

The Forced Safety Effect: How Higher Capital Requirements Can Increase Bank Lending

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Abstract

Government guarantees generate an implicit subsidy for banks. Even though a capital requirement reduces this subsidy, a bank may optimally respond to a higher capital requirement by increasing lending. This requires that the marginal loan generates positive residual cashflows in the states of the world where the bank just defaults. Since an increase in the capital requirement makes the bank safer, it makes the shareholders internalise such cash flows. We dub this mechanism, which we argue is empirically relevant, the *forced safety effect*.

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1 Introduction

Since the global financial crisis, bank capital requirements have been substantially tightened.¹ The merits of these reforms have been fiercely debated. Higher capital requirements, the typical refrain goes, raise banks' costs of funds, thereby reducing credit provision and dampening economic activity.² However, the academic literature has pointed out that bank capital is unlikely to be socially costly. Hence, increases in banks' private costs of funds are irrelevant from a normative perspective (see, e.g. Hanson et al. (2011) and Admati et al. (2013)). Nonetheless, the idea that such increases result in less lending has seeped into conventional wisdom.

In this paper, we challenge such conventional wisdom. We develop a model where capital is costly from a bank's perspective due to an implicit subsidy from a government guarantee. A higher capital requirement always reduces the value of this subsidy. Hence, it is correct that the bank's costs of funds go up (at a given level of lending). However, the total value of the subsidy is not what is relevant for the bank's lending decision. What matters is the marginal subsidy – i.e., the extent to which the marginal loan is subsidised. We show that an increase in the requirement can increase the marginal subsidy. Then, facing a lower *marginal* costs of funds, the bank increases lending.

Our model has a single period in which a representative bank faces a capital requirement and finances loans with a mix of liabilities that can be interpreted as deposits and capital. The bank starts with existing loans and can make new ones. All loans mature at the end of the period. New loans present diminishing marginal returns, which are not necessarily perfectly correlated with those on legacy loans. Deposits and capital are supplied perfectly elastically by risk-neutral households.

The bank maximises the expected payoff of initial shareholders. Deposits are insured by the government with no fee; hence, they are implicitly subsidised. This has two implications. First, the capital requirement is binding in equilibrium: the

¹Specifically, minimum tier one capital requirements were raised from 4 to 6% of risk weighted assets, but additional extra "buffers" were created to adjust, inter alia, for the systemic importance of the institution, the economic cycle, and to prevent accidental breaches of the minimum. Effective requirements for large global banks are now in the double digits of risk-weighted assets.

²See, for instance, Institute of International Finance (2011) on page 10.

bank chooses lending and adjusts capital to meet the requirement. Second, the objective function can be written as the sum of the economic surplus from lending and a term that captures the value of the implicit subsidy (as in Merton (1977)). The derivative of the subsidy with respect to lending, the marginal subsidy, is a wedge in the first order condition. This wedge captures the underlying moral hazard problem arising from the guarantee.

We study how marginal changes in the capital requirement affects the equilibrium level of lending (we refer to the response to an increase as the *lending response*). By definition, such changes do not directly affect economic surplus. Hence, if an increase in the requirement increases the marginal subsidy, the bank increases lending as a result.

Increasing the capital requirement has two effects on the marginal subsidy. First, a smaller fraction of the marginal loan is financed by deposits. This generates an intuitive composition effect: the bank substitutes subsidised deposits with capital. This effect captures exactly how the capital requirement raises the bank's average funding costs.³

However, the change in the capital requirement also affects whether the bank defaults or not in any given state. To go further, it is useful to define the residual cashflow associated with the marginal loan. This variable, which we denote Z , is what is left from the marginal loan's realised payoff after deducting the cost of deposits raised to finance it. In the states where the bank survives, Z comes as an addition to the shareholder's payoff. But if the bank defaults, Z accrues to the taxpayer.

The second effect, which we argue is overlooked by conventional wisdom, goes as follows. Consider the default boundary – that is, the set of states where the bank can just repay depositors. Increasing the requirement increases the buffer against losses and shifts the default boundary. There are now more states where Z accrues to shareholders. In particular, increasing the capital requirement makes the shareholders internalise the expected value of Z along the default boundary. This second effect captures that the requirement forces the bank

³The change in average funding costs is relevant to determine the impact on the bank's profit. See Kisin and Manela (2016) for a quantification.

towards safety.

Because the bank could have initially chosen to be safer and internalise these cashflows (by operating at a higher capital ratio than the requirement), but preferred not to, we dub the second effect the *Forced Safety Effect* (FSE).

If, in expectation, the residual cashflows along the default boundary are positive, the bank is internalising cashflows that increase the shareholders' payoff. In this case, the FSE is positive and it increases the value of the marginal subsidy. If the cashflows are negative, the FSE makes the bank internalise more losses. Hence, it decreases the value of the marginal subsidy and reinforces the composition effect.

Our main contribution is to show that: (i) the FSE can be positive (ii) the FSE can dominate the composition effect, which is why lending can increase with the capital requirement.

We solve an extended version of the model numerically to explore the empirical relevance of our analytical results.⁴ We find a positive lending response in plausible conditions – namely, in a calibration that targets the situation facing a global bank in 2017. However, at levels of capital requirements prevailing before the global financial crisis, lending responses are more likely negative. Overall, our sensitivity analysis reveals that lending responses are likely to exhibit substantial variation both in the cross section of banks and in the time series. Hence, one should not necessarily expect an homogeneous relationship between the capital requirement and bank lending. We argue, in Section 5.3, that this may help reconcile results in the empirical literature, and we discuss the empirical predictions that arise from our analysis in Section 6.2.

In the literature, the idea that tighter capital requirements raise banks' average costs of funds and prompt a credit contraction has been formalised by Thakor (1996), among others. As Suarez (2010) discusses, a usual way to capture such an effect is to assume an exogenous cost of issuing outside equity, in a way that makes the capital requirement, in effect, a tax on lending. A negative lending response then emerges naturally, as in recent quantitative studies (e.g., Corbae

⁴Specifically, we model competition to microfound a downward sloping demand for loans, introduce tax-deductible interest payments on deposits, and consider ex-ante heterogeneity among banks.

and D Erasmo (2017) and Elenev et al. (2017)).⁵

However, general equilibrium effects can overturn this. For instance, higher capital requirements can reduce the aggregate supply of deposits. If deposits provide liquidity services, this can *decrease* banks' funding costs in equilibrium and can generate a positive aggregate lending response (Begenau (2018)). We highlight that a higher capital requirement can actually increase lending, despite *increasing* average funding costs (i.e., reducing the average subsidy) in equilibrium.

The key ingredient for a positive FSE is a form of asset heterogeneity. The residual cashflow of the marginal loan (i.e., Z) cannot always have the same sign as the residual cash flow of the average loan on the bank's balance sheet. Specifically, along the default boundary, in expectation, the percentage loss on the marginal loan must be smaller than that on the average loan.

Introducing a second type of assets on the balance sheet (which we do with legacy loans) makes a positive FSE possible. This second class of assets need not be very different from the marginal loan. In fact, the returns on the two can even be perfectly correlated, as long as percentage losses can differ. So all in all, the heterogeneity we need is rather mild, especially with regard to the heterogeneity of bank assets in reality.

Heterogeneity in residual cashflows generates another counterintuitive result. To expose it and put it in perspective, it is useful to first gather insights from different strands of literature.

Limited liability introduces a kink into the shareholders' payoff function. Alongside a friction, the resulting convexity can induce risk loving behaviour. For instance, a firm that has existing debt (with given interest payments) has an incentive to make excessively risky investments (Jensen and Meckling (1976)), and may value assets over their fundamental values (Allen and Gale (2000)). Such phenomena are usually referred to as risk-shifting problems. On the other hand, the presence of existing debt can also lead to under-investment (Myers (1977)) – a phenomenon referred to as a debt overhang problem.

⁵See also Estrella (2004), Repullo and Suarez (2013), Martinez-Miera and Suarez (2014), and Malherbe (2015) for analyses of the underlying mechanisms.

Government guarantees to banks also generate risk-shifting behaviour: Kareken and Wallace (1978) establish the counterpart of Jensen and Meckling (1976). Also, when assets are loans, overvaluation leads to overlending (McKinnon and Pill (1997)), which is the counterpart of Allen and Gale (2000).⁶

Our second contribution is to point out that the implicit subsidy from government guarantees can also lead to underlending. How can a subsidy lead to less lending? The answer is that while the total subsidy is always positive, the marginal subsidy can be negative and, therefore, act like a tax. This happens when the marginal new loan generates positive residual cashflows in default states. These cashflows then accrue to the taxpayer and, hence, reduce the value of the implicit subsidy.⁷ We argue that this is the counterpart of Myers (1977).⁸ As with the debt overhang problem, the bank undervalues new loans because of an implicit transfer from the shareholders to other stakeholders. However, the mechanism is different. It does not hinge on existing debt: it actually operates if the bank is initially 100% equity financed.⁹ We dub this phenomenon the *guarantee overhang* problem.¹⁰

Finally, a positive lending response and the guarantee overhang problem are linked: they both arise from the same moral hazard problem (i.e., the guarantee) and both require heterogeneity in residual cashflows. But we wish to stress that they are distinct phenomena: a positive lending response can arise when the bank overlends and a negative lending response can arise when the bank underlends.

To sum up, the punchline of this paper is that the lending response can be positive. This happens when the FSE more than offsets the composition effect,

⁶See also, e.g., Rochet (1992), Martinez-Miera and Suarez 2014 on bank risk-shifting, and Krugman (1999) and Malherbe (2015) on over-investment, which is ultimately the consequence of overlending.

⁷In a sense, positive residual cashflows in default provide insurance. But shareholders do not value insurance, as the benefits go to the taxpayer. Hence, when the bank refrains from issuing a positive NPV loan, this must be interpreted here as a way to take excessive risk.

⁸Papers that link bank underlending to the debt overhang problem include Hanson et al. (2011), Admati et al. (2018), and Jakucionyte and van Wijnbergen (2018).

⁹See Section 6.4 for more details.

¹⁰In Harris et al. (2017) and Martinez-Miera and Suarez (2014), banks face a menu of risks and could, in principle, hold assets with the relevant residual cashflow heterogeneity. However, in equilibrium, they choose not to. Underlending can happen in their models, but this is due to the scarcity of bank capital in the aggregate, not to the guarantee overhang effect.

which we argue can happen in plausible conditions. However, the main takeaway for the policy debate goes beyond the sign of the lending response. Indeed, in our calibrations, when the FSE does not dominate, it still makes the lending response substantially less negative than otherwise. Overlooking this effect is tantamount to confusing how average, rather than marginal, costs of funds are affected by changes in capital requirements.

2 The baseline model

2.1 The environment

There are two dates, 1 and 2. There is a bank and a continuum of households who own the bank's liabilities, and a government. Households are risk neutral and do not discount the future; they supply funds perfectly elastically with an opportunity cost of funding of 1. We focus on the date-1 decision of the bank. The random variable A captures the realised state of the economy at date-2. It is distributed according to a function $f(A)$ with positive support $[a_L, a_H]$ and with $E[A] = 1$. Figure 1 summaries the bank's balance sheet.

Predetermined variables. As of date-1, there are legacy loans on the bank's balance sheet. Their total book value is λ , and they generate a risky date-2 payoff $A\lambda$. Without loss of generality, the bank holds no cash. The bank has existing deposits that can be withdrawn at par at date-1 and, hence, for the analysis it is only necessary to consider the end date-1 level deposits. The book value of capital at the beginning of date-1 is denoted by κ .

Decision variables. The bank decides how much to lend. We denote the total amount of new lending by $x \geq 0$. For simplicity, new loans also mature at date-2. The bank has some market power over borrowers. We capture this with a payoff function $X(x)$, which is increasing and strictly concave in x , with $X(0) = 0$. We assume X is twice differentiable in the strictly positive domain and $\lim_{x \rightarrow 0} X_x(x) = \infty$. For now, the returns to new loans are deterministic. This helps isolate the

main mechanism and provide intuition. We introduce stochastic new loan returns in Section 4.1.

At the same time, the bank adjusts its liabilities: capital and deposits. We denote the change in capital c . The change in capital can be negative (as long as $\kappa + c > 0$). In this case, the change can be interpreted as a dividend payment or the value of a share repurchase. If c is positive, it should be interpreted as the bank raising more capital. In this case, an amount c is raised in exchange for date-2 cashflow rights. The corresponding total repayment is denoted C . This repayment is determined in equilibrium and can be contingent on any realised variable.

The bank's chosen level of deposits is denoted d . As we will see, the presence of an implicit subsidy makes deposits the most attractive source of funds for the bank. Hence, the capital requirement will be binding in equilibrium and the optimal choice for x will, effectively, pin down the liability side of the balance sheet.

Deposit insurance and the capital requirement. The government insures bank deposits with no premium: in the event the bank has insufficient cashflows to repay depositors in period 2, the government makes depositors whole, and breaks even via an ex-post lump-sum tax on households. This is the source of moral hazard in the model. Deposits pay no interest. If the bank defaults on deposits, no payment to any other liability is allowed.

The bank faces a capital requirement constraint that takes the form:

$$\kappa + c \geq \gamma(x + \lambda), \tag{1}$$

where $\gamma \in (0, 1)$ is a parameter (which we refer to as the *requirement*) set by the government, $x + \lambda$ is the book value of total assets on the balance sheet, and $\kappa + c$ is the bank's total capital at book value.

To be allowed to operate at date-1, the bank must satisfy the capital requirement; otherwise, the government shuts down the bank. In this case, initial shareholders walk away with 0 and we impose $x = c = 0$.

Figure 1: The bank's balance sheet

Assets	Liabilities
(new loans) x	$\kappa + c = \gamma(x + \lambda)$ (capital)
(legacy loans) λ	$d = (1 - \gamma)(x + \lambda)$ (deposits)

Notes: The parameter γ denotes the capital requirement, κ is existing capital and c is net issuance (can be negative). The capital requirement is always binding; see Section 2.2.

Finally, note that the assumption $E[A] = 1$ ensures that initial shareholders always find it profitable not to walk away at date-1. We explore date-1 bank closure in Section A.1.

2.2 Setting up the analysis

Date-2 default on deposits. If date-2 cashflows are too low to repay the depositors, the bank defaults on them. This happens when

$$d > X + A\lambda$$

promised repayment total cash flow

We can then define a_0 as the realisation for A below which the bank defaults on deposits:

$$a_0 \equiv \frac{d - X}{\lambda}.$$

We refer to a_0 as the default boundary. We can also define p , the probability that the bank does not default:

$$p \equiv \int_{a_0}^{a_H} f(A) dA.$$

Pricing of new capital. Investors act competitively, so that, in equilibrium, they just break even in expectation. First, we assume that the bank issues new capital (i.e., $c \geq 0$). Denoting $C(A)$ the contingent, date-2 repayment to new capital, we then have:

$$\int_{a_L}^{a_H} C(A) f(A) dA = c. \tag{2}$$

To be able to interpret c as capital issuance, the underlying securities should be junior to deposits. Hence, we impose

$$C(A) \leq 0, \forall A \leq a_0. \quad (3)$$

We also impose limited liability for investors, which implies $C(A) \geq 0$, and for initial shareholders, which implies $C(A) \leq X + A\lambda - d$. However, we do not restrict new capital to be a particular form of security. What matters is that capital is junior to deposits and will, therefore, absorb losses. In practice, one can, for instance, think of it as seasoned equity or subordinated debt.¹¹

Finally note that, even though they all refer to households, we use different terms for holders of different bank-issued liabilities. Initial shareholders own the initial (i.e., inside) equity, investors hold new capital, and depositors hold deposits.

Initial shareholders' payoff. If c is positive, the expected payoff to (or expected final *wealth* of) initial shareholders is:

$$w \equiv \int_{a_0}^{a_H} [X(x) + A\lambda - d - C(A)] f(A) dA$$

substituting break even condition (2) gives:

$$w \equiv \int_{a_0}^{a_H} [X(x) + A\lambda - d] f(A) dA - c$$

Now, if c is negative, the payoff is identical to the above as initial shareholders will receive $-c$ with certainty in period 1. In the absence of frictions affecting the contracting between initial shareholders and investors in new capital, the shadow

¹¹For subordinated debt, the interest payment should compensate the loss of capital that happens in some states. Assume the bank can fully repay deposits with probability p , then the expected return for the subordinated debt holders in these states should be $\frac{1}{p}$ (and zero otherwise). In the case of seasoned equity, the logic is the same, but the mapping goes as follows: at date-1 the bank starts with κ shares and issues κ' additional shares at a unit price v in exchange of $c = \kappa'v$. This gives investors the right to a payoff of $C = \frac{\kappa'}{\kappa + \kappa'} \max\{0, V\}$ where $V = X + A\lambda - d$. Hence, their break-even condition is $v = E \left[\frac{\max\{0, V\}}{\kappa + \kappa'} \right]$.

value of initial capital is equal to the price of new capital. Hence, it is unnecessary to treat positive and negative c as separate cases in what follows.¹² And, accordingly, there is no need to distinguish between the owners of different classes of bank capital. For simplicity, we refer to them (both the initial shareholders and the investors in new capital) collectively as “the shareholders”.

The problem of the bank If the bank is fully safe in equilibrium (i.e., $p = 1$), the capital ratio is irrelevant (in a Modigliani and Miller sense). In this case, the bank is locally indifferent between any mix of capital and deposits that satisfies the requirement. For most of the analysis, we focus on the cases where the bank defaults at date-2 with strictly positive probability in equilibrium. In these cases, the capital requirement always binds because, from the bank’s point of view, deposits are cheaper (depositors always break even, but sometimes at the expense of the taxpayer). Hence, the bank’s problem boils down to finding a level of lending x^* that solves

$$\max_{x \geq 0} \int_{a_0}^{a_H} [X(x) + A\lambda - (1 - \gamma)(x + \lambda)] f(A) dA - (\gamma(x + \lambda) - \kappa). \quad (4)$$

We refer to x^* as the equilibrium level of lending.

3 Analysis of the baseline model

Our main result is that x^* may increase with γ . In the case where the problem of the bank (4) admits a unique maximum, x^* can be expressed as a function of γ . Then, we formalise this result as follows:

Proposition 1. *For all $\gamma \in (0, 1)$ and an associated function $x^*(\gamma)$, if $p(x^*(\gamma), \gamma) < 1$, there exists $\gamma' > \gamma$ such that $x^*(\gamma') > x^*(\gamma)$.*

¹²In Appendix A.1, we relax the assumption that $E[A] = 1$. In that case, the initial shareholder’s participation constraint might bind. As a result, initial and new capital are not equivalent. One implication of this is that when $E[A]$ is sufficiently low, an increase in capital requirement may result in the shareholders closing the bank instead of raising capital.

Proof. All proofs are in Appendix A. (Generalising this result to the case of multiple maxima only complicates notation.) \square

To study situations where the requirement is relevant, we assume $p^*(x^*(\gamma), \gamma) < 1$ for what follows. For better readability, we often omit function dependencies on x and γ . Additionally, we use subscripts for partial derivatives in these two variables and stars to indicate where functions are evaluated in equilibrium. For instance: $p_x^* \equiv p_x(x^*, \gamma)$.

3.1 The first order approach

We derive intuition by focusing on small increases in γ and assume that the first order condition uniquely pins down a function $x^*(\gamma)$.

The implicit subsidy decomposition

Lemma 1. *The bank's objective function can be rewritten:*

$$w(x) = \underbrace{X - x}_{\text{economic surplus}} + \underbrace{\int_{a_L}^{a_0} ((1 - \gamma)(x + \lambda) - X - A\lambda) f(A) dA + \kappa}_{\equiv s(x, \gamma), \text{ i.e. the implicit subsidy}} \quad (5)$$

The first term in equation (5) captures, intuitively, the economic surplus generated by new loans.¹³ The second term integrates, over all the default states, the difference between the promised repayment to the depositors, $(1 - \gamma)(x + \lambda)$, and the total cashflow available to the bank, $X + A\lambda$. Under unlimited liability, this term would be the expectation of how much, ex-post, the shareholders would have to pay into the bank to make depositors whole. But instead, here, the taxpayer is footing the bill. This is why s should be interpreted as the implicit subsidy to the bank's shareholders arising from the government guarantee. The implicit subsidy is the source of a moral hazard in the model.

Remark 1. The implicit subsidy corresponds to the expected net worth of the bank, when it is negative. As Merton (1977) has shown, deposit insurance can be

¹³Since we assume $E[A] = 1$, legacy loans are valued on the balance sheet at their expected value. Hence, $(E[A] - 1)\lambda = 0$, and they do not appear in (5).

interpreted as a (free, or at least mispriced) put option on the bank equity with a strike price of 0. The implicit subsidy is therefore equal to the value of such an option.¹⁴

The sign of the lending response The first order condition can be written as:¹⁵

$$\underbrace{(X_x^* - 1)}_{\text{surplus maximisation}} + \underbrace{(1 - p^*)[(1 - \gamma) - X_x^*]}_{\equiv s_x^*, \text{ i.e. the marginal subsidy}} = 0. \quad (6)$$

The first term represents economic surplus maximisation. Absent the implicit subsidy, the bank would choose a level of lending x_{MM} such that $X_x(x_{MM}) = 1$. This would correspond to Proposition 3 in Modigliani and Miller (1958): equilibrium investment (here lending) only depends on the opportunity cost of funds in the economy. We therefore sometimes refer to x_{MM} as the Modigliani and Miller (or MM) level of lending.

The function $x^*(\gamma)$ is generally not explicit. We can, however, establish three of its properties. The first is that when γ is sufficiently large, $x^* = x_{MM}$. To see this, note that when γ is large enough, $a_0^* \leq a_L$. Hence, $p^* = 1$, and the distortion disappears ($s^* = s_x^* = 0$). The second property is that, for lower values of γ such that the bank defaults with positive probability in equilibrium, $x^* < x_{MM}$. To see this, rewrite the first order condition as

$$X_x^* = (1 - \gamma) + \frac{\gamma}{p^*},$$

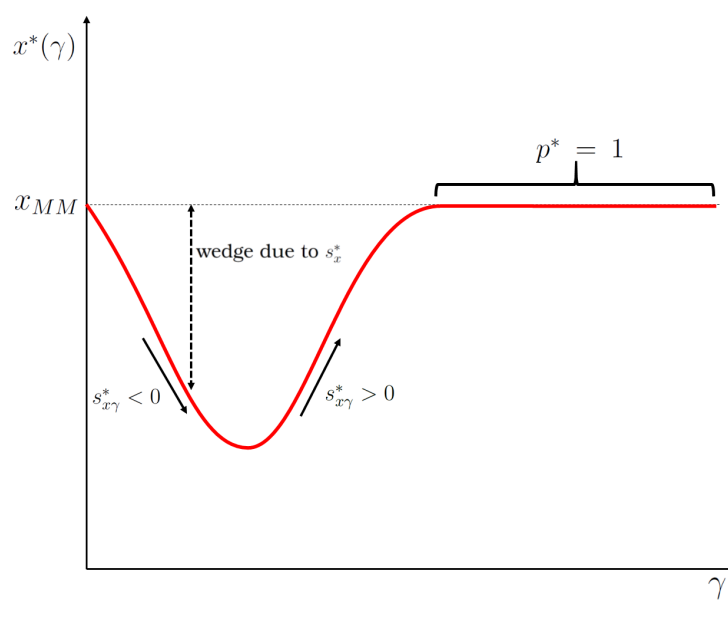
and note that $p^* < 1$ implies $X_x^* > 1$. These two properties are in fact sufficient to establish that there will be values of γ where $x^*(\gamma)$ is increasing. The third property is that when $\gamma = 0$, we also have $x^* = x_{MM}$. The intuition is that the marginal subsidy is, in this special case, proportional to the marginal economic surplus, and hence it does not distort the bank's decision.¹⁶

¹⁴If there were not government guarantees, depositors would require an interest payment that would, in equilibrium, just compensate for the value of s . Hence, the objective function would boil down to $X - x$.

¹⁵See the proof of Proposition 1.

¹⁶The first order condition shrinks here to $p(X_x - 1) = 0$. This result is specific to having

Figure 2: Example of $x^*(\gamma)$ in the baseline model



Notes: The solid red line provides an example of $x^*(\gamma)$, which is obtained when X is isoelastic and A is uniformly distributed. The dotted line is the MM level of lending. Based on a numerical analysis, we conjecture that well behaved U-shapes are also obtained for general X functions and single peaked distributions, so long as the median of A is greater than $1 - \gamma$.

An example for how these three properties play out is given in Figure 2, where $x^*(\gamma)$ is a well behaved U shape. This enables the visualisation of Proposition 1. Either we are in an upward sloping region, and the result applies to small increases in γ . Or, we are in a downward sloping region, and a sufficiently large increase in γ is needed to increase lending.

Figure 2 is also useful to understand our next proposition. Since economic surplus maximisation is not affected by γ , the gap between x_{MM} and x^* is solely due to s_x^* , which is the wedge in the first order condition. This wedge is negative, which is something we will return to in Section 3.3. For now, what is important is not the sign of the wedge, but whether it increases with γ . Intuitively, if an increase in γ increases the extent to which the marginal loan is subsidised, the *lending response* is positive (that is: $\frac{dx^*}{d\gamma} > 0$). Formally:

deterministic returns on new loans. As we will see in Section 4.1, when new loans bear risk, $x^*(0)$ is generally different from x_{MM} .

Proposition 2. (The sign of the lending response)

$$\frac{dx^*}{d\gamma} \begin{matrix} \leq \\ \geq \end{matrix} 0 \Leftrightarrow s_{x\gamma}^* \begin{matrix} \leq \\ \geq \end{matrix} 0$$

Bank capital ratio and profitability At this point, it is useful to point out that an increase in capital requirement unambiguously decreases shareholders' expected payoff: $\forall x, w_\gamma < 0$. This is because, for any level of lending x , total funding costs increase with γ (intuitively, the expected transfer from the taxpayer shrinks as the share of deposits in the bank's liabilities goes down). Formally:

$$w_\gamma = s_\gamma = -(1-p)(x+\lambda). \tag{7}$$

But this is not incompatible with the marginal subsidy being increasing in γ . Figure 3 proposes a simple illustration. The solid line depicts the payoff function associated with an initial capital requirement γ . That $w_\gamma < 0$ means that the payoff function associated with a higher capital requirement (denoted γ') is below the initial one. But it does not tell us whether it peaks to the left or to the right of the initial optimum. If $s_{x\gamma}^* > 0$, it peaks to its right. This is what happens when $x^*(\gamma)$ is upward sloping.

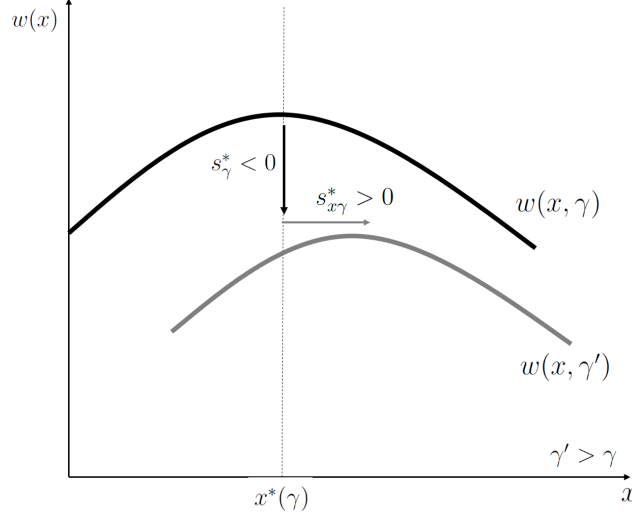
Finally, γ only sets a minimum. The bank is, therefore, allowed to operate at any capital ratio $\gamma' > \gamma$. If the bank did so, it would then choose the level of lending $x^*(\gamma')$. Such an option is, however, not optimal from the shareholders' perspective. They are always better off maximising leverage (and, in the example of Figure 3, choosing a lower level of lending than $x^*(\gamma')$).

3.2 The forced safety effect

We have established that the sign of the lending response is that of $s_{x\gamma}^*$, and we have shown that it is positive for some region of γ . Now, we turn to the underlying economic mechanism.

Property rights and the marginal residual cash flow Issuing the marginal loan affects the bank's cashflows. Irrespective of A , the bank's revenue increases

Figure 3: The effect of an increase in γ on the bank's objective



Notes: This diagram shows how the bank's objective, $w(x)$, is shifted by an increase in the capital requirement when the lending response is positive at the initial equilibrium, $x^*(\gamma)$. If, instead, the lending response is negative, $w(x, \gamma')$ peaks to the left of $x^*(\gamma)$.

by X_x , and the repayment due to depositors increase by $1 - \gamma$. Let us define Z , the residual (i.e., net of deposits) cash flow associated with the marginal loan, as the difference between the two:

$$Z(x) \equiv X_x - (1 - \gamma).$$

Since $X_x^* > 1$, the residual cash flow is positive in equilibrium: $Z^* > 0$.

Now, which stakeholder is entitled to Z^* depends on the realisation of A . If the bank survives, the shareholders are the residual claimants. But if the bank defaults, shareholders walk away with zero, and the taxpayer becomes, in effect, the residual claimant.¹⁷ What determines the bank's survival is the sign of the *total* residual cash flow ($A\lambda + X - (1 - \gamma)(x + \lambda)$), which is different from Z^* .

As we will see, Z^* plays a key role in our analysis. In the baseline model, it is deterministic, and always positive. This helps explain how the main mechanism works and will also be useful to gain intuition in the full version of the model in

¹⁷Technically, the taxpayer is a claimant in the sense that there is reduction in the transfer needed to make depositors whole.

Section 4.

The cross-partial derivative of the subsidy The marginal subsidy is $s_x^* = (1 - p^*) [(1 - \gamma) - X_x^*]$. Deriving with respect to γ and using the definition above, we obtain:

$$s_{x\gamma}^* = \underbrace{-(1 - p^*)}_{\text{composition effect}} + \underbrace{p_\gamma^* Z^*}_{\text{Forced Safety Effect}}. \quad (8)$$

The cross-partial derivative is the sum of two components. The first term is negative. Raising γ reduces the portion of the marginal loan that is financed with (subsidised) deposits. Since the bank must substitute deposits (which it repays with probability p^*) with capital (which it always repays in expectation), this change in the composition of liabilities reduces the marginal subsidy. We dub this effect *the composition effect*.

The second component captures that, keeping x^* constant, an increase in γ makes the bank safer: $p_\gamma^* > 0$. This corresponds to a shift in the default boundary, a_0^* , which means that there are states of the world where the bank would have defaulted if not for the extra capital. In these states, the rights to the residual cashflow from the marginal loan (Z^*) switch from the taxpayer to the shareholders. In expectation, this raises the marginal subsidy by $p_\gamma^* Z^*$. Since this term stems from the fact that the bank is forced to be safer (something it could have always chosen to do), we dub this effect *the forced safety effect*. To the best of our knowledge, this paper is the first to highlight such a mechanism.

Remark 2. The forced safety effect is completely different from the mechanism behind the usual risk premium argument based upon the second proposition in Modigliani and Miller (1958). In short, that argument is: a higher capital ratio decreases the volatility of the return to capital (equity, for instance) and, therefore, its required return. In our model, all agents are risk neutral, and the equivalent notion of required return to capital is always 1, irrespective of γ .¹⁸

¹⁸Note that our analysis could equally be done in terms of a risk-neutral probability measure. So, introducing risk aversion would not affect the logic behind the forced safety effect.

Average versus marginal subsidy and conventional wisdom The composition effect is behind the claim (made repeatedly by bank lobbies and sometimes by policymakers (see for instance Brooke et al. (2015), p5)) that increasing capital requirements would: (i) increase bank funding costs; and (ii) naturally lead to less lending.

In the context of our model, the first part of this claim is correct. As we have established above – the total subsidy decreases with the capital requirement, that is $s_\gamma < 0$. Of course this also applies to the average subsidy:

$$\frac{\partial}{\partial \gamma} \left(\frac{s}{x + \lambda} \right) = \frac{s_\gamma}{x + \lambda} = -(1 - p). \quad (9)$$

Note that this effect corresponds exactly to the composition effect.

However, the second part of the claim, regarding lending, is incorrect. Our interpretation is that it confounds the effect of the capital requirement on the *average* and the *marginal* subsidy.¹⁹ Comparing equations (8) and (9) makes clear that making this mistake is equivalent to ignoring the forced safety effect.

Positive lending response Equations (8) also makes clear that, for the lending response to be positive, the FSE needs to more than offset the composition effect. Proposition 1 establishes that, in our baseline model, there *always* is a range of γ where it is case. The general properties of $x^*(\gamma)$ make this clear: the bank generally underlends ($x^* < x_{MM}$), but chooses $x^* = x_{MM}$ for sufficiently large values of γ . How is it possible then that an implicit subsidy distorts the outcome towards *less* lending? This is the question we explore next.

3.3 A subsidy or a tax?

To reconcile the idea of an overall positive subsidy, occurring alongside a distortion toward less lending, one can think of the subsidy as the sum of two compon-

¹⁹Imagine if, instead of guaranteeing deposits, the government explicitly subsidised them at a flat rate of δ . In this case, the total subsidy would be $(1 - \gamma)(x + \lambda)\delta$. Since this subsidy is linear in x , average and marginal subsidy coincide, and are affected in the same, negative way by an increase in γ .

ents – a lump-sum subsidy and a tax on lending.

The bank has legacy loans that are funded with deposits. These legacy loans are risky and, in certain states of the world, generate negative residual cashflows that need to be covered by the taxpayer. Specifically, the expected transfer from the taxpayer (i.e., the total subsidy) if the bank does not issue any new loans (i.e., $x = 0$ hence $a_0 = (1 - \gamma)$) is:

$$s(0) \equiv \int_{a_L}^{(1-\gamma)} ((1 - \gamma) \lambda - A\lambda) f(A) dA > 0. \quad (10)$$

As established above, issuing new loans generates positive residual cashflows. Formally: $\forall x \leq x^*$, $Z(x) > 0$ (since it is true for the marginal loan, it is also true for infra-marginal ones). When the bank goes bust, these residual cashflows accrue to the taxpayer. This reduces the transfer needed to make the deposits that financed the legacy assets whole. Hence, new loans reduce the size of the total subsidy; the marginal subsidy is negative. Formally, $\forall x \leq x^*$, $s_x(x) = (1 - p(x))Z(x) < 0$.

Since a negative marginal subsidy can be interpreted as a tax, we can write the total subsidy as the sum of a lump-sum subsidy and a tax that increases with lending:

$$s^* = s(0) + \int_0^{x^*} \underbrace{s_x(x)}_{<0} dx.$$

The total value of the subsidy is always positive, but it is maximised at $x = 0$. Hence, the bank faces the following trade off: while issuing loans increases economic surplus, it also decreases the total value of the subsidy. This is why the bank's lending is below x_{MM} .

A guarantee overhang: similarities and differences with the debt overhang problem To gain further intuition it is also useful to draw some parallels to the debt overhang problem (Myers (1977)).

In Myers (1977), a firm has existing risky assets and existing debt on which it will default in some states of the world. The firm considers raising capital to

finance a positive net-present-value project. The problem is that the cashflow from the project will go to the existing debtholders in the default states. Hence, there is an implicit transfer, and the firm does not capture the full expected surplus from the project. As a result, the firm may find it preferable to pass on this investment opportunity.

Our baseline model shares this outcome: in equilibrium, the bank passes on positive net-present-value loans. Likewise, an implicit transfer is involved. When the bank goes bust, the residual cashflows from the marginal loan decreases the transfer needed from the taxpayer to make depositors whole. Hence, in expectation, part of the residual cashflows generated by the marginal loan is transferred to the taxpayer (which resonates with our tax interpretation above), and is not captured by the bank shareholders. This is an overhang problem, in the sense that the existing balance sheet affects the decision to make an otherwise independent investment.

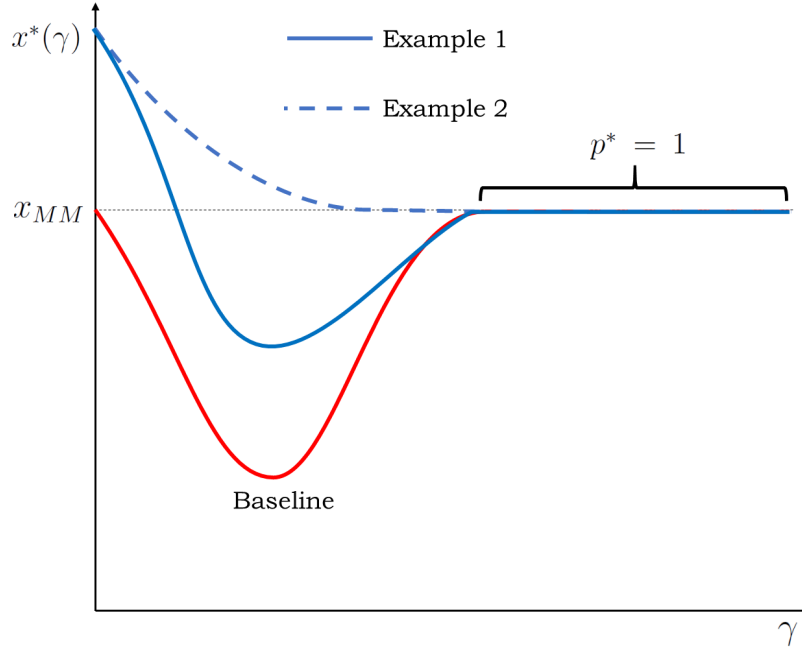
However, it is not a *debt* overhang problem. The simplest way to illustrate this is to consider a bank that is, initially, fully funded with capital (i.e., $\kappa = \lambda$). Nothing would change in the main analysis; in particular, $x^*(\gamma)$ would be unchanged. Rather than being due to the presence of existing debt, the inefficiency is due to the implicit subsidy that arises from government guarantees, which applies to all deposits, *existing and new*. In this sense, there is a *guarantee overhang*.

To our knowledge, this paper is the first to make the point that the implicit subsidy from government guarantees can act as a tax at the margin and, as a result, generate such an overhang problem.

4 Extensions and the full model

In this section, we extend the baseline model to derive further insight into the determinants of the lending response. We then combine all these ingredients in a full model that we calibrate in Section 5.

Figure 4: Two examples of $x^*(\gamma)$ when new loans are risky



Notes: This Figure provides two examples of $x^*(\gamma)$ when new loans are also risky (blue solid and blue dashed lined). See the main text for a description of how these two examples come about. The U-shaped red solid line is an example of $x^*(\gamma)$ in the baseline model for comparison. The dotted line is the MM level of lending.

4.1 Risky new loans

4.1.1 Preamble

When new loans are risky, $x^*(\gamma)$ can take different shapes. To set the stage for this section, let us illustrate this idea with two new examples (see Figure 4).

Example 1, depicted by the solid blue line, captures the pattern that we typically find in our calibrations (See Section 5). It resembles the U-shape of the baseline mode, but it exhibits overlending ($x^* > x_{MM}$) and a strongly negative lending response at low levels of γ . Example 2, depicted by the dashed blue line, captures a somewhat more extreme case: the bank overlends, and the lending response is negative for all γ . We obtain such a shape if, for instance, legacy loans are safe, and $A = 1$.

In what follows, we show that it is the structure of residual cashflows drives the shape of $x^*(\gamma)$. In particular, what is needed for the existence of a positive

lending response is a form of residual cashflow heterogeneity. To establish this concretely, we revisit the concepts of FSE and guarantee overhang to (i) formally show that a positive lending response and/or underlending can still occur when new loans are risky ; (ii) identify the conditions under which these will or will not arise; and (iii) stress that these are disjoint phenomena: a positive lending response can arise when the bank overlends, and vice versa.

4.1.2 The first order approach with a second source of risk

To make new loans risky, we introduce a new random variable B , with positive support $[b_L, b_H]$ and $E[B] = 1$. The payoff function for new loans is now $BX(x)$. Let the joint density between A and B be given by $f(A, B)$. We can then define two functions, either of which can be used to define the default boundary:

$$a_0(B) = \frac{(1 - \gamma)(x + \lambda) - BX}{\lambda}, \quad (11)$$

$$b_0(A) = \frac{(1 - \gamma)(x + \lambda) - A\lambda}{X}, \quad (12)$$

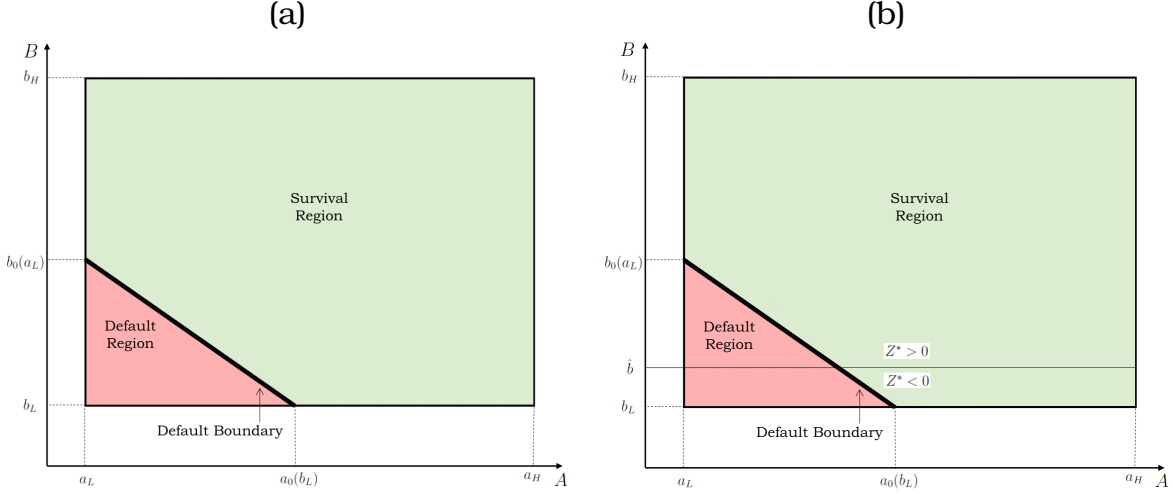
and the probability that the bank does not default as $p(x, \gamma) = \int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} f(A, B) dA dB$.

Note that $E[B] = 1$ preserves the definition of economic surplus, and the implicit subsidy now reads:

$$s(x, \gamma) = \int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} ((1 - \gamma)(x + \lambda) - BX(x) - A\lambda) f(A, B) dA dB.$$

We first emphasise that the bank's first order condition remains $X_x^* - 1 + s_x^* = 0$, and Proposition 2 still holds: $\frac{dx^*}{d\gamma} > 0 \Leftrightarrow s_{x\gamma}^* > 0$. However, the expression $s_{x\gamma}^*$ now reflects the fact that marginal residual cashflows are stochastic.

Figure 5: Default region and boundary



Notes: This Figure illustrates the default boundary and default region in the state space $\{[a_L, a_H] \times [b_L, b_H]\}$. The default boundary is the locus of points such that $BX^* + A\lambda - (1 - \gamma)(x^* + \lambda) = 0$; the default region corresponds to $BX^* + A\lambda - (1 - \gamma)(x^* + \lambda) < 0$. The term $Z^* = BX^* - (1 - \gamma)x^*$ is the equilibrium residual cashflow on the marginal loan. The threshold $\hat{b} = \frac{1 - \gamma}{X^*}$ is such that $B > \hat{b} \Rightarrow Z^* > 0$.

4.1.3 The FSE revisited

The cross partial derivative now reads:

$$s_{x\gamma}^* = \underbrace{-(1 - p^*)}_{<0} + \underbrace{p_\gamma^* z_{\Delta_0}^*}_{\text{FSE} \geq 0} \quad (13)$$

where

$$z_{\Delta_0}^* \equiv E[Z^* | A = a_0^*(B)] = \frac{\int_{b_L}^{b_0^*(a_L)} Z^* f(a_0^*(B), B) dB}{\int_{b_L}^{b_0^*(a_L)} f(a_0^*(B), B) dB}$$

is the expected marginal residual cash flow conditional on being on the default boundary.

Panel (a) in Figure 5 depicts the default boundary (the thick black line) and the whole default region (the red triangle) as subsets of the state space $\{[a_L, a_H] \times [b_L, b_H]\}$. The subscript Δ_0 in $z_{\Delta_0}^*$, refers to the fact that the default boundary is what pins down the triangular default region.

As in the baseline model, raising γ shifts the default boundary. Hence, at each

point on the initial boundary, Z^* now accrues to the shareholders. In expectation, this additional residual cash flow is worth $p_\gamma^* z_{\Delta_0}^*$. As before, $p_\gamma^* > 0$. Whether the FSE is positive or not therefore hinges on the sign of $z_{\Delta_0}^*$. We have:

Proposition 3. (i) $z_{\Delta_0}^*$ can be positive; (ii) this implies a positive forced safety effect; (iii) and can lead to a positive lending response: $s_{x\gamma}^* > 0$.

We mentioned above that a positive FSE requires some heterogeneity in residual cashflows. We can now be precise about what this means. By construction, on the default boundary, the residual cashflow on the bank's average loan is nil ($BX^* + A\lambda - (1 - \gamma)(x^* + \lambda) = 0$). If all loans were homogeneous, we would also have $Z^* = 0$ at all points on the boundary, and the FSE would be nil.

How should one interpret $z_{\Delta_0}^* > 0$? The condition means that that, in expectation along the default boundary, the marginal loan fares better than the average loan (which, by definition, have zero residual cashflows on the boundary). In Panel (b) of Figure 5, $Z^* > 0$ in all the states above the threshold \hat{b} . Hence, $z_{\Delta_0}^*$ will be positive if there is sufficient probability mass concentrated on the corresponding upper segment of the boundary.

Since X is concave, the marginal loan is the worst, *among new loans*. Hence, in our model, the marginal loan can only fare better than the average loan (i.e. average over the the whole balance sheet) if legacy loans generate negative residual cashflows in those states. This is why legacy loans play an important role in our model: they provide the necessary heterogeneity in residual cashflows.²⁰ That said, as we discuss in Section 6.1, there is nothing special about legacy loans: there are many potential other sources of residual cashflow heterogeneity. In general, the key necessary condition for $z_{\Delta_0}^* > 0$ to be possible is that the marginal loan sometimes does not perform too badly when the bank defaults.

How do the model parameters affect the FSE, and ultimately, the lending response? Both the default boundary and the marginal loan are endogenous objects. Tractability is an issue for further comparative statics, but we can still

²⁰If $A = 1$, as in Example 1 above, legacy loans always generate strictly positive residual cashflows. The marginal loan always fares worse, in expectation on the default boundary, than the average loan and a positive lending response is therefore not possible.

make a couple of points that will be useful for understanding the results of our calibration exercises below.

First, increasing the downside of legacy loans means that they will generate more negative residual cashflows. *Ceteris paribus*, decreasing the mean of A or increasing its variance is likely to increase the FSE.

Second, lower dependence between A and B makes it less likely that Z^* is negative when legacy loans perform badly. Hence, this can contribute to a large FSE too.

Before turning to the calibration exercise, let us also revisit the guarantee overhang and its subtle relation to the FSE.

4.1.4 The guarantee overhang revisited

The expression for the marginal subsidy now reads:

$$s_x^* = - (1 - p^*) \underbrace{z_\Delta^*}_{\geq 0},$$

where

$$z_\Delta^* \equiv E[Z^* | A \leq a_0^*(B)] = \frac{\int_{b_L}^{b_0^*(a_L)} \int_{a_L}^{a_0^*(B)} Z^* f(A, B) dA dB}{1 - p^*}$$

is the expected marginal residual cash flow over the entire default region (see Figure 5). As we note, z_Δ^* and, therefore, s_x^* can now have either sign.

If z_Δ^* is negative the marginal loan will increase, on average, the required transfer from the taxpayer to make depositors whole when the bank defaults. Hence, the marginal loan benefits from a positive subsidy, and the bank overlends relative to x_{MM} . Here, the marginal subsidy really acts as a subsidy.

In panel (b) in Figure 5, $z_\Delta^* > 0$ if there is sufficient probability mass concentrated in the area of the default region that is above the \hat{b} threshold. In that case, the subsidy again acts as a tax, and the equilibrium is symptomatic of the guarantee overhang problem that we identified in the baseline model. We therefore have a result that shares the logic of Proposition 1:

Proposition 4. *For all γ such that $s_x^*(\gamma) < 0$, there exists $\gamma' \geq \gamma$ such that $x^*(\gamma') > x^*(\gamma)$.*

Proposition 4 only expresses a sufficient condition. Perhaps counter-intuitively, equilibrium overlending – that is, $s_x^* > 0$ – is compatible with a positive FSE. One can even construct cases where the FSE dominates and the lending response is positive when the bank overlends. Figure 10 in Appendix C provides a numerical example. To generate this, we used a joint distribution, $f(A, B)$, with a lot of probability mass concentrated in the lower left corner of the default region – i.e., A and B have high tail dependency. But we restricted A and B to have a low correlation away from the lower tail, so the dependency between A and B is weak along the default boundary.

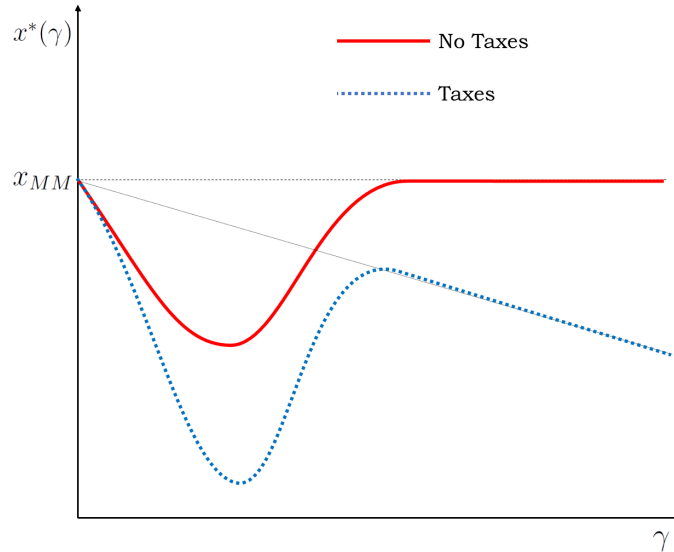
While such cases are a bit extreme, and less likely to be relevant empirically, they reinforce the point that z_{Δ}^* and $z_{\Delta_0}^*$ are different objects and can have different signs. Whether the bank under- or overlends does not dictate the sign of the lending response.

4.2 Taxes and tax shields

Another reason why banks may find capital relatively costly is the tax advantage of debt. To capture this, we assume that the bank faces a tax rate τ on positive profits, net of interest expenses on deposits. In order to introduce a meaningful tax shield, we also assume that households have an opportunity cost of funds $1 + \rho > 1$, so that the interest rate on deposits is ρ . To maintain our normalisation, we now set $E[A] = 1 + \rho$.

We describe how the tax interacts with the model formally in Appendix A.2. However, as an illustration, Figure 6 shows the two main ways the tax affects our results. The solid red line depicts $x^*(\gamma)$ without the tax ($\tau = 0$). This is the U-shape relationship of the baseline model. The blue dashed curve is the case with the tax. The net effect of taxes on the lending response is ambiguous: the U-shape relationship is still present, but it is (i) tilted clockwise and (ii) deeper than it would be in the absence of a tax. The tilt is intuitive, it comes from the tax deductibility: similar to the composition effect above, an increase in γ increases the average cost of funds. The deeper U-shape is more subtle. This is a result of the fact that the tax itself tends to reduce equilibrium lending and,

Figure 6: The shape of $x^*(\gamma)$ with corporate income tax



Notes: The diagram provides an example of $x^*(\gamma)$ in the baseline model (red solid line) and when the bank faces a corporate income tax τ (blue dotted line). As in Figure 2, this is obtained when X is isoelastic and A is uniform. Similar patterns arise with alternative functional forms and distributions. The horizontal dotted line is the MM level of lending. The downward sloping dashed line is a counterfactual level of lending where $s_x = 0$.

therefore, to increase the realised return to the marginal loan in all states. This applies in the default region, which tends to increase the transfer to the taxpayer, thereby making the overhang problem more acute. And it applies on the boundary too, which tends to reinforce the FSE. Identifying these effects is not only useful in helping to interpret the results of the sensitivity analysis in our calibration exercise below, but it also tells us that other frictions that reduce the equilibrium level of lending could have similar reinforcing effects.

4.3 The full model

We are now in a position to describe the general model. This model includes the two extensions above: risk on new and legacy loans and the corporate income tax. We also modify the capital requirement to takes into account that, in real world regulation, the requirements apply to risk-weighted assets. Accordingly, we

rewrite the constraint as:

$$\kappa + c \geq \gamma(\alpha x + \beta \lambda),$$

where α and β are risk weights parameters.

So far, we have focused on a single bank, facing a given downward sloping demand for loans. The loan demand was not affected by the capital requirement. In practice, however, the loan demand for a bank is affected by the loan supply of other banks and, therefore, by the capital requirements they face. A calibration of the model that aims at informing the debate on the overall level of capital requirements should take such bank interactions into account.

To do this, we first propose a Cournot competition extension of the baseline model.²¹ We take aggregate demand for loans as given, and consider a given number ν of identical banks that all face the same capital requirement γ . Up to a normalisation that we will introduce later, the payoff function of the representative bank takes the form:

$$BX(x) = Bx \left((x + x')^{-\eta} \right),$$

where x' captures the total lending by other banks (and is taken as given), and η is a parameter that captures the elasticity of aggregate loan demand.

Our equilibrium concept is a symmetric Nash equilibrium. Assuming it is unique, it corresponds to the fixed point (i.e. $x = x'$) that solves the representative bank's first order condition.²²

5 Empirical relevance

Our benchmark calibration aims at capturing a plausible situation facing a major international bank in 2017. Table 1 summarises this calibration.

²¹A Cournot approach is analytically convenient, but we also believe that it is particularly meaningful if one thinks that banks first choose their level of capital (which, given the capital requirement creates a capacity constraint) and then compete in price (i.e., in interest rate) in the market for loans. Schliephake and Kirstein (2013) have shown that this results in an elegant application of Kreps and Scheinkman (1983): the equilibrium outcome corresponds to that under Cournot competition. Other papers using Cournot competition for banks include Corbae and D Erasmo (2017) and Jakucionyte and van Wijnbergen (2018).

²²In our numerical explorations, we have not encountered multiple fixed points.

Table 1: Benchmark Calibration

Parameter	Value	Definition	Calculation	Source(s)
γ	0.13	capital requirement	Tier 1 Risk Based Minimum Capital Requirement of Globally Systemically Important Banks.	BCBS (2017) - Table B.4
α	0.5	risk weight on new loans	Average risk weights.	Mariathasan and Merrouche (2014)
β	0.5	risk weight on legacy loans	Normalisation.	
x_{MM}	1	MM level of lending	Average 1 year constant maturity US treasury yield.	Federal Reserve Board - Release H.15
ρ	0.012	interest rate	Target $p = 0.97$: annual frequency of banking crises in OECD countries 1970-2012.	
σ_A	0.041	standard deviation of $\log(A)$	$E[A] = 1 + \rho$, existing loans fairly valued.	Valencia and Laeven (2012)
σ_B	0.041	standard deviation of $\log(B)$	$E[B] = 1 + \rho$, implies $x_{MM} = 1$.	
μ_A	$\log(1+r) - 0.5\sigma_A^2$	expectation of $\log(A)$	x_{MM} normalised, and $x_{MM}/\lambda \implies 20\%$ of loans maturing per year.	van den Heuvel (2009)
μ_B	$\log(1+r) - 0.5\sigma_B^2$	expectation of $\log(B)$	Loan spread over deposit rate = 2%	Bernanke et al. (1999)
λ	4	book value of legacy loans	interest elasticity of demand on mortgage debt estimated from UK loan-to-value notches.	Best et al. (2015)
ν	12	number of banks	OECD average corporate tax rate 2005/2017.	OECD tax database
η	0.2	interest elasticity of demand		
τ	0.24	corporate tax rate		

Calibrating the capital requirement The capital requirement is not a straightforward object to calibrate. In the model, what matters is loss absorbing liabilities, as a percentage of the bank’s assets. Even accounting for risk weights, this is not necessarily the same object as the headline regulatory capital requirement that is the focus of the policy debate.

Relevant considerations include: (i) Banks have hybrid liabilities that both may or may not count towards the requirement and may or may not have implicit guarantees attached to them; (ii) there are different requirements for different types of capital; (iii) banks hold voluntary buffers above the requirements (for instance, to prevent small shocks from leading to violations); and (iv) requirements vary across jurisdictions, types of banks (for example, banks deemed to be globally systemic now face higher requirements), and with macroeconomic conditions (this is the role of counter-cyclical capital buffers).

To circumvent the issue, we present our results for a wide range of values of γ . That is, we display the $x^*(\gamma)$ functions. Still, we need a reference value to centre the calibration. For ease of interpretation, we consider a headline number of 13% of risk weighted assets for the requirement. Under Basel III, this roughly corresponds to the Tier 1 capital requirement (including systemic, conservation, and pillar 2 buffers) that globally systemically important banks face (see BCBS (2017) and EBA (2017) for recent assessments; Table A in BoE (2015) describes a breakdown of different requirements). This is the number we use for our calibration.

Average risk weights are typically around 50% (see Mariathasan and Merrouche (2014)), so this is the number we use for α and β .²³ Again, in practice, there is variation across banks, and over time.

Taxation and interest rates We calibrate τ to match the simple average statutory corporate tax rate among OECD countries; this corresponds to 24% in

²³Using the same value for α and β makes exploring a range of γ is equivalent to exploring a range for average risk weights.

2017. In our sensitivity analysis, we show the effect of cutting the tax rate from 35% to 21%, which are the percentage rates before and after the recent tax reform in the US. We interpret the period in our model as one year. Hence, we calibrate the interest rate ρ to match the average 1-year constant maturity US treasury bond yield: 1.2% in 2017.

Normalisation, competition, and elasticity of demand. We select parameters so that $x_{MM} = 1$ in the benchmark calibration and any alternatives presented. To this effect, we rescale the representative bank’s gross return function:

$$BX(x) = Bkx \left((x + x')^{-\eta} \right), \quad (14)$$

where $k = \nu^\eta / (1 - \frac{\eta}{\nu})$, and $E[B] = 1 + \rho$.

We calibrate η to match the interest elasticity of demand on residential mortgage debt estimated from UK loan-to-value notches (Best et al. (2015)). Last, we choose ν to target the average spread on new loans in the model $(E[B] \frac{x^*}{x^*} - 1 - \rho)$, which we calibrate at 2%, consistent with Bernanke et al. (1999). This gives $\nu = 12$.

We calibrate the book value of legacy assets (λ) to 4 such that, if the bank chooses $x = x_{MM}$, 20% of loans on the balance sheet were made in the current period. This is in line with values in the literature (see, for example, van den Heuvel (2009)). To abstract from bank closure (see Appendix A.1), we assume that $\kappa > \gamma\lambda$.

Risks and probability of default We model the joint distribution of $f(A, B)$ as a log-normal. We assume that legacy loans are held at fair value on the bank’s balance sheet; that is, $E[A] = 1 + \rho$. Finally, we assume that A and B have identical standard deviations, which we calibrate by targeting the bank’s equilibrium default probability $(1 - p^*$ in the model).

In the data, the appropriate calibration for $1 - p^*$ is the probability that the implicit subsidy is in the money and the bank benefits from a taxpayer transfer. Determining the value of p from the price of bank securities is challenging, precisely due to the need to strip out the value of any expected transfer. Instead,

we use realised frequencies. Specifically, Valencia and Laeven (2012) find that there have been 40 banking crises among the 34 OECD members over the period 1970-2012, which suggests a target value of $p^* = 0.97$ or a 3% annual probability of default (Martinez-Miera and Suarez (2014) use a similar value in their calibration).

Last, we set the correlation parameter between A and B equal to 0.5. This choice is arbitrary, and we run sensitivity analysis over it below.

5.1 Results

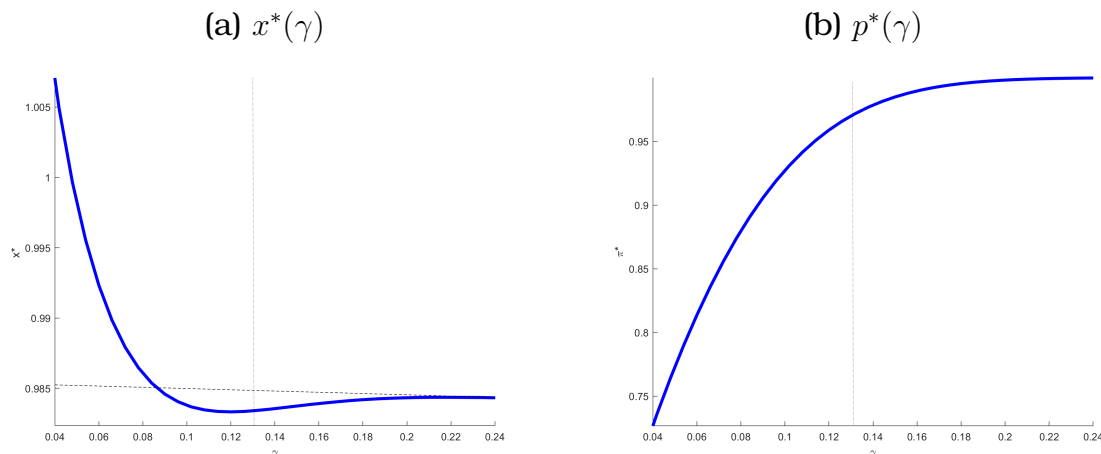
Benchmark example Figure 7 displays the $x^*(\gamma)$ curve (the left panel) and the associated probability of survival $p^*(\gamma)$ (right panel) for our benchmark calibration. As in Figure 6, the downward sloping dashed curve filters out the direct effect of s_x^* on $x^*(\gamma)$. The vertical dotted line indicates the reference value for γ .

As we can see, the representative bank is in an upward sloping region that extends from a requirement of about 11% to 21%. At lower values of γ , the slope is negative (and relatively steeper at very low values). From values 21% onward, the curve is downward sloping again: the bank is in fact very safe, and what dominates is the direct effect of taxation (as in Figure 6 in the subsection on taxes).

Discussion and sensitivity analysis The slope is positive at our reference capital requirement, $\gamma = 13\%$, but the response is economically small. For instance, a capital requirement increase from 13% to 14%, would generate an increase in lending of 0.02%. However, this is still very different from conventional wisdom and the typical concern that such a policy change would provoke a cut in lending. And the reason for the absence of a cut is the forced safety effect. Here, it is strong enough to more than offset the other forces in the model. The result also contrasts with those of recent quantitative studies that suggest a strongly negative lending response (e.g., Corbae and D Erasmo (2017) and Elenev et al. (2017)).

Now consider an 8% capital requirement. The curve is downward sloping, and

Figure 7: Equilibrium Lending and Survival Probability Under the Benchmark Calibration



Notes: The two panels show equilibrium lending and survival probability for different levels of the capital requirement under the benchmark calibration defined in Table 1. Panel a: equilibrium lending for the representative bank under the alternative levels of γ ; the downward sloping dashed line is the counterfactual level of lending where $s_x = 0$; the vertical dashed line denotes the reference level of the capital requirement. Panel b: equilibrium survival probability for the representative bank under the alternative levels of γ ; the vertical dashed line denotes the reference level of the capital requirement.

steeper, which is more in line with conventional wisdom. Given that this percentage is the one mandated by Basel I, the regulation in place in most countries in the 1990's and most part of the 2000's, this case constitutes a plausible situation facing the banks before the global financial crisis (note also the corresponding, higher probability of a bailout in the right panel).²⁴ In this case, going from a capital requirement of 7% to 8% causes a cut of lending of 0.2% and an increase in lending spread of 5bps.

We believe that our benchmark numbers for the calibration are plausible. However, they only constitute one example; we do not wish readers to take our results as a prediction that raising capital requirements today would necessarily cause most banks to increase lending. Nor would we want to claim, on the basis of the paragraph above, that all banks would have shown steep negative lending responses in the run-up to the global financial crisis.

We would rather argue that lending responses are likely to show a lot of vari-

²⁴Given the many loopholes and the amount of regulatory arbitrage at the time, one could even consider an effective capital requirement below the headline 8%.

ation, in both the times series and in the cross section, and in both signs and in magnitude. To illustrate this, Figure 8 shows how the lending response of the representative bank in the benchmark example changes when we alter parameter values one at a time. First, to generate a steeper positive response, one can, for instance, make the legacy loans overvalued (see Panel a) or decrease the correlation between A and B (Panel b). And vice versa: undervalued legacy loans and higher correlation make the lending response more negative, and relatively steep in some cases. Second, consistent with our analysis in Section 4.2, one can see in Panel c that higher tax rates are generally associated with lower lending, but they also affect the slope of the curve, as they tend to deepen the U-shape. Additionally, higher interest rates tilt the curve clockwise, as tax deductibility becomes more relevant.

5.2 Bank heterogeneity

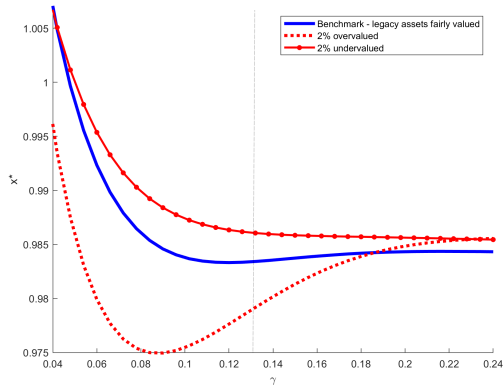
So far, our numerical exercise has considered ν identical banks all facing an identical capital requirement. In general, the lending of an individual bank is more sensitive to an idiosyncratic change in its individual capital requirement (see Figure 9 in Appendix C). This is simply because an individual bank faces a much shallower residual demand curve for loans than the total banking system (Kisin and Manela (2016) make a similar point).

It is also the case that heterogeneity among banks can substantially alter how banks respond to a change in aggregate capital requirements. To illustrate this, take one potential dimension in which banks could be heterogeneous: legacy loan valuation. Imagine that, instead of all banks having fairly valued legacy loans, as in the previous subsection, there are two equally sized groups of banks: strong banks, whose assets are in fact better than their valuation (i.e. they are undervalued) and weak banks that have overvalued legacy loans (or unrecognised losses). Still, on average in the economy, legacy loans are fairly valued.

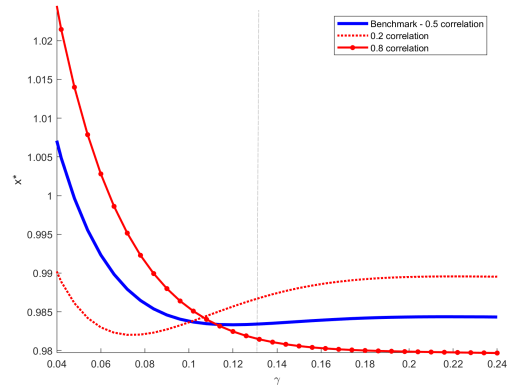
The two groups of banks will typically have different levels of lending. Broadly speaking, strong banks will be safer and so should suffer from less of a guarantee overhang, but they should also have less incentive to overlend. So either group

Figure 8: Sensitivity analysis on equilibrium lending

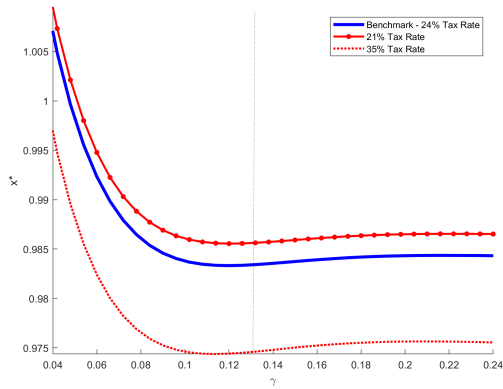
(a) Legacy Loan Valuation



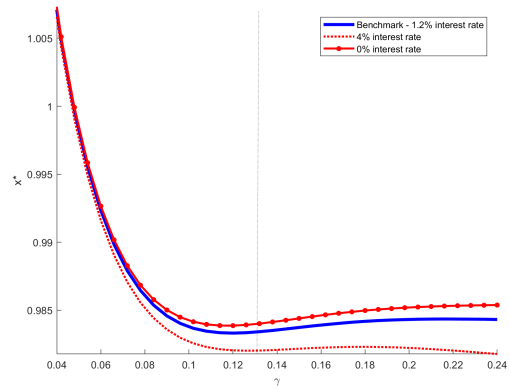
(b) Correlation



(c) Taxes



(d) Interest Rates



Notes: The panels show equilibrium lending for the representative bank under the alternative levels of γ when the benchmark calibration defined in Table 1 is modified in a single dimension. All panels: the blue line denotes benchmark calibration; the vertical dashed line denotes the reference level of the capital requirement. Panel a: the red dotted line is legacy loans undervalued by 1% ($E(A) = 1.01(1 + \rho)$), the red dashed line is legacy loans overvalued by 1% ($E(A) = 0.99(1 + \rho)$). Panel b: the red dotted line is when $\log(A)$ and $\log(B)$ have 0.8 correlation, red dashed line is when $\log(A)$ and $\log(B)$ have 0.2 correlation. Panel c: the red dotted line is $\tau = 0.21$, the red dashed line is $\tau = 0.35$. Panel d: the red dotted line is $\rho = 0.0$, the red dashed line is $\rho = 0.04$.

could lend more than the other.

The banks will also respond differently to a change in their capital requirement. Put differently, each group has a different “U-shape” and, at a given γ , these shapes have different slopes with potentially different signs. As we describe below, competition further complicates the situation.

Such environment is highly non-linear. So aggregate lending, and the aggregate lending response to a change in the capital requirement, is affected by heterogeneity. It turns out that, compared to our representative bank benchmark case, introducing heterogeneity can either magnify, mitigate, or even flip the sign of the aggregate lending response.

Table 2 provides an example of how heterogeneity can play out. It shows the change in lending response following a capital requirement increases of 1 and 2 percentage points respectively, with weak (strong) banks having legacy loans that are 1% overvalued (undervalued). All other parameters are in line with Table 1, and the initial capital requirement is 13%.

The responses for the representative bank are both small (they correspond to our benchmark result, see Figure 7). It is immediately obvious that, with heterogeneity, the two groups of banks are each more sensitive to the requirement. Our interpretation of what happens is the following. Weak banks are on the upward sloping portion of their U-shape. Therefore, their individual response is to increase lending. Strong banks’ legacy assets are less likely to generate negative residual cash flows. Their FSE is weaker. For the sake of simplicity, assume that their individual response is nil. Now we can consider the effect of competition. The lending increase by weak banks decreases the residual demand facing strong banks. As a result strong banks cut lending. But this increases the residual demand facing weak banks, so they lend more (which also makes them safer and feeds the FSE), and so on and so forth. In equilibrium, we end up with polarised responses.

While this qualitative description is accurate for both columns in 2, between the two cases the magnitudes and aggregate response differ markedly. When capital requirements are increased by 2 percentage points, this has a dramatic effect on weak banks: they become much safer, and end up lending much more. Com-

Table 2: Heterogeneous lending responses to a capital requirement Increase

	(1) 1% <i>increase</i>	(2) 2% <i>increase</i>
Representative Bank Response (%)	0.021	0.034
Weak Banks: 1% Overvalued (%)	0.084	2.873
Strong Banks: 1% Undervalued (%)	-0.046	-0.642
Aggregate Lending (%)	-0.014	0.062

Notes: The table shows the response of lending to a 1 percentage point (column 1) and 2 percentage point (column 2) capital requirement increase starting at $\gamma = 13\%$. The first row shows how the representative bank responds under the benchmark calibration. The final three rows considers how the responses are altered by heterogeneity among banks. Specifically, we assume that half of banks have legacy loans that are 1% overvalued ($E(A) = 0.99(1 + \rho)$), and half have legacy assets that are 1% undervalued ($E(A) = 1.01(1 + \rho)$). We denote these weak and strong banks respectively and the second and third columns present the lending response in each group. The final row presents the aggregated response of lending by both types of banks to the capital requirement increase. Note that at the initial value of $\gamma = 13\%$, strong banks lend more hence the aggregate lending response places more weight on the reaction of strong banks. All other parameters are calibrated as in Table 1.

petition amplifies this further and we see a 2.9% increase in lending. While this crowds out strong bank lending, the net effect is to boost aggregate lending by 0.06%, double the increase in the representative case. In contrast, a 1 percentage point increase gives a smaller boost to weak banks. Since strong banks are initially on a downward sloping portion of their U-shape their response actually dominates, and overall aggregate lending falls.

5.3 Link to the empirical literature

To sum up, the exercises above highlight that the determinants of bank behaviour under capital requirements are many and complex. There are circumstances where a bank's lending will respond positively to a change in capital requirements and, we have argued, these circumstances are neither extreme nor implausible. Furthermore, a change in the bank's circumstances could substantially alter that response. One should not necessarily expect a stable relationship between capital requirements and lending.

This may explain inconsistencies in the recent empirical literature. For example, Gropp et al. (2018) and Bassett and Berrospide (2017) study stress test

induced increases in the capital requirements in Europe in 2011 and the United States in 2013-2016, respectively. Both use types of difference-in-differences estimators where size based stress test eligibility criteria determines treatment. Despite the similar settings, the conclusions are very different: Gropp et al. (2018) find a reduction in lending, and Bassett and Berrospide (2017) find that, if anything, lending goes up. Yet both are consistent with our model: inspecting Figure 7, our calibration suggests that times where banks have low probability of failure tend to be associated with flat or positive responses to capital requirements. In contrast, the downward sloping region coincides with a higher default probability. Given the stressed European banking sector facing the teeth of a sovereign debt crisis in 2011²⁵ versus the recovering US in 2013-2016, our model perhaps helps reconcile the two findings.

Jiménez et al. (2017) also find that wider economic conditions matter for the lending response to capital requirement changes. Looking at reforms to dynamic provisioning in Spain they find that aggregate credit supply contracts by less in good times versus than bad. As with our model, this indicates that attempts to extrapolate evidence from specific settings or time periods can be problematic.

Empirical evidence from pre-crisis sample periods generally points to a negative lending response (see for instance, Hancock and Wilcox (1994); Francis and Osborne (2012); Aiyar et al. (2014a,b)). Our model predicts that a negative lending response is more likely at low *levels* of the capital requirement. It is, therefore, conceivable that future empirical research, with sample periods under the stricter requirements of the new Basel III regime, will have different findings.

Another strand of the empirical literature focuses on the response of different types of lending to heterogeneous capital requirements or different risk weights. For instance, Behn et al. (2016) provides evidence on whether capital requirements are binding and how certain types of borrowers would be affected. However, the response of specific asset holdings to their capital charge cannot be extrapolated to the whole balance sheet. Looking at our model, imagine that the bank could make two types of new loans, and that the risk weight was raised on

²⁵De Jonghe et al. (2016) and Fraise et al. (2017) also report that tighter capital requirements reduced lending in European in samples during the crisis.

just one of them. The bank would do less of that type of lending relative to the other type but if this makes the bank safer enough, the bank could still expand its whole balance sheet, which would reflect a strongly positive FSE.

Finally, a related strand of the empirical literature focuses on the lending response to shocks to the *level* of bank capital (Bernanke and Lown (1991); Berrospide and Edge (2010)) or to losses on the bank's existing assets (Peek and Rosengren (1997); Puri et al. (2011); Rice and Rose (2016)) . While these questions are related, they are different from asking how the capital *requirement* affects lending. However, through the lens of our model we can say that an unrecognised loss to the bank's existing assets (i.e., legacy loans are overvalued and $E(A) < 1$) will typically reduce lending (see Figure 8a). A recognised loss, where the book value of legacy loans is marked down to a fair value, is equivalent to an equal reduction in κ and λ , holding $E[A]$ fixed. However, as we will discuss next, a change in κ , considered in isolation, typically has no effect on lending in our model. Still, from an empirical standpoint, it is challenging to separate recognised from unrecognised losses.

6 Residual cashflows

6.1 Residual cashflow heterogeneity

A key feature of our model is that, when valuing the marginal loan, the bank will consider both its contribution to economic surplus and its effect on the implicit subsidy. Put differently, the bank does not price assets independently, rather it values them in relation to its whole balance sheet. In this subsection we discuss the role of legacy loans, and how this pricing mechanism would play out under alternative assumptions.

Legacy loans and the interpretation of λ We highlighted in Section 4 that some residual cashflow heterogeneity is needed for the existence of a positive lending response. In our model, this gives a key role to λ , which we interpret as legacy loans. However, λ could equally be interpreted as a portfolio of securities,

or another business division. Note also, as we will discuss below, that our mechanism still applies if λ is exempt of capital charges (e.g., an OECD sovereign debt exposure) or even if the exposure is not on the balance sheet (litigation risk for instance).

Since banks are going concerns and have, at all times, a stock of existing loans on the balance sheet, interpreting λ as legacy loans is a natural thing to do. Nevertheless, we want to stress that it is not necessary that residual cashflow heterogeneity comes from legacy loans.

Loan sales at fair price In our model, loans cannot be sold. What would happen if they could be?

First, note that in the baseline model, the bank has no incentive to sell legacy loans at a fair price (i.e., at a unit price $E[A]$). The reason is precisely that these loans carry an implicit subsidy when they are held on the bank's balance sheet. By decreasing its exposure at the margin, the bank would simply reduce the value of the subsidy. Selling all legacy loans is not in the bank's interest either. It is true that this would fully alleviate the guarantee overhang: without legacy loans on the balance sheet, the bank would choose $x^* = x_{MM}$. But it would be better off choosing $x^* = x_{MM}$ while keeping the legacy loans.

This intuition is in line with the reluctance of a distressed bank's shareholders to clean up their balance sheet by selling risky assets at market prices (see, for instance, Philippon and Schnabl (2013)).

When new loans are risky, selling legacy loans may become attractive for the bank. However, since we assume legacy loans to be (among themselves) perfectly correlated, the bank will either choose to keep them all, or to sell them all. Which option is best depends on the overall risk structure. The logic of our analysis still holds, though, and, at the margin, the effect of a change in the capital requirement on the bank's choice will still be captured by the cross-partial derivative of the subsidy.

Starting a new bank (i.e., selling the real option to lend) In our model, issuing new loans generally reduces the subsidy attached to legacy loans. If we

allowed the shareholders to start a second, separate bank in which they could issue the new loans, they would do it. The reason is that, while economic surplus is additive, the rents they get from government guarantees are not. In particular, the sum of the values of the two implicit subsidies exceeds the value of the implicit subsidy on an integrated portfolio. This is basic option theory: the value of the implicit subsidy is that of a put option (Merton (1977)), and the value of a portfolio of options exceeds the value of an option on the portfolio.

6.2 Residual cashflow alignment

A conclusion from our calibration exercise is that whether a bank's lending response is positive or negative depends, on a complex way, on many variables. Since these relationships are captured by the third cross-partial derivative of the value of a put option, this is perhaps not too surprising. On the one hand, this means that the model can help reconcile apparently conflicting findings. On the other hand, this also means that the model does not provide a crisp set of empirical predictions. Still, we can formulate a useful general prediction regarding how government guarantees affect bank behaviour.

To do this, it is useful to think of the bank's general asset pricing problem. The bank's relevant pricing kernel is binary: it values residual cashflows linearly in the states it survives, and does not value them at all in the states it defaults. As a result:

Financial institutions that benefit from government guarantees will overvalue loans (and assets) that have negative residual cashflows in the states of the world where the bank defaults; and will undervalue assets that have positive residual cashflows in those states.

This means that banks' loan and asset portfolio decisions are distorted towards aligning the *sign* of their residual cashflow. We refer to such behaviour as Residual Cashflow Alignment (RCA).

Menu of heterogeneous new loans What would RCA behaviour mean if the bank could choose, with full flexibility, among a set of potential new loans? And

how would its choice be affected by the capital requirement?

Maintaining our assumption that legacy loans are not sold, what the bank would do is to issue the loans whose residual cashflows in survival states are at least $\gamma\%$ of face value, no matter their residual cashflows in default states. As a result, depending on the joint distribution of payoffs, the marginal loan in equilibrium (i.e., the one that the bank just did not make) may have positive or negative NPV in equilibrium. Whether a change in capital requirement makes this marginal loan more or less attractive would still crucially depend on the expected residual cash flow along the default boundary.

The same logic would apply, in equilibrium, to a bank that has no legacy assets on its balance sheet. Imagine such a bank faces a finite number of potential loans. In equilibrium, the subset of loans the bank would choose to finance would only include those with the residual cashflow structure described in the previous paragraph. And, again, the marginal loan would have either positive or negative NPV, and a change in capital requirement could make it more or less attractive.

Harris et al. (2017) study a related portfolio allocation problem. In their model, banks have different initial portfolios of loans, which are perfectly liquid, and they compete to make new loans to heterogeneous borrowers. What the authors find is that the general equilibrium allocation exhibits perfect residual cashflow sign alignment. That is, assuming a bank holds a given loan in its portfolio in equilibrium, then, it will have sold all loans (and not issued any loan) that generate, in any state, residual cashflows of a different sign than that of this loan.

In the author's terminology, banks are seeking downside risk correlation (see Landier et al. (2015) for a similar interpretation). We think that our terminology is more accurate, as it is the alignment of the signs of the residual cashflow that is relevant, not their correlation in default states. In fact, perfectly negative correlation *given default* is compatible with perfect RCA.

Concrete examples of prediction Formulating our prediction in terms of RCA is useful in several dimensions. First, we note that limited liability, in the pres-

ence of existing debt, also induces RCA behaviour. It is on this basis that we make the link between risk-shifting, debt overhang, and guarantee overhang problems in the introduction.

Second, building on four concrete cases that have been studied in the recent literature, we can formulate scenarios in which we are relatively comfortable making predictions on the sign of the lending response. (Note that the original papers do not consider changes in capital requirements).

First, consider the case of New Century Finance Corporation (NC). Landier et al. (2015) point out that the monetary tightening by the Fed in the spring of 2014 was a negative shock to NC's sub-prime loan portfolio. They argue that, NC reacted to this shock by resorting to deferred amortization loan contracts on a massive scale, precisely because they would perform badly if house prices were to fall. Imagine that a bank was in a similar situation; an increase in capital requirement would mitigate the problem, and reduce excessively risky lending. This is an intuitive case.

In general, however, banks have more heterogeneous assets on their balance sheets than such a specialised mortgage originator. Our next two scenarios reflect this. Consider a scenario similar to that studied in Puri et al. (2011): a German saving bank whose business is primarily to lend to domestic households and SMEs. The bank has, however, a substantial exposure to US sub-prime mortgages, which could generate sufficient losses to cause the bank to default in some states. Assuming the marginal loan on the domestic portfolio would not make dramatic losses in those states, our model predicts that the bank will cut domestic lending as the outlook on US sub-prime worsens. This is consistent with the findings of Puri et al. (2011). In addition, our model predicts that: (i) the FSE is positive; and (ii) that it dominates (i.e., the lending response is positive).²⁶

In our interpretation, the lending cut documented by Puri et al. (2011) reflects RCA behaviour. In the literature, the scaling down of a bank that has faced an adverse shock is sometimes interpreted as a sign of prudent behaviour, perhaps reflecting an increase in risk aversion (e.g., DeYoung et al. (2015) and Elenev et al.

²⁶This second point assumes that the subprime exposure is not too large.

(2017)).²⁷ Our interpretation is fundamentally different: we predict that a bank will pass on investment opportunities with misaligned residual cashflows. This is because misaligned residual cashflows provide a form of (default) insurance, whose benefits accrue to other stakeholders.²⁸ Rather than prudence, this should still be interpreted as a form of excessive risk taking. Furthermore, it is possible to build examples where, in order to align residual cashflows, a bank passes on what would appear in isolation to be a textbook risk-shifting opportunity.²⁹ But again, what motivates such behaviour is not the aim of improving efficiency or limiting risk exposure; the goal is simply to maximise expected payoff given limited liability.

The third scenario is, in a sense, a variant of the second. Consider an Irish bank that had been lending extensively to high LTV, urban mortgagors when the Central Bank of Ireland imposed restrictions on the issuance of such loans. The findings in Acharya et al. (2018) suggest that this policy would cause the bank to aggressively expand its issuance of loans to safer borrowers, in rural counties. The question then arises as to why the bank was passing on the second type of loans in the first place (and why it is now willing to issue them at a lower rate). One possibility is that the bank found these loans unattractive because they would have produced positive expected residual cashflows in default states in general, and on the boundary in particular. Then, the policy made the bank safer, which made the bank internalise those positive cashflows. Our prediction is

²⁷The argument is often based on Froot and Stein (1998), who have shown that, if raising capital is ex-post costly, banks will behave in a risk averse fashion (by building precautionary buffers, for instance) in an effort to avoid those costs. Froot and Stein (1998) make however clear that their analysis abstracts from the distortions created by government guarantees.

²⁸Fecht et al. (2015) also find that deposit insurance hinders banks' incentive to insure themselves using the interbank market.

²⁹Imagine that our bank faces a single new loan opportunity. This is a unit loan that repays either 1.12 or 0.8, with equal probability. Imagine that the capital requirement is 10%. Even though it is negative NPV, a new bank would finance this loan, because investing 0.1 in capital yields a payoff of 0.22 with 50% probability (and 0 otherwise). This would be a classic example of risk-shifting (Kareken and Wallace (1978)). But if the bank's portfolio includes a legacy loan that generates positive residual cashflows (of at least 0.02 in expectation) in the state where the new loan performs badly, the bank will refrain from making the loan. From the perspective of a new bank, the implicit subsidy makes the (intrinsically negative NPV) loan a positive NPV investment. But residual cashflow misalignment reduces the part of the payoff that is a rent (the marginal subsidy).

that such loans would also have become more attractive after an overall increase in capital requirements.

Finally, consider a market maker in a particular security business. These securities are relatively safe (e.g., US treasuries) and not closely related to the rest of the bank's balance sheet. RCA behaviour implies that the bank may pass on positive NPV trades, which is in line with the interpretation in Duffie et al. (2018). Assuming the bank is not in distress to start with, an increase in the capital requirement is likely to make the bank more willing to deal in those assets, and should narrow the bid ask spread.

We find these examples useful for illustrating the potentially wide range of applications for our results. Besides the concrete empirical predictions on the sign of the lending response, a key takeaway is that, while RCA reflects excessive risk-taking, the induced behaviour does not necessarily take the form of textbook risk-shifting behaviour.³⁰ This may help explain the lack of evidence that banks start lending more aggressively as their balance sheets deteriorate.

6.3 Multiple requirements

Imagine that the bank faces two capital requirements γ_1 and γ_0 , such that:

$$\kappa + c \geq \gamma_1 x + \gamma_0 \lambda.$$

We can now redefine the residual cashflows from the marginal loan as $Z = BX_x - (1 - \gamma_1)$. The expression for the marginal subsidy is then unchanged except that the default boundary is altered.³¹ As with a single capital requirement, the sign of the lending response depends on the derivative of the marginal subsidy with

³⁰This observation is related to Gollier and Pratt (1996). They show that, in the presence of background risk, a concave but piece-wise linear objective function can induce behaviour that is hard to interpret in terms of risk-aversion. In our context, limited liability makes the objective function convex and piece-wise linear. And our example in the footnote above is indeed the mirror of that proposed by Gollier and Pratt (1996).

³¹Specifically, we have $a_0(B) = \frac{1}{\lambda} ((1 - \gamma_1)x + (1 - \gamma_0)\lambda - BX)$ and $b_0(A) = \frac{1}{X} ((1 - \gamma_1)x + (1 - \gamma_0)\lambda - A\lambda)$.

respect to the requirement in question. Specifically, we have that for a change γ_1 :

$$s_{x\gamma_1}^* = -(1 - p^*) + p_{\gamma_1}^* z_{\Delta_0}^*.$$

The first term can be interpreted as a composition effect; the second term is a form of FSE. Starting from $\gamma_1 = \gamma_0 = \gamma$, the only difference in this expression from equation (13) is that $p_{\gamma}^* > p_{\gamma_1}^* > 0$. Raising the capital requirement on new loans generates a smaller shift in the default boundary than an increase in the requirement on all loans. The FSE is weakened but still present. Furthermore, the expression is unchanged if $\gamma_0 = 0$; in particular, the FSE can still be positive and dominate the composition effect even if no capital is held against legacy loans at all.³²

Now consider a change in γ_0 :

$$s_{x\gamma_0}^* = p_{\gamma_0}^* z_{\Delta_0}^*.$$

There is no composition effect: changing γ_0 has no impact on the composition of liabilities used to fund new loans. The sign of the lending response is determined solely by the (modified) FSE; whether or not the bank increases lending depends on the sign of the residual cashflows, in expectation, along the default boundary.

Now consider what happens if $\gamma_1 = 1$ such that new loans are fully financed by capital. This brings us very close to the classic debt overhang model of Myers (1977). The bank has existing assets that are partly debt financed (at a fixed interest rate, in our model, due to the guarantee), but new investments are wholly financed by capital. Now the (modified) FSE and the overhang problem go hand in hand. As residual cashflows are always positive, $Z = BX_x$, the bank always underlends ($s_x^* = -(1 - p^*)z_{\Delta}^* < 0$), and lending is always increasing in γ_0 . Furthermore, the tighter requirement increases lending precisely because it makes the overhang problem less severe.³³

³²Compared to $\gamma_0 = \gamma_1 > 0$, setting $\gamma_0 = 0$, influences the equilibrium values of p^* , $p_{\gamma_1}^*$ and $z_{\Delta_0}^*$. It will unambiguously make the bank riskier, strengthening the composition effect but not the fact that the default boundary shifts outward could increase both $p_{\gamma_1}^*$ and $z_{\Delta_0}^*$, strengthening the FSE.

³³This result stands in sharp contrast with the conclusion of Admati et al. (2018) that a firm

This extreme case serves to highlight some of the original features of our model. To generate the competing forces that are present in our model, one must account for the fact that capital requirements inherently influence both the residual cash flows on the marginal loan and the default boundary. This is why we can have, emerging from the same moral hazard problem, ambiguous predictions over the sign of the lending response, the sign of the FSE, and a disjunction between the two. This example also illustrates that even if the marginal new loan has an exactly identical payoff distribution as the average loan on the bank's balance sheet, one can still obtain residual cashflow heterogeneity if the capital requirements (or the risk weights) on different assets are heterogeneous.

7 Conclusion

We see our contribution to the policy debate as follows: that capital is costly for banks does not imply a negative lending response. Indeed, the Forced Safety Effect can counteract the liability composition effect, which overturns conventional wisdom.

The policy debate concerns both the long term effect of capital requirements (what is their socially optimal level?) and their effect in the shorter term (how should they be adjusted to the state of the economy?).

To maintain as much tractability as possible, we study a static model. Endowing the banks with legacy loans allows us to give banks a going-concern dimension. Crucially, it enables us to show that the way in which banks react to capital requirements is history dependent: existing assets on their balance sheet matters. Hence, our approach directly speaks to changes in capital requirements (which can be interpreted as time varying adjustments, such as Basel III's coun-

that faces a debt overhang problem and is forced to deleverage is more likely to sell assets than to raise capital. A key difference is that they consider zero NPV trades in an environment that is, essentially, scale invariant. Our approach allows us to study the effect of forced deleveraging starting from a level that is privately optimal for the firm, given the initial leverage restriction. When there is a guarantee overhang, the marginal trade (i.e., the marginal loan) has a strictly positive NPV. So, at the margin, an increase in capital requirement is unlikely to make a zero NPV loan attractive. However, given that the (modified) FSE is positive, it would make the (initially) marginal loan more attractive.

tercyclical buffers, or as gradual increases towards higher levels). Insights from our paper could, for instance, inform a regulator who wants to time increases in capital requirements in a way that minimises the impact on economic activity.

Our approach does not allow us to directly address the questions linked to the overall long term ideal level of capital requirements. Given that lending responses exhibit strong non-linearities, calibrating our model to average conditions and interpreting this as a proxy for a steady state would not be appropriate.³⁴ In fact, in light of our results, it is hard to think of a relevant concept of steady state. The fact that banks are going concerns, at any given time (excluding when the bank starts), they will always have legacy loans. How the structure of their residual cashflows relates to that of the marginal loan is a complicated question. All objects reflect endogenous decisions, and the interaction also depends on the regulatory regime.

From that point of view, solving a dynamic version of our model with heterogeneous banks could help draw new insights on the long-term and dynamic effects of capital requirements. However, keeping such a model tractable will be challenging. We leave this for future research.

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³⁴De Nicolò et al. (2014) simulate a dynamic model of a bank in partial equilibrium. To proxy for steady state, they look at unconditional moments. They find that lending is, on average, lower at a 12% capital requirement than at 4%. They also find that lending is lower for an unregulated bank. But this result is hard to interpret, as the bank operates, on average, with a negative book value of equity.

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A Additional Theoretical Results

A.1 Participation constraint and bank closure

In our model, the shadow value of initial capital is equal to the price of new capital. It is therefore irrelevant how much capital comes from new shareholders versus how much was already on the bank's books. This is why the value of κ does not affect the value of x^* .

Still, initial shareholders have the option to close the bank at date 1 and walk away with zero. This means that κ alters their participation constraint. To see this, consider again the baseline model, the participation constraint is:

$$w^* = \int_{a_L}^{a_H} (X^* - x^*) f(A) dA + \int_{a_L}^{a_H} (A\lambda - \lambda) f(A) dA + s^* + \kappa \geq 0. \quad (15)$$

Under our assumption that $E[A] = 1$, the second term disappears, and the constraint is always satisfied (all other terms are positive). However, if $E[A] < 1$, κ becomes relevant, and the participation constraint can be violated. First, consider $\kappa \geq \gamma\lambda$. Here, closing the bank is never the best option for shareholders. This is because, under limited liability, even operating at $x = 0$ gives the bank's shareholders a positive payoff in expectation. However, if $\kappa < \gamma\lambda$, shareholders must first raise new capital if the bank is to operate. When the option value of operating the bank is low (i.e., when $E[A]$ is low and new loans do not generate much surplus), operating may not be worth the cost of recapitalisation. The

participation constraint is then violated.³⁵

Now, the key point we want to make is that when $E[A] < 1$, increasing γ may make the bank close at date 1. Formally:

Proposition 5. *If $\kappa + (X^* - x^*) < (1 - E[A]) \lambda$, there exists a $\bar{\gamma} < 1$ such that for all $\gamma \geq \bar{\gamma}$, the bank closes at date 1.*

To understand Proposition 5, first note that a high γ makes it more likely that $\kappa < \gamma\lambda$. Second, γ reduces the subsidy (see Equation 7), so that $s^* = 0$ for some sufficiently large γ . Given $s^* = 0$, if initial equity plus the surplus on new loans is insufficient to cover the expected losses on legacy assets, then the bank will shut down.

Since a large γ is not viable for distressed banks, the logic behind Proposition 1 does not necessarily apply here. If $x^*(\gamma)$ is only upward sloping when $\gamma > \bar{\gamma}$, the bank would always choose to close rather than increase lending in response to a requirement increase.

A.2 The model with taxes

Consider the baseline version of the model. Adding the tax and assuming that households have an opportunity cost of funds $1 + \rho > 1$ means the bank's objective function now reads:

$$\begin{aligned}
 w = & \underbrace{X - (1 + \rho)x}_{\text{economic surplus}} + \underbrace{\int_{a_L}^{a_0} ((1 - \gamma)(1 + \rho)(x + \lambda) - X - A\lambda) f(A) dA}_{\equiv s(x, \gamma), \text{ i.e. the implicit subsidy}} \\
 & - \tau \underbrace{\int_{a_1}^{a_H} (X + A\lambda - (1 + (1 - \gamma)\rho)(x + \lambda)) f(A) dA}_{\equiv t(x, \gamma), \text{ i.e. the expected tax bill}} + \kappa,
 \end{aligned}$$

³⁵To see this, rewrite the participation constraint as:

$$\underbrace{\int_{a_0}^{a_H} (X^* + A\lambda - (1 - \gamma)(x^* + \lambda)) f(A) dA}_{\text{option value of operating}} \geq \underbrace{(x^* + \lambda)\gamma - \kappa}_{\text{new capital needed}}.$$

where $a_1 = \frac{1}{\lambda} ((1 + (1 - \gamma)\rho)(x + \lambda) - X)$ denotes the threshold such that the bank has positive taxable income if $A > a_1$. The expected tax bill is then given by $t(x, \gamma)$. All other terms are in line with Section 3, except that they now account for ρ .

The tax adds a new wedge in the first order condition:

$$X_x^* - (1 + \rho) + s_x^* - t_x^* = 0, \quad (16)$$

where $t_x = q\tau (X_x - (1 + (1 - \gamma)\rho))$ with $q = \int_{a_1}^{a_H} f(A)dA$ is the probability that the bank pays tax. Intuitively, $t_x^* > 0$. Ceteris paribus, the tax reduces equilibrium lending.

Now, the question we are interested in is how the tax, and the tax shield, affect the shape of the lending response. The reasoning follows the same logic as before: what happens depends on the cross-partial derivatives. But now, we have:

$$w_{x\gamma}^* = s_{x\gamma}^* - t_{x\gamma}^*.$$

The introduction of a tax has two effects on $w_{x\gamma}^*$:³⁶ a direct effect that materialises through the appearance of a term $t_{x\gamma}^*$, and an indirect effect that works through a change in the value of $s_{x\gamma}^*$.

To understand how these two different effects play out, first inspect Figure 6. The solid red line depicts $x^*(\gamma)$ without the tax ($\tau = 0$). This is the U-shape relationship of the baseline model. The blue dashed curve is the case with the tax. The net effect of taxes on the lending response is ambiguous: the U-shape relationship is still present, but it is (i) tilted clockwise and (ii) deeper than in the absence of a tax.

The tilt The tilt emerges because $t_{x\gamma}^* > 0$.³⁷ Intuitively, the wedge driven by taxation is increasing in the capital requirement due to the deductibility of interest on deposits. As a benchmark, note that setting $s_x = 0$ (i.e., using $X_x^* - (1 + \rho) - t_x^* = 0$ as a counterfactual equilibrium condition) would yield the downward sloping

³⁶The tax also affects w_{xx}^* , and therefore, the magnitude of the lending response. As before, its sign is solely determined by $w_{x\gamma}^*$.

³⁷ $t_{x\gamma}^* = q^*\tau\rho + q_\gamma^*\tau (X_x^* - (1 + \rho(1 - \gamma))) > 0$

dotted curve . It then follows that, as γ gets sufficiently high and the bank becomes safe, the level of lending converges to this line. Comparing this line to the x_{MM} level of lending makes the tilt more evident.

The deeper U-shape The deeper U-shape is due to the indirect effect of the tax on $s_{x\gamma}^*$. As in the baseline model, we have:

$$s_{x\gamma}^* = -(1 - p^*) + p_\gamma^* Z^*,$$

but the presence of the tax reduces x^* , which affects its equilibrium value. In the baseline model, we argued that the implicit subsidy works as a tax at the margin. The key implication was that it reduced equilibrium lending and made the marginal loan a positive contributor to the bank's residual cash flow. Now we have introduced an actual tax, which reduces lending further and reinforces this mechanism.

This has three effects. First, less (positive NPV) lending reduces profitability, which increases the probability of default $(1 - p^*)$. Second, it affects p_γ^* . Third, and crucially, it raises the equilibrium marginal return to lending X_x^* , which therefore raises Z^* and makes the forced safety effect stronger. The net effect is to deepen the U-shape. At low levels of γ , the presence of the tax makes the bank respond more negatively to an increase in γ , but at higher levels, it makes it respond more positively. As the bank becomes fully safe, $s_{x\gamma}^* = 0$, and only the direct effect is at work. The response becomes negative again.

The tax and tax shield It is important to differentiate between taxes and tax shields. Absent the deductibility of interest payments, $t_{x\gamma} = 0$, and the tax only matters for the lending response through its impact on $s_{x\gamma}^*$. Now, introducing a tax shield on deposits means both that $t_{x\gamma}^* > 0$ and that the marginal tax rate, t_x^* , is lower, which in turn potentially reduces the impact of the tax on $s_{x\gamma}^*$.

Adding a second source of risk Combining the corporate income tax (and the tax shield) with two sources of risk creates additional effects. Given that the

expression for $t(x, \gamma)$ is very similar to that for $s(x, \gamma)$, the derivations are very similar, and it should not be too surprising that counter-intuitive cases can arise for some sets of parameter values. In particular, it is possible that $t_{x\gamma}^* < 0$ (which would then contribute to a positive lending response) and $t_x^* < 0$.³⁸ However, these are specific cases that are unlikely to be relevant in practice: in our calibration exercise of Section 5, where there are two sources of risk, the effect of the tax on $x^*(\gamma)$ can essentially be understood and the basis of the tilt and the deeper U-shape that we explain above.

B Proofs

Proposition. 1. For all $\gamma \in (0, 1)$ and an associated $x^*(\gamma)$, if $p(x^*(\gamma), \gamma) < 1$, there exists $\gamma' > \gamma$ such that $x^*(\gamma') > x^*(\gamma)$.

Proof. First, set $\gamma' = 1$, and note that $x^*(\gamma') = x_{MM}$. Hence, it suffices to establish that $\forall \gamma \in (0, 1)$, $x^*(\gamma) < x_{MM}$. If $w(x)$ is continuously concave, Problem 4 is convex and the proof is straightforward. The first order condition is:

$$\frac{\partial a_0}{\partial x} \underbrace{(X_x^* + a_0^* \lambda - (1 - \gamma)(x^* + \lambda))}_{=0} + \int_{a_0^*}^{a_H} [X_x^* - (1 - \gamma)] f(A) dA - \gamma = 0.$$

Rewriting this as $X_x^* = (1 - \gamma) + \gamma/p^*$ makes clear that $\forall \gamma \in (0, 1)$, $X_x^* > 1$ if $p^* < 1$, which establishes the result.

However, $w(x)$ may not be well behaved. For a more general proof, we exploit the properties of the objective function on either side of a threshold \hat{x} such that $X_x(\hat{x}) = 1 - \gamma$.

First consider $x < \hat{x}$. Write the objective function as in Lemma 1 below, and note that $X - x$ is smooth and has a unique maximum at x_{MM} . Note that for all

³⁸A negative marginal tax requires that the marginal loan transfers cashflows from states where the rest of the bank's assets are generating a tax profit to states where there is a tax loss. Hence, the marginal loan then reduces the expected tax bill and, absent other frictions, the tax can lead to the bank making negative NPV loans. In turn, a change in the capital requirement shifts the boundary between a taxable losses and profits. If the marginal loan generates a large tax loss on this boundary, then it is possible that $t_{x\gamma}^* < 0$.

$x < \hat{x}$, and $\gamma \in (0, 1)$, both the upper limit of integration $a_0(x)$, and the integrand, are strictly decreasing functions of x . It follows that a point in the set $[x_{MM}, \hat{x}[$ cannot be a maximum of $w(x)$.

Second, consider $x \geq \hat{x}$. Write the objective as stated in Problem (4). Now, $a_0(x)$ is the lower limit of integration and is increasing in x . The integrand is also decreasing in x , and so is the third term. Hence, $w(x)$ is decreasing over the set $[\hat{x}, \infty$. Hence, for all $\gamma \in (0, 1)$, if $p^* < 1$ then $x^*(\gamma) < x_{MM}$. \square

Lemma. 1. *The objective function can be rewritten:*

$$w(x) = \underbrace{X - x}_{\text{economic surplus}} + \underbrace{\int_{a_L}^{a_0} ((1 - \gamma)(x + \lambda) - X - A\lambda) f(A) dA}_{\equiv s(x, \gamma), \text{ i.e. the implicit subsidy}} + \kappa \quad (17)$$

Proof. Use the assumption that $E[A] = 1$ and rearrange the objective function in equation (4). \square

Proposition. 2. (The sign of the lending response)

$$\frac{dx^*}{d\gamma} \begin{matrix} \leq \\ \geq \end{matrix} 0 \Leftrightarrow s_{x\gamma}^* \begin{matrix} \leq \\ \geq \end{matrix} 0.$$

Proof. This result follows directly from the implicit function theorem applied to the first order condition. \square

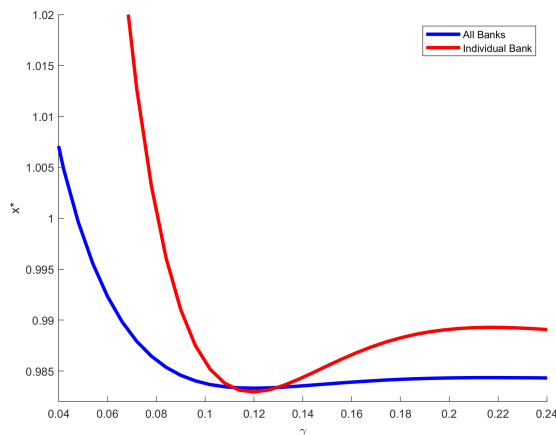
Proposition. 3 (i) $z_{\Delta_0}^*$ can be positive; (ii) this implies a positive forced safety effect; (iii) and can lead to a positive lending response: $s_{x\gamma}^* > 0$.

Proof. We provide examples of positive lending responses in Section 5. Upon request, we can provide examples with $\tau = 0$ (see also Appendix C). Now, having proved (iii) by example, (ii) must be true, and, therefore, (i), as well, since the composition effect is always negative. \square

Proposition. 4 For all γ such that $s_x^*(\gamma) < 0$, then there exists $\gamma' \geq \gamma$ such that $x^*(\gamma') > x^*(\gamma)$.

Proof. $s_x^*(\gamma) < 0 \Rightarrow x^*(\gamma) < x_{MM}$. Then, note that $x^*(\gamma' = 1) = x_{MM}$. \square

Figure 9: The lending response for an individual bank versus the banking system



Notes: We use the benchmark calibration as in Table 1. The blue line shows results for $x^*(\gamma)$ assuming all banks face the same capital requirement under the benchmark calibration. The red line shows the optimal lending decision for a single bank facing a change in its own capital requirement, and we modify equation (14) such that x' (lending at other banks) is held fixed at $x^*(\gamma = 0.13)$.

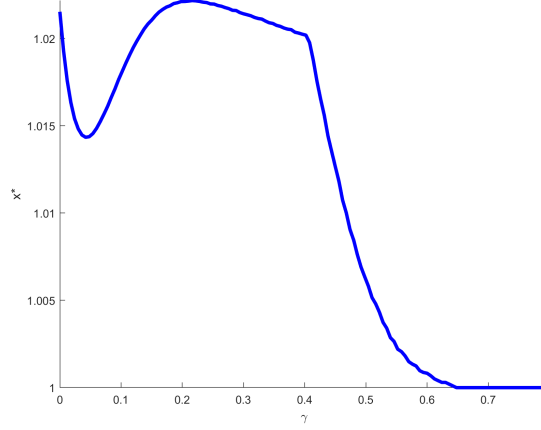
Proposition. 5. *If $\kappa + (X^* - x^*) < (1 - E[A]) \lambda$, there exists a $\bar{\gamma} < 1$ such that for all $\gamma \geq \bar{\gamma}$ the bank closes at date 1.*

Proof. Since $X'(0) = \infty$, if the bank decides to operate, it must be the case that $x^* > 0$. Given that in equilibrium the marginal loan satisfies $X_x^* = (1 - \gamma) + \frac{\gamma}{p^*} > 1$, all infra-marginal loans have positive net present value. Hence, $\int_{a_L}^{a_H} (X^* - x^*) f(A) dA > 0$. Since new loans generate some surplus, there exists a $\gamma < 1$ such that $p^* = 1$. But then, $s^* = 0$, and the participation constraint (15) simplifies to $(X^* - x^*) + (E[A] - 1) \lambda + \kappa \geq 0$. The condition in the Proposition ensures that it is violated. \square

C Additional figures

In our calibration exercise in Section 5, we run comparative statics over a symmetric equilibrium for ν identical banks facing the same capital requirement. As an alternative, Figure 9 presents, in red, the optimal level of lending for an individual bank when its requirement is changed, assuming all other banks lend

Figure 10: Example of a positive lending response when the bank overlends



Notes: This figure plots $x^*(\gamma)$ for an example in which the bank always overlends relative to x_{MM} but the equilibrium level of lending is increasing in γ for some range. Some pertinent details of the calibration are provided in the main text. In addition, we calibrate the parameters as follows: $\eta = 0.2$; $\nu = 1$; $\lambda = 1$; $\tau = 0$; $\rho = 0$; $\mu_A = 1$; $\sigma_A^2 = 0.063$; $\sigma_B^2 = 0.003$; and μ_B is set such that $x_{MM} = 1$.

at a rate given by the initial equilibrium when $\gamma = 0.13$. The blue line shows the symmetric equilibrium for comparison. As can be seen, the red curve is much steeper.

In Section 4.1, we claimed it was possible to generate examples where the lending response is positive and the bank overlends relative to x_{MM} . Figure 10 presents one such example. The crucial deviation from our benchmark calibration is that we assume that when A realises below its median value, there is a 1% chance that $B = 0$. Otherwise, the two random variables are distributed jointly log normal. We adjust the means such that expected values are not affected (specifically, $x_{MM} = 1$ in this example) and we set the correlation between the two to -0.9. The net effect of this change is to push some probability mass into very bottom corner of the default region in Figure 5. So A and B have high tail dependence but are negatively related for most of the $A \times B$ domain. The high tail dependence means that $z_{\Delta}^* < 0$: there is a set of default states where the marginal new loan performs very badly. But these states are typically not on the default boundary, so it can be true that $z_{\Delta_0}^* > 0$.