross-effects of time and age in liquidation intensity, which are difficult to observe when the fund lifetimes are separately analysed w.r.t. either time (see Figure 2), or age (see Figure 3). These cross-effects can be detected by means of the Lexis diagrams. Each liquidated fund is reported on the diagram by a dot with the date of death on the x-axis and its age at death on the y-axis. All the funds of the same cohort are represented by the 45^0 line passing through this dot. In particular, the intersection of this line with the x-axis provides the birth date of the funds in this cohort (see Figure 4).

[Insert Figure 4: Lexis diagram]

The Lexis diagrams for four management styles are provided in Figures 5-8. In these figures, each star represents a liquidation event in the time-age plan, and we look for concentration of stars in a band either parallel to the x-axis (age effect), or parallel to the y-axis (time effect), or parallel to the 45^0 line (cohort effect). For example, the Emerging Markets strategy represented in Figure 5 features a concentration of liquidation events around the age of 20 months, regularly spaced liquidation events for the cohort born in 1993, and another concentration during the crisis of 2008.

[Insert Figure 5: Lexis diagram for Emerging Markets]

The Lexis diagram for the Global Macro strategy (Figure 6) reveals two high time concentrations around 1998 (the LTCM crash) and 2008-2009 (the recent financial crisis), whereas the time concentrations are around January 2003 and the 2008 crisis for the Multi Strategy funds (Figure 7).

[Insert Figure 6: Lexis diagram for Global Macro]

[Insert Figure 7: Lexis diagram for Multi Strategy]

The strategy Managed Futures in Figure 8 shows essentially an age effect.

[Insert Figure 8: Lexis diagram for Managed Futures]

As illustrated above, the individual data on liquidation and their analysis are rather complex. We focus below on the dynamics of liquidation at the semi-aggregate level of the management style.

3 Contagion modeling

In this Section we introduce a dynamic model for the joint distribution of liquidation count histories of hedge funds of different management styles. This model accounts for the integer feature of the count dependent variables. It is also written to disentangle the initial exogenous common shocks (frailty) from the propagation of such shocks by the contagion phenomenons within and between management styles. Then, we compare our specification with the literature on financial contagion.

3.1 A multivariate dynamic Poisson model with frailty and contagion

For each month t and each management style $k = 1, ..., 9$, we observe the number $n_{k,t}$ of HF alive at the beginning of the month, and the liquidation count $Y_{k,t}$ during the month. The model specifies the joint conditional distribution of the vector of counts $Y_t = (Y_{1,t}, ..., Y_{9,t})'$ at the beginning of the period. The conditioning set includes the lagged counts and some unobserved common factor F_t. The counts are assumed conditionally independent across management styles, with Poisson distribution:

$$Y_{k,t} \sim \mathcal{P}\left[(n_{k,t}/n_{k,t_0})(a_k + b_k F_t + c_k' Y_{t-1}^*)\right], \quad k = 1, ..., 9, \tag{3.1}$$

where $Y_t^* = (Y_{1,t}^*, ..., Y_{t,9}^*)'$, with $Y_{k,t}^* = Y_{k,t}/n_{k,t}$ the liquidations frequency in style k, a_k and b_k are scalar coefficients and c_k is a vector of coefficients of dimension 9. This specification is inspired by the literature in epidemiology on contagion [see e.g. Anderson, Britton (2000)]. This literature goes back to the work of Sir Ronald Ross [Ross (1911)], who was awarded the Nobel Prize in Medicine in 1902, and of his students Kermack and McKendrick [Kermack and McKendrick (1927, 1932, 1933)]. The baseline intensity includes two components. The first one, $a_k + b_k F_t$, is the intensity of getting the disease via the exogenous factor represented by F_t and the vector $b = (b_1, ..., b_9)'$ of exposures to the factor. For instance, in the case of the Asian flu, the common factor is the contact with birds. This factor is exogenous, since there is no contagion from humans to birds. In analogy with Duffie et al.

(2009) in an application to credit risk, we refer to the unobservable common factor F_t as the "dynamic frailty". [6] The second component, $c_k' Y_{t-1}^* = \sum_{l=1}^{9} c_{k,l} Y_{l,t-1}^*$, is the intensity to get the disease via the contact with a sick human, in the same or in different subpopulations. The contagion is introduced with a lag effect to capture the propagation phenomenon, which is inherently a dynamic phenomenon. It is also required to adjust the model *i)* for the time-varying sizes of the subpopulations, via the adjustment term $n_{k,t}/n_{k,t_0}$, and *ii)* for the density of sick people in the subpopulations, via the use of Y_{t-1}^* instead of Y_{t-1} as explanatory variable.

The contagion effect is measured by means of the 9×9 contagion matrix C with rows c_k', $k = 1, ..., 9$. This modeling enables to account for contagion between as well as within management styles, since both nondiagonal and diagonal elements of the contagion matrix C can be nonzero.

As discussed in the Introduction, for HF liquidation the intensity specification (3.1) admits an economic interpretation that differs from the one of epidemiological models. The exogenous shocks are mainly shocks on liability due to large cash withdrawals of investors. This is the so-called redemption risk discussed in Diamond, Dybvig (1983), and Shleifer, Vishny (1997). The contagion goes through the asset component of the balance sheet. "Market liquidity and funding liquidity are mutually reinforcing leading to a liquidity spiral" [Brunnermeier, Pedersen (2009)].

The assumption of a conditional Poisson distribution for the liquidation counts in (3.1) can be motivated as a result of the aggregation of a microscopic contagion model specified at the level of the individual hedge funds. In such a microscopic model, the liquidation intensity of a fund in management style k at month t is proportional to $a_k + b_k F_t + c_k' Y_{t-1}^*$. The Poisson model for the counts is

[6] The notion of frailty has been initially introduced in duration models in Vaupel, Manton, Stallard (1979) and later used to define the Archimedean copulas [Oakes (1989)]. In this meaning, the frailty is an unobservable individual variable introduced to account for omitted individual characteristics and correct for the so-called mover-stayer phenomenon [see e.g. Baba, Goko (2006) for the introduction of an individual static frailty in the HF literature]. In our framework, F_t is indexed by time and common to all HF, which justifies the terminology "dynamic frailty".

obtained as the limit when the sizes of the management styles tend to infinity and the proportionality constants in the liquidation intensities tend to zero. Thus, the Poisson model can be seen as a macroscopic approximation for large classes of management styles with rather small monthly liquidation intensities [see e.g. Czado, Delwarde, Denuit (2005) and Gagliardini, Gourieroux (2013) for Poisson approximations in life insurance, and credit risk, applications, respectively]. In the microscopic model the proportionality constants can depend on the fund and capture the fund specific (idiosyncratic) components of the liquidation risk. These idiosyncratic effects are assumed eliminated by aggregation. The aggregation procedure has the advantage to focus on the common factor of interest (frailty). This explains why other data available in the TASS database, such as the HF return, are not introduced in the model [7].

3.2 The literature on financial contagion

It is often considered that there is “no consensus on what constitutes contagion and how it should be defined” [Forbes, Rigobon (2002); see also Bekaert, Harvey, Ng (2005)]. This lack of accordance on what contagion means is likely due to the fact that the financial literature has mostly focused on “contagion” between asset returns. Indeed, when the analysis is performed on asset returns, it is difficult to rely on the interpretation suggested by an epidemiological model. Following Eichengreen, Rose, Wyplosz (1996), Bae, Karolyi, Stulz (2003) and Boyson, Stahel and Stulz (2010) circumvent this difficulty by replacing the analysis of asset returns by the analysis of “risk infected” assets. An asset (hedge fund, or stock) is “risk infected”, i.e. “sick”, if its return is below some cutoff, such as the 10% quantile of the overall returns distribution.

The contagion models for asset returns considered in the literature differ in other respects, concern-

[7] By performing the analysis at the semi-aggregate level of the management style, we also avoid error-in-variable biases. Indeed, while the data on HF lifetimes are rather reliable up to the filtering described in Section 2.1, this is not the case for other individual HF data reported by the fund managers on a voluntary basis.

ing the explanatory variables introduced in the return equations. The differences are: *i)* the presence or not of common exogenous factors capturing part of the dependence across assets, also called fundamental dependence, or interdependence; *ii)* the observability of these common factors; *iii)* the presence or not of asset returns among the explanatory variables; *iv)* the fact that these returns are contemporaneous or lagged. For instance, Forbes, Rigobon (2002) and Dungey, Fry, Gonzalez-Hermosillo, Martin (2005) consider a contagion model with unobservable factor and contemporaneous returns among the explanatory variables. Boyson, Stahel, Stulz (2010), Section II.C, consider a model with observable factors, lagged "worst" return for the within-class contagion effects and contemporaneous "worst" return for the between-classes contagion effect. Ait-Sahalia, Cacho-Diaz, Laeven (2010) and Billio et al (2012) consider contagion models without common exogenous factor, whereas Boyson, Stahel, Stulz (2010), Section II.B, use a model with observable factors only.

As seen in the standard epidemiological model, it is important to introduce lagged dependent variables and not contemporaneous dependent variables among the regressors. The main reason is that contagion is a propagation effect. The dynamic modeling is required for prediction of future risk, for computation of reserves, as well as for stress testing (see Section 5). It is also important to introduce unobservable factors and not observable ones. First, in a model with observable factors it is not possible to predict the future risk, to compute the reserves, or to perform stress tests without completing the risk model by a model describing the future evolution of these factors, which means that they will also be considered as random. Second, the structural interpretation of contagion requires exogenous factors. Therefore, if observable factors with financial or economic interpretations are introduced ex-ante, it is necessary to test that they are exogenous in order to avoid a misleading interpretation of the contagion matrix (see Appendix 1 for a more complete discussion). This test of factor exogeneity is usually omitted in the literature.

Frailty and contagion both create correlation between the lifetimes of individual HF. In the limiting case of a static model, that is, with a serially independent frailty process (F_t) and simultaneous effects of observed liquidation counts, the two phenomena cannot be identified. This is the reflection problem highlighted in Manski (1993). In our dynamic framework, frailty and contagion can be disentangled, since the frailty has a contemporaneous effect while contagion is produced only after a time lag [see Gagliardini and Gourieroux (2012) for a discussion of identification in models with latent dynamic factors].

The models with common factors only (resp. asset returns only) as explanatory variables are special cases of the models including both types of covariates. Thus, these constrained specifications can be easily tested. In practice (see Section 4) these constrained specifications appear to be misspecified, which may imply significant biases in contagion analysis. Typically the contagion phenomena are overestimated, when common factors are omitted. When other asset returns are omitted, the contagion effects are hidden in the dependence between the residuals [see e.g. Boyson, Stahel, Stulz (2010), Section II.B].

To summarize, our paper is one of the first attempts to consider both unobserved frailty and contagion in a dynamic framework and to develop the appropriate modeling and statistical inference (see Appendix 3 for estimation and the supplementary materials for testing procedures).

4 Empirical analysis

Let us now move to the estimation of the dynamic Poisson model for contagion. We consider first a model with pure contagion. Then we also introduce an unobserved frailty with autoregressive gamma dynamics.

4.1 Model with contagion only

Let us first focus on a model with pure contagion:

$$Y_{k,t} \sim \mathcal{P}\left[n_{k,t}/n_{k,t_0}\left(a_k + c_k' Y_{t-1}^*\right)\right], \quad k = 1, ..., 9. \tag{4.1}$$

This specification corresponds to model (3.1) with $b_k = 0$ for any management style k. It is a multivariate Poisson regression model, with observed lagged adjusted counts as explanatory variables. The lagged counts capture the liquidation clustering effects and their diffusion between and within management styles. The model involves 9 intercept parameters a_k, $k = 1, ..., 9$, and a matrix of 81 contagion parameters $c_{k,k'}$, with $k, k' = 1, ..., 9$. As usual the parameters of the Poisson regression model are estimated by the Maximum Likelihood (ML) [Cameron, Trivedi (1998)]. As this model is a special case of Generalized Linear Model (GLM), the likelihood equations are easily solved by applying iteratively the weighted least squares in a Seemingly Unrelated Regression (SUR) model [McCullagh, Nelder (1989)]. The estimated values of the intercepts are given in Table 3 with standard errors in parentheses. The estimated contagion matrix is provided in Table 4, where we display only the statistically significant contagion coefficients at the 5% level.

[Insert Table 3: Estimated intercepts a_k in the pure contagion model]

[Insert Table 4: Estimated contagion parameters $c_{k,k'}$ in the pure contagion model]

In Tables 3 and 4, the estimates of the intercepts a_k and the rows c_k' of the contagion matrix differ significantly across management styles k. When another model with fund age and management style as the sole explanatory variables is fitted to the data, as in Figure 3 and Table 2, the variable age partly captures the effect of the time-varying lagged liquidation counts, that are the explanatory variables in model (4.1), with different impacts across the management styles. Thus, the results in Tables 3 and 4 are compatible with the findings in Figure 3 and Table 2, and support the evidence that a proportional hazard specification without time-varying explanatory variables is not appropriate for our dataset.

The contagion matrix is represented as a network in Figure 9, where an arrow from style k' to style k corresponds to a statistically significant estimate of parameter $c_{k,k'}$.

[Insert Figure 9: The contagion scheme for the pure contagion model]

All strategies are interconnected either directly, or indirectly through multistep contagion channels. Such a contagion scheme corresponds to a complete structure in Allen, Gale (2000) terminology. The structure of the contagion matrix provides interesting information on the contagion interactions and the possible model misspecification. We observe the special roles of the Fixed Income Arbitrage and Long/Short Equity Hedge strategies, which both influence directly most of the other strategies. However, some estimated contagion parameters likely indicate a misspecification of the model without frailty and lead possibly to misleading interpretations. For instance, we get a large value 0.67 of the contagion parameter from Fixed Income Arbitrage to Long/Short Equity Hedge. Such a causal effect is unlikely since the Fixed Income Arbitrage strategies are investing in bonds and, when the associated managers deleverage their portfolios, the impact on Long/Short Equity strategies invested in stocks is expected to be small.

In the model with pure contagion, the funding and market liquidity risks cannot be separated and are both captured by the lagged liquidation counts. To disentangle the effects of the two types of liquidity risk, in the next subsection we extend the model to include an exogenous shared dynamic frailty, that is expected to represent economy wide funding liquidity risks. This interpretation will be confirmed in Section 5.

4.2 Model with frailty and contagion

We assume that the frailty follows an Autoregressive Gamma (ARG) process. The ARG process is the time discretized Cox, Ingersoll, Ross process [Cox, Ingersoll, Ross (1985)]. The transition of this

Markov process corresponds to a noncentral gamma distribution $\gamma(\delta, \eta F_{t-1}, \nu)$, where $\nu > 0$ is a scale parameter, $\delta > 0$ is the degree of freedom of the Gamma transition distribution and parameter $\eta \geq 0$ is such that $\rho = \eta\nu$ is the first-order autocorrelation (see Appendix 2.1 for basic results on the ARG process). Since the factor is unobservable, it is always possible to assume $E(F_t) = 1$ for identification purpose. Then, the frailty dynamics can be conveniently parameterized by parameters δ and ρ. We get:

$$\begin{aligned} Y_{k,t} &\sim \mathcal{P}\left[(n_{k,t}/n_{k,t_0})\left(a_k + b_k F_t + c_k' Y_{t-1}^*\right)\right], \quad k = 1, ..., 9, \\ F_t &\sim \gamma\left(\delta, \frac{\rho\delta}{1-\rho} F_{t-1}, \frac{1-\rho}{\delta}\right). \end{aligned} \tag{4.2}$$

The ARG specification has several advantages. First, it ensures positive values for the factor, and thus a positive intensity if $b_k \geq 0$ and $c_{k,l} \geq 0$ for all k, l. Second, the joint model (4.2) for the liquidation counts and the factor is an affine model [see e.g. Duffie, Filipovic, Schachermayer (2003) for continuous time affine processes, and Darolles, Gourieroux, Jasiak (2006) for discrete time]. This allows for explicit expressions of nonlinear predictions at any horizon (see Appendix 2.2). The single factor assumption is convenient for tractability. It is also in line with empirical findings in the literature. For instance, Carlson, Steinman (2008) regress the aggregate liquidation count for the entire HF market on several time-dependent observable variables related to market conditions and on the lagged aggregate liquidation count, and find only one statistically significant variable. [8] The model in equation (4.2) involves 99 parameters a_k, b_k and $c_{k,k'}$ in the liquidation intensities, plus 2 parameters for the frailty dynamics, namely the degrees of freedom δ and the autocorrelation ρ. The large number of parameters is due to the introducing cross-effects between management style and frailty, and between management style and lagged liquidation counts.

The likelihood function of this multivariate autoregressive Poisson regression model with shared

[8]These market conditions are essentially the market index return and the market volatility, the significant observed variable being the S&P 500 return. However, the model in Carlson, Steinman (2008) includes no variable measuring liquidity features, such as measures of counterparty risk.

dynamic frailty involves a multi-dimensional integral and a large number of parameters. This makes the numerical implementation of the likelihood maximization via simulation based methods cumbersome [see e.g. Cappé, Moulines, Rydén (2005) for estimation methods based on Gibbs sampling in nonlinear state space models, and Duffie et al. (2009) for an application in credit risk]. We propose in Appendix 3 a new informative Generalized Method of Moments (GMM) approach for our estimation problem. The moment restrictions are based on the conditional Laplace transform and involve either the stationary distribution of the frailty (static moment restrictions), or the transition distribution of the frailty (dynamic moment restrictions).

In Table 5 we display the estimated intercept parameters a_k and factor sensitivities b_k along with their standard errors. The estimated contagion matrix is provided in Table 6, where we display only the statistically significant contagion coefficients at the 10% level. The estimated parameters of the frailty dynamics are $\hat{\delta} = 0.59$ (with standard error 0.34) and $\hat{\rho} = 0.74$ (with standard error 0.20). The standard errors of the GMM estimates are computed using the asymptotic distribution.

[Insert Table 5: Estimated intercepts a_k and factor sensitivities b_k in the model with contagion and frailty]

[Insert Table 6: Estimated contagion parameters $c_{k,k'}$ in the model with contagion and frailty]

The estimated factor sensitivities are all positive and statistically significant. As discussed in the Introduction, the unobservable frailty is likely a measure of economy wide funding liquidity risk. This interpretation is coming from a careful analysis of the estimated sensitivities across management styles. For a given management style, the liquidity features are twofold: the portfolio can be invested in more or less liquid assets, and the strategy can require a longer or shorter horizon to be applied. In this respect, Global Macro and Managed Futures portfolios are invested in liquid assets; they can offer to investors weekly, or even daily liquidity conditions, and have small sensitivity coefficients 0.33

and 0.63, respectively. At the opposite, the Event Driven strategies are essentially looking for positive outcomes in merger and acquisitions, which can only be expected in a medium horizon. They have less interesting, generally quarterly, liquidity conditions and the factor sensitivity coefficient 1.39 is among the highest ones. Similar remarks can be done for other management styles.

The discussion above is completed by comparing the factor sensitivities to the average redemption frequencies and leverage in each management style, which are displayed in Table 7.

[Insert Table 7: Redemption frequency, leverage and factor sensitivity]

Whereas the majority of funds across management styles allow for redemptions on a monthly basis or better, we observe some differences, especially for the Event Driven management style, with very often a redemption frequency close to 3 months. It has been observed that "hedge funds with favorable redemption terms differ significantly in terms of their appetites for liquidity risks" [see Teo (2011)]. We observe a significant negative link between the factor sensitivity and the proportion of hedge funds with a redemption frequency of one month or better (resp. of HF using leverage). This link is likely explained by the type of assets introduced in the HF portfolio. For instance, the funds in the management styles Managed Futures and Global Macro are invested in very liquid assets. Thus, they can easily propose good redemption frequencies and use leverage without being too sensitive to the common factor. However, to attract investors, some managers in other strategies may propose "favorable" redemption conditions and simultaneously post high returns obtained by taking an excessive liquidity risk. This is likely the case for some HF in the Long Short Equity category, where the favorable announced redemption and the high leverage are not in line with the very high exposure to the risk factor. The link between the frailty sensitivities and the redemption frequencies is confirmed by the correlation between the former and the proportion of favorable redemption conditions (less than one month) equal to -0.27, passing to -0.56 when the Long Short Equity category is not considered. To summarize, the sensitivity coefficients measure the funding liquidity risk exposures of the different management styles, and these exposures are related to the management of gates and leverage. This interpretation will be discussed later on when filtering the factor.

The sums $a_k + b_k = a_k + b_k E[F_t]$ in Table 5 are much larger than the coefficients a_k estimated in the pure contagion model (see Table 3). Thus, a large fraction of the common liquidation features is captured by the introduction of the frailty. This effect is balanced by a diminution in the estimated contagion parameters. Only 11 (resp. 13) estimated contagion parameters in Table 6 are statistically significant at 5% (resp. 10%) level. These numbers have to be compared with the 27 statistically significant contagion parameters at 5% in Table 4 obtained when the frailty component is omitted. The contagion scheme for the model including both contagion and frailty is displayed in Figure 10, where any estimated contagion coefficient in Table 6, that is statistically significant at 5% level, is represented by an arrow.

[Insert Figure 10: The contagion scheme for the model with contagion and frailty]

By comparing Figure 10 with Figure 9, as expected the introduction of the systematic risk factor F_t largely diminishes the perception of contagion phenomena. In particular, the key roles of Fixed Income Arbitrage and Long-Short Equity Hedge in Table 4 and Figure 9 were mostly due to the effect of the common factor, and the fact that these management strategies are rather sensitive to the factor, especially the second management style. Moreover, in Figure 10 we observe that contagion occurs along some specific directions, such as Multi Strategy $\rightarrow$ Equity Market Neutral $\rightarrow$ Event Driven $\rightarrow$ Fixed Income Arbitrage $\rightarrow$ Emerging Markets, without any evidence of contagion in the reverse direction.

Let us now discuss carefully the scheme in Figure 10. This estimated scheme shows a classification of management styles into four categories:

i) Funds mainly invested in fixed income products and using high leverage, that are Fixed Income Arbitrage, Managed Futures, Emerging Markets and Global Macro.

ii) Funds mainly invested in equities, such as Equity Market Neutral, Long/Short Equity Hedge and Event Driven.

iii) Funds in the Convertible Arbitrage management style, in which the convertible products have features of the corporate bonds and associated stocks.

iv) Funds in the Multi-Strategy management style, with portfolios including subprimes and equities.

The classification above clearly differs from other classifications in the literature such as the one introduced by Morningstar, in which the funds are classified into Directional Traders, Relative Value, Security Selection and Multiprocess, respectively [see MSCI (2006) and Agarwal, Daniel, Naik (2009), Appendix B]. This latter classification is based on the type of strategy followed by the fund. For instance, Security Selection managers take long and short positions in undervalued and overvalued securities, while trying to reduce the systematic market risk. However, they can invest in more or less liquid assets, and introduce different levels of leverage. By comparison, our classification is clearly funding liquidity risk oriented and revealed by the HF liquidation data.

The causal scheme in Figure 10 is likely due to the subprime crisis and the associated lack of market liquidity on the different classes of assets. In 2007, there has been an increase of the expected default rates for mortgages, and then an increase of margin calls for credit derivatives. The Multi-Strategy funds, largely invested in subprimes but also on liquid market neutral strategies, had to get cash in order to satisfy these margin requirements. It was then natural for them to liquidate the most liquid part of their portfolios, i.e. the equity strategies. This massive deleveraging had a direct effect on stock prices. At the beginning this effect cannot be observed on the stock indices, but mainly on the relative performances of the individual stocks: the high-ranked stocks becoming low ranked and vice-versa, since the strategies followed by Equity Market Neutral and Long/Short Equity Hedge funds, for instance, are orthogonal to the market. This dislocation effect on stock prices has impacted all the equity strategies, including the Event Driven funds. The associated M&A strategies have transformed the short-term shocks into long-term shocks. This explains the key (systemic) role of the Event Driven management style, which creates the link between the shocks on stock markets and the shocks on fixed income markets.

4.3 The relative importance of frailty and contagion

The relative effect of contagion and frailty on liquidation risk can be measured by using the variance decomposition derived in Appendix 2.4. The variance-covariance matrix of the liquidation count vector Y_t can be decomposed as:

$$\begin{aligned} V(Y_t) &= \underbrace{diag[E(Y_t)]}_{\text{Poisson}} + \underbrace{CV(Y_t)C'}_{\text{pure contagion}} + \underbrace{(1/\delta)bb'}_{\text{direct frailty}} \\ &\quad + \underbrace{(1/\delta)\rho C(Id-\rho C)^{-1}bb' + (1/\delta)\rho bb'(Id-\rho C')^{-1}C'}_{\text{propagation of the frailty by contagion}}, \end{aligned} \tag{4.3}$$

where $diag[E(Y_t)]$ denotes the diagonal matrix with diagonal elements corresponding to the elements of the vector of expected liquidation counts $E[Y_t] = (Id - C)^{-1}(a + b)$, and a and b denote vectors with elements a_k and b_k, respectively. The decomposition formula above is valid under stationarity conditions concerning the contagion matrix C and the autocorrelation ρ of the frailty. These conditions are: the eigenvalues of the contagion matrix C are smaller than 1 in modulus, and the autocorrelation ρ of the frailty is smaller than 1. Equation (4.3) provides a decomposition of the historical variance-covariance matrix of the liquidation counts. The first term $diag[E(Y_t)]$ in the right hand side corresponds to the variance in a Poisson model with cross-sectional independence. The sum of the first and second terms provides the expression of the variance in a model including contagion, but without frailty. The third term $(1/\delta)bb'$ captures the direct effect of the exogeneous frailty. The remaining terms accommodate its indirect effects through contagion, namely, the amplification of the frailty effect due to the network. This variance decomposition is written in an implicit form since the system (4.3) has to be solved to get the expression of $V(Y_t)$ as a function of the model parameters.

Let us now assess the magnitude of the terms in the variance decomposition above by using our estimated model. Let us consider a portfolio of Liquidation Swaps (LS) written on the individual hedge funds, which is diversified with respect to the management styles. The liquidation swap for management style k pays 1 USD for each fund of style k that is liquidated in month t. The payoff of the LS portfolio at month t is $e'Y_t$, where $e = (1, 1, ..., 1)'$ is a $(K, 1)$ vector of ones. To ensure the time-invariant diversification, the portfolio of LS has to be appropriately rebalanced when a liquidation occurs. By using the above decomposition of variance, we can evaluate the percentage of portfolio variance $e'V(Y_t)e$, due to the underlying Poisson shocks, contagion and frailty, respectively. The decomposition is displayed in Table 8.

[Insert Table 8: Decomposition of variance]

The largest contribution to the portfolio variance comes from the frailty process, either through a direct effect (64.30%), or through an indirect effect via the contagion network (24.06%). The remaining part of portfolio variance is explained by the underlying Poisson shocks (6.54%) and the direct contagion effects (5.10%). Even though the direct effect of contagion is modest, the network plays an important role in amplifying the effect of the exogenous frailty.

5 The funding liquidity factor

Three types of approaches have been followed in the literature to measure and analyze the funding liquidity risk and its evolution:

i) The first approach considers directly the refinancing costs. The Treasury-Eurodollar (TED) spread, equal to the difference between the 3-month Eurodollar LIBOR rate and the 3-month Treasury bill rate, is such a measure of refinancing cost frequently considered in the literature [see e.g. Gupta, Subrahmaniam (2000), Boyson, Stahel, Stulz (2010), Teo (2011)].

ii) Alternatively, we could consider a direct measure of market liquidity, such as a bid-ask spread, and use the link between funding and market liquidity emphasized in Brunnermeier, Pedersen (2009) [see e.g. Goyenko, Subrahmaniam, Ukhov (2011)].

iii) Finally, liquidity is analyzed by the way it affects asset prices. In this respect, Vayanos (2004) suggests to measure the liquidity premium between two assets of similar characteristics, but different liquidity. These assets can be for instance thirty-year Treasury bonds just issued (on-the-run), or issued three months ago. They have the same cash flows, but the on-the-run bonds are much more liquid [see e.g. Fontaine, Garcia (2012)]. Another example of such assets are the German bonds (i.e. the bunds) and those issued by the Kreditanstalt fur Wiederaufbau (KfW), a German agency whose bonds are explicitly guaranteed by the Federal Republic of Germany [see e.g. Gourieroux, Monfort, Pegoraro, Renne (2013)].

In our paper we follow another approach. Since the hedge funds are often invested in derivatives and use a high leverage, they can be very sensitive to the funding liquidity risks. Thus, we expect the common factor appearing in the analysis of hedge fund survival to be a proxy for funding liquidity risk. This section reinforces the funding risk interpretation of contagion developed in Subsection 4.2. We first filter the unobservable factor path. Then, we investigate how this factor is related with other funding liquidity proxies introduced in the literature.

5.1 Filtering of the factor

The Poisson model with both frailty and contagion is a nonlinear state space model, which requires appropriate methods to filter the unobservable factor (see Appendix 5 for a description of alternative filtering approaches). Since the joint process $(Y_t', F_t)'$ of observable and unobservable variables is affine (see Appendix 2), the Bates filter [Bates (2006)] could be used to update the conditional Laplace (or Fourier) transform of the filtering distribution. However, the implementation of the Bates filter requires, at each iteration, the evaluation of a numerical integral of dimension equal to the number of observable variables. In our model, the vector of observations (the liquidation counts) is nine-dimensional, which makes the implementation of the Bates filter numerically unfeasible. In Appendix 5 we build on Bates (2006) and propose a new filter for our model, which takes advantage of the affine property of the ARG frailty dynamics, but requires only a few one-dimensional numerical integrals to be computed at each iteration. The filter is based on the idea of approximating any conditional distribution of the frailty given the available information by means of a distribution in the gamma family.

The filtered path of the frailty is displayed in Figure 11.

[Insert Figure 11: The filtered path of the frailty]

The frailty features a rather stable path between 1996 and 2006, with spikes at the ends of years 2001, consequence of the 9/11 terrorist attack, 2002, the market confidence crisis due to the internet bubble, ... The frailty path features an upward trend over the years 2007 and 2008 (the recent financial crisis), and decreases rapidly afterwards.

5.2 The factor interpretation

The TED spread and the VIX are commonly used as measures of capital availability in the economy [see e.g. Goyenko (2012)]. As already mentioned, the TED spread is a measure of the standard refinancing cost on the clearing houses. It is introduced to capture a part of the so-called rollover funding liquidity risk. The VIX is a weighted average of the implied volatility in the S&P index options. This index measures the aggregate volatility of the stock market as well as the price of this volatility. In our framework, it is introduced to capture the magnitude and the cost of leverage. Currently the publicly available data on HF balance sheets are not sufficiently detailed, especially for very short term borrowing of repos, in order to know directly at any date the leverage used by each hedge fund. However, such data are available for other financial intermediaries and it has been noted that the (change in) leverage is highly related with the (innovation on the) VIX [see e.g. Adrian, Shin (2010)]. The VIX is also introduced to capture the cost of leverage, i.e., the margin funding liquidity risk. Indeed, for listed derivatives, but also for OTC derivatives, the margin calls depend on the volatility of the underlying index (and are independent of the riskiness of the fund). Thus, an increase of the volatility, proxied by the VIX, implies an increase of the margin calls and an increase of the cost of leverage. Moreover, in crisis periods HF managers turn to VIX futures for volatility hedging and their short positions on VIX futures were for instance so extreme on February 27, 2013, that the market was close to a significant squeeze. The margin calls on VIX futures are directly written on the VIX itself. The time series of the TED spread and the VIX are displayed in Figure 12. The data are obtained from the Federal Reserve Board's website for the TED spread, and from the Chicago Board Options Exchange (CBOE) website for the VIX.

[Insert Figure 12: Observable liquidity indicators]

We estimate the regression:

$$\begin{aligned} \hat{F}_t &= \beta_1 + \beta_2 TED_t + \beta_3 TEDL_t + \beta_4 VIX_t + \beta_5 VIXL_t + \beta_6 SPR_t \\ &\quad + I(VIX_t \geq c)\left(\gamma_1 + \gamma_2 TED_t + \gamma_3 TEDL_t + \gamma_4 VIX_t + \gamma_5 VIXL_t + \gamma_6 SPR_t\right) + e_t, \end{aligned} \tag{4.4}$$

where the explained variable $\hat{F}_t$ is the filtered value of the frailty. In addition to the current values of the TED spread and the VIX, the regression includes lagged observations via the average value of the TED spread in the previous quarter (TEDL) and the average value of the VIX in the previous 12 months (VIXL), to potentially capture the impacts of the associated innovations. The regression also includes the spread SPR between the BAA and AAA yields from the FRED database at the Federal Reserve Bank of St. Louis. It is often stated that part of this spread is unrelated to credit risk, but is due to the lower liquidity of the corporate bonds in the more risky rating class. Even if the empirical literature focuses mostly on linear relationships between the unobservable factor and the observed explanatory variables, we expect nonlinear effects. Indeed, it is widely believed that hedge funds provide liquidity to markets, especially for assets with high degree of information asymmetry [see e.g. Agarwal, Fung, Loon, Naik (2007), Brophy, Ouimet, Sialm (2009)]. However, this occurs in the standard situation of reasonable funding liquidity costs. This is the "good equilibrium", in which hedge funds provide liquidity and are invested in rather illiquid assets with high leverage. But, as noted in Ben-David, Franzoni, Moussawi (2010), when the refinancing costs increase, hedge funds reallocate their portfolios, reduce their equity holdings and try to diminish their leverage in order to anticipate the consequences of possible outflows. In this "bad equilibrium", they are liquidity demanders. This double equilibrium is captured by the threshold on the VIX, with the estimated value $c = 25$. We get a switching regression allowing for different coefficients in the low and high volatility regimes. The estimates of the coefficients in regression (4.4), as well as in some restricted specifications, are displayed in Table 9.

[Insert Table 9: Regression of the frailty on observable variables]

The estimated regressions show that a large fraction of the common factor, about 65%, is explained by the proxies for funding liquidity risk introduced in the specifications. In the "good equilibrium" both the TED and the unexpected innovation on the TED, namely TED-TEDL, matter, whereas only the expected VIX is significant in the "bad equilibrium". TED and expected TED are no longer significant. We also observe a change of sign of the effect of the spread SPR between the two regimes.

6 Stress-tests

The estimated model with dynamic frailty and contagion can be used for portfolio management of a fund of funds, for computation of reserves, etc. In this section we illustrate how to implement stress-tests for liquidation risk. We consider a portfolio of HF with fixed category sizes and compare the distribution of the future liquidation counts in the unstressed and stressed situations. The future counts are subject to a double uncertainty, that is the drawing in the Poisson distribution, but also the stochastic evolution of the exogenous dynamic frailty. The stress can be designed as follows:

i) We can stress the current factor value by setting $F_t = q_\alpha$ in the conditioning set, where q_α is the quantile of the estimated stationary distribution of the frailty F_t at level α. By choosing $\alpha = 95\%$, or 99%, we consider an extreme scenario with a large transitory shock on the underlying funding liquidity risk factor at month t.

ii) We can change some parameters values, by either "increasing" the matrix of contagion C, or by increasing the value of the frailty persistence parameter ρ. This stress scenario will increase the liquidation risks by amplifying the impact of the exogenous shock by contagion and by introducing some exogenous shocks clustering, respectively.

At month t the stress scenario is characterized by the observed lagged liquidation counts, and the possibly stressed factor value for month t and parameter values. Then, the future paths of both the factor and the liquidation counts could be simulated and used to compute the term structures of expected liquidation counts, and of the volatility and overdispersion of these counts. These term structures can be derived in closed form by using the exponential affine property of the joint process of frailty and liquidation counts (see the supplementary materials). Our stress test analysis is dynamic, as it fully accounts for both liquidation counts and exogenous frailty dynamics. Therefore, it sharply differs from the stress test analysis in models with time-varying observable variables, in which a crystallized scenario for the future factor path is assumed. Such a stress test would neglect the liquidation risk dependence induced by the exogenous factors.

We consider three sets of stress scenarios:

S.1: The current factor value F_t is increased from the median, i.e. $\alpha^s = 0.50$, to the 95% quantile, i.e. $\alpha^s = 0.95$, of the historical distribution. The parameter values correspond to the estimates of Section 4.2. By conditioning on an extreme event on the systematic risk factor, the expected liquidation counts are in line with the measures for systemic risk as the CoVaR [Adrian, Brunnermeier (2011)], or the marginal expected shortfall [Acharya et al. (2010)]. The main difference is in the definition of the conditioning set including the unobservable factor and the observable liquidation counts, instead of including the return of a market portfolio.

S.2: The contagion matrix is changed from $\hat{C}$ to $2\hat{C}$, where $\hat{C}$ is the estimate of Section 4.2. The other parameter values are kept constant, equal to the estimates of Section 4.2. The current factor value F_t is the median of the historical distribution.

S.3: The frailty autocorrelation is increased from $\rho = 0.74$ (corresponding to the estimate in Section 4.2) to $\rho = 0.90$. The other parameter values are kept constant, equal to the estimates of Section 4.2. The current factor value F_t is the median of the historical distribution.

For all stress scenarios, the vector of liquidation counts Y_t in the conditioning set is equal to the observations on the liquidation counts in the last month of the sample, i.e. June 2009. In Figures 13, 14 and 15 we display for the nine management styles the impact of stress scenarios S.1, S.2 and S.3, respectively, on the term structures of the conditional expectations of liquidation counts. [9]

[Insert Figure 13: Term structure of expected liquidation counts when stressing the current factor value]

[Insert Figure 14: Term structure of expected liquidation counts when stressing the contagion matrix]

[Insert Figure 15: Term structure of expected liquidation counts when stressing the frailty persistence]

In each figure, the squares represent the term structures of expected liquidation counts before stress,

[9] Figures with the term structures of conditional volatility and overdispersion of liquidation counts are provided in the supplementary material.

that are the same in each scenario. As the horizon increases, the term structure converges to the unconditional expectation of the liquidation count, for each management style. The term structures are upward sloping since the current month, i.e. June 2009, corresponds to a period with few liquidation events compared to the historical average in any style. The circles represent the term structures of expected liquidation counts after the shock. We observe that the three types of shocks have very different effects on the term structures. The shock to the factor value in stress scenario S.1 is a transitory funding liquidity shock, with different impacts in the short run with respect to the management style (see Figure 13). Its effect decays rather quickly and disappears after about 12 months. The results of these stress tests are compatible with the liquidity interpretation of the unobservable factor. We observe an immediate effect of the shock on the highly exposed strategies of the Long/Short Equity Hedge management style, whereas the effect is clearly lagged, and indirectly due to contagion, for the Fixed Income Arbitrage strategies. The magnitude of the impact on the term structure depends on the conditioning information. There can be months when the sensitivity to shocks are larger, depending on the lagged liquidation counts at the month of the stress. In stress scenario S.2, the change in the contagion matrix is a permanent shock. In Figure 14, there is no important effect in the short run, but the long run behaviors of the models with and without the shock in the contagion matrix significantly differ for all styles, except for Global Macro. Indeed, the elements in the row of the estimated contagion matrix (Table 6) for that management style are zero. We conclude that there is no contagion effect impacting the Global Macro style. Therefore, the stress in scenario S.2 is irrelevant for the distribution of liquidation counts in that management style. Finally, when the frailty persistence parameter is shocked in stress scenario S.3, we observe an increase in the time at which the long run expected values of the liquidation counts are attained (Figure 15).

A large part of the literature on financial contagion applied to asset returns defines contagion as revealed by an increased correlation of asset returns during crisis periods [see e.g. King, Wadhwani

(1990)]. Then, the idea is to test for significant changes in some parameters between the crisis and non-crisis periods [see e.g. Forbes, Rigobon (2002)]. The above stress-tests analysis shows that there exist different ways to obtain extreme liquidation counts in some months. This can be due to an extreme exogenous shock possibly amplified by contagion. It can also be due to a standard exogenous shock and a significant change in the contagion matrix. These two situations are clearly different. In the first case, we get an "exogenous crisis" and this exogenous crisis can arise without modifying the matrix C, that is, the speed of propagation of the shocks. In the second case, there is a change in the structure and speed of contagion, which is more in line with the above mentioned financial literature. Such a change, which is currently not taken into account in the contagion model (4.2), can be due to either a change in the behaviour of the fund managers, or of the regulator for systemic risk. Let us consider for instance the behaviour of such a regulator. She will use a contagion model of type (4.2) in order to determine a reasonable level of contagion to avoid systemic risk and then introduce some regulation to be close to this level. To summarize, the model considered in this paper provides a framework to highlight that we may have to distinguish between exogenous crises, due to exogenous shocks, and endogenous crises, due to changes in the contagion matrix. To diminish the probability of an exogenous crisis, the regulator has to control the extreme exogenous risks, that is, the distribution of factor F_t. To come back to the analogy with the Asian flu, the authorities in charge of health supervision will demand to control the population of birds. To diminish the probability of endogenous crises, they will try to limit the contacts between humans.

7 Concluding remarks

In this paper we develop a new methodology to analyse the dynamics of liquidation risk dependence in the hedge fund industry. The autoregressive Poisson model with dynamic frailty is especially appealing, since it allows for distinguishing the effect of exogenous shocks, which affect directly the liability component of the balance sheet, and the endogenous contagion effects, which pass through the asset component. The common factor, the sensitivities to this factor, and the contagion scheme all get interpretations in terms of liquidity risks. The underlying factor is related in a nonlinear way to standard proxies of rollover and margin funding liquidity risks with two endogenous regimes (equilibria). In the first regime, when aggregate funding liquidity is not tight, the hedge funds are liquidity providers, while in the second regime, when aggregate funding liquidity is tight, hedge funds become liquity demanders. The sensitivities to the factor are providing the funding liquidity risk exposures of the different management styles and are linked with the redemption and gate management by the fund managers. Finally, the causal scheme captures a part of the spiral effect highlighted in Brunnermeier, Pedersen (2009), in which market liquidity and funding liquidity are mutually reinforcing, with a shock to one management style propagating into the others. Such a specification with both frailty and contagion is required to perform relevant stress tests. Indeed, the recent regulation has to define the reserve to cover extreme systematic risks. This requires to stress the systematic factor without stressing the hedge fund specific risk factors, but also to account in the appropriate way for contagion phenomena to derive the term structure of the reserves for different horizons and to detect the systematically important management styles. The frailty and contagion components could also be disentangled in a more general (parametric) dynamic framework. The model can include more than one factor, lagged effects of these factors on the liquidation intensity, and an autoregressive order larger than one. The relative magnitude of the frailty and contagion components will depend on the selected number of factors and lags.

It is known that systemic risk can be due to a significant shock on a common factor amplified by contagion. Models with both frailty and contagion are required to understand what is the main channel of systemic risk, namely the exogenous shock and/or contagion, and to develop accurate strategies to avoid systemic crises. Concerning hedge funds, such an analysis would have to be performed jointly for market risk, that is on returns, and liquidation risk, that is on lifetimes. In this respect, our work completes the systemic risk analysis based on returns developed in a series of recent papers including Sadka (2010), Boyson, Stahel, Stulz (2010), Akay, Seniuz, Yoldas (2011), Brown et al. (2011), Billio et al. (2012), Bali, Brown, Caglayan (2012), and the analysis of comovements between inflows and outflows [Sialm, Sun and Zheng (2012)], especially since the common factor for HF liquidation differs from the factors usually exhibited for HF returns.

The existence of common liquidation risk and contagion has also other consequences for the standard practices in hedge fund analysis. For instance, it is known that liquidation can bias the performance analysis of hedge funds, as it is often a consequence of unexpected bad results. There exist an extensive literature on correcting the survivorship bias in individual mutual fund or HF performance [see e.g. Brown, Goetzmann, Ibbotson, Ross (1992), Carpenter, Lynch (1999) for mutual funds, and Fung, Hsieh (1997), (2000), Ackermann, McEnally, Ravenscraft (1999), Liang (2000), Baquero, Horst, Verbeek (2005) for hedge funds]. Liquidation risk dependence implies that the correction for survivorship bias should be performed jointly for all funds, and not individually fund by fund. The specification of a joint model for hedge fund returns and endogenous liquidation is an interesting avenue for future research.

Finally, the existence of a rather significant systematic liquidation risk has important consequences for investors in hedge funds. The literature on mutual funds has introduced the so-called smart money hypothesis. If the investors can correctly predict the future returns on the funds, they will invest in funds that outperform and redeem from funds that underperform [see e.g. Gruber (1996), Zheng (1999)]. Such a strategy and such a theory are likely not relevant for hedge funds, since the liquidation risk is significant and has also to be taken into account. In fact, by investing in hedge funds that outperformed in the past, the investor may invest in a fund with a high liquity risk appetite and a high probability of liquidation (or a high probability to raise gates to avoid liquidation due to outflows).

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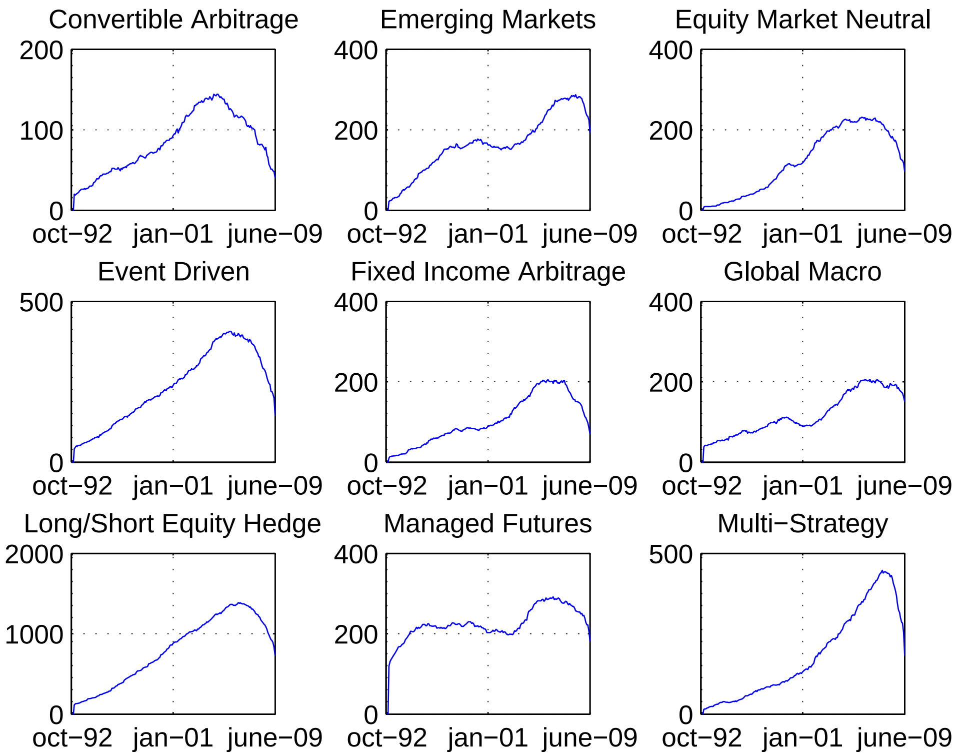
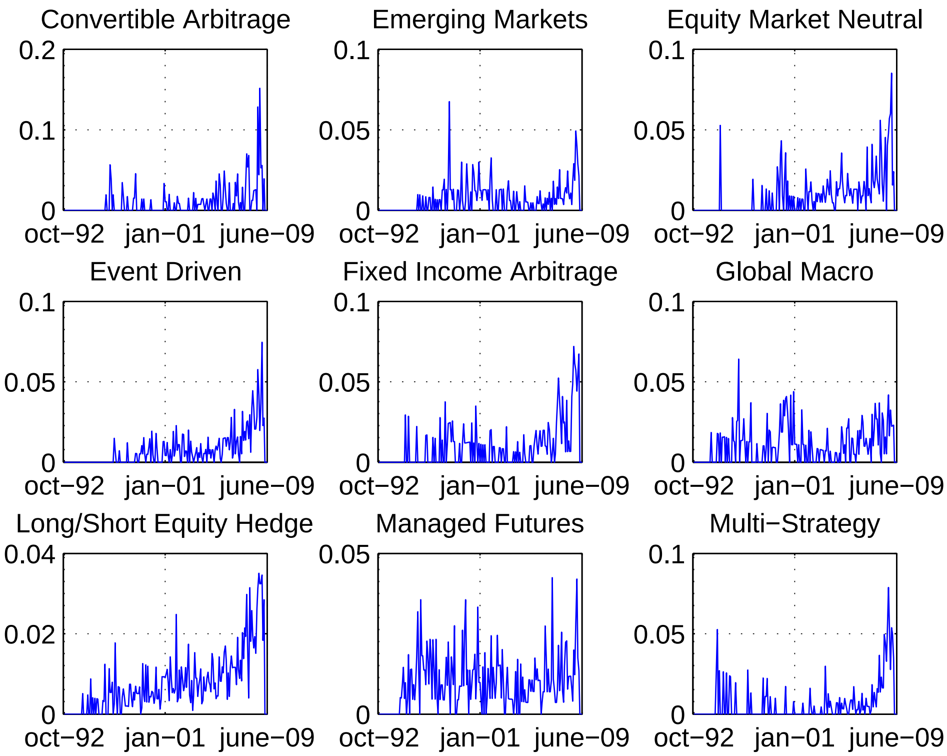
Figure 1: Subpopulation sizes of HF.



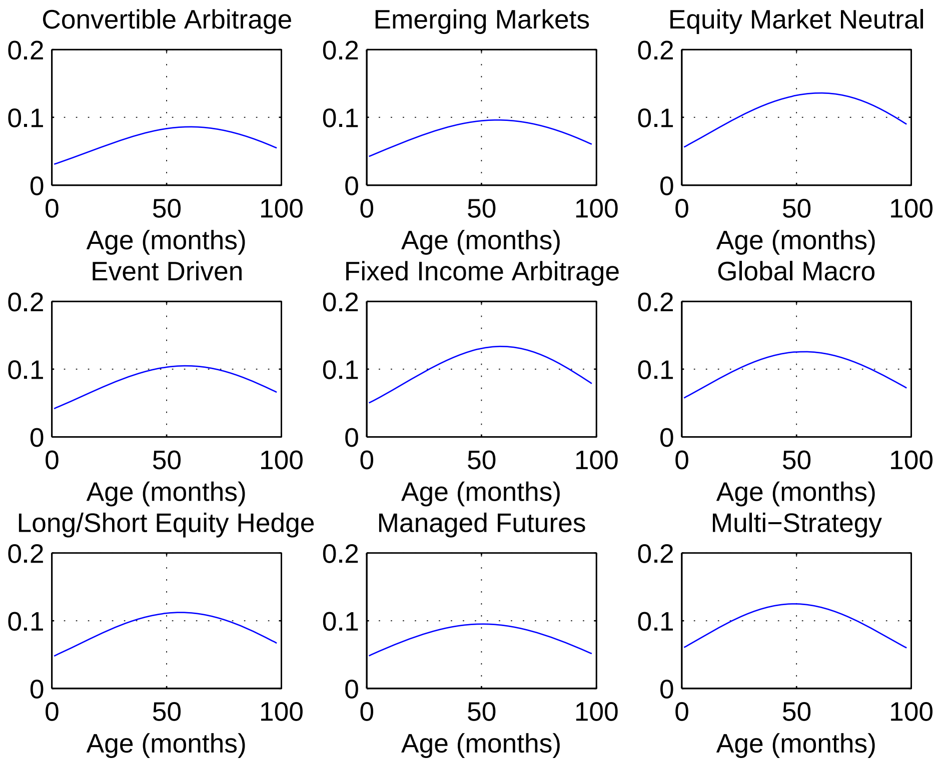
The figure displays the evolution of the population size $n_{k,t}$ between October 1992 and June 2009 for the nine management styles. Data are aggregated w.r.t. age.

Figure 2: Liquidation rates of HF.



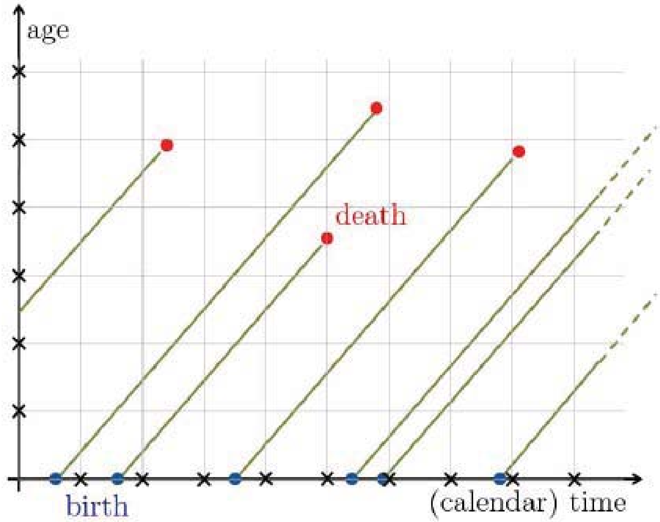
The figure displays the time series of liquidation rate $Y_{k,t}/n_{k,t}$ between October 1992 and June 2009 for the nine management styles. Data are aggregated w.r.t. age.

Figure 3: Smoothed estimates of liquidation intensity.



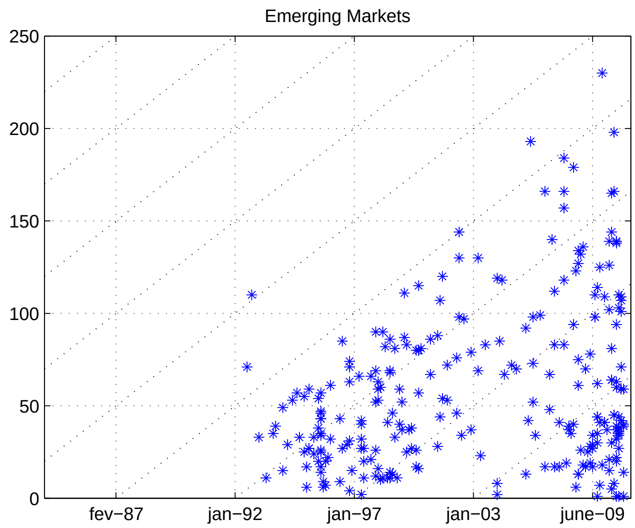
The figure displays the smoothed nonparametric estimates of the liquidation intensity as a function of age for the nine management styles. The estimates are based on the Kaplan-Meier estimators of the historical survival functions.

Figure 4: Lexis diagram.



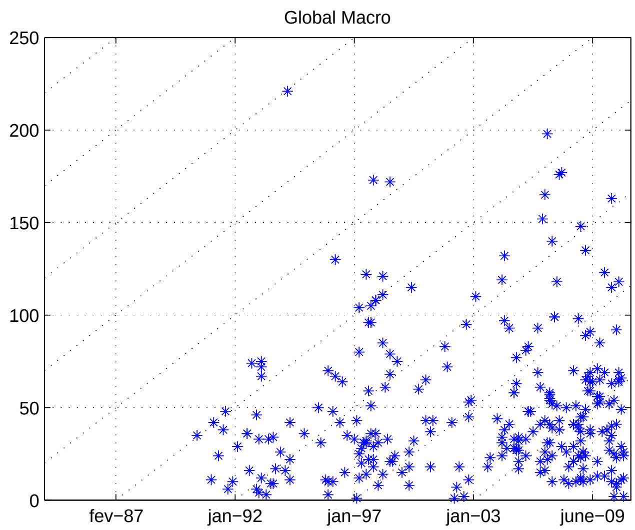
In the Lexis diagram, the liquidation of a HF is represented by a dot in the plane. The horizontal axis corresponds to the calendar time of the liquidation event, while the vertical axis displays the age of the fund at liquidation. The diagonal 45-degree lines correspond to funds in a same cohort.

Figure 5: Lexis diagram for Emerging Markets.



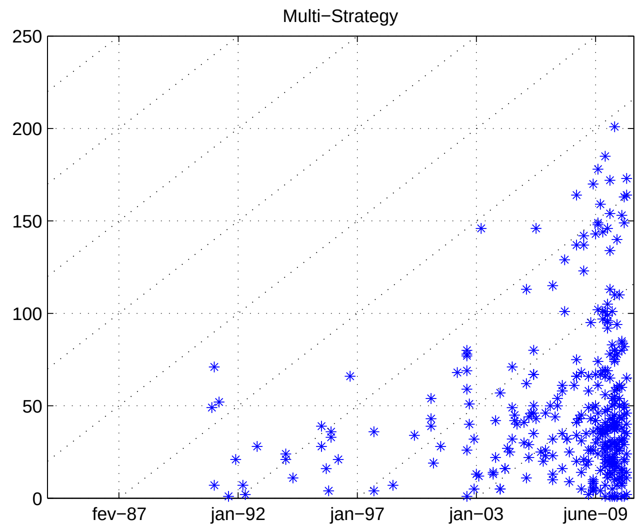
The figure displays the Lexis diagram for liquidation events of HF with management style Emerging Markets in the TASS database. The horizontal axis represents calendar time and the vertical axis represents age in months.

Figure 6: Lexis diagram for Global Macro.



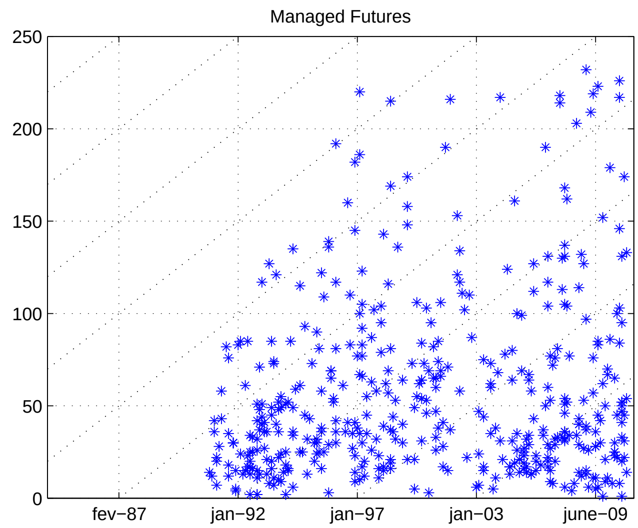
The figure displays the Lexis diagram for liquidation events of HF with management style Global Macro in the TASS database. The horizontal axis represents calendar time and the vertical axis represents age in months.

Figure 7: Lexis diagram for Multi Strategy.



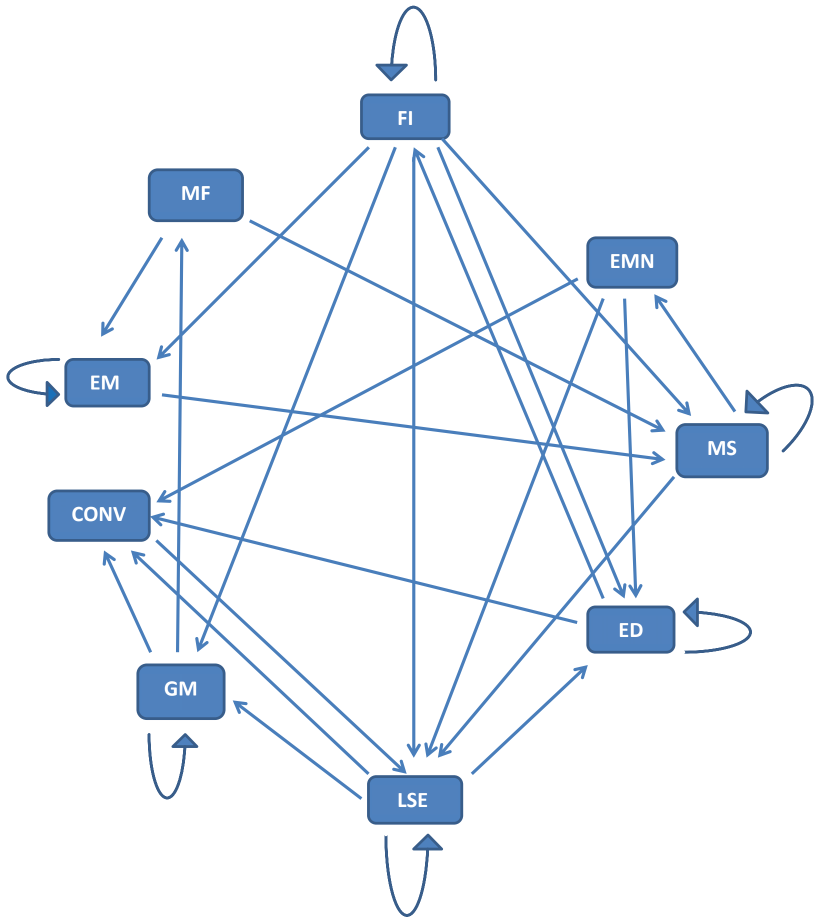
The figure displays the Lexis diagram for liquidation events of HF with management style Multi Strategy in the TASS database. The horizontal axis represents calendar time and the vertical axis represents age in months.

Figure 8: Lexis diagram for Managed Futures.



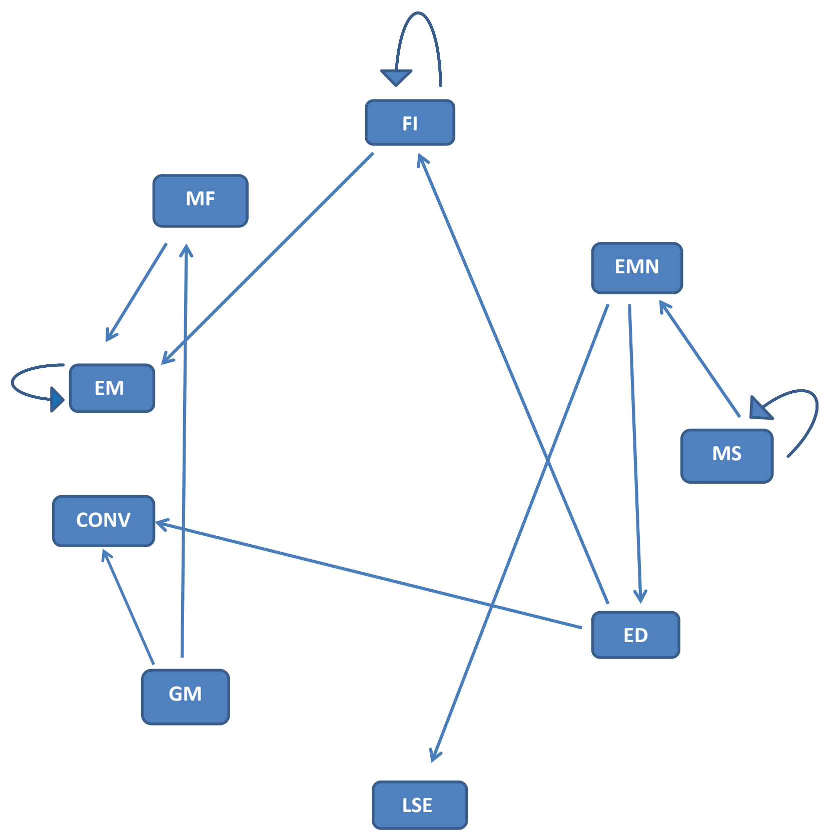
The figure displays the Lexis diagram for liquidation events of HF with management style Managed Futures in the TASS database. The horizontal axis represents calendar time and the vertical axis represents age in months.

Figure 9: The contagion scheme for the pure contagion model.



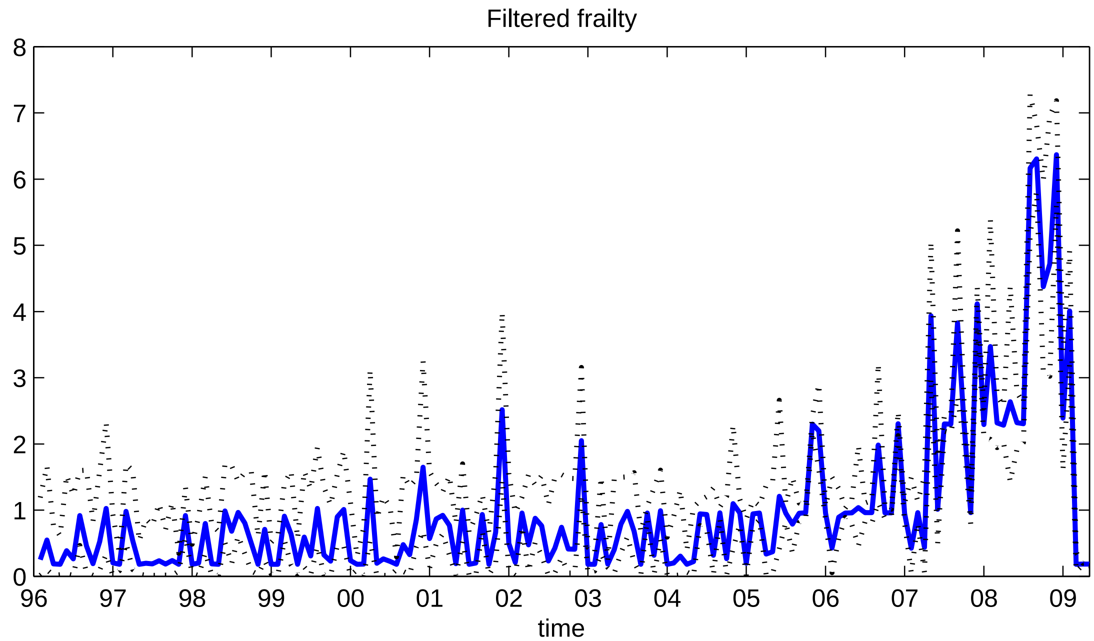
The figure provides the contagion scheme for the Poisson model with contagion only estimated on the TASS database. We display an arrow between two management styles, if the estimated contagion coefficient from the first style to the second style is statistically significant at 5% level. The estimated contagion matrix is provided in Table 4.

Figure 10: The contagion scheme for the model with contagion and frailty.



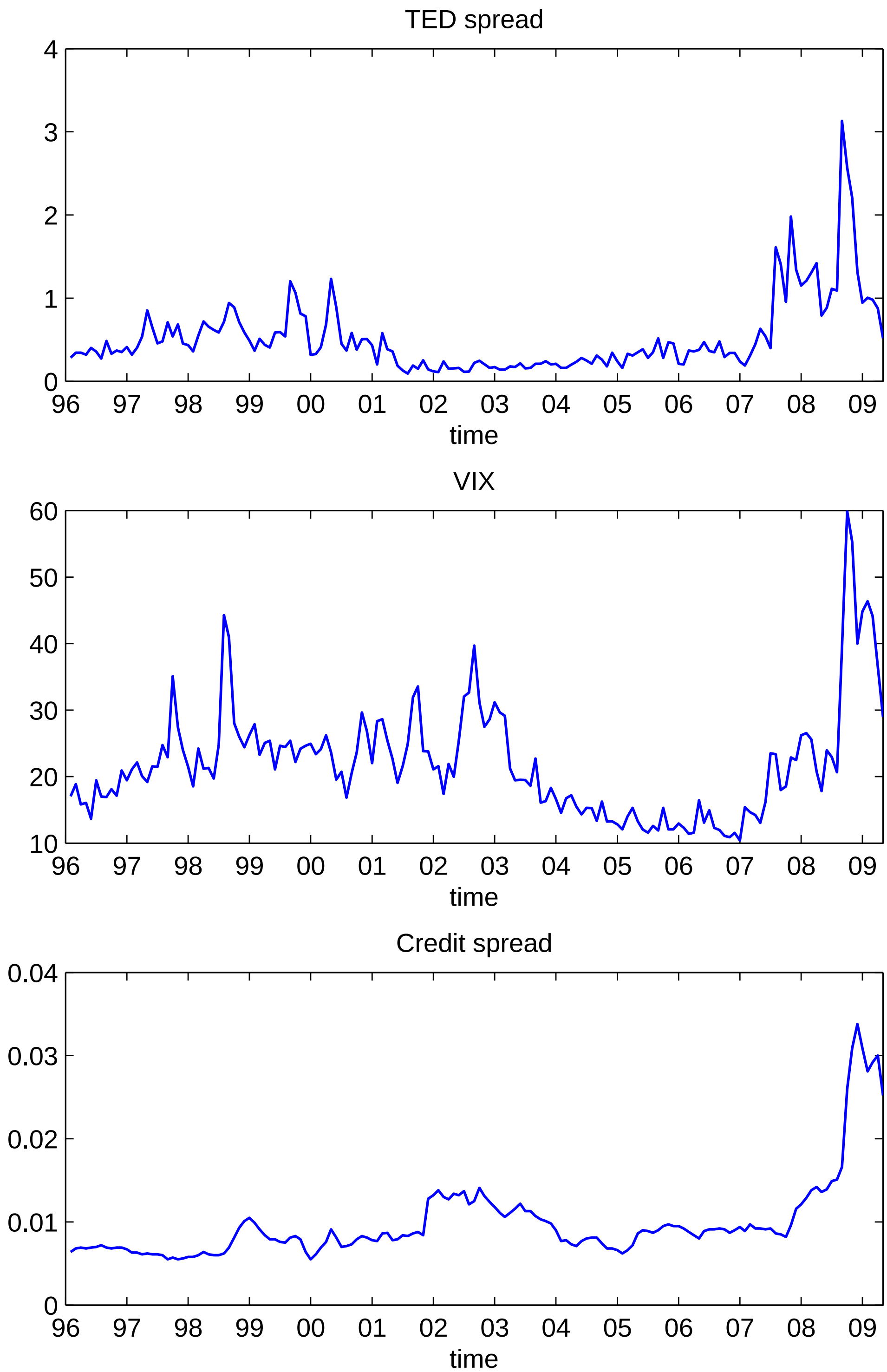
The figure provides the contagion scheme for the Poisson model with both frailty and contagion estimated on the TASS database. We display an arrow between two management styles, if the estimated contagion coefficient from the first style to the second style is statistically significant at 5% level. The estimated contagion matrix is provided in Table 6.

Figure 11: Filtered path of the frailty.



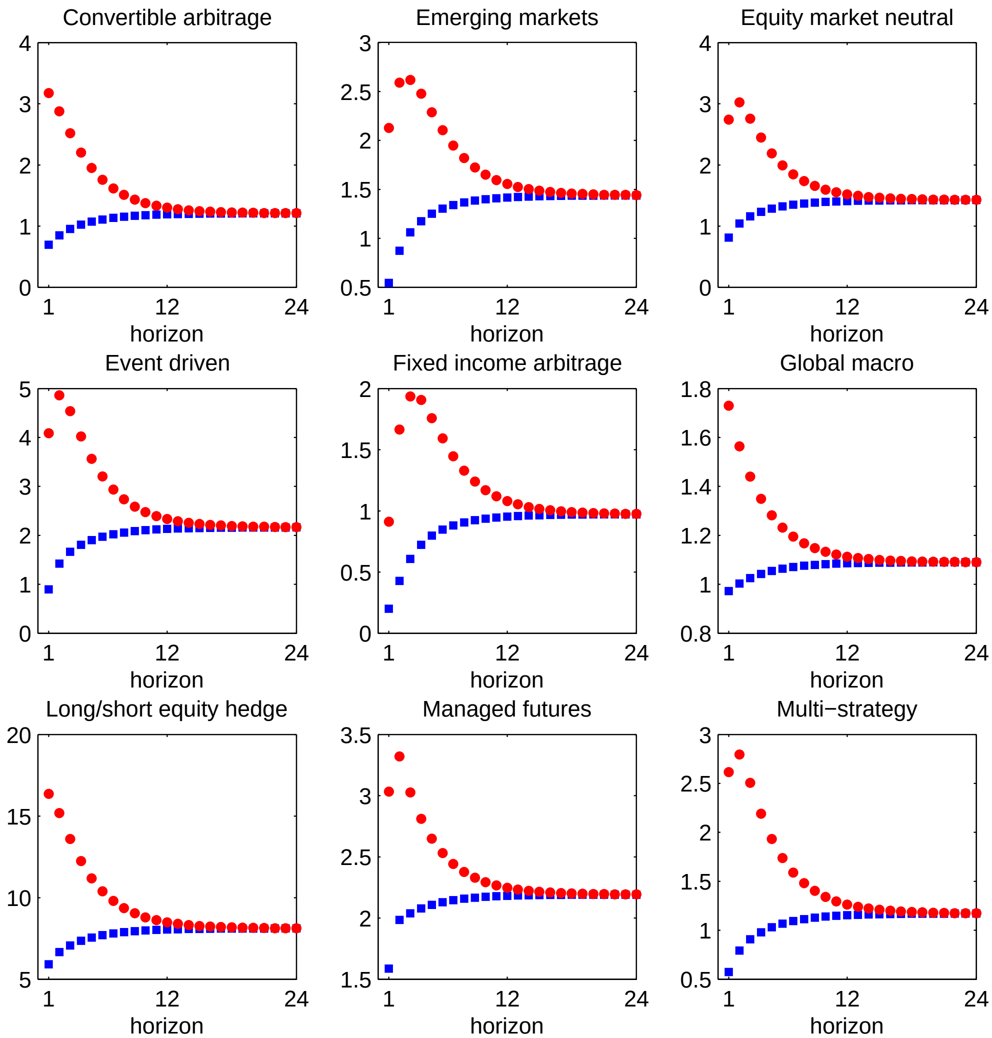
The figure displays the filtered path of the frailty (solid line) and the pointwise 95% confidence bands (dotted lines). The filtered value (resp. the lower and upper confidence bands) at a given month is the median (resp. the 2.5% and the 97.5% quantiles) of the filtering distribution of that month computed with the algorithm described in Appendix 5.

Figure 12: Observable liquidity indicators.



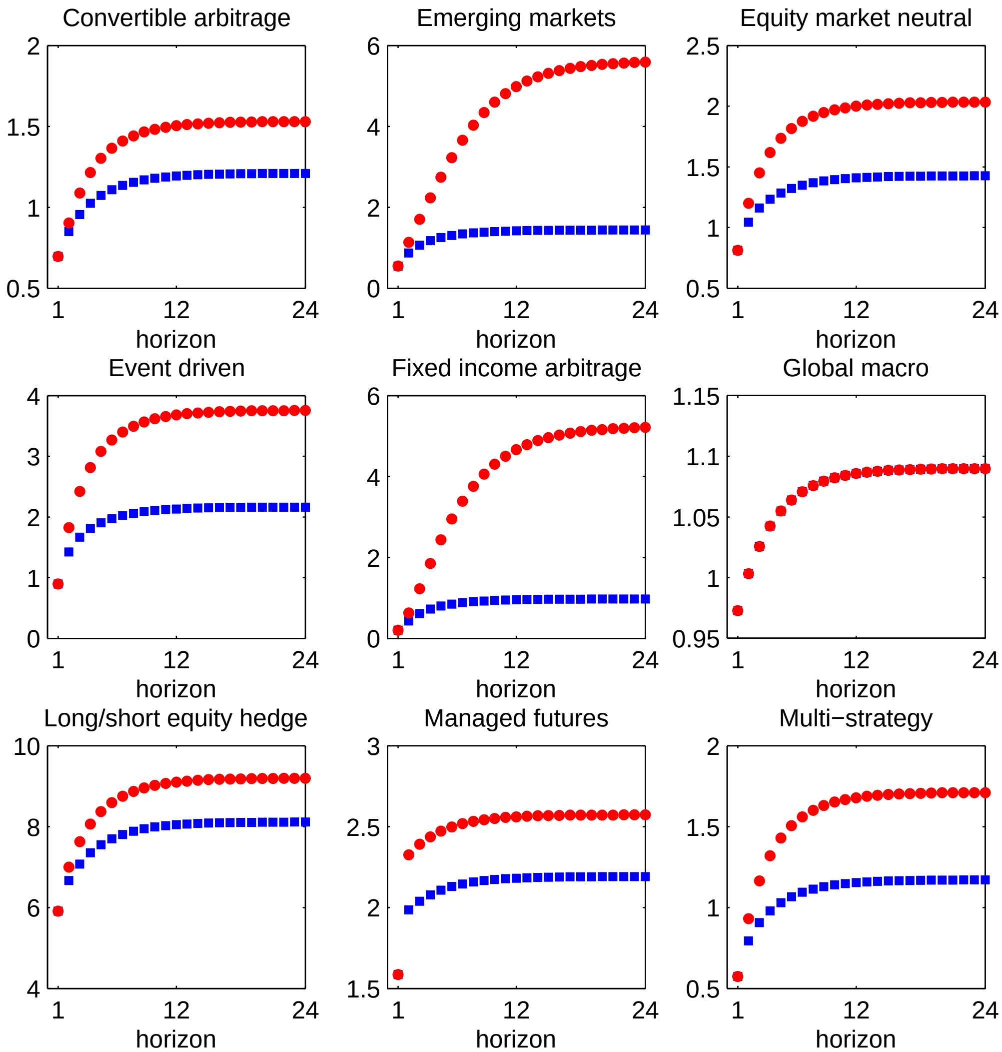
The figure displays the time series of the Treasury - Eurodollar (TED) spread, the volatility index VIX, and the credit spread, measured as the difference between the BAA and AAA yields.

Figure 13: Term structure of expected liquidation counts when stressing the current factor value.



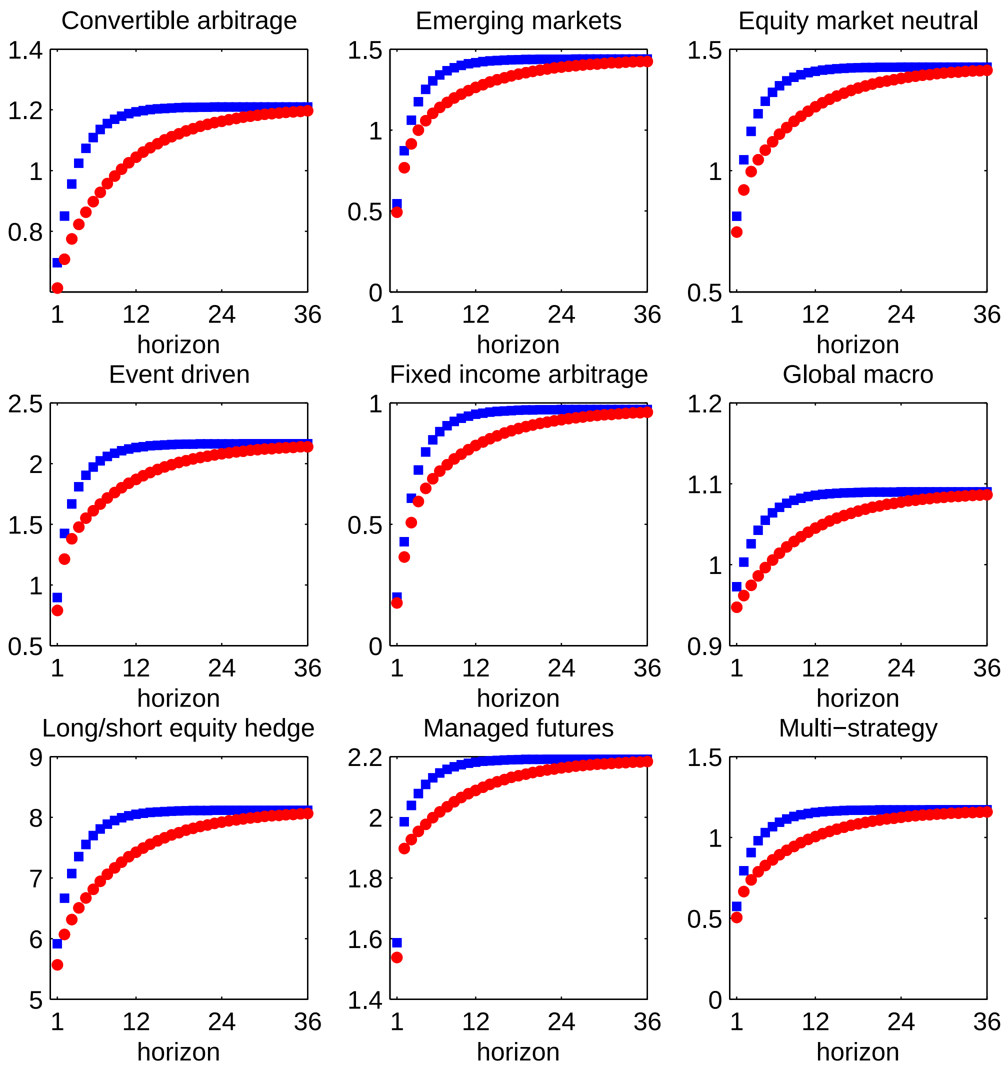
Term structure of the conditional expectation $E[Y_{k,t+\tau}|Y_t, F_t]$ of liquidation counts for horizon $\tau = 1, 2, ..., 24$ months, by management style k. Squares and circles correspond to conditioning sets with F_t equal to the median and the 95% quantile, respectively, of the stationary distribution of the frailty. The liquidation counts vector Y_t in the conditioning set corresponds to the observations in June 2009 for both curves. The model is the specification including frailty and contagion, with intensity parameters as in Tables 5 and 6, and frailty dynamic parameters $\delta = 0.59$ and $\rho = 0.74$, corresponding to the estimates of Section 4.2.

Figure 14: Term structure of expected liquidation counts when stressing the contagion matrix.



Term structure of the conditional expectation $E[Y_{k,t+\tau}|Y_t, F_t]$ of liquidation counts for horizon $\tau = 1, 2, ..., 24$ months, by management style k. Squares and circles correspond to models with contagion matrices $\hat{C}$ and $2\hat{C}$, respectively, where $\hat{C}$ is the matrix of estimates in Table 6. The intercepts and frailty sensitivities are as in Table 5, and the frailty dynamic parameters are $\delta = 0.59$ and $\rho = 0.74$, corresponding to the estimates of Section 4.2. The factor value F_t in the conditioning set corresponds to the median of the stationary distribution of the frailty, while the liquidation counts vector Y_t in the conditioning set corresponds to the observations in June 2009 for both curves.

Figure 15: Term structure of expected liquidation counts when stressing the frailty persistence.



Term structure of the conditional expectation $E[Y_{k,t+\tau}|Y_t, F_t]$ of liquidation counts for horizon $\tau = 1, 2, ..., 24$ months, by management style k. Squares and circles correspond to models with frailty autocorrelation $\rho = 0.74$ (corresponding to the estimate in Section 4.2) and $\rho = 0.90$, respectively. The intensity parameters are as in Tables 5 and 6, and the parameter characterizing the stationary distribution of the frailty is $\delta = 0.59$, corresponding to the estimate of Section 3.4. The factor value F_t in the conditioning set corresponds to the median of the stationary distribution of the frailty, while the liquidation counts vector Y_t in the conditioning set corresponds to the observations in June 2009 for both curves.

Table 1: The database.

		Alive funds	(%)	Liquidated funds	(%)	**Total**	(%)
CONV	Convertible Arbitrage	45	2.00%	66	4.30%	**111**	2.90%
EM	Emerging Markets	227	10.00%	111	7.30%	**338**	8.90%
EMN	Equity Market Neutral	126	5.50%	139	9.10%	**265**	7.00%
ED	Event Driven	216	9.50%	129	8.50%	**345**	9.10%
FI	Fixed Income Arbitrage	95	4.20%	75	4.90%	**170**	4.50%
GM	Global Macro	162	7.10%	102	6.70%	**264**	6.90%
LSE	Long/Short Equity Hedge	885	38.80%	546	35.90%	**1431**	37.70%
MF	Managed Futures	224	9.80%	230	15.10%	**454**	12.00%
MS	Multi-Strategy	299	13.10%	122	8.00 %	**421**	11.10%
	Total	**2279**	**100.00%**	**1520**	**100.00%**	**3799**	**100.00%**

The table provides the distribution of alive funds on June 2009, and funds liquidated prior to June 2009, across the nine management styles.

Table 2: Maximum and boundary values of the liquidation rates.

	Lower Boundary	Maximum	Upper Boundary
Convertible Arbitrage	0.022	0.075	0.051
Emerging Markets	0.034	0.082	0.051
Equity Market Neutral	0.045	0.124	0.084
Event Driven	0.032	0.093	0.058
Fixed Income Arbitrage	0.040	0.114	0.066
Global Macro	0.045	0.108	0.066
Long/Short Equity Hedge	0.038	0.102	0.062
Managed Futures	0.041	0.086	0.045
Multi-Strategy	0.052	0.119	0.058

The table provides the value of the estimated liquidation intensity at the lower and upper bounderies of the age domain, as well as the maximum estimated value of the liquidation intensity. The lower and upper boundaries of the age domain are 0 months and 100 months from inception, respectively.

Table 3: Estimated intercepts in the pure contagion model.

	Intercept a_k
Convertible Arbitrage	0.05
	(0.11)
Emerging Markets	0.51***
	(0.15)
Equity Market Neutral	0.63***
	(0.17)
Event Driven	0.59***
	(0.21)
Fixed Income Arbitrage	0.30**
	(0.13)
Global Macro	0.73***
	(0.13)
Long/Short Equity Hedge	4.56***
	(0.46)
Managed Futures	1.65***
	(0.22)
Multi-Strategy	0.21**
	(0.10)

The table provides the Maximum Likelihood (ML) estimates of the intercept parameters a_k for the Poisson model with contagion only. Standard errors are provided in parentheses. Stars $*$, $**$ and $***$ denote significance at the 10%, 5% and 1% level, respectively.

Table 4: Estimated contagion parameters in the pure contagion model.

	Convertible Arbitrage	Emerging Markets	Equity Market Neutral	Event Driven	Fixed Income Arbitrage	Global Macro	Long/Short Equity Hedge	Managed Futures	Multi-Strategy
Convertible Arbitrage			-0.13^{**}	0.11^{**}		0.20^{***}	0.05^{**}		
			(0.06)	*(0.05)*		*(0.07)*	*(0.02)*		
Emerging Markets		0.19^{***}			0.31^{***}			0.17^{***}	
		(0.07)			*(0.09)*			*(0.05)*	
Equity Market Neutral									0.24^{***}
									(0.09)
Event Driven			0.35^{***}	0.16^{**}	0.36^{***}		0.08^{***}		
			(0.11)	*(0.07)*	*(0.13)*		*(0.03)*		
Fixed Income Arbitrage				0.08^{**}	0.32^{***}				
				(0.04)	*(0.08)*				
Global Macro					0.20^{***}	0.23^{***}	-0.04^{**}		
					(0.07)	*(0.08)*	*(0.02)*		
Long/Short Equity Hedge	0.42^{**}		0.42^{**}		0.67^{***}		0.21^{***}		0.78^{***}
	(0.17)		*(0.20)*		*(0.25)*		*(0.06)*		*(0.23)*
Managed Futures						0.27^{**}			
						(0.11)			
Multi-Strategy		-0.16^{***}			0.15^{***}			-0.07^{**}	0.47^{***}
		(0.04)			*(0.06)*			*(0.03)*	*(0.07)*

The table provides the Maximum Likelihood (ML) estimates of the contagion parameters $(c_{k,k'})$ for the Poisson model with contagion only. Rows and columns correspond to target and source of contagion, respectively. Standard errors are provided in parentheses. Stars $**$ and $***$ denote significance at the 5% and 1% level, respectively. The estimates that are not statistically significant at 5% level are not displayed.

Table 5: Estimated intercepts and factor sensitivities in the model with contagion and frailty.

	Intercept a_k	Sensitivity b_k
Convertible Arbitrage	0.00	1.08**
	(0.15)	*(0.55)*
Emerging Markets	0.10	0.69**
	(0.23)	*(0.27)*
Equity Market Neutral	0.27	0.84**
	(0.30)	*(0.40)*
Event Driven	0.00	1.39**
	(0.15)	*(0.70)*
Fixed Income Arbitrage	0.00	0.31**
	(0.20)	*(0.13)*
Global Macro	0.76***	0.33***
	(0.14)	*(0.12)*
Long/Short Equity Hedge	2.98***	4.55**
	(1.10)	*(1.92)*
Managed Futures	1.18	0.63**
	(0.75)	*(0.25)*
Multi-Strategy	0.00	0.89**
	(0.17)	*(0.41)*

The table provides the estimates of the intercepts a_k and frailty sensitivities b_k for the Poisson model with frailty and contagion. The estimates are obtained by the Generalized Method of Moments (GMM) estimator of Appendix 3. Standard errors are provided in parentheses. Stars $*$, $**$ and $***$ denote significance at the 10%, 5% and 1% level, respectively.

Table 6: Estimated contagion parameters in the model with contagion and frailty.

	Convertible Arbitrage	Emerging Markets	Equity Market Neutral	Event Driven	Fixed Income Arbitrage	Global Macro	Long/Short Equity Hedge	Managed Futures	Multi-Strategy
Convertible Arbitrage						0.15^{**}			
						(0.07)			
Emerging Markets		0.17^{***}			0.21^{**}			0.20^{***}	
		(0.06)			*(0.08)*			*(0.05)*	
Equity Market Neutral									0.10^{**}
									(0.05)
Event Driven			0.35^{***}	0.10^{*}					
			(0.09)	*(0.06)*					
Fixed Income Arbitrage				0.09^{**}	0.22^{***}				
				(0.04)	*(0.07)*				
Global Macro					0.20^{*}				
					(0.11)				
Long/Short Equity Hedge			0.39^{**}						
			(0.16)						
Managed Futures						0.27^{**}			
						(0.11)			
Multi-Strategy									0.29^{***}
									(0.06)

The table provides the estimates of the contagion parameters $(c_{k,k'})$ for the Poisson model with frailty and contagion. Rows and columns correspond to target and source of contagion, respectively. We set equal to zero the contagion parameters, that are not statistically significant at 5% in the model with contagion only (Table 4). The free parameters are estimated by the Generalized Method of Moments (GMM) estimator of Appendix 3. Standard errors are provided in parentheses. Stars $*$, $**$ and $***$ denote significance at the 10%, 5% and 1% level, respectively. The estimates that are not statistically significant at 10% level are not displayed.

Table 7: Redemption frequency, leverage and factor sensitivity.

	Red. freq. ≤ 1 m	1 m $<$ Red. freq. ≤ 3 m	Red. freq. > 3 m	Leverage	Sensitivity b_k
Convertible Arbitrage	49%	47%	4%	77%	1.08
Emerging Markets	66%	31%	3%	57%	0.69
Equity Market Neutral	68%	28%	4%	58%	0.84
Event Driven	36%	48%	15%	54%	1.39
Fixed Income Arbitrage	46%	48%	6%	68%	0.31
Global Macro	80%	18%	2%	70%	0.33
Long/Short Equity Hedge	56%	37%	7%	57%	4.55
Managed Futures	92%	7%	1%	79%	0.63
Multi-Strategy	67%	29%	4%	50%	0.89

The second, third and fourth columns display the percentages of hedge funds with redemption frequency smaller or equal to 3 month, between 1 month and 3 months, and larger than 3 months, respectively, for the nine management styles. The fifth column displays the percentage of hedge funds announcing some use of leverage. The available information concerns only whether leverage is used or not, and not its amount. For comparison purpose, the last column of the table displays the sensitivities to the frailty (see also Table 6).

Table 8: Decomposition of the variance

	Underlying Poisson	Contagion	Frailty (direct effect)	Frailty (propagated by contagion)
Percentage of variance	6.54 %	5.10 %	64.30 %	24.06 %

The table provides the decomposition of the variance of a portfolio of liquidation swaps written on the individual hedge funds. The liquidation swap for management style k pays 1 USD for each fund of style k that is liquidate in a given month.

Table 9: Regression of the frailty on observable variables.

Model	C	TED	$TEDL$	VIX	$VIXL$	SPR	I	$TED \cdot I$	$TEDL \cdot I$	$VIX \cdot I$	$VIXL \cdot I$	$SPR \cdot I$	R^2
1	0.12 (1.11)	1.69 (10.71)											0.42
2	− 0.00 (−0.03)	1.09 (4.65)	0.85 (3.32)										0.46
3	0.10 (0.40)			0.04 (3.86)									0.09
4	1.36 (3.95)			0.08 (6.09)	− 0.10 (−4.92)								0.21
5	0.90 (3.15)	0.91 (3.61)	1.03 (3.87)	0.01 (0.82)	− 0.05 (−2.93)								0.50
6	− 0.18 (−1.06)					119.85 (7.61)							0.27
7	0.84 (3.20)	1.04 (4.52)	0.43 (1.60)	− 0.00 (−0.35)	− 0.07 (−4.04)	97.14 (5.47)							0.58
8	1.38 (3.77)	0.98 (2.49)	0.81 (1.79)	− 0.06 (−1.68)	− 0.01 (−0.50)		− 0.33 (−0.33)	− 0.04 (−0.08)	0.41 (0.73)	0.08 (1.93)	− 0.06 (−1.39)		0.53
9	− 0.31 (−0.74)	0.65 (1.87)	0.77 (1.94)	− 0.00 (−0.16)	− 0.06 (−2.45)	208.42 (6.15)	3.58 (2.70)	0.32 (0.70)	− 0.27 (−0.48)	0.00 (0.08)	− 0.11 (−2.15)	−105.50 (−2.13)	0.65
10	− 0.34 (−0.79)	1.11 (4.27)		0.01 (0.19)	− 0.06 (−2.50)	209.66 (6.11)	4.02 (3.41)	0.04 (0.12)		− 0.01 (−0.32)	− 0.13 (−2.54)	−76.35 (−1.73)	0.63

The table displays the estimated coefficients in the regression of the frailty on some sets of observed variables, which include: the constant (C), the Treasury - Eurodollar (TED) spread, the average value of the TED spread over the previous quarter (TEDL), the volatility index (VIX), the average of the VIX over the previous 12 months (VIXL), and the credit spread (SPR), measured as the difference between the BAA and AAA yields. These variables are interacted with the dummy I, that is 1 when the VIX is above the value 25. The interaction yields a switching regression allowing for different coefficients in the high and low volatility regimes. The last column provides the R^2 of the regressions.

Appendix 1: The contagion model with observable factors

In this Appendix we discuss the inclusion of observable common factors in the specification of the the liquidation intensity. The Poisson model with contagion becomes:

$$Y_{k,t} \sim \mathcal{P}[(n_{k,t}/n_{k,t_0})(a_k + b_k' X_t + c_k' Y_{t-1}^*)], \quad k = 1, ..., K,$$

where $X_t = (X_{1,t}, ..., X_{L,t})'$ denotes the observations at date t of L factors, and K denotes the generic number of management styles ($K = 9$ in our empirical study). As mentioned in the main body of the paper, it is necessary to complete this specification by a dynamic model for X_t, if we want to use the model for prediction of future risk, computation of the reserves, stress testing, but also to test for factor exogeneity. For expository purpose, let us assume a linear dynamic multivariate model for X_t such as:

$$X_t = \gamma + \Phi X_{t-1} + \Psi Y_{t-1} + u_t,$$

where the error terms (u_t) satisfy standard assumptions for time series regression. Then, the analysis has to be performed along the following steps:

Step 1: Estimate the parameters of the Poisson model a_k, b_k, c_k, for $k = 1, ..., K$, and the parameters characterizing the factor dynamics γ, Φ, Ψ.

Step 2: Derive the actual set of factors by performing the singular value decomposition of matrix B with rows b_k', $k = 1, ..., K$. Its rank provides the actual number of factors. The actual factors are obtained by selecting linearly independent combinations among the $b_k' X_t$, for $k = 1, ..., K$.

Step 3: Finally, we have to test for the factor exogeneity. The required exogeneity property concerns the actual factors, and not necessarily the initial factors themselves. Thus, we have to test the null hypothesis: $B\Psi = 0$.

It might be inappropriate to introduce a large number L of observable factors. First, this complicates the implementation of the above procedure. Second, this can be useless since the number of actual factors is necessarily smaller than K, that is in general rather small.

Appendix 2: The Poisson model with contagion and frailty

In this Appendix we analyze in more depth the Poisson model with contagion and frailty defined in Section 3. We focus on the Autoregressive Gamma specification for the frailty dynamics (Section A.2.1), the affine property of the joint process of liquidation counts and frailty (Section A.2.2), we derive the stationarity conditions for this joint process (Section A.2.3) and its unconditional moments (Section A.2.4).

A.2.1 The Autoregressive Gamma (ARG) process

In this Section we review the main properties of the ARG(1) process [see Gourieroux, Jasiak (2006)].

i) The conditional distribution

The ARG(1) process (F_t) is a Markov process with conditional distribution the noncentral gamma distribution $\gamma(\delta, \eta F_{t-1}, \nu)$, where δ, $\delta > 0$, is the degree of freedom, ηF_{t-1}, $\eta > 0$, the noncentrality parameter and ν, $\nu > 0$, a scale parameter. Its first- and second-order conditional moments are:

$$E(F_t|F_{t-1}) = \delta\nu + \eta\nu F_{t-1}, \quad V(F_t|F_{t-1}) = \nu^2\delta + 2\eta\nu^2 F_{t-1}. \tag{a.1}$$

The ARG(1) process is a discrete-time affine process, that is, the conditional Laplace transform is an exponential affine function of the lagged variable:

$$E[\exp(-uF_t)|F_{t-1}] = \exp[-\alpha(u)F_{t-1} - \beta(u)], \tag{a.2}$$

where functions α and β are given by:

$$\alpha(u) = \frac{\eta\nu u}{1+\nu u}, \quad \beta(u) = \delta\log(1+\nu u), \tag{a.3}$$

for any real value of the argument $u > -1/\nu$.

ii) The state space representation

The ARG(1) process admits a state space representation, which is especially convenient for simulating the trajectories of the process. To get a simulated value of F_t given F_{t-1}, we proceed as follows:

a) We draw an intermediate value Z_t^s in a Poisson distribution $\mathcal{P}(\eta F_{t-1})$;

b) Then, F_t is drawn in the centered gamma distribution $\gamma(\delta + Z_t^s, 0, \nu)$.

iii) Stationarity condition and stationary distribution

The ARG(1) process is stationary if $\rho = \nu\eta$ is such that $\rho < 1$. Then, the stationary distribution is a centered gamma distribution $\gamma(\delta, 0, \frac{\nu}{1-\nu\eta})$. In particular, we get the unconditional moments:

$$E(F_t) = \frac{\nu\delta}{1-\nu\eta}, \quad V(F_t) = \delta\left(\frac{\nu}{1-\nu\eta}\right)^2. \tag{a.4}$$

From the first equation in (a.1) it is seen that parameter ρ is the first-order autocorrelation of process (F_t).

iv) Normalization and reparameterization

When the $ARG(1)$ process is used as a latent frailty, the scale of the process can be absorbed in the sensitivity parameters b_k of the intensity function. Then, the process (F_t) can be normalized to have $E(F_t) = 1$. Thus, from the first equation in (a.4), the parameters are such that $\nu\delta = 1 - \nu\eta = 1 - \rho$. It follows that the model can be parameterized in terms of δ and ρ, while the remaining parameters are given by:

$$\nu = \frac{1-\rho}{\delta}, \qquad \eta = \frac{\rho\delta}{1-\rho}. \tag{a.5}$$

The stationary distribution is $\gamma(\delta, 0, 1/\delta)$, with Laplace transform $E[\exp(-uF_t)] = (1 + u/\delta)^{-\delta}$, for $u > -\delta$. Moreover, the stationary variance is $V(F_t) = 1/\delta$.

A.2.2 The affine property

In this section we show that the joint process of liquidation counts and frailty defined in System (4.2) is affine. For completeness, we give the definition of the process in the next Assumption A.1.

Assumption A.1: *The joint distribution of the liquidation counts Y_t and frailty F_t given the past values $\underline{Y_{t-1}} = (Y_{t-1}, Y_{t-2}, ...)$ and $\underline{F_{t-1}} = (F_{t-1}, F_{t-2}, ...)$ is such that:*

$$\begin{aligned} Y_{k,t}|\underline{Y_{t-1}}, \underline{F_t} &\sim \mathcal{P}\left[(n_{k,t}/n_{k,t_0})\left(a_k + b_k F_t + c_k' Y_{t-1}^*\right)\right], \text{ independent across } k = 1, ..., K, \\ F_t|\underline{Y_{t-1}}, \underline{F_{t-1}} &\sim \gamma\left(\delta, \eta F_{t-1}, \nu\right), \end{aligned}$$

where parameters η and ν are given in (a.5).

The liquidation counts of different management styles are conditionally independent given the past values of the liquidation counts and the current and past values of the frailty. Moreover, the conditional distribution of the frailty F_t is independent on the past liquidations counts. This condition corresponds to the exogeneity property of the frailty process, which follows a ARG(1) process.

Let us now show that the joint process (Y_t, F_t) is affine. By the moment generating function of the Poisson distribution, we deduce that the conditional Laplace transform of the current liquidation counts vector Y_t given $\underline{F_t}$ and $\underline{Y_{t-1}}$ is given by:

$$\begin{aligned}\psi_t(u) &= E\left[\exp(-u'Y_t)|\underline{F_t}, \underline{Y_{t-1}}\right] = \prod_k E[\exp(-u_k Y_{k,t})|\underline{F_t}, \underline{Y_{t-1}}] \\ &= \prod_k \exp\left\{-\gamma_{k,t}\lambda_{k,t}[1-\exp(-u_k)]\right\} \\ &= \exp\left\{-\sum_k [1-\exp(-u_k)]\gamma_{k,t}a_k - \sum_k [1-\exp(-u_k)]\gamma_{k,t}b_k F_t - \sum_k [1-\exp(-u_k)]\gamma_{k,t}c_k' Y_{t-1}^*\right\},\end{aligned} \tag{a.6}$$

where u is a vector with nonnegative components $u_{k,t}$, and we use the notation $\lambda_{k,t} = (n_{k,t}/n_{k,t_0})(a_k + b_k F_t + c_k' Y_{t-1}^*)$ for the conditional intensity and $\gamma_{k,t} = n_{k,t}/n_{k,t_0}$ for the size adjustment in management style k. Moreover, by the exogeneous ARG(1) dynamics of the frailty process, we have:

$$E[\exp(-uF_t)|\underline{Y_{t-1}}, \underline{F_{t-1}}] = \exp[-\alpha(u)F_{t-1} - \beta(u)], \tag{a.7}$$

where functions α and β are given in (a.3). Then, from (a.6)-(a.7) and the Law of iterated expectation, we get the conditional Laplace transform of the joint process $(Y_t', F_t)'$ given its past:

$$\begin{aligned}\psi_t(u,v) &= E[\exp(-u'Y_t - vF_t)|\underline{F_{t-1}}, \underline{Y_{t-1}}] \\ &= E\left\{E\left[\exp(-u'Y_t)|\underline{F_t}, \underline{Y_{t-1}}\right]\exp(-vF_t)|\underline{F_{t-1}}, \underline{Y_{t-1}}\right\} \\ &= \exp\left\{-\sum_k \gamma_{k,t}a_k[1-\exp(-u_k)] - \sum_k [1-\exp(-u_k)]\gamma_{k,t}c_k' Y_{t-1}^* \right. \\ &\qquad \left. -\alpha\left(\sum_{k,} \gamma_{k,t}b_k[1-\exp(-u_k)] + v\right)F_{t-1} - \beta\left(\sum_k \gamma_{k,t}b_k[1-\exp(-u_k)] + v\right)\right\}.\end{aligned} \tag{a.8}$$

This joint conditional Laplace transform is exponential affine in lagged values Y_{t-1} and F_{t-1}, that is, it is of the form:

$$\psi_t(u,v) = \exp\{-\alpha_{1,t}(u,v)'Y_{t-1} - \alpha_{2,t}(u,v)F_{t-1} - \beta_t(u,v)\}, \text{ say,} \tag{a.9}$$

where the sensitivity coefficients $\alpha_{1,t}$, $\alpha_{2,t}$ and β_t depend on the class sizes and are time dependent in general. Thus, process $(Y_t', F_t)'$ is a time-heterogenous affine Markov process. The closed form exponential affine expression of the conditional Laplace transform of process $(Y_t', F_t)'$ simplifies the computation of the predictive distributions at any prediction horizon [see e.g. Darolles, Gourieroux, Jasiak (2006)] and the filtering of the latent factor [see Bates (2006)]. The closed form expression of the conditional Laplace transform also provides informative moment restrictions, which are the basis of estimation with the Generalized Method of Moments (see Appendix 3). While we have kept the frailty process one-dimensional with ARG dynamics for simplicity, the affine property of the joint process $(Y_t', F_t)'$ remains valid as long as the frailty admits a (multidimensional) affine dynamics.

A.2.3 Stationarity

In some applications such as stress testing (see Section 5), it is appropriate to consider a given portfolio structure with respect to the management style, which is held fixed through time in the analysis. For this purpose, we consider the next Assumption A.2 in the rest of this Appendix.

Assumption A.2: *The size adjustments are constant and equal to* 1*:* $\gamma_{k,t} = 1$*, for any* k, t*.*

Thus, the sizes of the categories are fixed through time, and are assumed homogeneous across management styles for expository purpose.

Let us now derive the stationarity conditions for the joint process $Z_t = (Y_t', F_t)'$ under Assumptions A.1 and A.2. From equation (a.8), the conditional Laplace transform of the Markov process Z_t is given by:

$$E\left[\exp\left(-w'Z_t\right)|Z_{t-1}\right] = \exp\left(-A(w)'Z_{t-1} - B(w)\right),$$

where $w = (u', v)' \in \mathbb{R}^K \times \mathbb{R}$, functions $A(w)$ and $B(w)$ are given by:

$$A(w) = \left[\sum_{k=1}^{K} c_k'(1-e^{-u_k}), \alpha\left(v + \sum_{k=1}^{K}(1-e^{-u_k})b_k\right)\right]',$$

and:

$$B(w) = \sum_{k=1}^{K}(1-e^{-u_k})a_k + \beta\left(v + \sum_{k=1}^{K}(1-e^{-u_k})b_k\right),$$

and functions α and β are given in (a.3). Thus, process (Z_t) is a time-homogeneous affine process. From Proposition 2 in Darolles, Gourieroux, Jasiak (2006), process (Z_t) is strictly stationary if:

$$\lim_{\tau\to\infty}\left[\frac{\partial A(0)}{\partial w'}\right]^{\tau} = 0. \tag{a.10}$$

Now, by using that $\dfrac{\partial A(0)}{\partial u_k} = [c_k', b_k d\alpha(0)/du]' = [c_k, \rho b_k]'$, for $k = 1, ..., K$, and $\dfrac{\partial A(0)}{\partial v} = [0', d\alpha(0)/du]' = [0', \rho]'$, we get:

$$\frac{\partial A(0)}{\partial w'} = \begin{pmatrix} C & 0 \\ \rho b' & \rho \end{pmatrix}.$$

Thus, condition (a.10) is satisfied if, and only if, the first-order autocorrelation of the frailty is such that $\rho < 1$ and the eigenvalues of the contagion matrix C have modulus smaller than 1.

A.2.4 Unconditional moments

In this section we derive the first- and second-order unconditional moments of the liquidation count process Y_t under the stationarity conditions derived in the previous section.

i) Moments of order 1

We have:

$$E_{t-1}(Y_t) = E_{t-1}[E_{t-1}(Y_t|F_t)] = E_{t-1}(a + bF_t + CY_{t-1}) = a + bE_{t-1}(F_t) + CY_{t-1},$$

where E_{t-1} denotes expectation conditional on the past histories of liquidation counts $\underline{Y_{t-1}}$ and factor $\underline{F_{t-1}}$. By taking expectation of both sides of the equation, and using the stationarity of process (Y_t) and the normalization $E(F_t) = 1$, we get:

$$E(Y_t) = a + b + CE(Y_t) \quad \Leftrightarrow \quad E(Y_t) = (Id - C)^{-1}(a + b). \tag{a.11}$$

ii) Moments of order 2

Let us first consider the covariance between the liquidation counts and the frailty. We have:

$$\begin{aligned} E_{t-1}(F_tY_t) &= E_{t-1}[E_{t-1}(F_tY_t|F_t)] = E_{t-1}[F_t(a + bF_t + CY_{t-1})] \\ &= aE_{t-1}(F_t) + bE_{t-1}(F_t^2) + CE_{t-1}(F_t)Y_{t-1} \\ &= aE_{t-1}(F_t) + bE_{t-1}(F_t^2) + C(1 - \rho + \rho F_{t-1})Y_{t-1}, \end{aligned}$$

from equations (a.1) and (a.5). By taking the expectation of both sides of the equation, we get:

$$E(F_tY_t) = a + b(1+\sigma^2) + (1-\rho)C(Id-C)^{-1}(a+b) + \rho CE(F_tY_t),$$

where $\sigma^2 = V(F_t) = 1/\delta$ denotes the variance of the frailty [see Section A.2.1 iv)]. We deduce that:

$$\begin{aligned} E(F_tY_t) &= (Id-\rho C)^{-1}\{b\sigma^2 + [Id + (1-\rho)C(Id-C)^{-1}](a+b)\}, \\ &= (Id-\rho C)^{-1}b\sigma^2 + (Id-C)^{-1}(a+b). \end{aligned}$$

Thus:

$$Cov(Y_t, F_t) = \sigma^2(Id-\rho C)^{-1}b. \tag{a.12}$$

Let us now consider the variance-covariance matrix of the liquidation counts vector Y_t. We have:

$$\begin{aligned} E_{t-1}(Y_tY_t') &= E_{t-1}(E_{t-1}[Y_tY_t'|F_t]) = E_{t-1}[V_{t-1}(Y_t|F_t) + E_{t-1}(Y_t|F_t)E_{t-1}(Y_t|F_t)'] \\ &= E_{t-1}[diag(a + bF_t + CY_{t-1})] + E_{t-1}[(a + bF_t + CY_{t-1})(a + bF_t + CY_{t-1})'] \\ &= diag[a + bE_{t-1}(F_t) + CY_{t-1}] + bb'V_{t-1}(F_t) \\ &\quad + [E_{t-1}(a + bF_t + CY_{t-1})][E_{t-1}(a + bF_t + CY_{t-1})]' \\ &= diag[a + bE_{t-1}(F_t) + CY_{t-1}] + bb'V_{t-1}(F_t) \\ &\quad + [a + b(1-\rho+\rho F_{t-1}) + CY_{t-1}][a + b(1-\rho+\rho F_{t-1}) + CY_{t-1}]'. \end{aligned}$$

By taking the expectation of both sides, we deduce:

$$\begin{aligned} E(Y_tY_t') &= diag[a + b + C(Id-C)^{-1}(a+b)] + bb'E[V_{t-1}(F_t)] + V[b\rho F_{t-1} + CY_{t-1}] \\ &\quad + E[a + b(1-\rho+\rho F_{t-1}) + CY_{t-1}]E[a + b(1-\rho+\rho F_{t-1}) + CY_{t-1}]' \\ &= diag[(Id-C)^{-1}(a+b)] + \sigma^2(1-\rho^2)bb' + V[b\rho F_{t-1} + CY_{t-1}] + E(Y_t)E(Y_t'), \end{aligned}$$

where we used that $E[V_{t-1}(F_t)] = \sigma^2(1-\rho^2)$. Therefore, the variance-covariance matrix of Y_t satisfies the recursive equation:

$$\begin{aligned} V(Y_t) &= CV(Y_t)C' + diag[(Id-C)^{-1}(a+b)] + \sigma^2 bb' \\ &\quad + \rho bCov(F_t, Y_t)C' + \rho CCov(Y_t, F_t)b'. \end{aligned} \tag{a.13}$$

Equation (4.3) is obtained by substituting the expression (a.12) of $Cov(Y_t, F_t)$.

iii) Autocovariance at order 1

We have:

$$\begin{aligned} Cov(Y_t, Y_{t-1}) &= Cov[E_{t-1}(Y_t), Y_{t-1}] = Cov[E_{t-1}(a + bF_t + CY_{t-1}), Y_{t-1}] \\ &= Cov[a + b(1 - \rho + \rho F_{t-1}) + CY_{t-1}, Y_{t-1}]. \end{aligned}$$

Therefore:

$$\begin{aligned} Cov(Y_t, Y_{t-1}) &= Cov(b\rho F_{t-1} + CY_{t-1}, Y_{t-1}) = b\rho Cov(F_{t-1}, Y_{t-1}) + CV(Y_t) \\ &= CV(Y_t) + \sigma^2 \rho bb'(Id - \rho C')^{-1}. \end{aligned} \tag{a.14}$$

Appendix 3: GMM estimation of the Poisson model with frailty and contagion

In this Appendix we develop an estimation approach for the parameters of the Poisson model with contagion and frailty that is based on the Generalized Method of Moments (GMM). We start by considering moment restrictions that involve the liquidation intensity parameters and the parameters in the stationary distribution of the frailty.

i) Static moment restrictions

The moment restrictions are based on the special form of the conditional Laplace transform in equation (a.6). For management style k we have:

$$E[\exp\left(-u_k Y_{k,t}\right) | \underline{F_t}, \underline{Y_{t-1}}] = \exp\left\{-\gamma_{k,t}\left(a_k + b_k F_t + c_k' Y_{t-1}^*\right)\left(1 - e^{-u_k}\right)\right\}, \tag{a.15}$$

for any argument $u_k \in [0, \infty)$. These conditional moments are appropriate for analyzing risk parameters. Indeed, the left hand side of the above equation is simply the expected utility function for an investor with a portfolio totally invested in the liquidation events of style k, and an absolute risk aversion equal to u_k. By considering the associated set of moment restrictions, we consider all types of investment, for all values of risk aversion. Therefore, the associated moment method will calibrate the unknown parameters on the whole set of expected utilities.

The equations in (a.15) can be rewritten as:

$$E\left[\exp\left\{-u_k Y_{k,t} + \gamma_{k,t}\left(a_k + c_k' Y_{t-1}^*\right)(1-e^{-u_k})\right\} | \underline{F_t}, \underline{Y_{t-1}}\right] = \exp\left\{-\gamma_{k,t} b_k (1-e^{-u_k}) F_t\right\}, \qquad \text{(a.16)}$$

for $u_k \in [0,\infty)$. Thus, we obtain nonlinear transforms of the observable liquidation count variables, whose conditional expectation depends on the frailty only. As equation (a.16) holds for all real positive arguments u_k, we can consider a time-dependent argument u_k. The time dependence is selected such that the RHS of equation (a.16), and thus the LHS as well, is stationary. More precisely, let $u_{k,t}$ be such that $1 - e^{-u_{k,t}} = v/\gamma_{k,t}$, for given $v \in \mathcal{V}_k$, i.e. $u_{k,t} = -\log\left(1 - v/\gamma_{k,t}\right)$. In order to obtain a well-defined $u_{k,t}$ in $[0,\infty)$, the real interval $\mathcal{V}_k$ has to be a subset of $[0, \inf_t \gamma_{k,t})$. Then, equation (a.16) becomes:

$$E\left[\exp\left\{\log(1 - v/\gamma_{k,t}) Y_{k,t} + v\left(a_k + c_k' Y_{t-1}^*\right)\right\} | \underline{F_t}, \underline{Y_{t-1}}\right] = \exp\left(-v b_k F_t\right), \ \ \forall v \in \mathcal{V}_k. \qquad \text{(a.17)}$$

These moment restrictions are conditional on factor path $\underline{F_t}$ and cannot be used directly for estimation since the factor is unobservable. Therefore, we integrate out the latent factor by taking expectation on both sides of the equation w.r.t. the gamma stationary distribution $\gamma(\delta, 0, 1/\delta)$ of factor process F_t [see Appendix 2.1 iv)]. We get a continuum of unconditional moment restrictions:

$$E\left[\exp\left\{\log(1 - v/\gamma_{k,t}) Y_{k,t} + v\left(a_k + c_k' Y_{t-1}^*\right)\right\}\right] = \frac{1}{\left(1 + v b_k/\delta\right)^{\delta}}, \quad \forall v \in \mathcal{V}_k. \qquad \text{(a.18)}$$

These static moment restrictions involve the intensity parameters a_k, b_k, c_k for any type k, as well as parameter δ characterizing the stationary distribution of the frailty, but do not allow to identify the frailty persistence parameter ρ.

ii) Dynamic moment restrictions

In order to derive moment restrictions that allow for estimation of parameter ρ, let us consider equation (a.17) and multiply both sides by $\exp(-\tilde{u}_{l,t-1} Y_{l,t-1})$, where $\tilde{u}_{l,t-1} = -\log(1 - \tilde{v}/\gamma_{l,t-1})$, for some type l and any $\tilde{v} \in \mathcal{V}_l$. We have:

$$E\left[\exp\left\{-u_{k,t} Y_{k,t} - \tilde{u}_{l,t-1} Y_{l,t-1} + v\left(a_k + c_k' Y_{t-1}^*\right)\right\} | \underline{F_t}, \underline{Y_{t-1}}\right] = \exp\left(-v b_k F_t - \tilde{u}_{l,t-1} Y_{l,t-1}\right),$$

for all $v \in \mathcal{V}_k, \tilde{v} \in \mathcal{V}_l$, where $u_{k,t} = \log(1 - v/\gamma_{k,t})$. By taking the conditional expectation given $\underline{F_t}$ and the liquidation counts history $\underline{Y_{t-2}}$ up to month $t-2$ on both sides of the equation, we get:

$$\begin{aligned}
& E\left[\exp\left\{-u_{k,t}Y_{k,t} - \tilde{u}_{l,t-1}Y_{l,t-1} + v\left(a_k + c_k'Y_{t-1}^*\right)\right\} | \underline{F_t}, \underline{Y_{t-2}}\right] \\
= \quad & \exp\left(-vb_kF_t\right) E\left[\exp\left(-\tilde{u}_{l,t-1}Y_{l,t-1}\right) | \underline{F_t}, \underline{Y_{t-2}}\right] = \exp\left\{-vb_kF_t - (a_l + b_lF_{t-1} + c_l'Y_{t-2}^*)\tilde{v}\right\}.
\end{aligned}$$

By rearranging terms, and computing the unconditional expectation of both sides, we get:

$$E\left[\exp\left\{-u_{k,t}Y_{k,t} - \tilde{u}_{l,t-1}Y_{l,t-1} + v\left(a_k + c_k'Y_{t-1}^*\right) + \tilde{v}\left(a_l + c_l'Y_{t-2}^*\right)\right\}\right] = E\left[\exp\left(-vb_kF_t - \tilde{v}b_lF_{t-1}\right)\right],$$

for all $v \in \mathcal{V}_k, \tilde{v} \in \mathcal{V}_l$. The expectation in the right-hand side involves the joint distribution of the frailty values F_t and F_{t-1} on two consecutive months, and hence depends on the frailty autocorrelation parameter ρ. In fact, by the exponential affine property of the ARG process (see Appendix 2.1) we have:

$$\begin{aligned}
& E\left[\exp\left(-vb_kF_t - \tilde{v}b_lF_{t-1}\right)\right] \\
= \quad & E\left[E[\exp\left(-vb_kF_t\right)|F_{t-1}]\exp\left(-\tilde{v}b_lF_{t-1}\right)\right] = \exp\left(-\beta(vb_k)\right) E\left[\exp\left(-[\alpha(vb_k) + \tilde{v}b_l]F_{t-1}\right)\right] \\
= \quad & \frac{1}{[1 + (1-\rho)vb_k/\delta]^{\delta}} \frac{1}{[1 + [\alpha(bv_k) + \tilde{v}b_l]/\delta]^{\delta}} = \frac{1}{[1 + (vb_k + \tilde{v}b_l)/\delta + (1-\rho)v\tilde{v}b_kb_l/\delta^2]^{\delta}},
\end{aligned}$$

where we use $\beta(u) = \delta\log(1 + (1-\rho)u/\delta)$ and $\alpha(u) = \dfrac{\rho u}{1 + (1-\rho)u/\delta}$ from equations (a.3) and (a.5). Thus, we get the continuum of unconditional dynamic moment restrictions:

$$\begin{aligned}
& E\left[\exp\left\{\log(1 - v/\gamma_{k,t})Y_{k,t} + \log(1 - \tilde{v}/\gamma_{l,t-1})Y_{l,t-1} + v\left(a_k + c_k'Y_{t-1}^*\right) + \tilde{v}\left(a_l + c_l'Y_{t-2}^*\right)\right\}\right] \\
= \quad & \frac{1}{[1 + (vb_k + \tilde{v}b_l)/\delta + (1-\rho)v\tilde{v}b_kb_l/\delta^2]^{\delta}}, \quad \forall v \in \mathcal{V}_k, \tilde{v} \in \mathcal{V}_l.
\end{aligned} \tag{a.19}$$

These dynamic moment restrictions, written for all pairs of management styles (k, l), involve all model parameters. For $\tilde{v} = 0$, these moment restrictions reduce to the static moment restrictions (a.18).

iii) GMM estimators

The above moment restrictions can be used is different ways to define GMM estimators. We consider below three possible GMM approaches.

a) We can use the static moment restrictions (a.18) written for all types k to define a first-step GMM estimator of parameters a_k, b_k, c_k, for all management styles k, and parameter δ. Then, a second-step GMM estimator of parameter ρ can be defined by using some subset of the dynamic moment restrictions (a.19) and replacing parameters a_k, b_k, c_k and δ with their first-step GMM estimates.

b) Alternatively, the dynamic moment restrictions (a.19) can be used to estimate jointly all model parameters $(\theta', \varphi')'$, where $\theta = (a_k, b_k, c_k', k = 1, ..., K,)'$ and $\varphi = (\delta, \rho)'$, in one step. More precisely, consider the dynamic moment restrictions (a.19) written for all management styles k, with $l = k$, and for a given grid of values $v, \tilde{v}$. [10] They define a set of unconditional moment restrictions $E\left[h_{k,t}(a_k, b_k, c_k, \varphi)\right] = 0$, for $k = 1, ..., K$, say. Then, the GMM estimator $(\hat{\theta}', \hat{\varphi}')'$ minimizes the criterion:

$$Q_T(\theta, \varphi) = \sum_{k=1}^{K} \|\hat{h}_{k,T}(a_k, b_k, c_k, \varphi)\|^2, \tag{a.20}$$

where $\hat{h}_{k,T}(a_k, b_k, c_k, \varphi) = \frac{1}{T}\sum_{t=1}^{T} h_{k,t}(a_k, b_k, c_k, \varphi)$ is the sample average of the orthogonality function. By including only the moment restrictions with $k = l$, and using a weighting matrix that is diagonal across management styles, we simplify considerably the optimization problem. [11] In fact, for given value of the bivariate frailty parameter $\varphi = (\delta, \rho)'$, the GMM criterion (a.20) is additive w.r.t. the parameters of the styles. Therefore, the minimization can be performed by concentrating w.r.t. parameter θ. Namely, for given φ, the criteria for the different styles k are separately minimized w.r.t. parameters a_k, b_k, c_k by a Newton-Raphson algorithm. Then, the concentrated criterion is minimized by a bi-dimensional grid search over parameters (δ, ρ).

c) We can also consider the GMM estimators that exploit the full set of dynamic moment restrictions (a.19) written for all pairs of management styles (k, l). Such GMM estimators are expected to be more efficient than the GMM estimator minimizing criterion (a.20). However, such GMM estimators are computationally less convenient, because they involve an optimization over a large-dimensional parameter space that can not be dealt with by concentration easily. In this paper we apply method *b)*, which provides a trade-off between tractability and efficiency.

[10]It might also be possible to apply an efficient GMM taking into account the continuum of moment restrictions (a.19) [see e.g. Carrasco et al (2007)].

[11]For expository purpose the weighting matrix is set equal to the identity matrix for each type.