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Reevaluation of the capital charge in insurance after a large shock: empirical and theoretical views

Abstract: Motivated by the recent introduction of regulatory stress tests in the Solvency II framework, we study the impact of the re-estimation of the tail risk and of loss absorbing capacities on post-stress solvency ratios. Our contribution is threefold. First, we build the first stylized model for re-estimated solvency ratio in insurance. Second, we solve a new theoretical problem in statistical theory: what is the asymptotic impact of a record on the re-estimation of tail quantiles and tail probabilities for classical extreme value estimators? Third, we quantify the impact of the re-estimation of tail quantiles and of loss absorbing capacities on real-world solvency ratios thanks to regulatory data from EIOPA. Our analysis sheds a first light on the role of the loss absorbing capacity and its paramount importance in the Solvency II capital charge computations. We conclude with a number of policy recommendations for insurance regulators.

Keywords: Insurance, Extreme Value Theory, Financial Regulation, Solvency II, Solvency Capital Requirement, Loss Absorbing Capacities, Stress Tests, Enterprise Risk Management.

JEL Classification: G22, G18, C13 & C15

Réévaluation de la charge en capital en assurance après un choc important : points de vue théoriques et empiriques

Résumé : Dans le contexte de la récente introduction de stress tests dans le cadre Solvabilité II, nous étudions l'impact de la réévaluation du risque extrême et des capacités d'absorption dans le calcul de l'exigence en capital et du ratio de couverture juste après un choc. Nous apportons trois contributions à ce problème. Tout d'abord, nous construisons le premier modèle stylisé pour réévaluer les ratios de couverture en assurance. Deuxièmement, nous résolvons un problème de théorie statistique : quel est l'impact asymptotique de l'ajout d'un nouveau choc important sur l'évaluation des quantiles extrêmes et des distributions de probabilité dans le cas de l'utilisation d'estimateurs classiques de statistique des valeurs extrêmes ? Troisièmement, nous quantifions l'impact de la réévaluation des quantiles extrêmes et des capacités d'absorption des pertes en utilisant des données régulateurs (EIOPA). Notre analyse conclut à une hausse possible du SCR suite à un choc défavorable, mais met surtout en évidence le rôle important de la baisse des capacités d'absorption des pertes pour le calcul des exigences en capital dans le cadre Solvabilité II. En conclusion, plusieurs recommandations à l'endroit des régulateurs du secteur de l'assurance sont proposées.

Mots clés: Assurance, Théorie des valeurs extrêmes, Régulation financière, Solvabilité II, Exigence en capital de solvabilité, Capacités d'absorption des pertes, Gestion des risques d'entreprise

Non-Technical Summary

Research question

In the Solvency II framework governing the E.U. insurance sector, the solvency capital requirement (“SCR”) is roughly determined for all risks covered by the regulation via an estimate of the bicentennial losses followed by an aggregation. Solvency is obviously a pre-requisite for the on-going business continuity of an insurance company and its assessment is monitored by the supervisory authority. In addition, risk assessment exercises of insurance companies are organized regularly (e. g. stress test, ORSA). and the question of the prospective solvency of these entities seems rather natural. The issue here is whether an insurance company would meet its regulatory requirements right after a large shock. In the 2014 stress test Exercise, EIOPA assumed that the denominator of the SCR coverage ratio remained constant and focused only on the evolution of the numerator since the shocks were assumed instantaneous. This raised both empirical and theoretical questions: how would a new record loss modify the tail quantiles and the tail probability distribution?

The SCR is defined as the difference between, on the one hand, the gross solvency capital requirement (BSCR) – directly related to the risk assessment, thus to the extreme quantiles – and, on the other hand, the loss absorbing capacities (LAC). In such a context, we study which of these two components is the most affected by a large shock/record.

Results

Using a linear stylized model for the estimation of the solvency capital requirement, we decompose the SCR into the two aforementioned components, and consider a unidimensional shock. Reformulating the re-estimation of the solvency capital requirement in the occurrence of a large shock’s problem from a statistical point of view, we use non-life insurance techniques (Nat Cat) and Extreme Value Theory to determine a generic estimation of the effect of adding/discarding a new large record on the tail quantile and tail probability distribution. We also illustrate the re-estimation effect with Solvency II related quantities. Using data from the 2014 EIOPA stress test data, we show that the downward revision of the LAC actually contributes the most to the upward revision of the SCR.

Résumé non technique

Question de recherche

Selon la directive Solvabilité II encadrant le secteur des assurances dans l'Union Européenne, l'exigence en capital de solvabilité, dite « SCR », est schématiquement déterminé pour chaque type de risque couvert par la réglementation, via une estimation des pertes bicentennaires suivie d'une agrégation. Cette évaluation de la solvabilité d'une compagnie d'assurance conditionne la capacité d'exercice de l'activité et peut avoir lieu à tout moment, notamment en dehors de la clôture des comptes. En outre, dans le cadre de la supervision du secteur des assurances ont lieu des exercices d'évaluation du risque des entreprises (eg. *stress tests*, ORSA) et la question de la solvabilité prospective de ces entreprises apparaît naturelle. Dit autrement, en cas de choc, l'entreprise pourrait-elle faire face à ses exigences réglementaires ? Au cours des précédents exercices Stress tests, l'EIOPA, agence européenne de supervision des assurances, a réalisé cette évaluation en considérant constant le SCR, en raison du caractère instantané des chocs étudiés. Cela soulève une question tout autant empirique que théorique : si l'historique des chocs – importants – observés est augmenté d'un élément, comment les quantiles extrêmes sont-ils modifiés ?

Par ailleurs, le SCR est en réalité obtenue par la différence entre, d'une part, son évaluation brute (BSCR, pour « Basic SCR ») – directement liée au risque, donc aux quantiles extrêmes – et, d'autre part, les capacités d'absorption des pertes, ou « LAC » (pour « Loss Absorbing Capacity », c'est-à-dire des facteurs d'atténuation des pertes en cas de chocs notamment via la fiscalité, ou du fait de dispositifs inclus dans les contrats). Dans ce contexte, nous étudions si l'effet d'une réestimation porte plutôt sur la réévaluation de l'exigence brute ou bien sur celles des capacités d'absorption dans le cas de l'adjonction d'un nouveau choc de grande taille.

Résultats

En utilisant un modèle stylisé et linéaire d'évaluation des exigences en capital nous parvenons d'une part à interpréter la décomposition du SCR entre contribution brute et capacités d'absorption et, d'autre part, à justifier l'intérêt de l'approche uni-dimensionnelle à l'aune des données de l'exercice stress test EIOPA 2014. En reformulant le problème de la réévaluation de l'exigence en capital à la suite de la survenance d'un choc important de manière théorique, nous utilisons des techniques d'assurance non-vie (Cat Nat) et de théorie des valeurs extrêmes pour déterminer, en toute généralité, l'impact de l'effet d'adjonction et/ou d'oubli sur l'évaluation des quantiles extrêmes et des queues de distributions de probabilité. Au total, l'effet négatif sur la valeur du portefeuille en cas de choc de marché défavorable est en partie compensé par un choc à la hausse sur les quantiles extrêmes. Nous étudions si le mécanisme que nous avons mis en évidence est vérifié dans la pratique en considérant les stress tests coordonnés par EIOPA en 2014. Nous montrons qu'en fait c'est la baisse des LAC qui conduit à la hausse du niveau du SCR.

1 Introduction

Modern financial regulation frameworks are designed to take into account the actual risks faced by financial institutions. This precision in evaluating the risks comes at a cost since improving accuracy tends to be pro-cyclical. As a response to the potential increase of systemic risk, stress tests have increasingly become a common tool for insurance and banking supervision. In a nutshell, supervisors check the consequences of adverse shocks on the solvency, liquidity and stability assessment of undertakings. Since Basel I, financial regulation is based on the assessment of capital requirement and its coverage by undertakings. In this respect, undertakings would typically undergo assets and own-fund downfall after the simulation of the shock. Some companies pass the test and still hold enough capital after the stress test while some others do not.

This type of financial stability tests are suited for supervision. On the one hand, it helps monitor financial stability on the basis of a horizontal and cross-sectional analysis of individual responses. On the other hand, it can include a forward looking perspective. Moreover, some supervisors almost only rely on the outcome of such exercises. Even if the use of such tests is more recent in the insurance sector than the banking sector, they come more and more on top of the agenda, see for example NAIC and EIOPA’s recommendations arising after such exercises (e.g. [European Insurance and Occupational Pensions Authority, 2014](#)). Different aspects of stress test exercises need to be clarified: why stress testing? How should such exercises be organized to optimize supervision efficiency? How should the scenarios be selected and at which (quantile) level? How should the framework of the exercises be designed, e.g. which simplifying assumptions should be made?

In this study we only focus on the latter aspect with a glimpse on the European insurance stress test since those exercises are part of the more general Solvency II regulatory framework which has become fully applicable since January 2016. Since the CEIOPS quantitative impact studies performed in 2011, a consensus emerged in the European Union insurance supervisory community: the absence of Solvency Capital Requirement (SCR) reassessment after a shock was regarded as a prudent hypothesis. Indeed, it is often believed that the SCR is very likely to be smaller after the stress test is applied than initially, for example after an adverse shock leading to a decrease in the market value of the portfolio. Keeping the SCR constant would therefore correspond to a cautious strategy.

This rationale seems natural when looking at a shock on the financial markets: if stock prices would fall by 40%, say, then a second 40% shock would only correspond to a 24%-decrease with respect to the initial stock price. Besides, some countercyclical measures like the equity dampener¹ may reinforce this phenomenon.

However, as far as natural or man-made catastrophes in P&C risks (“Cat P&C risks”)

¹For more explanations on how the equity dampener is set up, see the consultation paper CP-14-058 on “the proposal for draft Implementing Technical Standards on the equity index for the symmetric adjustment of the equity capital [charge](https://eiopa.europa.eu/Publications/Consultations/EIOPA-CP-14-058_ITS_Equity_dampener.pdf)https://eiopa.europa.eu/Publications/Consultations/EIOPA-CP-14-058_ITS_Equity_dampener.pdf

are concerned, if some extreme scenario occurs, then it is likely that the tail distribution of the corresponding risk has to be re-evaluated. A scenario with a return time of 150 years can, upon occurrence and after re-estimation in the light of the new data, become a scenario with a 90 year return time, as observed empirically by [Mornet *et al.* \(2016\)](#) for storm risk in France. This may of course lead to an increase in the SCR.

In addition, the loss absorbing capacities generated by deferred tax or technical provisions have limitations. After a large adverse event, these capacities may be strongly reduced, and this would lead to an increase in the SCR.

In this paper, we aim at explaining these opposite effects and quantify their combined impacts on the SCR in a simplified model and also with regulatory data. Our contribution is threefold. First, we build the first stylized model for re-estimated solvency ratio in insurance. Second, this leads us to solve a new theoretical problem in statistics: what is the asymptotic impact of a record on the re-estimation of tail quantiles and tail probabilities for classical extreme value estimators? Third, we quantify the impact of the re-estimation of tail quantiles and of loss absorbing capacities on real-world solvency ratios thanks to regulator data from *EIOPA* featuring cases where re-computing leads to an increase in the SCR. Another striking outcome of our study is the importance of loss-absorbing capacity on solvency capital ratios.

Our paper is organized as follows. In Section 2, we explain how the Solvency Capital Requirement (SCR) is computed in Solvency II. In particular, we describe regulatory stress tests and loss absorbing capacity mechanisms. In Section 3, we present our simplified model for SCR re-estimation. Section 4 quantifies the asymptotic underestimation when one neglects a record with a theoretical extreme value analysis point of view. In Section 5, we provide orders of magnitude of the different effects using EU stress test data. In the conclusion, we give some policy implications and we introduce some future research questions.

2 Solvency capital, stress tests and loss absorbing capacity in Solvency II

2.1 Prudential balance sheet of European insurers

In the insurance sector, estimating liabilities can be very tricky since no actual market value exists for in-force businesses. Generally, only model-based valuations are available: producing the balance sheet of an insurer is already a difficult task for life insurers, involving simulations. Technical provisions in the Solvency II framework ([EU Parliament and Council, 2009](#)) consist in an actualization of the projection of cash flows made by the undertaking. The calculation methodologies of the best estimate is defined in the Article 28 of the Delegated Regulation ([Commission, 2015](#)) and is completed in the *EIOPA* guide-

lines on Technical Provisions (European Insurance and Occupational Pensions Authority, 2015). The overall shape of the balance sheet is given by Figure 1.

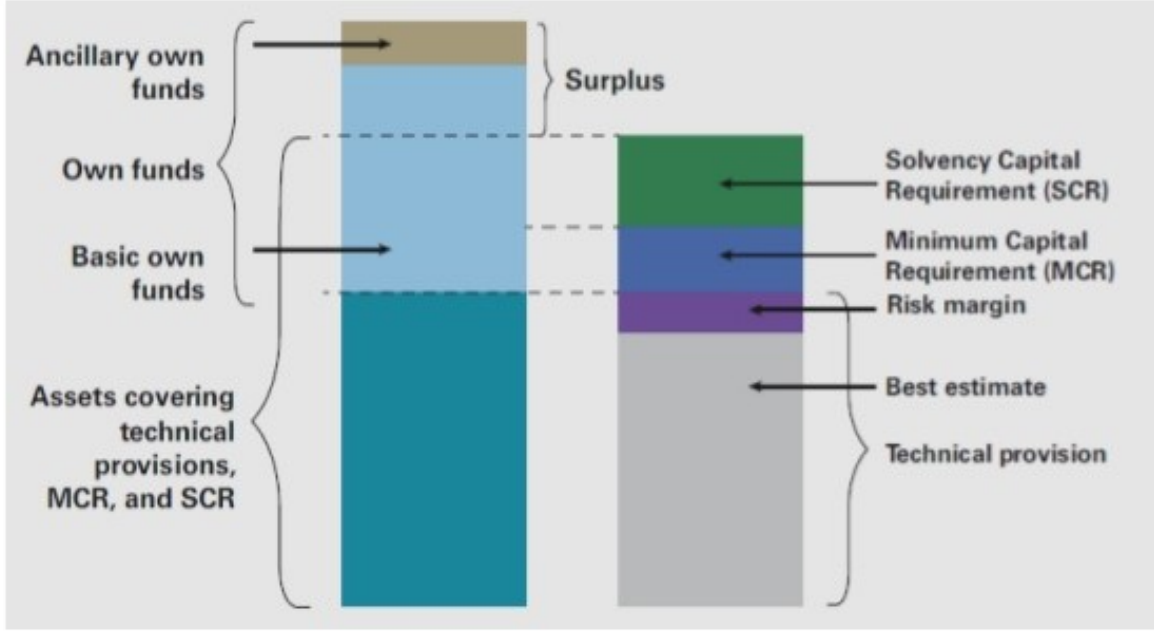


Figure 1: Insurer simplified Balance Sheet (Solvency II) (source: UK actuaries)

The Solvency II directive defines the Solvency Capital Requirement (“SCR”) as the aggregation of different risk modules. In turn, these modules are defined by sub-modules or even sub-sub-modules, see Figure 2.

For each sub-module (or sub-sub-module), a set of risk factors is considered and there exist two methods. Either a formula is used, e.g. the premium risk of the non-life underwriting risk sub-module or the result of a mono factor “stress test”, or an alternative method via VaR determination is employed. This alternative method consists in stressing a parameter (interest rates, stock indices, mortality tables, etc.), up to a 99.5% quantile level, and then compute the net-asset value (“NAV”) to infer the value of the gross and net sub-module depending on whether the different diversifications effects (see “loss absorbing capacities” in Subsection 2.2) are taken into account. The Δ -NAV is actually the difference of Basic Own Funds between the stressed and baseline situations, see Figure 3.

In 2014, EIOPA (“European Insurance and Occupational Pensions Authority”) led a pan-European insurance stress test. This exercise was composed of a core exercise applied to 167 insurance groups of the EU market² which included the 30 largest companies in

²This represents 55% of all gross written premiums. NCAs were allowed to add solo undertakings when unable to reach the 50% threshold with only the groups acting domestically.

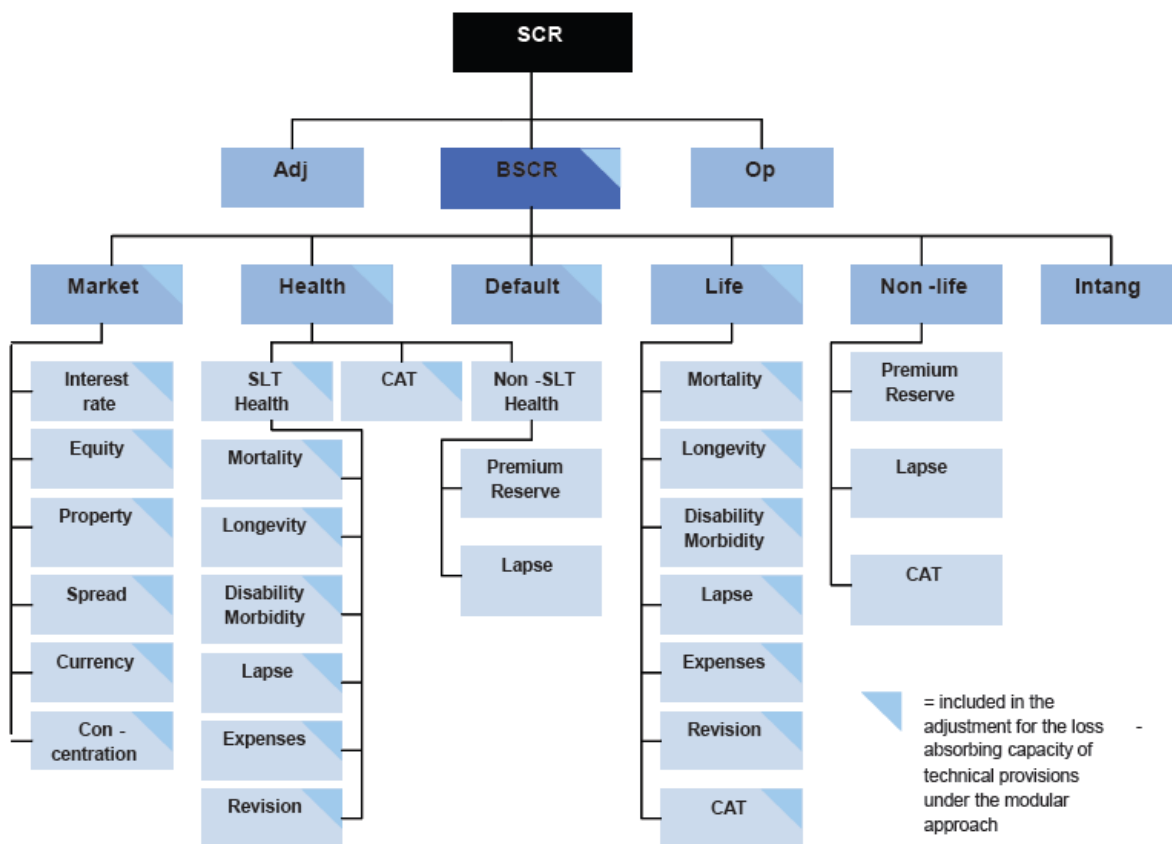


Figure 2: SCR: risk modules breakdown (Source: EIOPA)

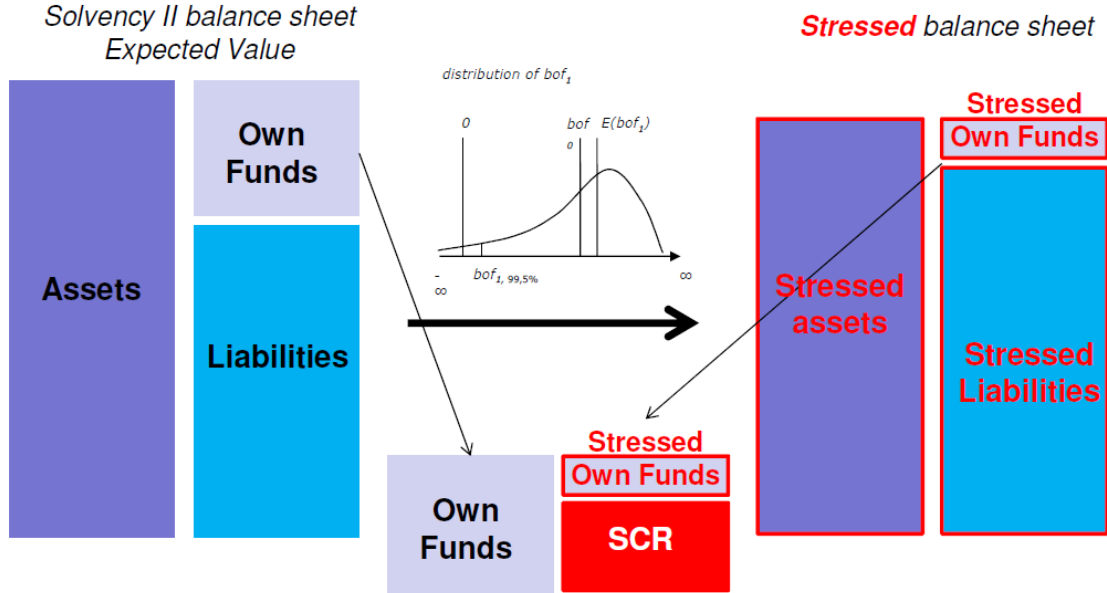


Figure 3: SCR risk sub-modules calculation (Source: ACPR)

Europe. Baseline figures revealed that life technical provisions are predominant within this scope. As a consequence, market risk is actually the most important module in the aggregated SCR, see Figures 4 and 5. For this reason and to simplify the calculations, we will assume henceforth that the insurance company only depends on a single risk factor.

2.2 Loss absorbing capacities

Before the launch of Solvency II, CEIOPS³ was responsible for determining which risk-measure should be best suited to insurance industry⁴. Different approaches were tested for the liability valuation⁵ and already at this level the impact of the future bonuses seemed to be material. The insurance industry is characterized by risk mitigation and so, Solvency II, being risk based, had to take this feature into account unlike Solvency I, which was based on fixed/all-inclusive calculations. In this regard, CEIOPS progressively introduced the concept of “loss absorbing capacity”: at first in the QIS 2 specifications one could find the “risk absorbing proportion of TP_{Benefits} ” or the “risk absorption” property of the future profit sharing only related to the discretionary nature of profit-sharing in almost all jurisdiction:

$$RPS = k \times TP_{\text{Benefits}},$$

³Committee of European Insurance and Occupational Pensions Supervisors, the predecessor of the European authority for insurance supervision, “EIOPA”

⁴The results of this analysis, called “QIS” for Quantitative Impact Studies, can be seen on the EIOPA website: <https://eiopa.europa.eu/publications/qis>

⁵Approaches tested included the best estimate, the 60th, 75th and 90th percentiles, and the company view.

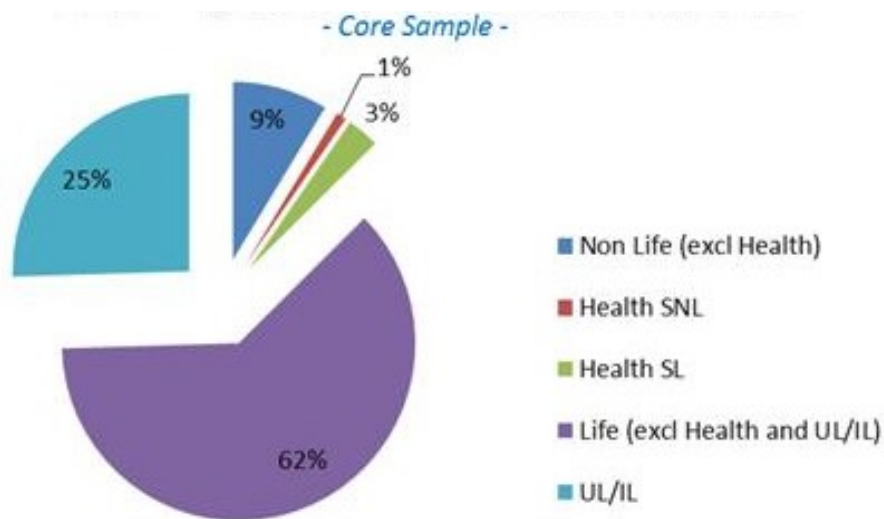


Figure 4: Technical provisions breakdown (source: EIOPA Stress Test 2014)

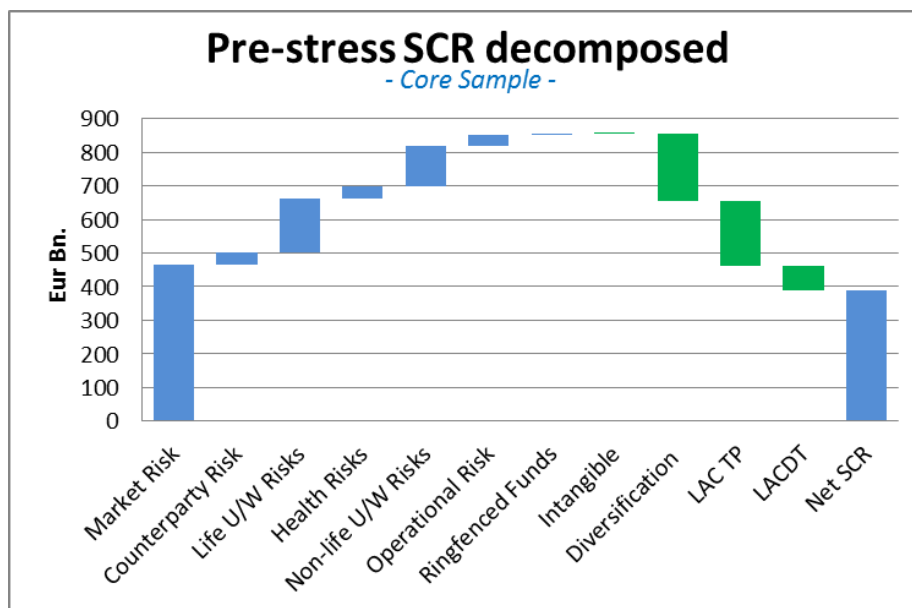


Figure 5: SCR Decomposition (source: EIOPA Stress Test 2014)

assuming a linear relation between the Reduction for Profit-Sharing (RPS) and the technical provisions (TP) which relates to the future discretionary profits, and k was the risk-absorbing proportion of those technical provisions. QIS 3 was only mentioning the “loss absorbing capacity” for the purpose of the valuation of contingent capital but confirmed the key role played by future bonuses granting those mechanisms some “risk absorption” abilities or properties. The QIS 2 linear relation was still mentioned but a more complex mechanism, called “three-step approach” was introduced: for each risk sub-module two calculations should be performed: a net SCR module, denoted by nSCR_{Mod} , and a gross one, denoted by gSCR_{Mod} . The difference, KC_{Mod} , between those two quantities is the “risk absorption ability” at the risk module level:

$$\text{KC}_{\text{Mod}} = \text{gSCR}_{\text{Mod}} - \text{nSCR}_{\text{Mod}}.$$

Then those KCs⁶ needed to be aggregated at the level of the five largest risk modules (Life module, Non-Life, Market, Counterparty and Health) with the same correlation matrices of the sub-risk-modules so as to produce KC_{Life} , KC_{NL} , KC_{Mkt} and so on. In a final step, those coefficients were eventually aggregated with the same correlation matrices as their risk-module counterparts. With this approach, the loss absorbing capacities were not assumed to be directly comparable to a specific balance-sheet element such as the with-profits technical provisions. As a consequence, this modular calculation made it unpredictable to any movement in the balance-sheet, were it on the liability or asset side. The QIS 4 specifications only refined this approach by defining more precisely what “loss absorbing capacities” (LAC) were, whether it be linked to an asset or a liability element, insisting on the role played by deferred tax (LAC_{DT}) and absorbing capacities by the technical provisions (LAC_{TP}). Finally, the Solvency II directive gave legal perspective to the concept of loss absorbing capacity in its Articles 103 and 108; Article 111 let the implementing measures give more details on how to compute those loss absorbing capacities.

As explained in the previous section, the core of the whole prudential balance sheet is the best estimate. For any simulated sample path used for the projection of the liabilities entering in the valuation of the best estimate, an undertaking might gain or lose some risk absorbing ability. As an illustration, in the life business, depending both on the market conditions (interest rates, stock prices, etc.) and on the level of the minimum guarantees granted to the insured, the undertaking running the best-estimate simulation might gain or lose some leeway with respect to the discretionary bonuses. In the end, any of the SCR sub-module (netto) whose calculation depends on a best estimate calculation will strongly be affected by these technical provisions’ absorbing mechanisms. Finally, all those sub-module loss absorbing capacities coming from technical provision or future discretionary benefits are gathered at the level of the SCR to account for a global diversification effect.

How does the mitigation actually work? In QIS 1 and 2, the risk-reduction mechanisms were initially designed and conceived by all the supervisors and regulators as constant elasticities to with-profit participations. In the final version of the regulatory texts, those

⁶KC stands for vector of risk mitigation correlations.

mechanisms are not straightforward especially for the calculation of a modular risk module (scenario-based calculations). At first, the insurance company needs to compute the SCR net of all effects, which means that the amount of the risk-mitigation techniques are taken into account in the different best-estimate evaluations (baseline and module shock) and can change on a sample path basis. Then on a second round one has to evaluate the Gross SCR. For this purpose, all the computations need to be made while assuming only the cash flows coming from the guaranteed benefits are rediscounted when the relevant scenario affects the interest rate term structure. In the gross calculation phase, the cash flows arising from the future discretionary benefits are supposed to be constant.

Considering market risk as an example, the lower the value of the assets, the lower the risk. Besides, after a large financial shock one would expect net SCR sub-modules linked to market risk to decrease when risk exposure decreases so that any SCR reevaluation after a large shock would benefit the undertaking thanks to a proportionality effect.

However, this one-to-one correspondence is not actually observed in the 2014 Stress test data ([European Insurance and Occupational Pensions Authority, 2014](#)): although very few undertakings reassessed their SCR post-stress – less than 30%, the reassessment was optional – a significant share (more than 40%) of the undertakings underwent an increase of their global net SCR in at least one of the market scenarios.

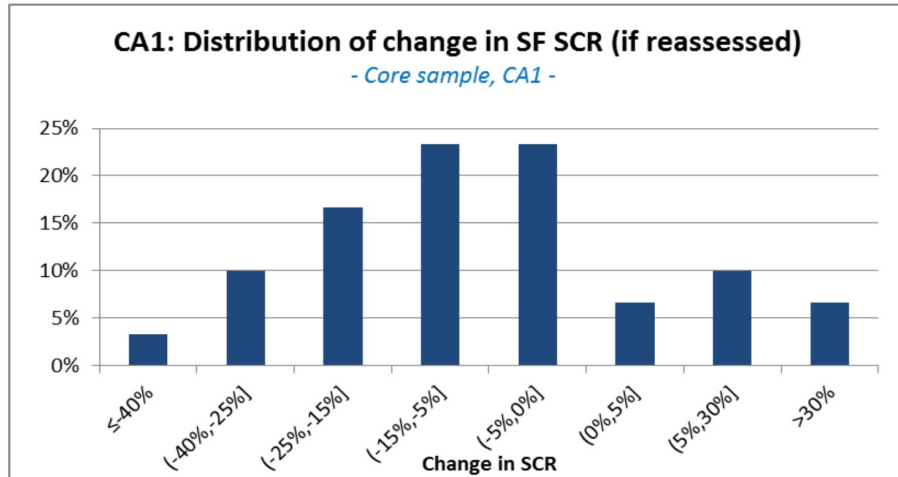


Figure 6: Distribution of reassessed SCR (source: EIOPA Stress Test 2014)

Indeed, taking a closer look at Figure 6, we observe that diversification effects can present some non-linearities, maybe due to the “modular” nature of their estimation. A very naive explanation to this counter-intuitive result could be that the post-stress reduction in the diversification abilities would be more significant than the reduction of risk exposure. Another simple idea would be that the addition of an extreme point changed the global shape of the underlying loss distribution. Interpretations based on both effects are developed in the following sections.

2.3 Stress tests and Solvency II

As explained in Subsections 2.1 and 2.2, most of the SCR risk sub-modules for life under-taking are estimated by means of stress tests on specific risk factors. The stress applied is supposed to represent a 1-in-200 year shock. For example, the Interest Rates sub-module is one of the most important ones as part of the Market Risk module. EIOPA used limited historical data, following four datasets to calibrate the different regulatory shocks to apply:

- EUR government zero coupon term structures (1997 to 2009),
- GBP government zero coupon term structures (1979 to 2009),
- and both Euro and GBP LIBOR/swap rates (1997 to 2009).

With this regulatory framework in place, other risk dimensions or quantiles of different levels are not covered. For this reason, EIOPA can run dedicated exercises complementing the regular Solvency assessment. These exercises allow data collection and analyses to test and measure the resilience and vulnerabilities of the insurance market. In 2014, the EIOPA stress tests ran two exercises: a core exercise, aiming at testing large groups, and a satellite exercise, challenging solos with prolonged low rate environments. In this article we have a closer look at the results of the market shocks of the “Core module”. These were two adverse financial market scenarios designed in cooperation with the ESRB which implemented a so-called “double hit”, meaning both an increase in the value of the liabilities, due to a prolonged low yield environment, and a decrease in the asset values for equity but also for sovereigns with a widening of the spreads⁷.

3 A simplified model for post-stress SCR

In this simplified model, we consider that the SCR is given by

$$\text{SCR} = [\text{VaR}_{99.5\%}(X) - E(X) - b(X)]_+, \quad (3.1)$$

where X is a random variable corresponding to the 1-year random loss the insurer may face. Here, for simplification purposes, we consider only one risk factor, which can be financial or P&C cat. Of course, in the real world, there are many risk factors, aggregated either with the standard formula or by means of an internal model. We shall discuss the impact of diversification on our results in the sequel. The parameter b plays an important role: it corresponds to the loss absorbing capacity, and it is likely to be affected if a large event occurs.

⁷For a more detailed view on the scenarios the description of the scenario can be downloaded on the EIOPA website: https://eiopa.europa.eu/Publications/Surveys/Note_on_market_adverse_scenarios_for_the_core_module_in_the_2014_EIOPA_stress_test.pdf

After a shock, b is transformed into b' and X is transformed into

$$X' = a\tilde{X}, \quad (3.2)$$

where a is a factor accounting for the change in the exposure, and \tilde{X} is the revised version of X after taking the last shock into account.

If one considers mass lapse risk or pandemic risk, then the portfolio size is smaller after the first shock, so that $a < 1$. Similarly, if stock prices go down by 40%, then it is natural to consider $a = 60\% < 1$, even in absence of countercyclical measures. For P&C disasters, the situation is less clear: on the one hand, some buildings might be partly or fully destroyed, which makes the exposure temporarily decrease ($a < 1$) as there is less to be potentially destroyed by a second event. On the other hand, a first event might also cause some frailty and make the consequences of a second event potentially more severe, for example in the case of floodings or earthquakes where some cumulative effect or some replicas may be disastrous ($a > 1$).

If an event like a major, unprecedented earthquake, hurricane or terror attack occurs, then the probability and potential severity of such an event will automatically be re-evaluated by cat models like RMS, EQECAT or AIR or by internal models, following Bayesian techniques. For most events, the impact on high-level Value-at-Risk is very likely to be much more important than the impact on the average. Therefore, we model this as a change from $\text{VaR}_{99.5\%}(X)$ to $\text{VaR}_{99.5\%}(\tilde{X})$, but for the sake of simplicity we do not update the average, considering that the impact on the average can be neglected: we assume that $E(X) = E(\tilde{X})$.

Of course, this assumption might be inappropriate in some cases, particularly for regime switching models like 3-state Hardy stock models or self-excited processes, in which the best estimate and the volatility tend to move in adverse directions when things go bad, and for mean-reverting models, where some mitigation is present when things go bad. For some other risks like sovereign risk or foreign exchange risk, some shocks may occur as jumps (CHF/EUR exchange rate in January 2016). The two types of risks that we consider in this paper, market shocks and large P&C claims, are thus both relevant for our study.

The parameter b , accounting for the loss absorbing capacity, can be transformed into b' after a large event for several reasons. The loss absorbing capacity thanks to differed tax and thanks to technical provisions is not infinite, and it may happen that the new loss absorbing capacity after a large event is much smaller than before, that is, $b' \ll b$.

Besides, reinsurance might become too costly, reinstatements might be exhausted, or protection from Insurance Linked Securities could be strongly imputed, which would again lead to $b' < b$ for other reasons. Even if we focus here on loss absorbing capacity in the Solvency II framework, the analysis that we develop could be adapted to loss absorbing capacity through risk transfer in a more general Enterprise Risk Management approach.

In contrast, some countercyclical mechanisms like the equity dampener might have a favorable impact by reducing the SCR if a downward shock occurs. This tends to increase

the value of b' .

For the sake of simplicity, we have focused on a single risk factor. Nevertheless, due to the complexity of risk aggregation and diversification, a large event might affect the diversification benefit if one risk becomes smaller or larger than other ones. The fact that part of the diversification benefit disappears has of course a negative impact on the SCR. On the other hand, one has already profited from mitigation of the initial shock thanks to diversification.

Three effects are present: the ones of a in (3.2), of b in (3.1), as well as of the tail quantile re-estimation. From a theoretical point of view, the impact of the first two ones is quite straightforward. The tail re-estimation effect, however, has not yet been studied in the literature and is a bit more technical. Therefore, in the next section, we quantify the change from $\text{VaR}_{99.5\%}(X)$ to $\text{VaR}_{99.5\%}(\tilde{X})$ after a record occurs in a P&C framework, in absence of loss absorbing capacity and for $a = 1$. As this is currently not taken into account, we formulate this as the underestimation of high quantiles when one ignores the record that has just occurred.

4 Pre-record estimation bias of tail estimators

4.1 Notation and framework

We take a P&C view on the random loss X underlying the SCR calibration. Let X_1, X_2, \dots be i.i.d. random variables corresponding to observations of X . For simplicity, assume that their common distribution is continuous. Denote the ascending order statistics of X_1, \dots, X_n by $X_{n:1} < \dots < X_{n:n}$.

Consider statistics of the type

$$T_n = t_n(X_1, \dots, X_n),$$

where $t_n : \mathbb{R}^n \rightarrow \mathbb{R}$ is a permutation invariant function. Think of T_n as an estimator of some tail-related quantity: a tail quantile, a return level, \dots . The statistic T_n depends on the data only through the order statistics:

$$T_n = t_n(X_{n:1}, \dots, X_{n:n}).$$

We want to understand the consequences of not reestimating the risk distribution in a stress test associated to an extreme shock. We focus on the case where the shock is unprecedented: the very recent loss corresponds to a record, like for example the Bar-le-Duc claim in 1976 for motor third party liability or Lothar in 1999 for storm risk in France. In practice, such events might be relevant for different sub-risk-modules of Solvency II (underwriting, cat, \dots) and their impact might be diluted with attritional claims during the year. To simplify, we assume here that X corresponds to the random variable whose

quantile is used to derive the Solvency Capital Requirement. We assume that at a given time instant, a *record* occurs: the new observation is larger than what has been observed before. When should we compute the statistic: right before or right after the record?

First, assume that the record occurs at “time” n , that is, $X_n > X_{n-1:n-1}$, or, in other words, the rank of X_n among X_1, \dots, X_n is equal to n . At a given sample size, the vector of order statistics is independent of the vector of ranks. We find that

$$[T_n \mid X_n > X_{n-1:n-1}] \stackrel{d}{=} T_n. \quad (4.1)$$

That is, computing the statistic right *after* a record does not lead to any distortion.

Second, assume that we compute the statistic right *before* a record occurs. Specifically, suppose that X_{n+1} is a record: $X_{n+1} > X_{n:n}$. How does the occurrence of that event affect the distribution of T_n ?

If X_{n+1} is a record in the stretch X_1, \dots, X_{n+1} , then $X_i < X_{n+1}$ for all $i = 1, \dots, n$, and the vector of order statistics $(X_{n:1}, \dots, X_{n:n})$ is equal to the vector $(X_{n+1:1}, \dots, X_{n+1:n})$. It follows that

$$[(X_{n:1}, \dots, X_{n:n}) \mid X_{n+1} > X_{n:n}] \stackrel{d}{=} (X_{n+1:1}, \dots, X_{n+1:n}). \quad (4.2)$$

Equation (4.2) implies that

$$[T_n \mid X_{n+1} > X_{n:n}] \stackrel{d}{=} t_n(X_{n+1:1}, \dots, X_{n+1:n}). \quad (4.3)$$

Computing the statistic right *before* the occurrence of a record has a clear impact on its distribution: compare (4.1) and (4.3).

The size of the effect depends on the function t_n . If T_n is a tail estimator, then the impact of omitting the largest observation could be potentially quite large. We work out two relevant cases for our initial problem in the following subsections.

4.2 Tail probability estimation error

We first investigate the question of tail probability re-estimation. After an extreme event, the CEO of an insurance company could ask the cat-modeling team: “What is the return period of yesterday’s event?”. The cat-modelers could in fact reply: “Well, two days ago I would have answered 200 years (tail probability $1/200$), but today I’d rather say 120 years!”. One can imagine the CEO’s reaction...

The following example quantifies the change in the tail probability estimate.

Example 1 (Tail probability). Let u be a high level. Aim is to estimate the tail probability $p = 1 - F(u)$. Note that the return level is equal to $1/p$. The simplest possible estimator is the empirical one,

$$T_n = \frac{1}{n} \sum_{i=1}^n I(X_i > u).$$

Clearly, the estimator is unbiased:

$$\mathbb{E}[T_n] = p.$$

However, if we ignore the information that at time $n + 1$, a new record occurred, then

$$\begin{aligned} \mathbb{E}[T_n \mid X_{n:n} < X_{n+1}] &= \mathbb{E} \left[\frac{1}{n} \sum_{i=1}^n I(X_{n+1:i} > u) \right] \\ &= \mathbb{E} \left[\frac{1}{n} \sum_{i=1}^{n+1} I(X_{n+1:i} > u) - \frac{1}{n} I(X_{n+1:n+1} > u) \right] \\ &= \frac{n+1}{n} p - \frac{1}{n} \{1 - (1-p)^{n+1}\}. \end{aligned}$$

The expected relative error is therefore

$$\frac{1}{p} \mathbb{E}[T_n \mid X_{n:n} < X_{n+1}] - 1 = \frac{1}{n} - \frac{1 - (1-p)^{n+1}}{np}.$$

If $u = u_n \rightarrow \infty$ in such a way that $np = np_n = n\{1 - F(u_n)\} \rightarrow \tau \in (0, \infty)$, i.e., if $p \sim \tau/n$, then the expected relative error converges to a nonzero limit:

$$\frac{1}{p} \mathbb{E}[T_n \mid X_{n:n} < X_{n+1}] - 1 \rightarrow -\frac{1 - e^{-\tau}}{\tau}, \quad n \rightarrow \infty. \quad (4.4)$$

The asymptotic expected relative error is negative and depends on the limit of the expected number of exceedances, τ , over the level u .

4.3 Tail-quantile error estimation

The fact that a 200-year event might become a 120-year event implies that the new 200-year event is much more severe after the extreme event. Motivated by the SCR re-estimation question, we now investigate the impact of a record on tail-quantile estimators.

Example 2 (Tail-quantile estimator). Let Q be the quantile function of F . The aim is to estimate a tail quantile, $Q(1-p)$, where the tail probability, $p \in (0, 1)$, is small. Assume that F is in the domain of attraction of the Fréchet distribution with shape parameter $\alpha \in (0, \infty)$. We will only use classical tools of extreme value theory. The interested reader may consult for example the book of [Beirlant et al. \(2006\)](#) for a presentation of the Fréchet domain of attraction. Let $\gamma = 1/\alpha$ be the extreme-value index. Let $k \in \{1, \dots, n-1\}$ be such that $p < k/n$. A common estimator is based on the approximation

$$Q(1-p) \approx Q(1-k/n) \{(k/n)/p\}^\gamma.$$

On a logarithmic scale, the estimator takes the form

$$\log \hat{Q}_{n,k}(1-p) = \log X_{n:n-k} + \hat{\gamma}_{n,k} \log \{(k/n)/p\}, \quad (4.5)$$

where $\hat{\gamma}_{n,k}$ is an estimator of the extreme-value index γ , for instance the Hill estimator

$$\hat{\gamma}_{n,k} = \frac{1}{k} \sum_{i=1}^k \log X_{n:n-i+1} - \log X_{n:n-k}. \quad (4.6)$$

(We implicitly assume that $X_{n:n-k} > 0$.)

Combining (4.5) and (4.6), we find that the tail quantile estimator is linear in the order statistics $Y_{n:n-k} < \dots < Y_{n:n}$, where $Y_i = \log X_i$. Identity (4.3) then permits in principle to calculate its conditional distribution on the event that $X_{n:n} < X_{n+1}$:

$$\begin{aligned} & [\log \hat{Q}_{n,k}(1-p) \mid X_{n:n} < X_{n+1}] \\ & \stackrel{d}{=} \log X_{n+1:n-k} + \left(\frac{1}{k} \sum_{i=1}^k \log X_{n+1:n-i+1} - \log X_{n+1:n-k} \right) \times \log\{(k/n)/p\}. \end{aligned}$$

To evaluate the impact of ignoring a known record, let us compute the expectation of the estimator under the simplifying assumption that the random variables X_i are iid Pareto with shape parameter α , that is, $F(x) = 1 - x^{-\alpha}$ for $x \geq 1$. Equivalently, the random variables Y_i are iid Exponential with expectation equal to γ . In that case, $\log Q(1-p) = \gamma \log(1/p)$. A well-known representation of the order statistics from an exponential distribution yields

$$\mathbb{E}[Y_{n:n-j+1}] = \gamma \left(\frac{1}{n} + \frac{1}{n-1} + \dots + \frac{1}{j} \right), \quad j \in \{1, \dots, n\}. \quad (4.7)$$

Equation (4.7) yields the following expressions for the expectation of the estimator of the log tail quantile. Unconditionally, we have

$$\mathbb{E}[\log \hat{Q}_{n,k}(1-p)] = \log Q(1-p) + \gamma \left(\frac{1}{n} + \dots + \frac{1}{k} - \log(n/k) \right).$$

The second term on the right-hand side converges to zero relatively quickly. In contrast, conditionally on the occurrence of a record on the next day, we have

$$\begin{aligned} & \mathbb{E}[\log \hat{Q}_{n,k}(1-p) \mid X_{n:n} < X_{n+1}] \\ & = (1 - a_k) \log Q(1-p) + \gamma \left(\frac{1}{n} + \dots + \frac{1}{k} - (1 - a_k) \log(n/k) \right), \end{aligned}$$

where

$$a_k = \frac{1}{k} \sum_{j=1}^k \frac{1}{j+1}.$$

The sequence a_k tends to zero as k tends to infinity: $a_k \sim \log(k)/k$ as $k \rightarrow \infty$. Still, since the relative error occurs on the logarithmic scale, there is potentially a severe under-estimation of the tail quantile: indeed, we have $(1 - a_k) \log Q(1-p) = \log[\{Q(1-p)\}^{1-a_k}]$.

The relative error is thus given by $\{Q(1-p)\}^{-a_k} = (1/p)^{a_k\gamma}$. The larger the tail index γ and the smaller the tail probability p , the larger the relative error. The result remains valid for the more general Pareto distribution $F(x) = 1 - (x/\sigma)^{-\alpha}$ for $x \geq \sigma$, where $\sigma > 0$ is a scale parameter.

In the next section, we investigate the concrete impact of this phenomenon and of two other ones, risk exposure reduction and decrease in diversification elements, on real-world insurance regulatory capitals.

5 Illustration with real-world situations

In this section, we calibrate the three effects following two approaches: the first approach is related to actual risk levels used in financial regulations and the second one using the 2014 EIOPA stress test data. We first provide orders of magnitude of the re-estimation effect on SCR in the insurance industry, in absence of loss absorbing capacity effect and for $a = 1$ in (3.2). Then, motivated by the design of the market risk SCR, we investigate the case where $a = 0.6$ and calibrate b in (3.1) and b' from real data. Finally, we study the case where $a > 1$ and we identify regions where one effect dominates the other one. On top of these empirical illustrations, we highlight the problem of the risk margin valuation, which strengthens our main conclusions on the SCR with a view on the whole prudential balance sheet.

5.1 Tail re-estimation effect

Parameter τ – Tail probability estimation error. In the case of a natural catastrophe, the expected number of high-threshold exceedances, $\tau = np$, belongs to a broad range of values. In the case of a stress test, τ is close to 0. It is quite common to consider $\tau = 1$ over 200 theoretical observations which is a typical target used in the Solvency II framework ($n = 1$, $p = \frac{1}{200}$).

The formula (4.4) for the expected relative error of the estimated exceedance probability due to the omission of the most recent record value as a function of τ is illustrated in Figure 7. We see that $\tau = 10$ exceedances already give a 10% misvaluation of the tail probability. If the expected number of exceedances decreases to $\tau = 1$, the relative estimation error goes up to 63%.

These numbers highlight the impact of the pre-record estimation bias. The effect is striking but cannot account for the error on the SCR, which is expressed on the quantile scale. We now consider the quantile error.

Parameter γ – Quantile estimation error. As a first-order approximation we can use the formula illustrated in Subsection 4.3 for the expected negative relative error of the

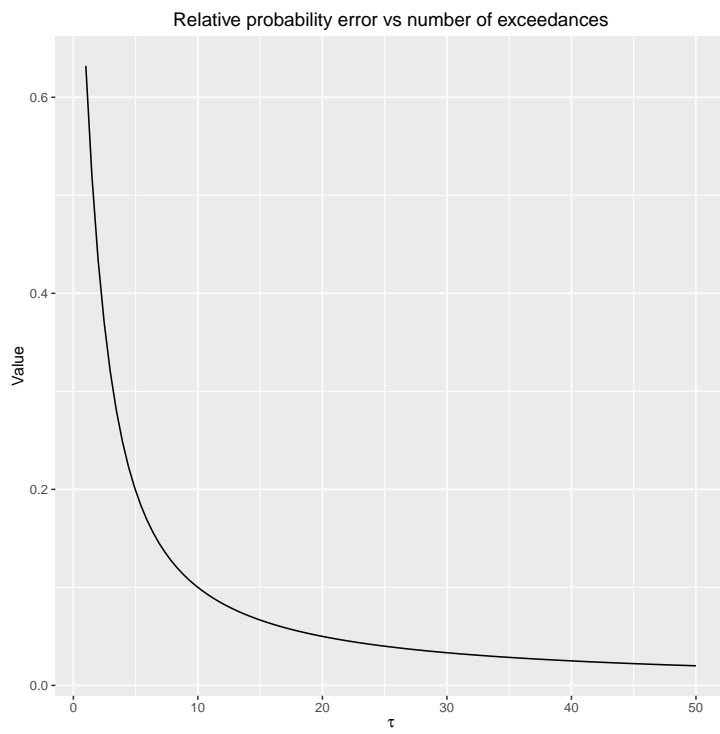


Figure 7: Relative probability error vs number of exceedances estimated with (4.4)

quantile estimate:

$$\delta_{p,k,\gamma} = (1/p)^{a_k \cdot \gamma} \quad (5.1)$$

with $a_k \approx k^{-1} \log(k)$.

In the context of the Solvency II, p should be equal to 0.005. For k , different values are plausible; the natural framework in Solvency II should be $k = 200$, since the current norm sets records up to 200 years of magnitude. In a Stress Test context, values of k in the range of 5 to 50 are also admissible.

As illustrated by the graphs in Figure 8, the difference between the actual quantile and its value just after the addition of a shock with magnitude γ times the expectation of the standard shocks can be very significant. For example, even with 200 records, the addition of an event 10 times larger than expected would lead to a quantile more than twice the initial value! Note that we implicitly assimilated the change in the estimated 99.5% VaR to the change in the SCR. This is not true in general as the SCR might be defined in a more complex way. Besides, the Best Estimate of Liabilities would also be impacted. Nevertheless, for reasonable values of n and k , the change in the estimated average of X is small in comparison to the change in the 99.5%-Value-at-Risk level. Therefore, for simplicity, we assume here that the Best Estimate of Liabilities can be neglected in this first study, and we leave it for further research to quantify the change in the best estimate.

In Figure 9, we focus on operational risk for banks, for which banking regulation imposes to compute the 99.9%-quantile of the one-year loss. Nešlehová *et al.* (2006) show that for banking operational risk, one cannot exclude that $\gamma > 1$, corresponding to infinite mean models. We therefore consider the impact of quantile re-estimation after a record: for finite mean models with γ close to 1, Figure 9 shows that the new result might be as large as 2.8 times the result without re-estimation. This shows that the phenomenon presented here deserves further research regarding banking supervision.

However, this first effect actually accounts only for changes in something equivalent to the gross BSCR (the “quantile error”) before diversification (not considered here). Let us now investigate the concrete effect of Loss Absorbing Capacity on the net SCR.

5.2 The case $a < 1$

The naive model introduced in Section 3 can be calibrated with the 2014 stress test data. An identification of the different terms on the right-hand side of (3.1) implies that $\text{VaR}_{99.5\%}(X) - E(X)$ is equal to the gross BSCR (adding Operational risk, denoted by gBSCR) and b is the sum of the different diversification and loss absorbing mechanisms, in particular the Loss-Absorbing Capacity with Technical Provisions and with Deferred Tax.

In absence of quantile re-estimation, after the shock, X becomes $X' = aX$ and the SCR becomes: $\text{SCR}(X') = a \times \text{gBSCR} - b'$. With this simplified setup, it appears very clearly why the risk could not depend on the scaling factor a and only on the potential increase of volatility of the profit and loss distribution. At this point, we emphasize that the desired

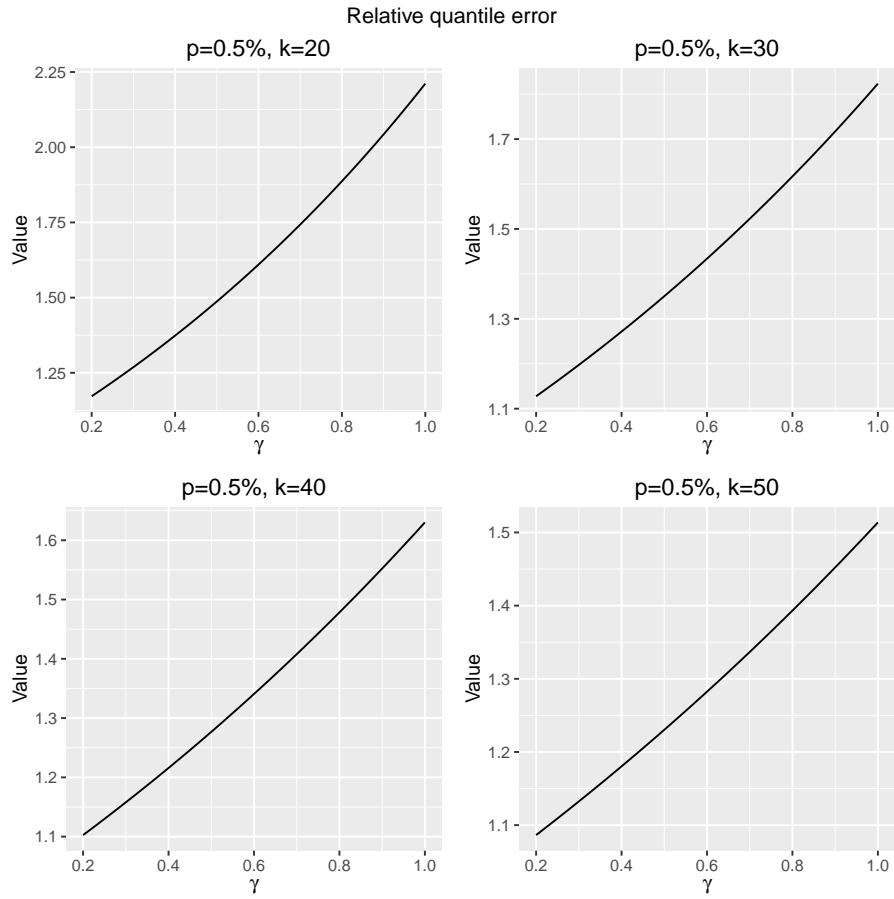


Figure 8: Relative quantile estimation error vs relative expected magnitude as a function of parameter γ for different values of p and k using equation (5.1)

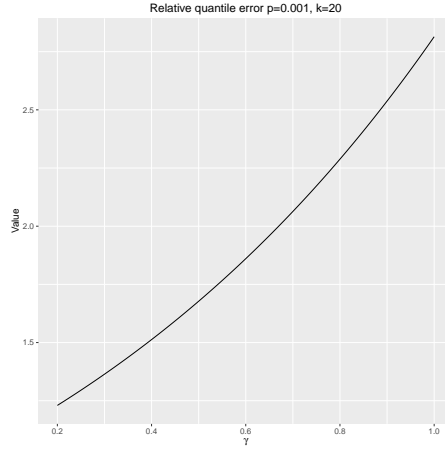


Figure 9: Relative quantile estimation error vs relative expected magnitude as a function of parameter γ (for $p = 0.001$ and $k = 20$) using equation (5.1)

quantile is not directly based on the exposure so that there might only exist a tenuous link between the risk exposure and the loss distribution.

The gross SCR is multiplied by a when $X' = aX$. Note that this property is very general and remains valid when the Solvency Capital is defined via a Tail-Value-at-Risk as in the Swiss Solvency Test, or when one uses any distortion risk measure for economic capital in Enterprise Risk Management. This positive homogeneity property is also valid in the practical approach adopted during the genesis of Solvency II: practitioners often approximate $\text{VaR}_{99.5\%}(X)$ with $E(X) + c\sigma_X$, where σ_X is the standard deviation of X and $2.5 \leq c \leq 5$ is a multiplier close to 3 in the lognormal case and closer to 4 or 5 for loss distributions with heavier tails.

To illustrate this setup, we create a company with 100M€ total balance sheet representative of the ST2014 data⁸ see [European Insurance and Occupational Pensions Authority \(2014\)](#) and [European Insurance and Occupational Pensions Authority \(2017\)](#) for the gross figures.

First remark: the diversification and loss absorbing mechanisms is larger than the net SCR in the CA1 case and more than 50% in CA2. This demonstrates its importance in the Solvency II framework. Another important consequence is that the variance of the profit & loss distribution plays a far greater role than the market risk exposure. Indeed, the a factor does not show up in the final estimation of the SCR. If we make another assumption

⁸The different prudential quantities in the table are computed from the companies which reassessed their SCR post-stress and had a positive increase in one of the financial stress scenarios, namely CA1 and CA2.

in M€	CA1	CA2
Liabilities	100	100
gBSCR	7.78	8.39
b	4.51	2.99
Net SCR	3.26	5.39

Table 1: Toy company, pre-stress situation for CA1 or CA2 scope of increase SCR reassessment's groups (source: ST 2014 figures)

in M€	ST ($a \approx 0.93$)	$a = 0.9$	$a = 0.8$
Liabilities'	95.1	92	81.8
gBSCR'	7.39	6.65	5.91
b'	3.6	2.86	2.12
Net SCR'	3.79	3.79	3.79

Table 2: Toy company, CA1 post-stress situation (source: ST 2014 figures, authors' calculations)

and assume a perfect correlation between market exposure and the P&L, we would get:

$$\text{SCR}(X') = a \times \text{gBSCR} - b',$$

with⁹ $a = 0.6$. In this simple model, the pre-stress net and gross SCR shown in Table 1 evolve¹⁰ after the stress as presented in Tables 2 and 3.

In fact, $a = 0.6$ corresponds to the pure shock for stocks and their spillovers. But given other risk modules and diversification and loss absorbing mechanisms it might be more consistent to choose¹¹ $a = 0.8$ or $a = 0.9$.

For the completeness of the analysis, the value of b' is deduced with the following equation (for $a = 0.9$ in the CA1 case, in M€):

$$b' = \text{gBSCR}' - \text{Net SCR}' = 6.65 - 3.79 = 4.45 = 0.63 \times b.$$

Also interesting is the joint evolution of TP and DT after a shock. In our simple model, b actually represents the sum of the diversification module, the LAC_{TP} and the LAC_{DT} .

⁹This corresponds to a 40% decrease of the value of stock, comparable to the shock of the first scenario of the 2014 Stress test.

¹⁰The value of the LAC post-stress and gBSCR' were not requested in the Stress Test exercise but could be reconstituted.

¹¹As an illustration, the value was $a = 0.93$ for the companies in the ST2014 first financial scenario ("CA1") sample used here.

in M€	ST	($a \approx 0.95$)	$a = 0.9$	$a = 0.8$
Liabilities'		93.4	90.4	80.3
gBSCR'		7.83	7.05	6.26
b'		1.83	1.05	0.27
Net SCR'		6.00	6.00	6.00

Table 3: Toy company, CA2 post-stress situation (source: ST 2014 figures, authors' calculations)

	CA1	CA2
b'/b	79.8%	61.2%
FDB' / FDB	8.5%	13.9%
DT' / DT	310%	286%

Table 4: Toy company, evolution of absorption capacities with constant diversification (source: ST 2014 figures and authors calculations)

Future discretionary benefits ("FDB") are a good proxy of the absorption capacities arising from technical provisions. The risk profile of the company does not instantaneously change because of the financial shock so that taking for granted a constant diversification through the shock is not a bold assumption. Moreover, a repartition 45%/45%/10% for Div / FDB / DT is in line with 5. In this context one can infer the proportion of DT after shock (see Table 4). Strikingly, on the smaller scope of groups which reassessed their SCR post-stress, the FDB decrease of 91.5% in the CA1 or 86.1% in the CA2. The effect illustrated in the previous parts empirically is quite substantial and our model could not be too conservative if FDB were to be used as a proxy for b .

As discussed in subsection 2.2, we observe in this simple example that the different diversification and loss absorbing mechanisms had to decrease much faster than the risk exposure. Regarding of their importance in the final assessment of an insurer's solvency, an estimation of the SCR and at least the different LAC component should be mandatory in any forward looking exercise (ORSA, Stress test, etc.) depending on their relevance to the scenarios. More generally, credibility of the different diversification modules should be checked thoroughly and be part of the annual risk review of any insurance supervisor.

5.3 The case $a > 1$

The case $a > 1$ corresponds to the situation where the risk exposure increases after the shock: for example after a first earthquake or some floodings, the next event might have more severe consequences if it occurs soon, because some buildings have become more

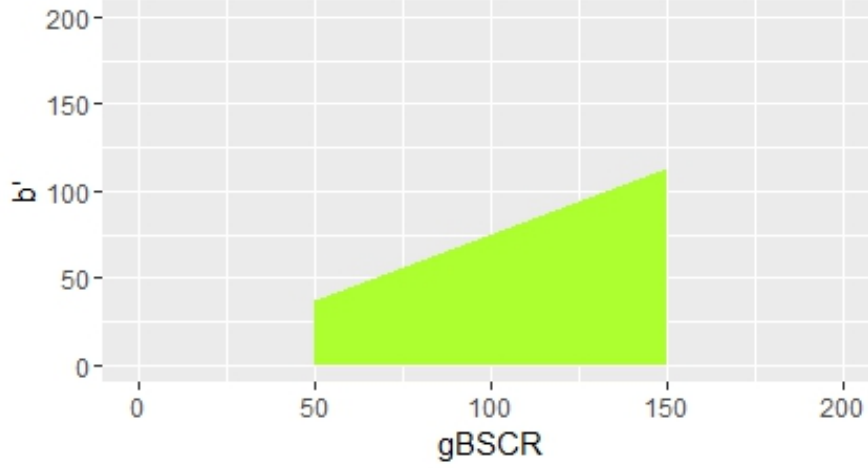


Figure 10: b' value with a positive increase of the net SCR

fragile or because the soil is already saturated with water. Another such situation, in the life insurance business, may occur in the case of mass non-lapse phenomenon, where remaining policyholders are more numerous than expected, for example if they benefit from a high guaranteed minimum interest rate in a low or negative interest rate context.

To illustrate this point, we choose for b a market average and $a = 1.2$. So far, this figure has been provided as a percentage of the aggregate basic solvency capital requirement both for the participants of the 2014 EIOPA ST ([European Insurance and Occupational Pensions Authority, 2014](#)) and their French counterparts ([Borel-Mathurin and Gandolphe, 2015](#)). The absorption capacity is $b = 38\% \times \overline{\text{gBSCR}}$ (resp. $b = 61\% \times \overline{\text{gBSCR}}$) for the whole setup of European groups participants (resp. the French groups), where we averaged over the corresponding samples. For values of gross BSCR ranging from 50% to 150% of the market average gross SCR, we plot in Figure 10 the sub-regions of the half-plane (b' , gross BSCR) where the re-evaluated SCR is larger than the initial one.

5.4 A potential scissors effect on SCR Coverage ratio

The surge in the post-stress SCR can also have unexpected consequences for solvency capital coverage ratios. A by-product of this SCR increase is the coincident effect on the Risk Margin (RM). Indeed, right after the stress, the $\text{SCR}(0+)$ or $\text{SCR}'(0)$ is the new basis for the calculation (assuming the cost of capital, CoC, generally set to 6%)¹²:

$$\text{RM}' = \text{CoC} \sum_{t \geq 0} \text{SCR}'(t).$$

To determine $\text{SCR}(t)$, one may either project at each timestep t and make a complete Best Estimate (BE) determination in future time, or use one of the simplified methodologies,

¹²Here, a simplified version is presented.

for example the proportional approach based on the Best Estimate¹³. We can then infer the future value of the SCR:

$$\text{SCR}(t) = \text{SCR}(0) \times \frac{\text{BE}(t)}{\text{BE}(0)} \quad \text{and} \quad \text{RM} = \text{CoC} \times \text{SCR}(0) \sum_{t \geq 0} \frac{\text{BE}(t)}{\text{BE}(0)}.$$

Assume that the proportions stay constant after stress, so that

$$\forall t \geq 0, \quad \frac{\text{BE}'(t)}{\text{BE}'(0)} = \frac{\text{BE}(t)}{\text{BE}(0)}.$$

Then

$$\text{RM}' = \text{CoC} \times \text{SCR}'(0) \sum_{t \geq 0} \frac{\text{BE}'(t)}{\text{BE}'(0)} = \frac{\text{SCR}'(0)}{\text{SCR}(0)} \text{RM}.$$

Therefore, the technical provisions would actually increase with even a quasi-constant $\text{BE}(0^+)$. In this context, the SCR coverage ratio would deteriorate even more since the SCR would increase and the available own funds would actually decrease as risk margin increases.

6 Conclusions and policy implications

The Solvency II framework is characterized by the estimation of loss quantiles based on historical data. This framework allows for diversification and loss absorbing mechanisms and absorption capacities, that is, the ability to transfer future risk to the policyholders. In this paper, we studied the implications of the records of large losses on the one hand and, on the other hand, the magnitude of diversification elements of the prudential balance sheet such as loss absorbing capacities using deferred taxes or the technical provisions. We computed the bias of estimators of tail probabilities and high quantiles of the loss distribution if the estimation is done immediately prior to the occurrence of a new record loss. We also proposed a stylized model to reassess the solvency capital requirement after a large record. The calibration using the data of the participants to the 2014 EIOPA Stress test confirms our theoretical arguments and showed the very prominent role of the loss absorbing capacities in the Solvency II framework. Based on our data and as far as our estimations are concerned, the decrease in the reassessment of the solvency capital requirement is in the range of 23% to 74%. One of the regular criticism addressed to the Solvency II framework is the one-year horizon used for the quantile calculations, as it could produce a lack of stability in the determination of the solvency capital requirement. In this regard, our work stresses the volatility-inducing potential of the absorption capacities. This feature emphasizes the importance of the future management actions and other means of diversification and risk mitigation while calculating the Best Estimate of the liabilities.

¹³There exist four different methods classified by level of simplification defined in Guidelines 61 and 62 in [EIOPA-BoS-14/166]. Here the fourth method is used for illustration.

Implications of our paper could have four facets: research, Enterprise Risk Management, supervision and regulation. As far as research is concerned, one might want to look ahead to a more advanced framework with a multi-dimensional setup. Insurance companies potentially undergo shocks from different risk factors simultaneously, the aggregation of which would introduce other effects to model. Another direction could be the use of these ideas in the banking sector, e.g. the calculation of the capital charge with VaRs such as Market risk in the Basel III framework.

Insurers, reinsurers and captives should take into account the impact of large events on their future ability to continue business. This study shows that re-evaluating the SCR after a shock should be part of a sound Enterprise Risk Management approach of risk measurement, risk controls and risk appetite determination.

The supervision duties should be modified in comparison to what was done in the Solvency I framework. Even in the standard formula, many levers exist and can be used while producing the prudential balance sheet. In this context, supervisory work should integrate the credibility checking of the projection hypotheses. Regarding prospective exercises, be it by the firm (e.g. ORSA) or the regulator (e.g. Stress Tests), we strongly recommend to always check the evolution of the solvency capital requirements after the occurrence of a shock, since letting these requirements remain constant cannot always be seen as a conservative assumption. Indeed, we showed in this paper that the risk exposure reduction does not necessarily decrease the value of the solvency capital requirement. In this regard we would strongly recommend that future exercises do not only specify the asset side but also the liability side and give guidance on the level of risk transfer to be operated with the technical provision.

Regarding banking supervision, our theoretical analysis and Figure 9 show that the re-estimation of the quantile of the operational loss is a very important question and deserves further research.

Finally, the regulatory bodies might have a closer look at the question of the accurate level of the chosen quantile and how to account for post-stress re-evaluation in the Solvency II framework. Such a study may motivate better ways to assess the prudential balance sheet figures estimations (SCR, MCR, etc.). A plausible response could consist of regulatory prescriptions such as floors or caps on the levels of the different diversification and loss absorbing mechanisms (LAC DT/TP, reinsurance or derivatives, and others). Moreover, the dynamic nature of capital requirements argues for simple multi-period stress tests instead of instantaneous ones.

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