Forecasting Excess Returns in the Housing Market with Local Cap Rates

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Abstract

We investigate the predictive power of rent-to-price ratios in an excess housing return equation. Relying on two large geo-coded databases related on the one hand to rents and on the other hand to selling prices in the Paris area from 1996 to 2007, we compile rent-to-prices ratios and price growth rates with individual modelings and localized imputation methods. Different sources of risk (price, rent or vacancy risk) are taken into account. We break down the contributions of this rent-to-price measurement on futures excess returns into different geographical scale contributions: from broad scale (city level) to small scale one (the land register unit level corresponding to a few building blocks). Comparing the forecasting power of rent-to-prices ratio at various spatial scales seems relevant when working on housing markets composed of illiquid assets with a large idiosyncratic component, but is not usually done by lack of data. The spatially-disaggregated forecasting equations are estimated with standard techniques for different forecast horizons (3 and 6 years). The time dimension of the sample being short, we analyse the impact of the small-sample bias on our estimates. We exhibit that rent-to-price ratios account for a substantial part of the forecasting error at medium term horizon, the largest share of it being captured by the smallest scale measure.

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A Introduction

Following a large literature in finance (see among others Fama and French, 1988), returns predictability in real estate has been intensively studied. The predictive power of various factors (construction costs, per capita income, variables related to demography, etc.) when forecasting real estate prices or returns has been assessed at several horizons and spatial scales. The role of the cap rate (i.e. the rent-to-price ratio or rental return) has been singled out due to the significant contribution generally evidenced for its stock market counterpart, the dividend-price ratio (Campbell and Shiller (2001)). Its predictive power has been evaluated mainly using metropolitan or city-level datasets with contrasted conclusions depending on the chosen metropolitan areas, spatial scale or asset type (housing, office, retail, etc.). Plazzi, Torous and Valkanov (2010) illustrate that cap rates performances vary greatly with the metropolitan area (hereafter MA) under study. However, real estate prices and rents largely vary at the infra-metropolitan level due to the existence of local markets (local housing stock characteristics, local amenities) so that trends for the whole metropolitan may not reflect local market rigidities at smaller scales (city-level, building block level, etc.) that have a direct impact on real estate price and rent dynamics. This neglected infra-MA heterogeneity may bias estimates because of a spatial aggregation bias, see Smith (2004). We here evaluate the predictability of local housing returns using properly measured cap rates at different spatial scales (from building block level to city level) as predictors. Our localized cap rate measures are indeed an average of individual rent-to-price ratios different from the average rent to average price ratio usually computed in the literature, measure subject to statistical bias affecting the quality and interpretation of the estimates in this kind of regressions.

When adopting an asset price perspective, the price of a real estate asset should equal the present value of its future expected rents. This implies that the dynamics in real estate prices mainly reflects variations in future expected rents or in future discount rates that vary across local markets depending on land availability, existing and planned amenities, etc. Recently, Campbell, Davis, Gallin and Martin (2009) proposed a variance decomposition of the rent-price ratio for 23 U.S. MAs housing markets. The rent-price ratio is split into the expected rents growth component and the expected real interest rates and housing premia components (these last two terms are part of the total expected return). They found significant time-variability of these components (as well as a significant correlation between these terms) which explain a substantial part of the cap rate heterogeneity.

This evidence of a large amount of heterogeneity in cap rates is in line with financial market observations that led to question the predictive power of dividend yields to forecast stock returns. A large literature in finance has been devoted to these problems\(^1\). The corresponding real estate literature is much smaller partly due to a lack of individual

\(^1\)See for example Fama and French, 1988, or Cochrane, 2008, for papers concluding to a significant returns predictability or Campbell
observations which implies the use of proxies that can affect the quality of empirical results. Mankiw and Weil (1989) or Case and Shiller (1990), working at different (large scale) levels, fail in detecting a significant relationship between rent-price ratios and subsequent changes in prices or excess returns. Meese and Wallace (1994) used time-series data on housing prices, rents and the user cost of capital for two Northern California counties (Alameda and San Francisco) and validate the housing present value model in the long run with data running from 1970 to 1988. Capozza and Seguin (1996) studied expectations of capital gains in the U.S. housing market. Using census data disaggregated by metropolitan areas, they show that cross-sectional cap rates have significant power in predicting 10 years capital gains. Clark (1995) using a methodology close to Capozza and Seguin (1996) and decennial census-tract level data from 1950 to 1980 finds a significant and negative relation between rent-price ratios and next 10 years’ rent growth rates. In a VECM framework controlling for the role of direct user costs, Gallin (2008) provides evidence of a significant long-run relationship between prices and rents for whole-US housing quarterly data from 1970 to 2005. Finally, in line with the present value model, current cap rates appear to be significantly linked negatively to future changes in rents and positively to future changes in prices.

Recently, Plazzi, Torous and Valkanov (2010) extended the previous studies to apartments and retail, industrial and office properties. They adopt a long horizon approach (similar to Gallin, 2008) and investigate whether the cap rate reflect fluctuations in expected returns and/or in rent growth rates, building on a version of Campbell and Shiller (1988)’ dynamic Gordon growth model. According to this model, high cap rates should reflect either higher future discount rates or lower expected rent growth rates. Using prices and cap rates for each property type on a quarterly basis from 1994 to 2003 for 53 U.S. metropolitan areas in a GMM framework, they estimate long-run predictive equations at different forecast horizons (1, 4, 8 and 12 quarters) controlling for inter-MA heterogeneity with various local demographic or economic factors. They provide evidence that higher cap rates predict higher future returns for the various types of real estate, office properties excepted. Their results also seem to confirm previous findings in the case of stocks (Fama and French, 1988): the predictive power of cap rates (or dividend yields) is stronger for long forecast horizons.

The most recent contributions to the literature of predictability in real estate find a significant power of cap rates for predicting returns. But, in most cases no individual estimation of rents is provided: some of the above papers use individual price data, but MA (or national) average for rents. These studies (except Clark, 1995) then evaluate the predictive power of cap rates at the MA level. However, we may reasonably expect that a large part of the information conveyed by cap rates is relevant at an infra-MA or infra-city level: due to the possibly high and Shiller, 2001, who did not find any significant forecasting power of the dividend-price ratio.
heterogeneity in the housing stock characteristics within a MA and its persistence, a large variability in buyers and sellers’ socio-demographic profiles may exist, thereby conducting to heterogeneity in expectations and then in local cap rates. Differently said, there may be more difference between the expected average discount rates (or risk premia) in a wealthy and low-income areas of a MA than between the wealthiest areas of two distinct MAs. Moreover, most of the real estate market actors collect information at a local level to base their decisions. Small-scale gaps in cap rates may then reflect difficulty in getting reliable informations (available data are noisy and involve lengthy and costly compiling processes) on future trends in rent (expected payoffs for a new owner) or in prices.

We give here a first micro-level account of the predictive power of cap rates on excess returns in checking if the relationship between current cap rates and future returns is valid at the local level, i.e. the convenient scale of market functioning and information availability, and in assessing its magnitude. A possible aggregation bias resulting from the use of ratio of aggregate indexes as proxy for cap rates has to be studied. We empirically illustrate that this bias is not constant and part of the dynamics in cap rates may result from it. At last, we can add that none of the preceding contributions take the vacancy risk into account. Such measures are only available for commercial real estate (see NCREIF or IPD indexes for example) and use appraisal-based data instead of transaction-based data. Consequently, these measures suffer from numerous well established shortcomings: oversmoothing (the true amount of volatility is underestimated) and lagging (time lag in detecting turning points), see Geltner (1997) for a comprehensive study on the limitations of these appraisal based measures.

Using two very large French databases, we produce a local measure of cap rates, capital appreciations, total returns and associated risks over the last housing boom that affected the French real estate market. We use the administrative registration by notaries of all the housing transactions between 1996 and 2007 (about 1,000,000 transactions) in inner Paris and a panel of about 27,000 rented flats or houses surveyed on a yearly basis in the same area. On the one hand, we estimate local hedonic price equations with the first database. It is combined with a repeat-sale type approach for a subset of about 7% of the sample in order to assess the average individual time correlation of the unexplained part of the hedonic equations. On the other hand, we estimate hedonic rent equations as well as occupation/vacancy spell equations with the second data base. These models allow us to impute local rent and occupation periods and measure real estate returns (cap rates and price growth rates). We provide estimates of local means of housing returns on Paris and its first suburbs for the 1996–2004 period.

We then estimate spatially-disaggregated forecasting equations with standard techniques for different forecast horizons (3 and 6 years). The time dimension of the sample being short, we analyze the impact of the small-sample bias on our estimates. Moreover, we break down the contributions of this rent-to-price measurement on futures excess
returns into different geographical scale contributions: from broad scale one (arrondissement or precinct for example) to smaller scale one (the land register unit level corresponding to a few building blocks). Our results suggest that cap rates may serve as a leading indicator of future excess returns in line with Plazzi, Torous and Valkanov (2010). They account for a substantial part of the forecasting error at medium term horizon, the largest share of it been captured by the more local measure. Most of the local future trends is conveyed by local – instead of global – current indicators.

This paper is organized as follows. Section 2 presents the rental market and our databases. Section 3 exposes models for rents and prices, as well as cap rates and capital price increases construction method. Section 4 presents the forecasting equations. Section 5 analyses the dynamic relationship between cap rates and capital appreciations (or excess returns) at different spatial scales. Section 6 concludes.

B Rental Market and Data Presentation

B.1 The rental market

The size of the rental housing stock, excluding the public sector, was almost one million dwellings\(^2\) in Paris Region on January 1\(^{st}\), 2008 according to OLAP (Observatoire des Loyers de l’Agglomération Parisienne – French observatory of rents for Paris Region) with 400,000 dwellings in Paris itself, 380,000 dwellings in the inner suburbs\(^3\) and 210,000 dwellings for Paris’s outer suburbs\(^4\). Hence, the rental estate is highly concentrated in the centre of the Paris region. During the last ten years, the size of the rental stock has decreased in Paris and increased in the suburbs due to growing urbanization, housing tax cuts, and subsidies for investors. For example, in the outer suburbs, almost half of the rental housing stock was built after 1975. Such recent buildings only account for 15\% of the rental housing stock in Paris and 32\% in the inner suburbs. In Paris, 66\% of the dwellings were built before 1949 (especially during the second half of the 19\(^{th}\) century, the Haussmann period) against 17\% in Paris’s outer suburbs.

\(^2\)This estimation is based on INSEE (French National Statistical Institute) census data

\(^3\)The inner suburbs consist of three départements (administrative units): the Hauts-de-Seine, the Seine-Saint-Denis and the Val-de-Marne.

\(^4\)The outer suburbs consist of four départements: the Seine-et-Marne, the Yvelines, the Essonne, and the Val d’Oise. We only consider that part of the outer suburbs contained within Paris’s metropolitan area.
The floor area of housing is correlated to the building construction period (for example, many Haussmann period buildings in Paris are quite small). Table 2 shows that the main part, 66.8%, of the housing stock in inner Paris is ‘studio’ or ‘one bedroom’. This part is lower in the inner suburbs (56.6%) and the outer suburbs (45.8%). The distribution of the rental housing stock according to the dwelling floor area is quite different from the distribution of the total housing stock: in 2006, the share of studios apartments in the total housing stock in inner Paris was 22.6% (30.8% for the rental housing stock) and the share of ‘more than 2 bedrooms’ dwellings was 24.1% (14% for the rental stock). Similarly, in Paris’s close periphery, the share of studio apartments in the total housing stock was 10.8% (22.4% for the rental sector) and the share of ‘more than 2 bedrooms’ dwellings was 36.9% (16.8% for the rental stock). Hence, small dwellings are more frequently on the rental market. This relative scarcity of large dwellings in the private rental housing sector might be responsible for their low vacancy rate (which will be evidenced below).

Table 2: Rental Housing Stock by Number of Rooms

<table>
<thead>
<tr>
<th>Bedrooms number</th>
<th>Paris</th>
<th>Inner Suburbs</th>
<th>Outer Suburbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (studio)</td>
<td>30.8%</td>
<td>22.4%</td>
<td>19.0%</td>
</tr>
<tr>
<td>1 bedroom</td>
<td>36.0%</td>
<td>34.2%</td>
<td>26.8%</td>
</tr>
<tr>
<td>2 bedrooms</td>
<td>19.2%</td>
<td>26.6%</td>
<td>27.0%</td>
</tr>
<tr>
<td>&gt; 2 bedrooms</td>
<td>14.0%</td>
<td>16.8%</td>
<td>27.2%</td>
</tr>
</tbody>
</table>

Consequently, we choose to focus our analysis on Paris itself due to the relatively small size of the rental sector in Paris’s suburbs (especially the outer suburbs). Moreover, since the outer suburban area is very large, the rental estate is irregularly spatially distributed which might lead to poor hedonic estimations. We also only consider second-hand apartments: new dwellings and houses only represent a small share of the total transactions for the Paris Region, and their price/rents and structural attributes differ greatly from those of second-hand apartments.

In the rental market, the rent of vacant housing is the result of a free bargaining between the owner and the tenant. However the evolution of the rent paid by the sitting tenant follows that of the national reference index (IRL—*Indice de Référence des Loyers*) based on the inflation rate and the growth of construction costs. The very
The large majority of lease contracts has a three-year duration (other types of contracts are excluded from our analysis). Contracts are renewable and rents are revised on an annual basis (according to the one-year growth rate of the IRL).

### B.2 Dataset for the rental market

Our dataset for the private rental sector in Paris Region comes from a survey carried out by OLAP of approximately 25,000 housing units over a representative sample of the rental market in inner Paris. Each year, more than 7,000 housing units in Paris are surveyed (the sampling rate is 1/80). This is a panel survey, each dwelling being regularly surveyed (every two or three years on average). Information is mainly gathered from the property manager (more than 80% of the whole dataset), rather than from the tenant or owner of the dwelling. This enables precise estimates of occupation or vacancy duration and rent evolution for each asset. The survey data includes information concerning the following: the current occupancy status of the housing unit (vacant/occupied), the current rent when the housing unit is occupied, the duration of residence of the current tenant (when occupied) as well as the date of the last rent revision, the duration of vacancy (when vacant) and the duration of residence (and rent evolution) of the preceding tenant.

Our empirical analysis is then based on a housing unit event-history sample. The frequency of the survey enables an almost complete (for more than 96% of the sample) reconstitution of the occupancy status and rent history for each housing units in the sample. Even if there has been more than one tenant change between the two survey’s dates, the missing information can generally be collected from the property manager.

The panel is regularly renewed, since some units may exit the survey (either because the surveyor did not find any respondent or because the unit is now occupied by the owner). The dataset also includes precise information on the unit structure type (floor area, floor level, construction period, number of rooms, elevator, number of bathrooms, number of garages, etc.). The attributes included in the duration and hedonic model final specifications will be further set out. Many other housing attributes regarding the comfort of the unit have been discarded because no corresponding item was available in the dataset on transaction prices. Moreover, detailed information regarding the location of the unit is available (here ranked from the larger to the finest geographic scale):

- **The postal code.** It gives the district (arrondissement) where the asset is located. Fig. 1 provides a map of Paris by district. The 9th district where some geographical refinements will be provided (Figs. 2 and 3) is in red.

- **The administrative precinct (quartier).** Each Parisian district is divided into four precincts (the smallest administrative units for Paris). Fig. 2 provides a map of the four precincts of the 9th district of Paris.
• The land register unit (section cadastrale) for each Parisian precinct. It is the lowest geographic scale available in the survey. Each land register unit is delimited by major streets and comprises approximately ten building blocks. Fig. 3 provides a map of the units of the 9th district of Paris. There are approximately 1388 land register units in inner Paris.

B.3 Dataset for sales

The dataset on housing unit sales comes from the Paris Region Chamber of Notaries (CINP—Chambre Interdépartementale des Notaires de Paris). In France, all property sales have to be registered by a notary, who collects the realty transfer fee to be paid to the Inland Revenue. The database includes information on the transaction price, along with detailed characteristics—the main variables are: floor area, floor level (for apartments, date of construction, number of garages, number of bathrooms, elevator/no elevator—precise location (postal code, precinct, land register unit) and transaction date (month) for each dwelling.

The global dataset consists of exactly 1,064,528 housing unit transactions for inner Paris. The coverage rate is approximately 90%. We restrict our sample to second-hand flat transactions. New flats and houses only represent a very small share of the total sales in Paris, and their price and structural attributes differ greatly from those of second-hand apartments. Moreover, transfer fees are not the same for new and second-hand properties.

C Returns construction methodology

The presentation of our model is divided into two parts: (1) the joint estimation of a model for rent and duration of occupancy and vacancy for the rental market, (2) the estimation of a model for transaction prices for the housing sales market. Notice that a complete model presentation is made in Gregoir et al. (2012).

C.1 Model for the rental market

The panel structure of our sample allows us to identify a set of specific effects: (i) We follow individual units for a long period and many have multiple spells which permits identification of an unobserved heterogeneity term (see Honoré, 1993, or Abbring and Van den Berg, 2001), (ii) we are also able to estimate the impact of the previous vacancy duration of a unit on the initial rent for the new tenant, (iii) the link between the current rent of an occupied unit and the probability of a transition to the vacant state is also estimated.
We simultaneously estimate two distinct discrete-time duration models for units in the vacant state \((e_{i\tau} = 0)\) or in the occupied state \((e_{i\tau} = 1)\) with \(i = 1, \ldots, N\) indexing the housing unit and \(\tau\) the calendar date. \(N\) is the total number of assets in the dataset.

C.1.1 Transition from vacant state

Let \(h_{v(i,\tau)}^v(t)\) be the exit rate from the vacant to the occupied state for unit \(i\) at the calendar date \(\tau\). \(t\) is the duration spell (in the vacant state). The exit rate is defined as follows:

\[
h_v^v(t \mid x_i, \omega_i^v) = \Pr (T_{v,i}^v = t \mid T_{v,i}^v > t - 1, x_i, \omega_i^v)
\]  

where \(x_i\) (including an intercept term) is a vector of physical, local and time attributes for asset \(i\) at date \(\tau\). Notice that \(x_i\) also includes spatial dummies \(q(i)\) (i.e., precincts). \(\omega_i^v\) is a fixed unobserved factor. \(T_{v,i}^v\) is the vacancy duration for unit \(i\) at date \(\tau\). Let us consider a model for capturing both duration dependence and business cycle effects:

\[
h_v^v(t \mid x_i, \omega_i^v) = G(x_i^\alpha + \gamma_v^v(t) + \omega_i^v)
\]  

where \(G(.)\) is the cumulative logistic distribution function. \(\gamma_v^v(t)\) is a term capturing duration dependence. The vector of parameters \(\alpha\) evaluates the impact of the components of vector \(x_i\) on the exit rate. In vacancy equation (C.2), the final specification for \(x_i\) will include the following variables: construction period, number of rooms, geographic and year dummies. Other variables (floor area, number of bathrooms, floor level, elevator) have been excluded with preliminary tests (see Gregoir et al., 2012 for a full presentation of covariates selection procedure). The business cycle effect is introduced via calendar dummies included in \(x\). Some housing units can differ in non observed attributes (i.e., not included in \(x\)). This could be responsible for spurious duration dependence effects in the model estimation. For example, dwellings in bad repair will stay vacant for longer periods than others, which can induce a negative duration dependence effect, i.e., \(\gamma_v^v(t)\) decreases as \(t\) grows. Exit rate functions are then estimated conditionally on an unobserved factor \(\omega_i^v\). The distribution of \(\omega_i^v\) is supposed to be independent of \(x\), but potentially linked to other heterogeneity terms (see below).

C.1.2 Transition from occupied state

Let \(h_o^v(t)\) be the exit rate from the occupied to the vacant state for unit \(i\) at calendar date \(\tau\). The interpretation of this function is similar to that of the transition rate from the vacant state and we keep the same functional form,
except for the introduction of a current rent term:

\[ h^*_i(t \mid x_{i\tau}, \omega^*_i) = G(x'_{i\tau} \beta + \gamma^*_i(t) + R_{i\tau} \theta + \omega^*_i) \]  

(C.3)

The interpretation of \( \beta_{j(i)} \), \( \gamma^*_i(t) \) and \( \omega^*_i \) is similar to that of \( \alpha_{j(i)} \), \( \gamma^*_i(t) \) and \( \omega^*_i \) in (C.2). The impact of the current rent (at calendar date \( \tau \)) \( R_{i\tau} \) for unit \( i \) on the transition probability is measured by \( \theta \). *Ceteris paribus* (\( x \) is similar to the vacancy equation and then include spatial and temporal dummies), the higher the current rent, the higher is the vacancy risk. The probability a tenant may leave their current dwelling increases when the rent is higher than those of neighboring flats with similar attributes (depending on local supply). We hence expect positive values for the estimators of \( \theta \).

C.1.3 Rent determination

For the estimation of the initial rent level, we rely on a usual hedonic model—i.e., including housing attributes as explanatory variables—and we add the past duration of vacancy. Let \( \bar{R}_{i\tau} \) be the initial rent (i.e., for a new lease with a new tenant) of unit \( i \) at date \( \tau \). This rent depends on observable physical attributes, the location of the dwelling, the date the lease was signed (included in \( x \)), the prior vacancy duration \( d_{i\tau} \), and unobserved heterogeneity factors. We choose a logarithmic specification:

\[ \log (\bar{R}_{i\tau}) = x'_{i\tau} \phi + \eta d_{i\tau} + \omega^R_i + \varepsilon_{i\tau} \]  

(C.4)

\( \phi \) are the usual hedonic parameters. \( \eta \) gives the contribution of the vacancy duration. This parameter captures two (opposite) effects. On the one hand, the longer a dwelling has been vacant, the lower the new rent to reduce non productive capital costs. On the other hand, if the dwelling was renovated during the vacancy period, the requested rent will be higher. \( \omega^R_i \) is an unobserved heterogeneity term with specification similar to \( \omega^\varepsilon_i \) and \( \omega^\omega_i \). \( \varepsilon_{i\tau} \) is a potentially heteroskedastic Gaussian error term, \( \varepsilon_{i\tau} \sim \mathcal{N}(0, \sigma^2_{\varepsilon, i, \tau}(x_i)) \). The specification of the variance-covariance matrix of innovations is detailed in Gregoir et al. (2012). Let \( g(\bar{R}_{i\tau} \mid x_{i\tau}, d_{i\tau}, \omega^\omega_i) \) be the density of the initial rent conditional on observed and non observed factors. Notice that the value of the current rent \( R_{i\tau+k} \) at calendar date \( (\tau + k) \) of a housing unit occupied for \( k \) periods is easily deduced from the value of the new rent \( \bar{R}_{i\tau} \). The ratio of these two rents is given by the cumulative growth rate \( \pi_{\tau, \tau+k} \) of the IRL (Indice de Référence des Loyers) between \( \tau \) and \( \tau + k \),

\[ R_{i\tau+k} = \pi_{\tau, \tau+k} \bar{R}_{i\tau}. \]
C.1.4 Estimation

Let $L_i(\omega_i^v, \omega_i^o, \omega_i^r)$ be the likelihood for asset $i$ conditional on $x$ (omitted from the argument of the function to keep notations simple) and on unobserved factors $\omega_i$. The joint distribution of the three heterogeneity terms of vector $\omega_i = (\omega_i^v, \omega_i^o, \omega_i^r)'$ (corresponding, respectively, to the duration model from the vacant state, from the occupied state, and to the hedonic rent model) is assumed to be normal $\omega_i \sim \mathcal{N}(0, \Omega)$. $\Omega$ is supposed to be time homogenous. We have to estimate the three variance terms $\sigma^2_\xi (\xi = v, o$ and $r)$ and three linear correlation terms $\rho_{ov}$, $\rho_{or}$ and $\rho_{rv}$. The time period of our monthly sample is [1996(1) ; 2007(12)]. Let $n_i$ be the total number of transitions (from vacancy to occupancy and conversely) of dwelling $i$ during that period. $\tau_i = \{\tau_{i,1}, ..., \tau_{i,n_i}\}$ is the set of calendar dates of transitions. $\tau_{i,0}$ is the date of the entry of dwelling $i$ in the database (\(\tau_{i,0} \geq 1996(1)\)) and $\tau_{i,n_i+1}$ is the date of exit (\(\tau_{i,n_i+1} \leq 2007(12)\)). We finally get the following formulation of the joint conditional likelihood $L_i(\omega_i^v, \omega_i^o, \omega_i^r)$:

$$
\prod_{k=1}^{n_i+1} \prod_{l=\tau_{i,k-1}+1}^{\tau_{i,k}-1} \left[1 - h_{il}^\prime (l - \tau_{i,k-1}, \omega_i^r) \right] \frac{dF_i(t, \omega)}{h_{il}(\tau_{i,k}, \omega_i^r)} \right]^{\eta_i(t, \omega)} \left(1 - h_{il}^\prime (l - \tau_{i,k-1}, \omega_i^r) \right]
$$

with $h_{il}(t, \omega) = h_{il}^\prime (t, \omega)^{1-c_i} \left[1 - h_{il}^\prime (t, \omega)\right]^{c_i}$ where $c_i$ is a variable indicating whether the observation is right-censored ($c_i = 1$) or not ($c_i = 0$). We deduce the contribution of housing unit $i$ to the joint non conditional likelihood:

$$
L_i = \int L_i(\omega_i^v, \omega_i^o, \omega_i^r) \, dF(\omega_i)
$$

where $F(.)$ is the cumulative normal distribution function with variance-covariance matrix $\Omega$. The joint likelihood for the whole sample is $L = \prod_{i=1}^n L_i$. The complete model of durations (occupation and vacancy) and rents is estimated with maximum likelihood techniques. The duration models are estimated using the occupational status history from January 1996 to December 2007 of 26,957 housing units located in Paris.

C.2 Sale price model

For the determination of transaction prices, we use a standard hedonic model. Let $P_{ir}$ be the (potentially theoretical) price of a housing unit $i$ at date $\tau$. Let $x_i$ be the vector of physical attributes of dwelling $i$. Note that this vector is not exactly similar to the one employed in the rental model $x_{i\tau}$ (C.4): some of these characteristics available in the OLAP dataset are not available (or differently recorded) in the notarial dataset. The price hedonic equation is

$$
\log (P_{ir}) = \psi_{k(i, \tau)} \bar{x}_{i\tau} + v_{i\tau}
$$

(C.5)
\( \psi_{k(i, \tau)} \) is the vector of hedonic parameters and \( k(i, \tau) \) is the spatial and temporal estimation area for the price model. Notice that the price estimation area differs from the one employed for the rent model, \( j(i) \). Indeed, the number of sale transactions in the Paris region is much higher than the number of new leases which enables geographical and temporal refinements. A separate model will then be estimated for each year \( \tau \) and each administrative district. \( u_{i\tau} \) is the zero mean error term. Its variance \( \sigma^2_{u_{i\tau}} \) depends on certain structural attributes (i.e. the number of rooms) and on the spatial and temporal estimation area of unit \( i \).

We choose a log-log specification (the floor area is the sole continuous explanatory variable and is specified in logarithm; other dummy variables are the number of rooms, the construction period, floor level, elevator, number of bathrooms, garage, time and location dummy variables and seasonality effects). Eq. (C.5) is estimated with two-stage least squares to control for heteroskedasticity.

### C.3 Cap rates and capital appreciation

For each apartment \( i = 1, \ldots, N \), with a land register unit localization and for each year of transaction since 1996, we calculate the housing returns (cap rates and capital appreciation are separately evaluated). Our measure of housing return is associated to the following strategy: a real estate investor buys a housing unit \( i \) at the beginning of the year corresponding to \( \tau_a \) (January, 1\textsuperscript{st} exactly). The transaction price is \( P_{i\tau_a} \). The investor puts the dwelling on the rental market. We consider only three-year leases (the most frequent on the French market). The ratio of received rent flows \( R_{i\tau} \) for \( \tau_a \leq \tau \leq \tau_a + 36 \) (taking vacancy risk into account) over the initial price \( P_{i\tau_a} \) is the \textit{cap rate} of the asset. Three years later in \( \tau_a + 36 \), i.e., the soonest point at which the lease could end or be renewed, the lessor wants to put the asset up for sale. The \textit{capital appreciation} will be the ratio between the sale price \( P_{i\tau_a} \) and the purchase price \( P_{i\tau_a + 36} \). However, such a sale is only possible at lease maturity. If at least one tenant’s change happened over the \([\tau_a, \tau_a + 36]\) period or if the owner did not find a tenant immediately (at date \( \tau_a \)), the apartment will not be available for sale on January 1\textsuperscript{st} of the year corresponding to \( \tau_a + 36 \). In such a case, the owner has two possibilities: (i) sell the occupied asset and transmit the lease, (ii) wait for the lease term or for an early departure of the current tenant and then put the (free) asset up for sale. In the first case, the rebate we observe for such sales in the database is on average large (approximately 20\% cheaper) and with a large variance. The notaries dataset does not permit a detailed evaluation of the impact of each housing attribute on this price gap. Consequently, we adopt strategy (ii) and suppose that the owner puts the asset for sale from the moment it becomes vacant.

We now present the methods for the evaluation of individual and localized cap rates and price gains (the full computational details are provided in Gregoir et al., 2012). For the calculation of cap rates, we propose the following
imputation technique: for each purchase date \( \tau_a \) and each housing unit attribute \( \pi_{i,a} \), let \( s(i) \) denote the land register unit where dwelling \( i \) is located. We use the estimated hedonic price equation and simulate 10 purchasing prices \( P_{i,\tau_a} \) by bootstrapping among residuals located in \( s(i) \). We then simulate 10 rental paths for housing unit \( i \) from purchase date \( \tau_a \) until the random selling date \( \tau_v \). For each path, we obtain occupation and vacancy periods with (C.2) and (C.3) and bootstrap newly bargained rents with (C.4) for each transition form vacancy to occupation. Notice that due to the reduced number of rent observations per land register unit, we cannot perform all drawings for unit \( i \) within the same register unit \( s(i) \). We enlarge the imputation area with a standard Spatial AutoRegressive SAR model estimated for each precinct \( q(i) \). With ten simulated rent paths and ten purchasing prices, we obtain 100 combinations for the cap rate \( \text{cap}_{i,\tau_a} \) of specific unit \( i \). The yields are then annualized. These returns are individual (same physical attributes for the simulated rent paths and purchasing price) and localized at the land register unit level.

For the capital appreciations of housing unit \( i \) and purchase date \( \tau_a \), we keep the 10 previously simulated purchasing prices \( P_{i,\tau_a} \) and now simulate resale prices \( P_{i,\tau_v} \) at random selling date \( \tau_v \). Notice that drawings of purchasing and resale prices should not be done independently, since they concern the same unit \( i \): the weak explanatory power of the hedonic price model implies that a remaining volatility could impact our measure of average returns. We then use an imputation technique in the spirit of the repeat-sales approach. We finally obtain individual and localized simulation of capital appreciation \( \Delta P_{i,\tau_a} \) of specific unit \( i \) at purchase date \( \tau_a \). The total return \( R_{i,\tau_a} \) is simply the sum of \( \text{cap}_{i,\tau_a} \) and \( \Delta P_{i,\tau_a} \).

C.4 Descriptive results

Over the considered period [1996-2007] and area (inner Paris), we show the existence of temporal and geographic disparities in cap rates and total returns. Thus, since 1996, we see a continued decline in cap rates on all Paris districts. For example, the average rental yield (in real terms and annualized) in Paris decreased from 5.14% in 1997 to 2.49% in 2004 (see Table 3). This is due to the sharp rise in transaction prices on the whole [1996–2007] period: more than 164% increase for second-hand apartments in Paris against only 40% for rents. The strong regulatory constraints on the evolution of private leases likely contributed to limit their growth, which may explain such a discrepancy with the trends in prices.

Meanwhile, total returns also experienced very marked movements over the period considered. In the mid-1990s, when prices rose only very slightly or stagnated in some districts, the total returns were relatively small, close to 10% in 1997. Following the sharp general rise in prices in the mid-2000s, capital yields rose sharply to reach levels close to 14% in 2002.
Table 3: Average cap rates and total returns in Paris per year (real, annualized)

<table>
<thead>
<tr>
<th>Year</th>
<th>cap$_{i,t}$</th>
<th>$R_{i,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997</td>
<td>5.14%</td>
<td>10.40%</td>
</tr>
<tr>
<td>1998</td>
<td>5.03%</td>
<td>12.14%</td>
</tr>
<tr>
<td>1999</td>
<td>4.23%</td>
<td>13.87%</td>
</tr>
<tr>
<td>2000</td>
<td>3.67%</td>
<td>12.21%</td>
</tr>
<tr>
<td>2001</td>
<td>3.22%</td>
<td>13.02%</td>
</tr>
<tr>
<td>2002</td>
<td>3.43%</td>
<td>13.83%</td>
</tr>
<tr>
<td>2003</td>
<td>2.70%</td>
<td>11.82%</td>
</tr>
<tr>
<td>2004</td>
<td>2.49%</td>
<td>9.08%</td>
</tr>
</tbody>
</table>

The cap rates $cap_{i,t}$ and total returns $R_{i,t}$ are also geographically very heterogeneous and these spatial disparities are quite persistent. Indeed, we observe significant differences between cap rates in the North East (see Figures 4a and 4c below for an example) in Paris (about 6% between 1997 and 2000) and those from central Paris (steadily below 5% over the same period for the first seven districts). In the late 2000s, these differences in local cap rates are still present and have not been dampened by the sharp rise in values: about 3% in the North East against only 2% in the center of Paris. Historically, the cheapest precinct/register units of Paris are also those where the rent-prices are higher, which stems from a great range of dispersion of transaction prices compared to rents.

The spatial heterogeneity of $R_{i,t}$ is equally marked (see Figures 4b and 4d for capital gains, recall that $R_{i,t}$ is the sum of the cap rates, $cap_{i,t}$, and capital gains): between 1997 and 2000, capital gains were high in the fourth, sixth and seventh districts, where the recovery of real estate values had already begun, while those gains were smaller (sometimes close to zero) in some districts of North East of the capital. Instead, over the period [2004-2007], our returns measures showed a reversal of this trend: the highest increases in selling prices have been observed in the North East of Paris, which resulted in capital gains well above those of the first seven districts. Inter-district differences in capital gains increased over time.

However, it appears that a simple comparison of yields by district in Paris is not enough. The spatial heterogeneity of returns to Paris is much thinner: working with averages of yields by district can lead to losing a lot of information on differences across precinct or small neighborhoods. Accordingly, we propose measures of cap rates and total returns for three different geographic levels: (a) by district, (b) by precinct (each district comprises four administrative precincts, (c) by land register unit (each district comprises between 25 and 150 land register units, each involving about ten/fifteen buildings). Table 4 below provides with the average standard deviations of returns (cap rates and total returns) across Parisian districts, then across precincts of the same district, and finally across all land register units of the same precinct. We present these results from 1996 to 2004.
Table 4: Standard deviation of cap rates and total returns per year and per geographic level

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>cap _i __ - district</td>
<td>0.6504</td>
<td>0.6681</td>
<td>0.7833</td>
<td>0.7739</td>
<td>0.7451</td>
<td>0.7553</td>
<td>0.6948</td>
<td>0.5513</td>
<td>0.4457</td>
</tr>
<tr>
<td>cap _i __ - precinct (same district)</td>
<td>0.3749</td>
<td>0.4088</td>
<td>0.4828</td>
<td>0.4810</td>
<td>0.4629</td>
<td>0.4553</td>
<td>0.4422</td>
<td>0.3419</td>
<td>0.2711</td>
</tr>
<tr>
<td>cap _i __ - land unit (same precinct)</td>
<td>1.0434</td>
<td>1.1186</td>
<td>1.1508</td>
<td>1.1783</td>
<td>1.1269</td>
<td>1.0608</td>
<td>1.0154</td>
<td>0.8289</td>
<td>0.7019</td>
</tr>
<tr>
<td>R _i __ - district</td>
<td>2.5432</td>
<td>2.8556</td>
<td>2.7890</td>
<td>2.0984</td>
<td>1.8899</td>
<td>2.5641</td>
<td>3.2919</td>
<td>3.7751</td>
<td>2.8193</td>
</tr>
<tr>
<td>R _i __ - precinct (same district)</td>
<td>1.1574</td>
<td>1.2507</td>
<td>0.9801</td>
<td>0.9909</td>
<td>0.9654</td>
<td>1.0660</td>
<td>1.7121</td>
<td>1.2994</td>
<td>0.9080</td>
</tr>
</tbody>
</table>

These results show that much of the variability of returns is at a sub-district and even sub-precinct level. In particular, in 2004, a large share of the total geographical variability of cap rates is due to differences across land register units of the same administrative precinct (a somewhat smaller fraction is due to differences between the four precincts in the same district). This variability at the land unit is even greater for total returns. These measures show how unobserved local factors (such as the presence of utilities, retails, etc.) can influence rental values and transactions prices and then potentially yields. Using a simple aggregate measure can lead (in addition to statistical bias that will be shown later) to smooth out these differences and thus lose some of the critical information needed to measure and predict changes in yields. We now turn to the illustration of the predictive power of this additional information (yields measured at the land register unit level) on future trends in excess returns.

## D Forecasting equations

### D.1 Framework

In line with the literature, we consider the following endogenous variable $y$: the excess housing return (see Case and Shiller, 1990, or Plazzi, Torous, Valkanov 2010). We set $y_{l,t} = \log(R_{l,t}) - \log(r_t)$ where $l$ stands for the land register unit and $t = 2002, ..., 2007$ for calendar year of resale. $R_{l,t}$ is the total housing return. $r_t$ is the real 10 years French treasury bond rate. Let $Y_t = \{y_{1,t}, ..., y_{n,t}\}$ be the vector of all excess returns at date $t$. $n$ is the size of the total set of land register units. In the rest of the paper for sake of notational simplicity, we assume that the panel structure is homogenous. There are $L$ land register units per precinct, $P$ precincts per district and $D$ districts. In practice, if $P$ is fixed, $L$ varies with the precinct.

The usual framework in finance to analyze the predictive power of a univariate variable $x_t$ to forecast a univariate
variable $y_t$ corresponds to the following set of equations

\[ y_t = \alpha + \beta x_{t-1} + u_t \]
\[ x_t = \gamma + \delta x_{t-1} + v_t \]

with various distributional or probabilistic assumptions on $(u_t, v_t)$ but in particular the existence of a non-zero contemporaneous correlation between the two error terms. This correlation generates a finite-sample bias problem for the estimation of $\beta$ that has been discussed in detail in particular when $(x_t)$ is near integrated (Nelson and Kim (1993), Stambaugh (1999), Amihud and Hurvich (2004), Lewellen (2004), Campbell and Yogo (2006) inter alios). We want to modify this framework to capture the influence of the different geographical scales. We thus have to complement a predictive regression with a set of equations stating the dynamics of the cap components associated to each scale. A direct extension of the above set of equations is under the assumption that $|\delta_l| < 1$, $|\delta_p| < 1$ and $|\delta_d| < 1$, for $t = 1, ..., T$

\[ y_{l,t} = \alpha + \beta_1 [x_{l,t-1} - x_{p,t-1}] + \beta_p [x_{p,t-1} - x_{d,t-1}] + \beta_d [x_{d,t-1} - x_{w,t-1}] + \eta_{1,l,t} + \varepsilon_{1,p,t} + \xi_{1,d,t} \]  (eq1)
\[ [x_{l,t} - x_{p,t}] = \delta_l [x_{l,t-1} - x_{p,t-1}] + \eta_{2,l,t} \]  (eq2)
\[ [x_{p,t} - x_{d,t}] = \delta_p [x_{p,t-1} - x_{d,t-1}] + \varepsilon_{2,p,t} \]  (eq3)
\[ [x_{d,t} - x_{w,t}] = \delta_d [x_{d,t-1} - x_{w,t-1}] + \xi_{2,d,t} \]  (eq4)

where $x_{z,t} = \log (\text{cap}_{z,t})$ is the average log of cap rates at the land register unit ($z = l$), precinct ($z = p$), district ($z = d$) or whole inner Paris ($z = w$) level. $[x_{l,t} - x_{p(t),t}]$ is the deviation of log cap rates at the land register unit level from its own precinct average (denoted $p$). $[x_{p,t} - x_{d(p),t}]$ and $[x_{d,t} - x_{w,t}]$ may be interpreted in a similar manner. $y_{l,t}$ is the annualized excess return between purchase date $t - 3$ and resale date $t$ and $x_{z,t-1}$ is the log of cap rates derived from flows of rents between $t - 4$ and $t - 1$ with $z = l, p, d$ or $w$. $(\eta_{1,l,t}, \eta_{2,l,t}, \varepsilon_{1,p,t}, \varepsilon_{2,p,t}, \xi_{1,d,t}, \xi_{2,d,t})$ is the six dimensional error term such that the three couples $(\eta_{1,l,t}, \eta_{2,l,t}), (\varepsilon_{1,p,t}, \varepsilon_{2,p,t})$ and $(\xi_{1,d,t}, \xi_{2,d,t})$ are not correlated for any couple of dates $(t, t')$, $(t, t'')$ or $(t', t'')$. The key identifying assumption is that there exists a non-zero contemporaneous correlation between the components of $(\eta_{l,1,t}, \eta_{l,2,t}), (\varepsilon_{p,1,t}, \varepsilon_{p,2,t})$ and $(\xi_{d,1,t}, \xi_{d,2,t})$, but not across spatial scales. An important point is that $y_{l,t}$ (which incorporates the cap rates) and $(x_{l,t-1}, x_{p,t-1}, x_{d,t-1}, x_{w,t-1})$ are overlapping on the $[t - 3, t - 1]$ period. $[x_{l,t-1} - x_{p,t-1}], [x_{p,t-1} - x_{d,t-1}]$ and $[x_{d,t-1} - x_{w,t-1}]$ are then endogenous in equation (eq1). We model this endogeneity through the correlation of $(\eta_{l,1,t}, \eta_{l,2,t}), (\varepsilon_{p,1,t}, \varepsilon_{p,2,t})$ and $(\xi_{d,1,t}, \xi_{d,2,t})$. 

16
A slightly different framework would have been a similar specification for the equations of \( y_t \) and \( x_t \) as follows

\[
\begin{align*}
y_{i,t} &= \alpha + \beta_t \cdot x_{i,t-1} + \beta_p \cdot x_{p,t-1} + \beta_d \cdot x_{d,t-1} + \eta_{1,i,t} + \varepsilon_{1,p,t} + \xi_{1,d,t} \\
x_{i,t} &= \gamma + \delta_t \cdot x_{i,t-1} + \delta_p \cdot x_{p,t-1} + \delta_d \cdot x_{d,t-1} + \eta_{2,i,t} + \varepsilon_{2,p,t} + \xi_{2,d,t}
\end{align*}
\] (D.7) (gen_cap)

We nevertheless choose to work on the set of equations (eq1, eq2, eq3, eq4) which corresponds to the set of within equations derived from \((D.7)\) (notice that implicitly the contemporaneous error terms at a given scale level have a sum equal to zero, this has to be taken into account in the derivation of small sample bias).

This modelling allows us to break down the contributions of cap rates on future excess returns at different geographical scales, the respective impact of the land register unit, precinct and district being measured by \( \beta_t, \beta_p \) and \( \beta_d \). If these impacts are equal, the first equation reduces to a standard one

\[
y_{i,t} = \alpha + \beta_t \cdot (x_{i,t-1} - x_{w,t-1}) + \eta_{1,i,t} + \varepsilon_{1,p,t} + \xi_{1,d,t}
\]

In the specification of the log cap rate dynamic equation, there is no intercept because we work with the deviations from the average of units of the same geographical sets. We are agnostic about the possible near-integratedness of the different log cap components. The empirical analysis will allow us to consider this question. This set of equations (eq1, eq2, eq3, eq4) does not correspond to a multivariate dynamic panel data model, each equation is related to a different level of observation and the associated sample size varies accordingly. The two first equations are associated to \( L \times P \times D \times T \) observations, the third one to \( P \times D \times T \) observations and the last one to \( D \times T \) observations.

The error terms are nevertheless correlated. We can deal separately with each equation in introducing in equation (eq1) the contemporaneous values of \( (x_{i,t} - x_{p,t}, x_{p,t} - x_{d,t}, x_{d,t} - x_{w,t})' \) which corresponds to the triangular representation of the three bivariate time series (Cholevsky factorization of each \((2 \times 2)\) variance covariance matrix of \( (\eta_{1,1,t}, \eta_{1,2,t}), (\varepsilon_{p,1,t}, \varepsilon_{p,2,t}) \) and \( (\xi_{d,1,t}, \xi_{d,2,t}) \)).

### D.2 Small sample bias correction

We therefore work on the following triangular autoregressive model

\[
\begin{align*}
y_{i,t} &= \alpha + \sum_{j=0}^{1} \alpha_{j,i} \cdot (x_{i,t-j} - x_{p,t-j}) + \sum_{k=0}^{1} \alpha_{k,p} \cdot (x_{p,t-k} - x_{d,t-k}) \\
&\quad + \sum_{i=0}^{1} \alpha_{i,d} \cdot (x_{d,t-i} - x_{w,t-i}) + \eta_{1,i,t} + \varepsilon_{1,i,t} + \xi_{1,d,t}
\end{align*}
\] (D.8)
where \( V \left( \eta_{1,l,t}^* \right) = \sigma_{l}^2, \ V \left( \varepsilon_{1,p,t}^* \right) = \sigma_{p}^2, \ V \left( \xi_{1,d,t}^* \right) = \sigma_{d}^2 \), and

\[
\begin{align*}
[x_{l,t} - x_{p,t}] &= \delta_l [x_{l,t-1} - x_{p,t-1}] + \eta_{2,l,t} \quad V \left( \eta_{2,l,t} \right) = \sigma_{l}^2 \\
[x_{p,t} - x_{d,t}] &= \delta_p [x_{p,t-1} - x_{d,t-1}] + \varepsilon_{2,p,t} \quad V \left( \varepsilon_{2,p,t} \right) = \sigma_{p}^2 \\
[x_{d,t} - x_{w,t}] &= \delta_d [x_{d,t-1} - x_{w,t-1}] + \xi_{2,d,t} \quad V \left( \xi_{2,d,t} \right) = \sigma_{d}^2
\end{align*}
\]

(eq2)

(eq3)

(eq4)

This dynamic set-up guarantees an orthogonality property of the shocks \( \eta_{1,l,t}^*, \varepsilon_{1,p(t),t}^* \) and \( \xi_{1,d(p(l)),t}^* \) with respectively \( \eta_{2,l,t}, \varepsilon_{2,p(l),t} \) and \( \xi_{2,d(p(l)),t} \).

In this set-up, we can develop an estimation strategy that allows us to take into account the small sample bias that affects the OLS estimates of \( \delta_l, \delta_p \) and \( \delta_d \). \((\widehat{\alpha}_{0,l}, \widehat{\alpha}_{1,l}, \widehat{\alpha}_{0,p}, \widehat{\alpha}_{1,p}, \widehat{\alpha}_{0,d}, \widehat{\alpha}_{1,d})\) are unbiased estimates of \((\alpha_{0,l}, \alpha_{1,l}, \alpha_{0,p}, \alpha_{1,p}, \alpha_{0,d}, \alpha_{1,d})\) that are linked to the parameters of interest as follows

\[
\begin{align*}
\alpha_{1,l} + \alpha_{0,l} \delta_l &= \beta_l \\
\alpha_{1,p} + \alpha_{0,p} \delta_p &= \beta_p \\
\alpha_{1,d} + \alpha_{0,d} \delta_d &= \beta_d
\end{align*}
\]

If we can produce bias-corrected estimates of \( (\widehat{\delta}_l, \widehat{\delta}_p, \widehat{\delta}_d) \), we can then derive bias-corrected estimates of \( (\beta_1, \beta_p, \beta_d) \). Biases of OLS estimates of \( \delta \)'s have been studied in a large literature. In case of a pure time series, an expansion is given in Kendall (1954), White (1961) and an exact expression in Sawa (1978). We follow Bao and Ullah (2007) to compute the second-order bias of the OLS estimators \( \widehat{\delta}_l, \widehat{\delta}_p \) and \( \widehat{\delta}_d \) when dealing with an AR(1) panel model with different panel structures. The detailed results are given in appendix. The formulas are derived under the assumption of Gaussian error terms. Under our assumption of a homogeneous autoregressive dynamics at each geographical scale, bias magnitude does not vary with the scale.

Our strategy is to run separately OLS estimation on each equation, then to compute first order bias-corrected estimates of \( (\widehat{\delta}_l, \widehat{\delta}_p, \widehat{\delta}_d) \) and use them to recover first order bias corrected estimates of the \( \beta \)'s. We first want to test for the significance of the parameters \( (\beta_1, \beta_p, \beta_d) \). We bootstrap under the null their distribution and compute the empirical p-value of the observed values.

D.3 Forecast variance decomposition

We second propose to compute a decomposition of the forecast error at various horizons. We indeed have for \( h > 1 \) when \( \mathcal{I} = \{ x_{l,T} - x_{p(l),T}; x_{p(l),T} - x_{d(p(l)),T}; x_{d(p(l)),T} - x_{w,T} \} \)
\[ ELy_{l,T+h} | \mathcal{I} = \alpha + \beta_1 \left[ \delta_l^{h-1} (x_l;T - x_{p(l);T}) \right] + \beta_p \left[ \delta_p^{h-1} (x_{p(l);T} - x_{d(p(l);T)}) \right] \]
\[ + \beta_d \left[ \delta_d^{h-1} (x_{d(p(l));T} - x_{w;T}) \right] \] (D.9)

\[ y_{l,T+h} - ELy_{l,T+h} | \mathcal{I} = \beta_1 \left[ \sum_{j=0}^{h-2} \delta_l^j \eta_{2,l,T+h-j} \right] + \beta_p \left[ \sum_{j=0}^{h-2} \delta_p^j \xi_{2,p(l);T+h-j} \right] \]
\[ + \beta_d \left[ \sum_{j=0}^{h-2} \delta_d^j \xi_{2,d(p(l));T+h-j} \right] + \eta_{1,l,T+h} + \alpha_0 \eta_{2,l,T+h} \]
\[ + \xi_{1,p(l);T+h} + \alpha_0 \xi_{2,p(l);T+h} + \alpha_0 \xi_{2,d(p(l);T+h} \] (D.10)

\[ + \varepsilon_{1,l,p(l);T+h} + \alpha_0 \varepsilon_{2,p(l);T+h} + \alpha_0 \varepsilon_{2,d(p(l);T+h} \] (D.11)

\[ + \varepsilon_{1,d(p(l);T+h} + \alpha_0 \varepsilon_{2,d(p(l);T+h} \] (D.12)

where we have decompose the error terms at date \( T + h \) into the contemporaneous shocks in the log cap equations and the orthogonal component. The variance of the forecast error can then be decomposed into the component associated to the local log cap dynamics, the log cap dynamics at the precinct level and the log cap dynamics at the district level.

For \( k \in \{l,p,d\} \), we can compute the share of variance associated to each geographic level \( \pi_{k,h} \):

\[
\pi_{k,h} = \frac{\left( \alpha_{0,k}^2 + 2\beta_k \alpha_{0,k} \varepsilon_{1,h} + \beta_k^2 \frac{1-\delta_k^2}{1-\delta_k} \right) \sigma_k^2}{\sum_{j \in \{l,p,d\}} \left( \alpha_{0,j}^2 + 2\beta_j \alpha_{0,j} \varepsilon_{1,h} + \beta_j^2 \frac{1-\delta_j^2}{1-\delta_j} \right) \sigma_j^2 + \sigma_j^2}
\]

E Results

In the following subsection, we present the results of the estimation of the forecasting model (eq1,eq2,eq3,eq4) with the excess return as endogenous variable. Then, we proceed to the simulation study of our approach to illustrate its performances.

E.1 Estimates

The whole set of estimates of the complete pooled forecasting model are summarized in Table 5.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \alpha_{0,l} )</th>
<th>( \alpha_{0,p} )</th>
<th>( \alpha_{0,d} )</th>
<th>( \alpha_{1,l} )</th>
<th>( \alpha_{1,p} )</th>
<th>( \alpha_{1,d} )</th>
<th>( \delta_d )</th>
<th>( \delta_p )</th>
<th>( \delta_l )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0935</td>
<td>0.0265</td>
<td>0.0263</td>
<td>0.0173</td>
<td>0.0173</td>
<td>-0.0056</td>
<td>0.0047</td>
<td>0.8105</td>
<td>0.7526</td>
<td>0.5044</td>
</tr>
<tr>
<td>(0.0017)</td>
<td>(0.0006)</td>
<td>(0.0026)</td>
<td>(0.0065)</td>
<td>(0.0065)</td>
<td>(0.0024)</td>
<td>(0.0058)</td>
<td>(0.00377)</td>
<td>(0.0217)</td>
<td>(0.0102)</td>
</tr>
</tbody>
</table>

First let us comments the results of the dynamics of cap rates: \( [x_{d,t} - x_{w,t}] \), \( [x_{p,t} - x_{d,t}] \) and \( [x_{l,t} - x_{p,t}] \) with respectively 120, 480 and 5,152 observations. All autoregressive parameters estimates are significantly above zero,
suggesting a large amount of persistence in cap rates. This was to be expected, since as detailed in the data section, information available to buyers and sellers is sticky and noisy even at the district level. Housing price dynamics are essentially present at low frequencies and part of the new rent dynamics is impacted by the regulated evolution on ongoing rents. Both effects contribute to persistent movements in cap rates.

Second, let us detail the estimates of equation (D.8). Almost all the parameters estimates concerning the impact of cap rates are significantly positive ($\alpha_{0,t} > 0$, $\alpha_{0,p} > 0$, $\alpha_{0,d} > 0$ ; $\alpha_{1,p}$ and $\alpha_{1,d}$ are negative but small). This confirms the positive impact of the cap rates on future excess returns already exemplified in other property markets, particularly the U.S. market. High rent-price ratios are precursory of excess returns above their historical average. The theoretical mechanisms presented in introduction seem to play a significant role. Notice also that each surge in local tightness (a precinct cap rate above its district average or a land register unit cap rate above its precinct average) positively contributes to future price growth rates. Each geographic scale plays a significant role and then provides valuable information. The respective magnitudes of these contributions will be compared in the variance decomposition subsection.

E.2 Simulation

We can illustrate a possible reason why the results we get are significantly above the usual order of magnitude found in the literature. The first is the specificity of the Paris market and the second is based on the geographical division of our methodology which allows a more complete integration of the information contained in the cap rates. Indeed, we construct an individualized measure of returns and then perform geographic averages. In other words, instead of calculating the ratios of average rents on average prices as is the case in the literature, we calculate averages of individual ratios rents on individual prices as explained in the methodology presentation. This technique, in front of taking into account vacancy spells of the property, produced significantly different results (see Gregoir et al., 2012).

Second, we highlight the crucial role played by the local accuracy of our forecast indicator (see Table 6). At a three years horizon, the cap rates dynamics at the lowest spatial scale (land register unit) explains over 27% of future returns, and more than 33% at 6 years forecast horizon. In comparison, the contributions of cap rates dynamics by administrative precinct and by district are smaller (around 4% and 2% respectively), but not negligible. The magnitude of the effect of cap rates on the future excess returns is rather huge: if we compare two neighboring and structurally similar (in terms of housing stock characteristics) land register unit, but suppose a difference of one percentage point in cap rates between the two units, then the difference in expected excess returns will be about 2.15% three years later. For comparison, the same difference between two administrative precincts (two districts respectively) contributes to
an average difference of 0.82% (0.86%, respectively) on the local excess returns.

<table>
<thead>
<tr>
<th>Share explained by:</th>
<th>$h = 3$ years</th>
<th>$h = 6$ years</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[x_{d,t} - x_{w,t}]$</td>
<td>2.21%</td>
<td>4.25%</td>
</tr>
<tr>
<td>$[x_{p,t} - x_{d,t}]$</td>
<td>2.28%</td>
<td>2.60%</td>
</tr>
<tr>
<td>$[x_{l,t} - x_{p,t}]$</td>
<td>27.19%</td>
<td>33.81%</td>
</tr>
</tbody>
</table>

Consequently, while disparities in rental yields in a relatively aggregated level (between precincts or between districts) provide relevant information on future trends in excess returns at a local scale, it nevertheless appears that these are the local tensions (gaps between neighborhoods in the same precinct) that explain the bulk of future trends in housing markets. In this sense, it does not seem sensible to try to predict local housing returns without having local indicators.

F Conclusion

Predicting property prices and returns is a difficult exercise: traded assets are unique and indivisible, transaction is costly, information on market conditions by location and structure of goods available to buyers and sellers is often noisy and asymmetric. Various factors affect the price. Some are macroeconomic: borrowing rates, income expectations, demographic pressures. Others are specific to the local markets. We able to evidence working at a small geographic scale that yields are in Paris as already shown in the U.S. a constituent factor of housing returns. At a 6 years horizon, they contribute for about half their variation.

References


Bias of the OLS estimates.

We now turn to the OLS expression of the estimates in the log cap equation. We present the general arguments of our results with the precinct level equation, but the same arguments can be used for the local level. We introduce some notations. \(I_{T+1}\) is the identity matrix of dimension \(T + 1\), \(D_1 = \begin{pmatrix} 0_{T \times 1} & I_T \end{pmatrix}\), \(D_2 = \begin{pmatrix} I_T & 0_{T \times 1} \end{pmatrix}\), \(B_{T+1}\) is the \(T + 1 \times T + 1\) matrix whose only non-zero coefficients are the \(B_{i,i-1}\)'s for \(i = 2, \ldots, T + 1\) that are equal to 1, \(e_K\) is a \(K \times 1\) vector whose components are equal to 1 and \(J_K = e_K e_K'\). For one unit \(p \subset d(p)\), let \((x_p - x_{d(p)})\) be the \((T + 1)\)-dimensional vector whose components are \(x_{p,t} - x_{d(p),t}\) for \(t = 0, \ldots, T\). We have

\[
D_1 \left( x_p - x_{d(p)} \right) = \delta_p D_2 \left( x_p - x_{d(p)} \right) + \varepsilon_{2,p}
\]

where \(V \varepsilon_{2,p} = \sigma^2_p I_T\). For the \(P\) units in the district \(d\), we pile up the equations, we denote \((x(d))\) the \((T + 1) P\)-dimensional vector obtained in piling up the vectors \((x_p)\) for all \(p \subset d\), notice that

\[
x(d) = \frac{1}{P} e_p' \otimes I_{T+1} x(d)
\]

and get

\[
\left( I_P - \frac{1}{P} J_P \right) \otimes D_1 x (d) = \delta_p \left( I_P - \frac{1}{P} J_P \right) \otimes D_2 x (d) + \varepsilon_2 (d)
\]

with similar notations for \(\varepsilon (d)\). We then pile up the data associated to all the districts and get with implicit notations

\[
I_D \otimes \left( I_P - \frac{1}{P} J_P \right) \otimes D_1 x = \delta_p I_D \otimes \left( I_P - \frac{1}{P} J_P \right) \otimes D_2 x + \varepsilon_2
\]

The OLS estimates of \(\delta_p\) is equal to

\[
\hat{\delta}_p = \frac{\sum_{d=1}^{D} \sum_{p \subset d} \sum_{t=1}^{T} \left( x_{p,t} - x_{d(p),t} \right) \left( x_{p,t-1} - x_{d(p),t-1} \right)}{\sum_{d=1}^{D} \sum_{p \subset d} \sum_{t=1}^{T} \left( x_{p,t-1} - x_{d(p),t-1} \right)^2}
\]

\[
= \frac{\sum_{d=1}^{D} x(d)' (I_P - \frac{1}{P} J_P) \otimes D_2 D_1 x (d)}{\sum_{d=1}^{D} x(d)' (I_P - \frac{1}{P} J_P) \otimes D_2 D_2 x (d)}
\]

which can be rewritten as

\[
\hat{\delta}_p = \frac{\sum_{d=1}^{D} x(d)' (I_P - \frac{1}{P} J_P) \otimes \frac{1}{2} (D_1 D_2 + D_2 D_1) x (d)}{\sum_{d=1}^{D} x(d)' (I_P - \frac{1}{P} J_P) \otimes D_2 D_2 x (d)}
\]

\[
= \frac{x' I_D \otimes (I_P - \frac{1}{P} J_P) \otimes \frac{1}{2} (D_1 D_2 + D_2 D_1) x}{x' I_D \otimes (I_P - \frac{1}{P} J_P) \otimes D_2 D_2 x (d)}
\]
and appear as a ratio of quadratic form. Bias of this OLS estimate has been analyzed in a large literature in relying on the properties of ratio of quadratic forms of random variables. Let us consider the stationary case for the AR(1) under study. This means that the first observation \((x_{p,0} - x_{d(p),0})\) is drawn in the stationary distribution whose mean is 0 and variance \(\frac{\sigma^2}{1 - \rho^2}\). Under the assumption that the error term is Gaussian, we can use the approximation introduced by Kendall (1954) and use the properties of the expectations of the products of quadratic forms due to Magnus (1978,1979), to state that if we denote \(B(\rho, T, P, D)\) the bias of the above OLS estimates, we have

\[
B(\rho, T, P, D) = 2 \left( \frac{TrHTG^2}{(TrG)^3} - \frac{TrHG}{(TrG)^2} \right) + O \left( \frac{1}{TPD} \right)
\]

with

\[
H = I_D \otimes \left( I_P - \frac{1}{P} J_P \right) \otimes \begin{pmatrix} \frac{1}{\sqrt{1-\rho^2}} & 0 \\ 0 & I_T \end{pmatrix} (I_{T+1} - \rho B_{T+1}')^{-1} \left( \frac{B_{T+1} + B_{T+1}'}{2} \right) (I_{T+1} - \rho B_{T+1})^{-1} \begin{pmatrix} \frac{1}{\sqrt{1-\rho^2}} & 0 \\ 0 & I_T \end{pmatrix}
\]

and

\[
G = I_D \otimes \left( I_P - \frac{1}{P} J_P \right) \otimes \begin{pmatrix} \frac{1}{\sqrt{1-\rho^2}} & 0 \\ 0 & I_T \end{pmatrix} (I_{T+1} - \rho B_{T+1}')^{-1} B_{T+1}' B_{T+1} (I_{T+1} - \rho B_{T+1})^{-1} \begin{pmatrix} \frac{1}{\sqrt{1-\rho^2}} & 0 \\ 0 & I_T \end{pmatrix}
\]

At last, we notice that as \(Tr(A \otimes B) = TrATB\), the result we get is similar for the different geographical levels.

**H Figures and Tables**
Figure 1: Paris by district. The 9th district where some geographical refinements will be provided (Figures 2 and 3) is in red.
Figure 2: Map of the 9th district of Paris by precinct. Precinct "Saint-Georges" is in yellow, precinct "Chaussée d'Antin" is in green, precinct "Faubourg-Montmartre" is in blue and precinct "Rochechouart" is in red.

Figure 3: Map of the 9th district of Paris by land register unit (delimited with red lines).
Figure 4a: Cap rates in 1997 in Paris (with legends)
Figure 4b: Capital Gains in 1997 for Paris (with legends)
Figure 4c: Cap Rates in 2004 for Paris (same legends as figure 4a)

Figure 4d: Capital Gains in 2004 for Paris (same legends as figure 4b)