Organizing insurance supply for new and undiversifiable risks *

David Alary[†] Catherine Bobtcheff[‡] Carole Haritchabalet[§]

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Abstract

This paper explores how insurance companies can coordinate to extend their joint capacity for the coverage of new and undiversifiable risks. The undiversifiable nature of such risks causes a shortage of insurance capacity and their limited knowledge makes learning and information sharing necessary. We develop a unified theoretical model to analyse co-insurance agreements. We show that organizing this insurance supply amounts to sharing a common value divisible good between capacity constrained and privately informed insurers with a reserve price. Coinsurance via the creation of an insurance pool turns out to operate as a uniform price auction with an "exit/re-entry" option. We compare it to a discriminatory auction for which no specific agreements are needed. Both auction formats lead to different coverage/premium tradeoffs. If at least one insurer provides an optimistic expertise about the risk, the pool offers higher coverage. This result is reversed when all insurers are pessimistic about the risk. Static comparative results with respect to the severity of the capacity constraints and the reserve price are provided. In the case of completely new risks, a regulator aiming at maximizing the expected coverage should promote the pool when the reserve price is low enough or when competition is high enough.

Keywords: Coinsurance, undiversifiable and new risks, common value divisible good auctions, competition, reserve price.

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[†]Toulouse School of Economics, Université Toulouse Capitole. Email: david.alary@tse-fr.eu.

[‡]Paris School of Economics, CNRS, Paris, France and CEPR. Email: catherine.bobtcheff@psemail.eu.

[§]Université de Pau et des Pays de l'Adour, E2S UPPA, CATT, Pau, France et Toulouse School of Economics. Email: carole.haritchabalet@univ-pau.fr.

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1 Introduction

Insurers often need to cooperate to cover certain large unconventional risks such as terrorism, nuclear power production, environmental protection or pandemic risks. Indeed, when the amounts of claims are very high, they exceed the financial capacity of insurers, equity and reinsurance included. In addition, the regulation of insurance companies requires a minimum level of capital and a target capital to absorb such risks (Solvability II in the European Union for instance): a single insurer can hardly fulfill this condition. Finally, these so-called unconventional risks are often poorly understood, the claim history being limited or even non-existent. All these characteristics explain the difficulty of their coverage by a standard insurance mechanism. This paper explores how insurance companies can coordinate to extend their joint capacity for the coverage of such risks.

One frequent practice in the insurance industry to achieve co-insurance is to set up cooperation between insurers. Such cooperations not only directly and significantly increases the financial capacity of participating insurers, but also allow to organize information sharing on the nature and intensity of the risks insured. The latter characteristic is crucial for the insurance of unknown and new risks since the sources of information are often dispersed, moving and heterogeneous. In the European Union, these cooperations were enhanced within the context of Article 5 of the the Insurance Block Exemption Regulation (IBER) of the European Commission. The IBER, that was established in 1992, authorized certain categories of agreements, decisions and concerted practices in the insurance sector "*to ensure the proper functioning of this sector and promote consumer interest*" [1]. In particular, Article 5 allows the practice of insurance companies to cooperate and jointly insure new risks, defined in the Regulation as "*risks which did not exists before, and for which insurance cover requires the development of an entirely new insurance product not involving an extension, improvement or replacement of an existing insurance product*" (European Commission [7]). Cooperations were formalized via the setting up of a pool, where a pool's leader takes most of the risk, and the remaining part of the risk is covered by follow insurers who are invited to cover that remainder. However, based on different commissioned studies (European Commission [8] and [9]), the European Commission took note on December 13, 2016 of the expiry of the IBER on March 31, 2017.

Indeed, the decision whether to maintain pools or not has been discussed for several years. This discussion was centered on the efficiency of such pools. What is the impact of this form of cooperation on the supply of insurance and on the pricing of insurance policies? To what extent is the performance of the insurance industry affected? Would

[1] Report from the commission to the European Parliament and the Council on the functioning of Commission Regulation (EU) No 267/2010 on the application of Article 101(3) of the Treaty on the functioning of the European Union to certain categories of agreements, decisions and concerted practices in the insurance sector

we observe a decrease in insurance capacity and thus less coverage for new risks without pools? According to insurance companies and their associations,[2] pools may be the only solution to provide insurance. Also, they argue that pools enable insurance companies to share knowledge and experience about certain less frequently occurring risks, which should, according to them, benefit to both sides of the market. The European Commission decided not to renew the exemption for pools, as it fears that pools generate a restriction of competition. The new policy is to provide a case by case analysis on the principle of self-assessment. The objective of the paper is to provide a theoretical framework to analyze the efficiency of such pools and to provide alternative cooperation schemes.

We propose to develop a unified theoretical model using auction theory to analyze these pooling agreements. Our theoretical model builds on a simplified representation of insurers' interactions based on empirical findings of the Ernst and Young report [8]. This report provides a detailed description of the procedures leading to pool agreements in several European countries. Even if some country-specific differences exist, they share some common features. Pools are constituted within a two-round auction, which defines a leading insurer and following ones. The leader's selection process may combine the following factors: capacity, premium, insurer's expertise or reputation, terms and conditions of the offer. As Ernst and Young [8] note, "*the followers are usually invited to either accept or decline or take a share of the risk on the same terms and conditions as the lead insurer*". Based on this, we propose to study a simplified scenario where pools operate as a uniform price auction with an "exit/re-entry" option and in which a pool leader is selected on the basis of the more competitive bid premium.

Following the observations drawn up, we incorporate several key ingredients to our modelling strategy. First, the undiversifiable nature of the risks causes a shortage of insurance capacity. These capacity constraints may come either from legal solvency regulation constraints and capital requisites that are imposed to prevent from insurers' bankruptcy or from the characteristics of the risk itself ("*extraordinary large risks that occur in irregular intervals but may lead to very large damage claims*", European Commission [9], for instance). These capacity constraints imply that a single insurance company cannot insure such risks. Second, when contemplating insuring new risks whose knowledge is limited, insurers use their own expertise to evaluate them. Each insurer has its own learning on the occurrence probability of the risk. At the industry level, this creates informational asymmetries not only between insurers and insureds, but also between insurers. The contracts' terms will depend on insurers' beliefs on the probability of the risk, but also on the functioning rules of the coinsurance agreement. Third, we will deal with the most general case in which the insurance of such risk is not mandatory. Policyholders may remain

[2]The Report from the commission to the European Parliament and the Council relates the results of a public consultation carried out from August to November 2014 and in December 2014 among pools, customers, intermediaries' federations brokers and mutual insurance associations.

uninsured or partially insured if insurance is too costly (see Kousky and Cooke [16] for instance). In terms of modelling, we will assume that insureds have a reserve price.

The game we consider is then a particular auction of a common value divisible good between capacity constrained agents who have private information in presence of a reserve price. We characterize the equilibrium risk premium of the pool and the resulting insurance capacity offered. We then compare the outcome of the pool to a discriminatory auction in which each insurer offers its own conditions (capacity and premium). We choose the discriminatory auction because it fits the situation where a broker meets each insurer individually so that no specific agreement between insurers is needed.This leads us to compare different auction pricing rules and to understand the role of the re-entry option. These auction formats are compared with respect to premiums and coverage, taking into account the impact of different markets and characteristics (intensity of competition and risk aversion via the reserve price). We then provide two kinds of results: some results directly help to the discussion of the European Commission and other complete the auction literature.

Let us first discuss the outcome of the pool. We determine the unique equilibrium in symmetric and strictly increasing bidding strategies. Conditional on bidding, the equilibrium strategy completely reveals the signal an insurer observed. The reserve price implies the existence of a maximum signal determining the participation to the first round of the auction. The equilibrium then exhibits both a complete market failure (no insurance) when both insurers have private pessimistic evaluations (above the threshold) and a partial market failure (partial insurance) when only one insurer is pessimistic about the risk. The re-entry option impacts these market failures in two ways: insurers refrain from bidding ex-ante (increasing the no-insurance region) but an insurer always has the possibility to re-enter the auction ex-post if he discovers that his opponent received a good signal (increasing the full insurance region). All these market failure regions are affected by the parameters of the model: intensity of the capacity constraints and reserve price. Increasing competition has two opposite effects on coverage: partial coverage is more likely but the proportion of uninsured risks decreased. A larger reserve price unambiguously increases insurance coverage: full coverage is more likely, and in case this latter is not achieved, partial coverage occurs more often when the reserve price increases.

Let us now turn to the discriminatory auction. We show that the equilibrium in symmetric and strictly increasing bidding strategies is semi-separating or separating and also involves some complete and partial market failure regions. The nature of the equilibrium and the maximum signal determining the participation depend on the parameters: intensity of competition or reserve price. The more intense the competition (weak capacity constraints), the smaller the biding regions. Similarly, the smaller the reserve price, the smaller the bidding region. Also, the bidding regions in the discriminatory auctions are always larger than in the pool : we say that the pool leads to more conservative bidding strategies. However, the pool auction offers the possibility to re-enter : we may observe full insurance with the pool and partial insurance in the discriminatory auction when the leader is optimistic enough about the risk occurrence. The follower's position is essential to understand the efficiency of a given auction format. In the pool, the follower does not take any risk, but only enjoys relatively low profits (because of uniform pricing). On the contrary, in the discriminatory auction, the potential negative profit of the follower is counterbalanced by higher premiums. It must be noted that the comparison of the equilibrium bidding strategies differ from the comparison of the premiums. If pricing is uniform in the pool, the leader and the follower offers different premiums in the discriminatory auction, the follower's premium being larger. The difficult comparison of the equilibrium bidding strategies makes the comparison between the premiums quite involved.

Both auction formats then lead to different coverage/premium tradeoffs and the analysis shows that ex-ante there is no clear dominance of one auction format. When we compare these two auctions ex-post (for any possible realization of the two insurers' signals), we show that if at least one insurer provides an optimistic expertise about the risk, the pool offers higher coverage. At the opposite, if all insurers receive pessimistic information, the discriminatory auction offers a better coverage. If insurers agree about the ex-post evaluation of risk, the re-entry option has less scope so that the size of the bidding region is key to determine the insurance coverage. The discriminatory auction therefore offers more coverage. On the contrary, if insurers receive opposite evaluations, the exercise of the re-entry option allows to increase insurance coverage.

We provide some results about the ex-ante comparison of auctions for the case of new risks (insurers' signals are therefore independent). We show that a regulator aiming at maximizing the expected coverage should promote the pool when the reserve price is low enough or when competition is high enough. Indeed in these two cases, the bidding regions of the two auction formats converge to the same region. As a result, the re-entry option offered by the pool allows to increase coverage as the follower can re-enter ex-post.

Contribution to the Literature. This paper is related to two different parts of the literature: insurance of catastrophic and undiversifible risks and auction theory. On important concern of the literature on catastrophic risk consists in explaining the failure of the purchase of disaster insurance. Many reasons are evoked: behavioral biases (Kunreuther [17]), the role of solvency constraints (Kousky and Cooke [16]), the presence of default risk (Charpentier and Le Maux [5]). Louaas and Picard [19] highlight the intrinsic determinants of demand and supply in the insurance market for catastrophic risks. A model of the supply side is proposed that takes into account the default risk by introducing collateral. However, capacity constraints are not taken into account.

The insurance of large risks thanks to a co-insurance mechanism is relatively limited. The literature on undiversifiable risks has mainly focused on the risk sharing problem between insurers and policyholders. This risk sharing problem is analyzed for instance in Doherty and Dionne [6] who introduce a new form of insurance contract called Decomposed Risk Transfer contract (DRT contract) defined by an insurance policy packaged with a residual claim on the insurance pool. They show that this contract increases policyholders welfare. They characterize the optimal coverage and the risk premium as a function of the cost of risk bearing derived from asset pricing models. Our setting builds on such a two dimensional contract (a risk premium and a coverage) but we do not discuss any risk sharing issue associated with the undiversifiable risk. We consider instead that because of the particular competition emerging from the pool, the pool risk premium (paid by the policyholders) may differ from the actuarial rate (paid by the insurer). Mahul and Write [20] examine catastrophic risk sharing arrangements within a pool when in presence of default risk from the consumers' perspective. Inderst [12] proposes a detailed description of coinsurance pools. However, the microeconomic analysis of the supply side of co-insurance mechanisms under solvency constraints is not treated to our knowledge.

The organization of insurance supply amounts to sharing a common value divisible good between capacity constrained agents who have private information in presence of a reserve price. The auction literature is abundant in this topic and incorporates some of these characteristics. In presence of reserve price and the existence of secondary markets for the goods being sold, Haile [11] analyzes a second price-price auction between two bidders with imperfect information about their valuations. A symmetric equilibrium exhibiting some pooling exists when the reserve price is sufficiently far below the maximum valuation. Jehiel and Moldovanu [13] analyze a more general setting by introducing positive or negative externalities in a standard second-price auction in presence of reserve price. They show that there must be some pooling at the reserve price in presence of positive externalities (as a resale opportunity for instance). Lizzeri and Persico [18] prove existence and uniqueness of equilibrium for a general class of two player bidding games in presence of a reserve price and interdependent values. In our paper, the divisibility of the good (the risk) makes the analysis quite different from these three papers, but we also prove the existence of a separating or semi-separating equilibrium in the discriminatory auction depending on reserve price. Moreover, we provide some comparative statics with respect to the strength of competition highlighting the role of the strength of the budget constraints on the equilibrium outcome. This allows to extend part of the results of Lizzeri and Persico [18] to the multi-unit auction setting in case of the discriminatory auction.

The analysis of the pool introduces the possibility for the follower to exit or to re-enter ex-post, after the first bidding round occurred. By analyzing such two round auction setting, we are part of the study of sequential auctions that has developed very quickly since the 2000's. Caillaud and Mezzetti [4] analyze sequential auctions in which bidders have correlated valuations for multiple units and where the seller can freely set the reserve price at the beginning of each auction. Haile [11] consider a two-stage model in which an auction in the first stage is followed by a resale auction, held by the first-stage winner. Our work is between these two settings because we consider a two round auction of a multi unit good. Moreover, the second round only concerns the follower. The originality of our approach is to provide a theoretical model a practice used in the insurance industry and to compare it to more standard auction settings.

The question of agreeing on a common coverage of a risk is akin to the one of exchanging Treasury debt and other divisible securities (where bonds are usually exchanged through a uniform auction or through a discriminatory auction). However, there is a need to develop a theoretical model. The rules of the pool are indeed specific to the insurance industry (the pool is constituted via a uniform price auction with an exit/re-entry option). Moreover, the nature of the good that is exchanged (reserve price and capacity constraints) differs from the nature of Treasury bonds. Finally, our objective slightly differs from the literature on Treasury auctions : if existing studies mainly compare the auctions with respect to revenues (as Back and Zender [2], Ausubel and Cramton [1] or Klemperer [15] for instance), we emphasize the ability of each auction to provide full insurance coverage.

The paper is organized as follows. We present the model in section 2. We then solve the equilibrium of the pool in section 3. In section 4, we introduce the discriminatory auction. Section 5 is devoted to the comparison between the two auction formats. All proofs are relegated to the appendix.

2 The model

2.1 Risk, insurers and contract

There exists a *new* and *undiversifiable* risk in the economy. It is characterized by a loss of size L occurring with probability p. Insureds ask for an exogenous coverage βL where $\beta \in [0,1]$ is the proportion of the risk that is covered. We assume that insureds have a reserve premium rate $\overline{P}$ that can reflect the existence of an outside opportunity (either no insurance or the access to other means of sharing risk).[3]

Two identical risk neutral insurers, i and j, propose the coverage of this risk with linear contracts completely defined by the coverage β and the premium rate P.

As *undiversifiable* risks may affect a large number of victims at the same time, there exist solvency regulation and capital requisites for their coverage (see for instance Kousky and Cooke (2012)). As a consequence, insurers are capacity constrained. A single insurer cannot offer more than a proportion $\overline{\beta}_i \leq \beta$ of the risk, with $\overline{\beta}_i = \overline{\beta}_j = \overline{\beta}$. We also assume that the market is too small to absorb the full capacity of the two insurers, i.e. $\beta \leq 2\overline{\beta}$.[4]

[3] Appendix A provides microfoundations for this insurance demand.

[4] Assuming sufficiently large capacities can avoid a monopoly outcome and allows to focus on the most

Insurers i and j compete for the coverage of this risk by choosing the premium rate at which they provide insurance and the quantity they insure. The minimum risk premium that they are willing to accept for this coverage is the actuarial premium rate p so that insurers offer coverage if and only if $p \leq P \leq \overline{P}$. Insurers' net expected benefit of such a contract, $\beta L\,(P-p)$, is linear with respect to β. As a consequence, an insurer chooses to insure its maximum capacity $\overline{\beta}$.

To measure the strength of competition on the insurance market, we define

$$\kappa = \frac{2\overline{\beta} - \beta}{\overline{\beta}} \in [0,1], \tag{1}$$

which can be interpreted as the relative excess supply that varies as $\overline{\beta}$ varies. When $\kappa = 0$ ($\overline{\beta} = \beta/2$), the two insurance companies may sell their entire capacity so that there is no competition. On the contrary, when $\kappa = 1$ ($\overline{\beta} = \beta$), a unique insurer could satisfy the whole demand leading to intense competition.[5]

2.2 Insurers' expertise

As the risk is *new*, its occurrence probability p is not perfectly known by the insurers. However, they can use their expertise in the evaluation of risks to infer p. Therefore, we assume that insurers obtain a costless signal related to the true occurrence probability. Signal S_i (resp. S_j) is the signal privately observed by insurer i (resp. j).

The actuarial premium is updated according to all events conveying valuable information about the signals. We assume that the actuarial premium rate can be expressed as a function of insurers' private information. It is identical for the two insurers and is a symmetric function of all insurers' signals.

$$p(s_i, s_j) = p(s_j, s_i) \equiv \mathbb{E}\left[p | S_i = s_i, S_j = s_j\right]. \tag{2}$$

We impose the following regularity assumptions on the actuarial premium rate.

Assumption 1 *The actuarial premium rate p satisfies the following properties.*

(i) *Function p is twice continuously differentiable and strictly increasing in the two variables;*

(ii) $\mathbb{E}\left[p(S_i, 0)\right] < \overline{P} < \mathbb{E}\left[p(S_i, 1)\right]$.

A high value of s_i or s_j signals a risk that is assumed to be more costly to insure and some risks cannot be insured. Indeed, Assumption 1(ii) means that if insurer j observes

interesting cases.

[5]Note that equation (1) implies that the proportion of the total demand an insurer can satisfy by itself, $\overline{\beta}/\beta$, equals $1/(2-\kappa)$.

the best (resp. worst) possible signal, covering the risk is always (resp. never) profitable for insurer i.

Signals S_i and S_j are distributed according to the same continuous distribution on the interval $[0, 1]$. Let $g(.|s)$ denote the (symmetric) probability distribution function of an insurer's signal conditional on the other insurer having observed signal s. We assume that the monotone likelihood ratio property (MLRP) is satisfied.

Assumption 2

$$\forall s_i' > s_i \text{ and } s_j' > s_j, \frac{g(s_i'|s_j')}{g(s_i'|s_j)} \geq \frac{g(s_i|s_j')}{g(s_i|s_j)}. \tag{3}$$

Let us also define signal $\widetilde{\sigma}$ as the maximal signal for which the two insurance companies accept to cover the risk in case they observe the same signal and function α that can be interpreted as an isocost curve evaluated at the maximal premium $\overline{P}$.[6]

Definition 1

(i) $\widetilde{\sigma}$ *is implicitly defined by*

$$p\left(\widetilde{\sigma}, \widetilde{\sigma}\right) = \overline{P}. \tag{4}$$

(ii) α *is implicitly defined by*

$$p(\alpha(x), x) = \overline{P} \ \forall x \in [0, 1]. \tag{5}$$

Given our assumptions, insurer i's net expected benefit of providing one unit of coverage writes[7]

$$P - \mathbb{E}\left[p(s_i, S_j)\right]. \tag{6}$$

2.3 Insurers' syndication

The organization of insurance supply amounts to the problem of sharing a common value divisible good between capacity constrained agents with a reserve price. The insurance industry has its own practices to provide coverage for undiversifiable risks under capacity constraints. As we described in the introduction, such arrangements are named co(re)insurance pools or co(re)insurance agreements.

The objective of this paper is to analyze different auction rules to constitute the syndicate, namely the pool (or the co(re)insurance agreements) and the more standard discriminatory auction. Each auction determines a game of incomplete information among the insurers: we look for a symmetric Bayesian Nash equilibrium that is increasing in the bidding strategies of each resulting game.

[6] According to Assumption 1(ii), α is a decreasing function. Moreover, the symmetry of α with respect to its arguments implies that $\alpha^{-1} = \alpha$.

[7] In what follows, we normalize L to 1.

3 Analysis of the pool

3.1 Description of the pool

In this section, we model co(re)insurance pools and co(re)insurance agreements, a representative organization of the insurance sector that we will refer to as the *pool*. The European Commission with the help of Ernst and Young [8] provides a detailed description of the procedures leading to agreements in several European countries. Even if country-specific differences exist, they share common features that we decide to highlight. The pool premium is unique and equals the lowest bid among insurers. Ernst and Young [8] also notes that "*the followers are usually invited to either accept or decline or take a share of the risk on the same terms and conditions as the lead insurer*".

We summarize these features with the following rules. After insurers receive their private signals s_i, a first price auction determines the pool risk premium. If at least one insurer submits a bid $P_i \leq \overline{P}$, the insurer that submitted the smallest premium is the pool leader and sells $\overline{\beta}$ at rate P^P; the other insurer is the follower and observes P^P. It decides whether it sells $\beta - \overline{\beta}$ at price P^P or not. In this particular syndication, the follower has both an exit and a re-entry option. Indeed, the rules state that the follower can join or exit the pool whatever its initial choice to submit a bid ex-ante. A follower has the possibility to exit the pool after having observed P^P: in this case, the leader is still committed to serve its announced capacity and there is partial insurance. Also, if a player is too pessimistic to submit a bid ex-ante, it may still re-enter and participate ex-post if the leader's bid reveals a good risk.

3.2 Separating equilibrium

We look for an equilibrium in strictly increasing and symmetric bidding strategies characterized by a threshold $\widehat{\sigma}^P$. When $s_i \leq \widehat{\sigma}^P$, firm i bids according to a strictly increasing bidding strategy $P^P(s_i)$ with $P^P(\widehat{\sigma}^P) = \overline{P}$. When $s_i > \widehat{\sigma}^P$, firm i is willing to participate only ex-post. In such a separating equilibrium, the bid an insurer submits unambiguously reveals the signal it observes. The profit per unit of coverage of firm i that observed a signal s_i and bids a premium $P^P(s_i)$ reads

$$\Pi^P(s_i) = \begin{cases} \frac{1}{2-\kappa}\left(1 - G(s_i|s_i)\right)\mathbb{E}\left[P^P(s_i) - p(s_i, S_j)|S_j > s_i\right] & \\ \quad + \frac{1-\kappa}{2-\kappa} G(s_i|s_i)\,\mathbb{E}\left[\left(P^P(S_j) - p(s_i, S_j)\right)_+ |S_j < s_i\right] & \text{for } s_i \leq \widehat{\sigma}^P \quad (7a) \\ \frac{1-\kappa}{2-\kappa} G\left(\widehat{\sigma}^P|s_i\right)\mathbb{E}\left[\left(P^P(S_j) - p(s_i, S_j)\right)_+ |S_j < \widehat{\sigma}^P\right] & \text{for } s_i > \widehat{\sigma}^P \quad (7b) \end{cases}$$

When $s_i \leq \widehat{\sigma}^P$, insurer i submits a bid ex-ante. The first term of equation (7a), *the* *leader's value of firm i*, corresponds to the case where insurer i observes the lowest signal. This happens when $S_j > s_i$, an event of probability $1 - G(s_i|s_i)$. In such case, firm i proposes the lowest premium and becomes the pool leader, serving the proportion $\frac{1}{2-\kappa}$ of the demand at its proposed rate $P^P(s_i)$.[8] The second term of equation (7a) corresponds to the case where insurer i observes the highest signal (this happens with probability $G(s_i|s_i)$). Firm i proposes the highest risk premium and becomes the pool follower, serving the proportion $\frac{1-\kappa}{2-\kappa}$ of the demand at firm j's price $P^P(s_j)$.[9] Note that if firm i's payoff turns out to be negative, it can withdraw from the pool (hence the subscript "$_+$"). Therefore, this term is named *the exit option value of firm i*. Equation (7b) corresponds to the case where firm i observes a signal greater than $\widehat{\sigma}_P$ and therefore does not want to participate ex-ante whereas its opponent submits a bid smaller than $\overline{P}$. Firm i as the pool follower agrees to re-enter in case it is profitable. In such case, it serves the remaining proportion $\frac{1-\kappa}{2-\kappa}$ of the demand at firm j's proposed rate $P^P(s_j)$. We refer to this term as *the re-entry option value of firm i*. Submitting a bid in the pool does not commit to stay in the auction for the highest bidder. This feature is unusual in the auction literature. This implies a strong asymmetry between the leader and the follower.

The participation constraint requires that bidders with signals greater than $\widehat{\sigma}^P$ prefer not to bid to submitting the bid $\overline{P}$

$$\widehat{\sigma}^P = \inf\{\sigma \in [0,1] : (1 - G(\sigma|\sigma))\, \mathbb{E}\left[\overline{P} - p(\sigma, S_j)|S_j > \sigma\right] \leq 0\}. \tag{8}$$

Lemma 1 *The threshold $\widehat{\sigma}^P$ exists and is unique. Moreover, $\widehat{\sigma}^P < \widetilde{\sigma}$.*

By continuity and with the results of Lemma 1, $\widehat{\sigma}^P$ is the unique root of the following equation

$$\left(1 - G\left((\widehat{\sigma}^P|(\widehat{\sigma}^P\right)\right) \mathbb{E}\left[\overline{P} - p(\widehat{\sigma}^P, S_j)|S_j > \widehat{\sigma}^P\right] = 0 \tag{9}$$

Given the specific rules of the pool (uniform pricing, options to exit and to re-enter), the follower profit is the same whatever the choice to participate to the auction ex-ante. At $s_i = \widehat{\sigma}^P$, the exit option value exactly compensates the re-entry option value: an insurer is indifferent between entering as a follower ex-ante round or re-entering ex-post. Therefore, the threshold $\widehat{\sigma}^P$ only matters for the leader's strategy and is determined to guarantee that the unit maximum net expected benefit is non negative which refrains from bidding when signals are too high. Strategies adopted by insurers in the pool are therefore said to be *conservative*.

We look for an equilibrium strategy such that firm i has an incentive to submit a bid according to its true signal. This implies that the equilibrium bid $P^P(s_i)$ satisfies the

[8] Note that $\frac{\overline{\beta}}{\beta} = \frac{1}{2-\kappa}$

[9] Note that $\frac{\beta-\overline{\beta}}{\beta} = \frac{1-\kappa}{2-\kappa}$.

following differential equation[10]

$$P^{P'}(s_i) = \kappa \frac{g(s_i|s_i)}{1 - G(s_i|s_i)} \left(P^P(s_i) - p(s_i, s_i) \right), \quad \forall s_i \leq \widehat{\sigma}^P. \tag{10}$$

The differential equation (10) is solved with the boundary condition that $P^P(\widehat{\sigma}^P) = \overline{P}$. In order the bidding strategy to be strictly increasing, a necessary condition is that $\widehat{\sigma}^P \leq \widetilde{\sigma}$ (it is satisfied, see Lemma 1). We then obtain the following equilibrium strategy.

Proposition 1 *There exists a unique symmetric Nash equilibrium in strictly increasing equilibrium bidding strategies where*

$$P^P(s) = \overline{P}(1 - L(\widehat{\sigma}^P|s)) + \int_s^{\widehat{\sigma}^P} p(x, x) dL(x|s) \;\; \forall \;\; s \leq \widehat{\sigma}^P \tag{11}$$

with

$$L(x|s) = 1 - \exp\left(-\kappa \int_s^x \frac{g(\tau|\tau)}{1 - G(\tau|\tau)} d\tau \right) \tag{12}$$

and $\widehat{\sigma}^P$ *defined by equation (9).*

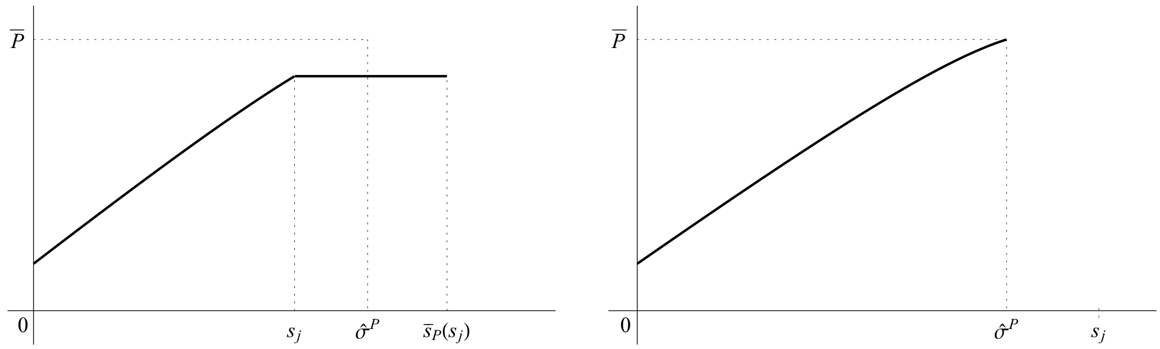
The expression of the equilibrium bidding strategy is standard in the auction literature: the first term takes into account the reserve price whereas the upper bound in the integral of the second term reflects the exit/re-entry option.[11] As for the pool premium that is indeed paid by the policyholders, it depends on the signal of the two insurers. In Figure 1(a), insurer i turns out to be the leader when $s_i \leq s_j$ in which case the pool premium corresponds to its bid. But when $s_i > s_j$, the premium corresponds to insurer j's bid. When $s_i > \widehat{\sigma}^P$, observe that insurer i decides to re-enter ex-post when $P^P(s_j) > p(s_i, s_j)$. Let us define function $\overline{s}^P(s_i)$ for $s_i \in [0, \widehat{\sigma}^P]$ that takes value on $[\widehat{\sigma}^P, 1]$.[12] It is implicitly defined by

$$P^P(s_i) = p(s_i, \overline{s}^P(s_i)). \tag{13}$$

Therefore, insurer i decides to re-enter ex-post if $s_i < \overline{s}^P(s_j)$. In Figure 1(b), insurer i's opponent observes a signal higher than $\widehat{\sigma}^P$, so that it is absent from the pool ex-ante and insurer i is always the pool leader.

One of the specificities of the pool is that insurance coverage can be full even if only one insurer submits a bid ex-ante (when the re-entry option value is exerted). Similarly, when the exit option value is exerted, insurance coverage can be partial even if the two insurers submit a bid ex-ante. The following lemma tells us that, under an additional assumption, an insurer that submits a bid ex-ante never wants to withdraw ex-post.

[10]The details of the equilibrium analysis is in the Appendix, Section B.1.
[11]Observe that $x \mapsto L(x|s)$ is an increasing function with $L(s|s) = 0$.
[12]This defines a function since p is increasing in each of its argument.



(a) $s_j < \widehat{\sigma}^P$. (b) $s_j > \widehat{\sigma}^P$.

Figure 1: Insurer i's premium for different insurer j's signal values.

Assumption 3 *Function p is supermodular,*

$$\frac{\partial^2 p(s_i, s_j)}{\partial s_i \partial s_j} \geq 0, \forall (s_i, s_j) \in [0,1]^2; \tag{14}$$

Supermodularity implies that some form of positive correlation exists between the two signals values.[13]

Lemma 2 *Under Assumption 3, an insurer that submits a bid ex-ante never wants to withdraw ex-post.*

$$P^P(s_j) - p(s_i, s_j) > 0 \;\; \forall \; s_j < s_i \leq \widehat{\sigma}^P. \tag{15}$$

Lemma 2 implies that there will always be full coverage when the two insurers submit a bid ex-ante. Indeed, the specific rules applying to the follower makes the leader position less enviable. The tradeoff is between insuring a large capacity with the risk of ex-post negative profit and insuring a smaller capacity at no risk of loss. This refrains insurers from bidding for high signal values in the first round.

Figure 2 describes insurance coverage for all signals' values. The diagram is symmetric with respect to the 45 degree line and three possible cases appear: the "full coverage region", the "partial coverage region" and the "no coverage region". Let us describe them in case insurer i observes the smallest signal. There is full coverage when both insurers initially receive a signal smaller than $\widehat{\sigma}^P$, but also when one insurer who decided not to submit a bid ex-ante decides to re-enter ex-post ($s_i \leq \widehat{\sigma}^P$ and $\widehat{\sigma}^P \leq s_j \leq \overline{s}^P(s_i)$, so that $P^P(s_i) > p(s_i, s_j)$). In the "partial coverage region", insurer i bids ex-ante and insurer j does not participate to the pool, so that only capacity $\overline{\beta}$ is provided, leading to a partial market failure. Observe that in this region, the leader's payoff is negative because of the

[13]Note that this additional assumption is compatible with MLRP. Assume for instance that the true probability is a function of the two signals and a third continuous random variable Y with a pdf f_Y distributed on an interval $[a, b]$ and independent of the signals such that $p = \psi(S_i, S_j, Y)$. In this case $p(s_i, s_j) = p(s_j, s_i) = \int_a^b \psi(s_i, s_j, y) f_Y(y) dy$. Assumption 3 is satisfied if $\int_a^b \frac{\partial^2}{\partial s_i \partial s_j} \psi(s_i, s_j, y) f_Y(y) dy \geq 0$.

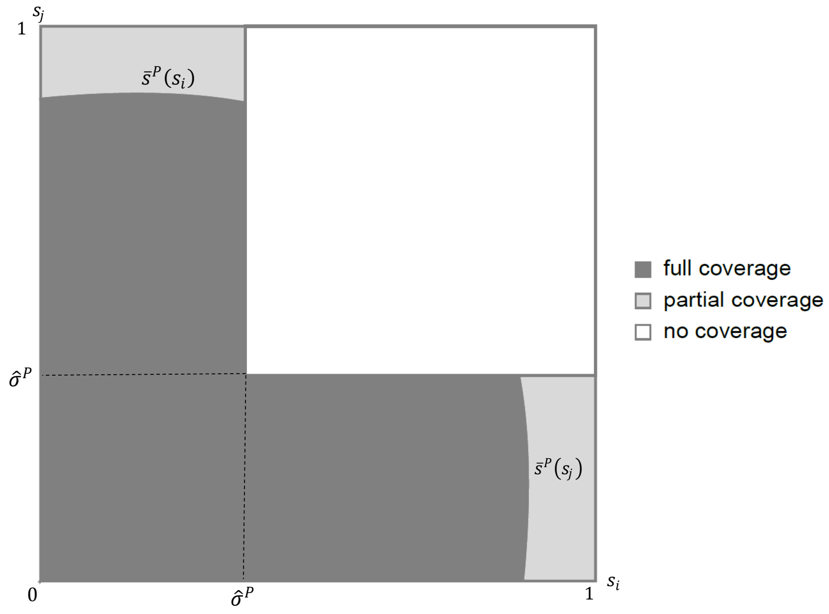


Figure 2: The different insurance coverages for all signals' values.

winner's curse. The boundary between the full coverage and the partial coverage regions is $\left\{(s_i, s_j) \in [0,1]^2 | P^P(s_i) = p(s_i, s_j)\right\}$ and corresponds to $\overline{s}^P(s_i)$: insurer j is indifferent between entering ex-post and never participating to the pool. According to Lemma 2, this boundary is greater than $\widehat{\sigma}^P$. Note that $\overline{s}_j^P(s_i)$ might be non-monotonic with respect to s_i. In particular, as $\frac{\partial^2 P^P(s_i)}{\partial s_i \partial \kappa} \geq 0$, the higher κ, the steeper $P^P(s_i)$. Therefore, if $s_i \mapsto P^P(s_i) - p(s_i, s_j)$ is a decreasing function of s_i when $\kappa = 0$, it might be a non monotonic function of s_i when κ is close to 1 as the Figure 2 illustrates. In the no coverage region, the two insurers observe a signal greater than $\widehat{\sigma}^P$, none of them submits a bid and no trade occurs leading to a complete market failure.

3.3 Equilibrium properties

Increasing competition. Competition is captured by the parameter κ in our model. Remember that $\kappa = \frac{2\overline{\beta} - \beta}{\overline{\beta}}$. Holding the insurance demand β fixed, an increase in $\overline{\beta}$ generates more competition and therefore an increase in κ. Interval $[0, \widehat{\sigma}^P]$, that represents the region of the signal values for which insurers decide to submit a bid ex-ante, is independent of the strength of competition κ (see equation (9) defining $\widehat{\sigma}^P$). However, the value of κ modifies the equilibrium bid P^P which in turn affects the follower decision to enter or not ex-post.

Proposition 2 *When competition increases, the equilibrium bidding strategy P^P decreases and the full coverage region decreases.*

The equilibrium bidding strategy is represented in Figure 3 for two values of κ. Competition unambiguously lowers premiums. When competition increases, the pool more often fails in offering complete coverage: partial market failure is more likely. However the proportion of the risk insured ($\frac{\bar{\beta}}{\beta} = \frac{1}{2-\kappa}$) increases. Therefore, increasing competition has two opposite effects on coverage: partial coverage is more likely but the proportion of uninsured risk decreases.

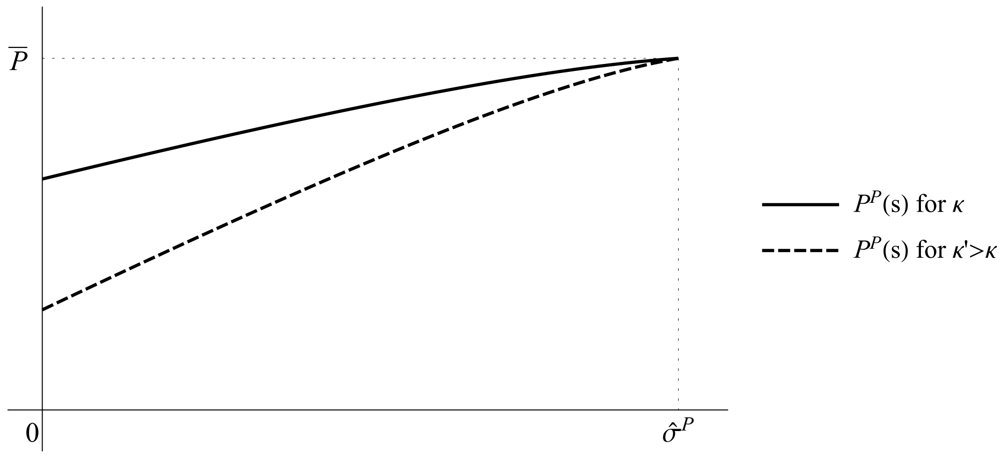


Figure 3: The equilibrium bidding strategy $P^P(s)$ for two values of κ.

Let us discuss these results in the context of the IBER. Note that the IBER imposes the existence of a limit for the market share of the pools' members (20% or 25%, see European Commission [7]). If this limit comes from the willingness of the European Commission to maintain a sufficiently high level of competition between pool members, our analysis highlights that such a limit generates low commercial premiums and a relatively high proportion of partial market failure. Indeed, even if $\widehat{\sigma}^P$ is independent of κ, remember that the boundary s^P decreases with κ. However, for a given insurance demand β, the proportion of uninsured risks decreases when κ increases.

Modifying the reserve price. According to the insurance demand model developed in Appendix A, an increase in the reserve price can be interpreted as an increase in insureds' risk aversion. It may also result from an increase in the risk.

Proposition 3 *When the reserve price increases, $\widehat{\sigma}^P$ increases and the equilibrium bidding strategy P^P increases.*

If a higher reserve price unambiguously increases the bidding regions, it has an ambiguous effect on the equilibrium bidding strategy. Indeed, on the one hand, for a given bidding region, a greater reserve price tends to increase the premium (direct effect). But on the other hand, as bidding regions increase ($\widehat{\sigma}^P$ increases with $\overline{P}$) and because the equilibrium is in strictly increasing strategies, this tends to lowers the premiums for a given signal value (indirect effect). However, Proposition 3 shows that the direct effect always dominates so that the higher the reserve price and the higher the equilibrium bidding strategy as Figure 4 illustrates. As a result, a larger reserve price unambiguously increases insurance coverage. Not only is full coverage more likely, but in case it is not achieved, partial coverage is more likely, too.

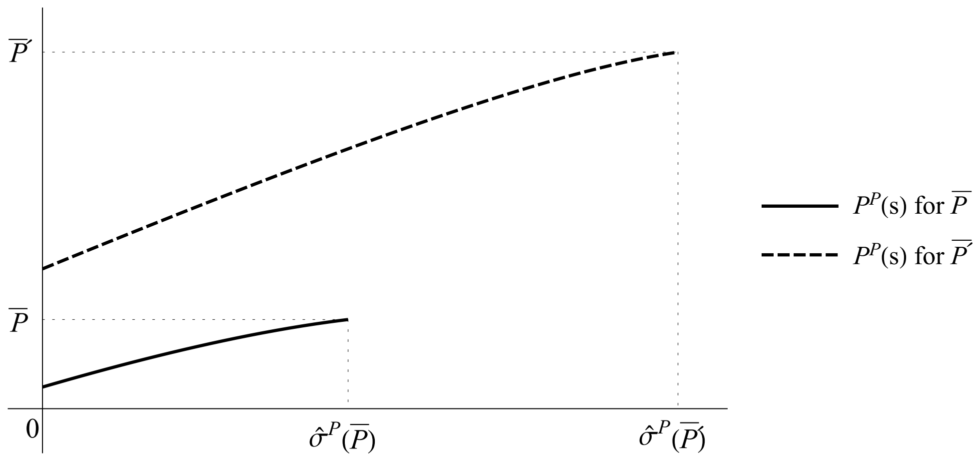


Figure 4: The equilibrium bidding strategy $P^P(s)$ for two values of $\overline{P}$.

In increase in the reserve price can be interpreted as an increase in the insureds' risk aversion. But it can also be viewed as a strengthening of the participation constraint for the insureds. In this case, making insurance mandatory (high reserve price) trivially increases insurance coverage, but increases premiums too.

4 A market alternative: a discriminatory auction

As an alternative to the pool, we model an agreement between insurance companies with heterogenous premiums. This corresponds to the case where a broker collects all the information and leaves the insurer with the lowest signal in the syndicate's leadership. In this bidding process, each firm proposes a risk premium ex-ante according to the private signal it received. Unlike the pool, the leader and the follower (if any) sell insurance coverage at their announced premium. Typically, this corresponds to a discriminatory auction.

4.1 Equilibrium analysis

Separating equilibrium We look for an equilibrium in strictly increasing and symmetric bidding strategies. Proceeding as for the pool, assume there exists a threshold $\widehat{\sigma}^D$ such that when $s_i \leq \widehat{\sigma}^D$, firm i bids according to a strictly increasing bidding strategy $P^D(s_i)$ with $P^D(\widehat{\sigma}^D) = \overline{P}$ and when $s_i > \widehat{\sigma}^D$, firm i does not participate anymore. The profit per unit of coverage of firm i that observed a signal s_i and bids a risk premium $P^D(s_i)$ reads

$$\Pi^D(s_i) = \begin{cases} \frac{1}{2-\kappa}\left(1 - G\left(s_i|s_i\right)\right) \mathbb{E}\left[P^D(s_i) - p(s_i, S_j)|S_j > s_i\right] \\ \quad + \frac{1-\kappa}{2-\kappa} G\left(s_i|s_i\right) \mathbb{E}\left[P^D(s_i) - p(s_i, S_j)|S_j < s_i\right] & \text{for } s_i \leq \widehat{\sigma}^D \quad (16a) \\ 0 & \text{for } s_i > \widehat{\sigma}^D. \quad (16b) \end{cases}$$

As for the pool, the *leader's value* (first term of equation (16a)) correspond to the case where firm i proposes the smallest risk premium. Unlike the pool, the *follower's value* (second term of equation (16a)) now depends on the follower's premium and not on the leader's premium. It tends therefore to be greater than the follower's value in the pool.

Incentive compatibility requires that bidders with signals greater than $\widehat{\sigma}^D$ prefer not to bid to submitting the bid $\overline{P}$.

$$\begin{aligned} \widehat{\sigma}^D &= \inf\{\sigma \in [0,1] : \left(1 - G\left(\sigma|\sigma\right)\right) \mathbb{E}\left[\overline{P} - p(\sigma, S_j)|S_j > \sigma\right] \\ &\quad + \left(1-\kappa\right) G\left(\sigma|\sigma\right) \mathbb{E}\left[\overline{P} - p(\sigma, S_j)|S_j < \sigma\right] \leq 0\}. \end{aligned} \tag{17}$$

Contrary to equation (9) that defined the pool threshold, the follower's payoff (the second term of equation (19)) matters. The leader's expected payoff is negative at the threshold making the winner's curse more intense. Indeed, an insurer bids until $\widehat{\sigma}^D$ in the expectation of being the follower rather than the leader. As a consequence, $\widehat{\sigma}^D$ may be larger than $\widetilde{\sigma}$ depending on the parameters of the model as the following lemma explains. The comparison between $\widehat{\sigma}^D$ and $\widetilde{\sigma}$ necessitates the introduction of a new vatiable κ^*.

Definition 2

$$\kappa^*\left(\overline{P}\right) = \max\left(\frac{\mathbb{E}\left[\overline{P} - p(\widetilde{\sigma}, S_j)\right]}{G\left(\widetilde{\sigma}|\widetilde{\sigma}\right) \mathbb{E}\left[\overline{P} - p(\widetilde{\sigma}, S_j)|S_j < \widetilde{\sigma}\right]}, 0\right). \tag{18}$$

Lemma 3 *The threshold $\widehat{\sigma}^D$ exists on $[0, \widetilde{\sigma}]$ and is unique if and only if $\kappa \geq \kappa^*\left(\overline{P}\right)$.*

When $\kappa \geq \kappa^* \left(\overline{P}\right)$, $\widehat{\sigma}^D$ is the unique root on $[0, \widetilde{\sigma}]$ of equation

$$\begin{aligned} &\left(1 - G\left(\widehat{\sigma}^D|\widehat{\sigma}^D\right)\right) \mathbb{E}\left[\overline{P} - p(\widehat{\sigma}^D, S_j)|S_j > \widehat{\sigma}^D\right] \\ &+ (1-\kappa)\, G\left(\widehat{\sigma}^D|\widehat{\sigma}^D\right) \mathbb{E}\left[\overline{P} - p(\widehat{\sigma}^D, S_j)|S_j < \widehat{\sigma}^D\right] = 0. \end{aligned} \tag{19}$$

As for the pool, incentive compatibility constraints imply that the equilibrium bid $P^D(s_i)$ satisfies the following differential equation solved with the boundary condition $P^D(\widehat{\sigma}^D) = \overline{P}$

$$P^{D\prime}(s_i) = \frac{\kappa g(s_i|s_i)}{1 - \kappa G(s_i|s_i)} \left(P^D(s_i) - p(s_i, s_i)\right). \tag{20}$$

It must be the case that $\widehat{\sigma}^D \leq \widetilde{\sigma}$ in order the bidding strategy to be strictly increasing.

Semi-separating equilibrium. If $\kappa < \kappa^* \left(\overline{P}\right)$, there does not exist a threshold $\widehat{\sigma}^D \leq \widetilde{\sigma}$. Therefore, we must look for another equilibrium strategy that involves pooling for some values of the signal. More precisely, we look for an equilibrium in symmetric and increasing bidding strategy that is characterized by two thresholds $\underline{\sigma}^D$ and $\overline{\sigma}^D > \underline{\sigma}^D$ such that: *(i)* when $s_i \in [0, \underline{\sigma}^D]$, firm i bids according to a strictly increasing bidding strategy $P^D(s_i)$ with $P^D(\underline{\sigma}^D) = \overline{P}$; *(ii)* when $s_i \in [\underline{\sigma}^D, \overline{\sigma}^D]$, firm i bids $\overline{P}$; and *(iii)* when $s_i > \overline{\sigma}^D$, firm i does not participate anymore. The equilibrium is thus separating when $s_i \in [0, \underline{\sigma}^D]$ and it is pooling when $s_i \in [\underline{\sigma}^D, \overline{\sigma}^D]$. The profit per unit of coverage of firm i that received a signal s_i and proposes a risk premium $P^D(s_i)$ is reads

$$\Pi^D(s_i) = \begin{cases} \frac{1}{2-\kappa}\left(1 - G(s_i|s_i)\right) \mathbb{E}\left[P^D(s_i) - p(s_i, S_j)|S_j > s_i\right] & \\ \quad + \frac{1-\kappa}{2-\kappa} G(s_i|s_i)\, \mathbb{E}\left[P^D(s_i) - p(s_i, S_j)|S_j < s_i\right] & \text{for } s_i \leq \underline{\sigma}^D \quad \text{(21a)} \\ \frac{1}{2-\kappa}\left(1 - G\left(\overline{\sigma}^D|s_i\right)\right) \mathbb{E}\left[\overline{P} - p(s_i, S_j)|S_j > \overline{\sigma}^D\right] & \\ \quad + \frac{1}{2}\left(G\left(\overline{\sigma}^D|s_i\right) - G\left(\underline{\sigma}^D|s_i\right)\right) \mathbb{E}\left[\overline{P} - p(s_i, S_j)|\underline{\sigma}^D < S_j < \overline{\sigma}^D\right] & \text{for } \underline{\sigma}^D < s_i \leq \overline{\sigma}^D \\ \quad + \frac{1-\kappa}{2-\kappa} G\left(\underline{\sigma}^D|s_i\right) \mathbb{E}\left[\overline{P} - p(s_i, S_j)|S_j < \underline{\sigma}^D\right] & \text{(21b)} \\ 0 & \text{for } s_i > \overline{\sigma}^D. \quad \text{(21c)} \end{cases}$$

Contrary to (16), there is a new intermediate case where firm i bids $\overline{P}$ (equation (21b)). The first term corresponds to the *leader's value*, the last to the *follower's value*. As for the second term, it corresponds to the case where the two firms bid $\overline{P}$ so that they equally share the market. Incentive compatibility requires that insurers with signal in $[\underline{\sigma}^D, \overline{\sigma}^D]$ prefer submitting $\overline{P}$ to not participating and to submitting any lower bid. Moreover, insurers with signals greater than $\overline{\sigma}^D$ prefer not to bid to submitting the bid $\overline{P}$. The two thresholds are thus defined by the following system.

$$\begin{cases} \left(G\left(\overline{\sigma}^D|\underline{\sigma}^D\right) - G\left(\underline{\sigma}^D|\underline{\sigma}^D\right)\right)\mathbb{E}\left[\overline{P} - p(\underline{\sigma}^D, S_j)|\underline{\sigma}^D < S_j < \overline{\sigma}^D\right] = 0 & \text{(22a)} \\ \left(1 - G\left(\overline{\sigma}^D|\overline{\sigma}^D\right)\right)\mathbb{E}\left[\overline{P} - p(\overline{\sigma}^D, S_j)|S_j > \overline{\sigma}^D\right] \\ + \left(1 - \frac{\kappa}{2}\right)\left(G\left(\overline{\sigma}^D|\overline{\sigma}^D\right) - G\left(\underline{\sigma}^D|\overline{\sigma}^D\right)\right)\mathbb{E}\left[\overline{P} - p(\overline{\sigma}^D, S_j)|\underline{\sigma}^D < S_j < \overline{\sigma}^D\right] \\ + (1-\kappa)\,G\left(\underline{\sigma}^D|\overline{\sigma}^D\right)\mathbb{E}\left[\overline{P} - p(\overline{\sigma}^D, S_j)|S_j < \underline{\sigma}^D\right] = 0. & \text{(22b)} \end{cases}$$

It must also be checked that an insurer bidding $\overline{P}$ when it observes a signal comprised between $\underline{\sigma}^D$ and $\overline{\sigma}^D$ does not have an incentive to underprice. This comes down to checking that[14]

$$\left(G\left(\overline{\sigma}^D|s_i\right) - G\left(\underline{\sigma}^D|s_i\right)\right)\mathbb{E}\left[\overline{P} - p(s_i, S_j)|\underline{\sigma}^D < S_j < \overline{\sigma}^D\right] \leq 0 \;\; \forall s_i \in [\underline{\sigma}^D, \overline{\sigma}^D].$$

When the separating equilibrium exists ($\kappa \geq \kappa^*\left(\overline{P}\right)$), the system ((22a)-(22b)) has a unique solution $\underline{\sigma}^D = \overline{\sigma}^D = \widehat{\sigma}^D$ involving no pooling region. We can then state the following proposition that characterizes the equilibrium strategy.[15]

Proposition 4 *If $\kappa \geq \kappa^*\left(\overline{P}\right)$, the unique equilibrium in increasing strategy is the separating equilibrium.*

If $\kappa < \kappa^*\left(\overline{P}\right)$, the unique equilibrium in increasing strategy is the semi-separating equilibrium. In this case, the following ranking holds

$$\alpha(\overline{\sigma}^D) \leq \underline{\sigma}^D < \widetilde{\sigma} < \alpha(\underline{\sigma}^D) \leq \overline{\sigma}^D \leq \widehat{\sigma}^D.$$

Corollary 1 *In the separating equilibrium, the strictly increasing bidding equilibrium strategies read*

$$P^D(s) = \overline{P}(1 - K(\widehat{\sigma}^D|s)) + \int_s^{\widehat{\sigma}^D} p(x,x)dK(x|s) \;\; \forall s \leq \widehat{\sigma}^D. \tag{23}$$

with $\widehat{\sigma}^D$ defined by equation (19).

In the semi-separating equilibrium, the increasing bidding equilibrium strategies read

$$P^D(s) = \begin{cases} \overline{P}(1 - K(\underline{\sigma}^D|s)) + \int_s^{\underline{\sigma}^D} p(x,x)dK(x|s) & \text{for } s \leq \underline{\sigma}^D \quad \text{(24a)} \\ \overline{P} & \text{for } \underline{\sigma}^D < s \leq \overline{\sigma}^D. \quad \text{(24b)} \end{cases}$$

with $\underline{\sigma}^D$ and $\overline{\sigma}^D$ defined by equations (22a) and (22b).

[14]This is checked in the proof of Lemma 4.
[15]The complete characterization of the equilibrium is presented in Appendix B.2.

Function K equals

$$K(x|s) = 1 - \exp\left(-\int_s^x \frac{\kappa g(\tau|\tau)}{1-\kappa G(\tau|\tau)} d\tau\right). \tag{25}$$

Note that this results complements the literature on auction. We prove the existence of a separating or a semi-separating equilibrium depending on the reserve price or the competition level in a common value auction with affiliated values.

4.2 Equilibrium properties

This section provides a comparative static analysis of the discriminatory auction equilibrium where we emphasize the role of competition and of the reserve price.

Increasing competition. The characterization of the equilibrium already highlighted the role of competition on the nature of equilibrium that emerges. Therefore, contrary to the pool, the region in which insurance companies submit bids now depends on the competition strength.

Lemma 4 *The following comparative static results hold for the different thresholds:*

$$\frac{\partial \widehat{\sigma}^D}{\partial \kappa} \le 0,\ \frac{\partial \underline{\sigma}^D}{\partial \kappa} \ge 0 \text{ and } \frac{\partial \overline{\sigma}^D}{\partial \kappa} \le 0.$$

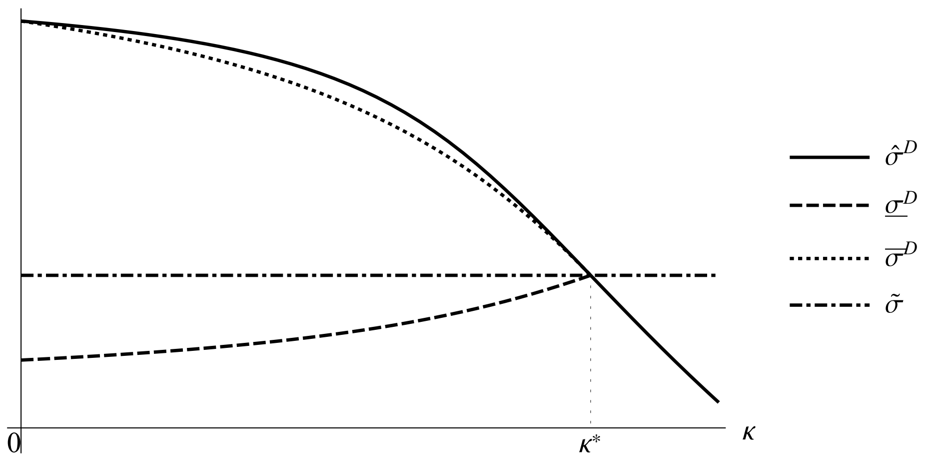


Figure 5: The different thresholds as a function of κ.

In Figure 5, we retrieve the results of Proposition 4 that the equilibrium is separating when $\kappa \ge \kappa^*\left(\overline{P}\right)$ and semi-separating when $\kappa < \kappa^*\left(\overline{P}\right)$. The more intense the competition, the smaller the bidding regions. The perspective of a tougher competition makes insurers more prudent when bidding. Note that when the equilibrium is semi-separating,

$\overline{\sigma}^D$ decreases with κ and $\underline{\sigma}^D$ increases with κ. This implies that the reserve premium rate $\overline{P}$ is reached for higher signal's values and that the region for which insurers bid $\overline{P}$ shrinks.

The results on the bidding regions are closely related to the way the bidding strategies evolve with respect to κ. Holding the bidding regions constant, an increase in competition has the direct effect of reducing the bidding strategies. However, as we just saw, an increase in κ also affects the bidding regions introducing an indirect effect.[16] When $\kappa < \kappa^* \left(\overline{P}\right)$, as $\underline{\sigma}^D$ increases, $\overline{P}$ is reached for higher signals' values so that the direct and the indirect effect go in the same direction, implying that bidding strategies decrease with competition. On the contrary, when $\kappa \geq \kappa \left(\overline{P}\right)^*$, $\widehat{\sigma}^D$ decreases: the indirect effect therefore tends to increase bidding strategy and the total effect is ambiguous.

Modifying the reserve price. Observe first that, in the general case, it is not straightforward to determine whether the equilibrium is separating or semi-separating when the reserve price increases. Indeed, the dependance of $\kappa^*(\overline{P})$ with respect to $\overline{P}$ is difficult to analyze. This is in part due to the dependence of $\widetilde{\sigma}$ with respect to $\overline{P}$ that intervenes both directly (through $p(S, \widetilde{\sigma})$) and indirectly (through affiliation and the distribution function) in the definition of $\kappa^*(\overline{P})$.

As for the comparative static of the thresholds with respect to the reserve price, the following result holds.

Lemma 5 *It holds that*

$$\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}} \geq 0 \text{ and } \frac{\partial \overline{\sigma}^D}{\partial \overline{P}} \geq 0.$$

As for the pool, a higher reserve price unambiguously increases the bidding regions. There is more surplus to extract from the insureds so that insurance companies continue to bid even if they are less optimistic about the risk occurrence. However, the comparative statics of the bidding strategy does not lead to direct results.

An illustration is provided in Figure 6. In this example, the equilibrium is separating if and only if the reserve price is smaller than some threshold $\overline{P}^*(\kappa)$. For a large reserve price, there is more surplus to extract from the insureds so that insurance companies can afford bidding less aggressively. Therefore, the equilibrium is semi-separating.

[16] This indirect effect writes:

$$-\frac{\partial \min(\widehat{\sigma}^D, \underline{\sigma}^D)}{\partial \kappa} \left(\overline{P} - p(\min(\widehat{\sigma}^D, \underline{\sigma}^D), \min(\widehat{\sigma}^D, \underline{\sigma}^D))\right) \frac{dK(x|s)}{dx}\bigg|_{x=\min(\widehat{\sigma}^D, \underline{\sigma}^D)}. \tag{26}$$

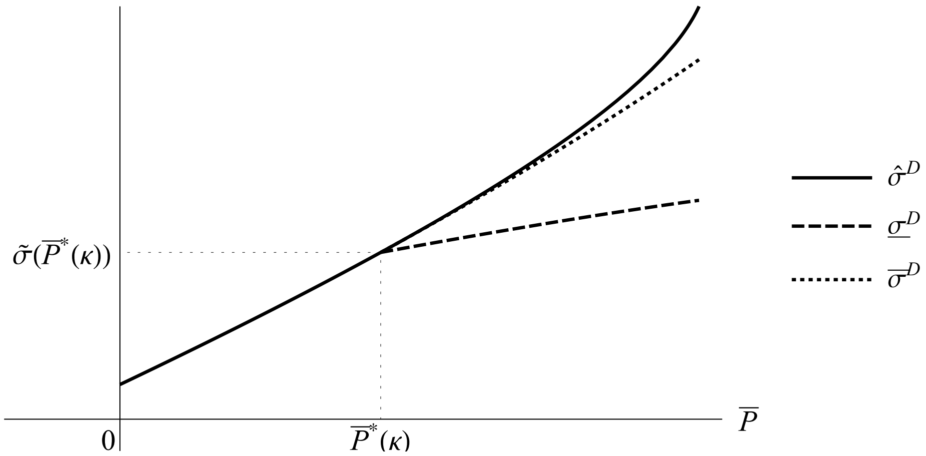


Figure 6: The different thresholds as a function of $\overline{P}$.

5 Which auction for the coverage of these risks?

We evaluate the efficiency of the pool with respect to two dimensions: premiums' levels and coverage.

5.1 Pool vs discriminatory auction

Both auctions deal with different risk/return tradeoff for the follower. The follower's position is essential to understand the efficiency of a given auction format. In the pool, the follower does not take any risk, but only enjoys relatively low profits (because of uniform pricing), whereas in the discriminatory auction the potential negative profit of the follower is counterbalanced by higher premiums.

Lemma 6 *It holds that*

$$\widehat{\sigma}^P \leq \min\left(\widehat{\sigma}^D, \underline{\sigma}^D\right).$$

A direct consequence of this lemma is that the discriminatory auction offers a positive coverage (partial or full) of the risk whereas the pool offers no coverage when the leader signal is between $\widehat{\sigma}^P$ and $\widehat{\sigma}^D$. However, in order to be able to compare coverage when the leader's signal is smaller than $\widehat{\sigma}^P$, it is necessary to compare the boundary between the full coverage region and the partial coverage region of the pool to $\widehat{\sigma}^D$. To do that, a comparison of the premiums in the two settings is necessary. Unfortunately, this very general setting does not allow us to have a clear result.

Proposition 5 *The bidding strategies in the pool and in the discriminatory auction cross at most one.*

- *Either* $P^D(s) < P^P(s)$ *for all* $s \in [0, \widehat{\sigma}^P]$*,*

- *Or,* $P^P(s) < P^D(s)$ *and then* $P^P(s) > P^D(s)$*, when* s *increases from 0 to* $\widehat{\sigma}^P$*.*

When the leader's signal $s \leq \widehat{\sigma}^P$ *is such that* $P^D(s) < P^P(s)$*, the pool always offer more coverage than the discriminatory auction.*

The classic tradeoff between quantity and price is highlighted as being the one driving the differences between the two organizations. Observe that there are only two regions of the signals values for which the coverage in the two organizations differs.

- When the leader's signal belongs to $[\widehat{\sigma}^P, \min\left(\widehat{\sigma}^D, \underline{\sigma}^D\right)]$, no coverage is offered in the pool whereas either full of partial coverage is offered in the discriminatory auction (depending on the follower's signal);

- When the leader's signal is smaller than $\widehat{\sigma}^P$, the follower's signal determines whether coverage is partial or full. When the boundary $\overline{s}^P$ is larger than σ^D, the pool offers full coverage whereas only partial coverage is offered in the discriminatory auction. The reverse holds when $\overline{s}^P$ is smaller than σ^D. We show in the proof of Lemma 5, that when $P^D(s) < P^P(s)$ then $\overline{s}^P$ is larger than σ^D. However, in the general comparing the boundary $\overline{s}^P$ to $\widehat{\sigma}^D$ is difficult and $\overline{s}^P$ may cross $\widehat{\sigma}^D$.

As a consequence, when one of the insurance companies is optimistic about the risk (the leader's signal is smaller than σ^P) and $P^D(s) < P^P(s)$, the pool tends to offer more coverage at a larger price. However, if the forecasts about the risk occurrence are rather pessimistic (when the most optimistic insurance company is already quite pessimistic, that is when the leader's signal belongs to $[\widehat{\sigma}^P, \min\left(\widehat{\sigma}^D, \underline{\sigma}^D\right))$, only the discriminatory auction allows to offer a positive coverage (partial or full coverage).

However, as it has already been highlighted, the comparison of the equilibrium bidding strategies differ from the comparison of the premiums. If pricing is uniform in the pool, the leader and the follower offers different premiums in the discriminatory auction, the follower's premium being larger. The difficult comparison of the equilibrium bidding strategies makes the comparison between the premiums even more tricky.

This analysis shows that ex-ante there is no clear dominance of one auction format. However, we can highlight two messages depending on the realization of the signals. If insurers agree about the ex-post evaluation of risk (s_i close to s_j), the re-entry option has less scope so that the size of the bidding region is key to determine the insurance coverage. The discriminatory auction therefore offers more coverage. On the contrary, if insurers receive opposite evaluations, the exercise of the re-entry option allows to increase insurance coverage.

As Proposition 5 underlines, the generality of our model makes the comparison of the two organizations quite involved (the fact that the bidding strategies cross or not depends on the comparison of $P^P(0)$ with $P^D(0)$ which involves all the parameters of the model). Therefore, as an illustration, we propose to solve explicitly an example in the next subsection.

5.2 The case of independent signals and linear premium rate

We consider the case in which the two insurance companies receive independent signals that are distributed according to a uniform distribution on $[0, 1]$. this corresponds to the case where the risk is new so that each insurer gathers together its own information on the occurrence probability. The cost function is moreover assumed to be linear in the two signals

$$p(s_i, s_j) = \frac{s_i + s_j}{2}.$$

In this case, Assumption 1(ii) implies that $\overline{P} \in [1/4, 3/4]$. Note also that $\widetilde{\sigma} = \overline{P}$. All the computations are detailed in the Appendix D. In particular, we do not detail here the equilibrium strategies that are explicitly computed in the Appendix. In this illustration, we restrict ourselves to the case where the separating equilibrium exists in the discriminatory auction. This implies that we restrict κ to be greater than $\kappa^*(\overline{P}) = \max\left(\frac{2\overline{P}-1}{\overline{P}^2}, 0\right)$. Observe that when $\overline{P} \in [1/4, 1/2]$, the equilibrium is separating for any value of κ ($\kappa^*(\overline{P}) = 0$).

Proposition 6 *If $\kappa \geq \kappa^*(\overline{P})$, there exists a unique $\widehat{\overline{P}}$ such that*

- *if $\overline{P} < \widehat{\overline{P}}$, $P^P(s) > P^D(s)$, $\forall s \in [0, \widehat{\sigma}^P]$*
- *if $\overline{P} \geq \widehat{\overline{P}}$, $P^P(s) < P^D(s)$ and then $P^P(s) > P^D(s)$, when s increases from 0 to $\widehat{\sigma}^P$.*

When the reserve price is low, the pool offers higher premiums and higher coverage than the discriminatory auction. All these results are ex-post as they depend on the signals' realization. This example allows us to have a look at an ex-ante analysis. To do that, we compute both the expected profit and the expected coverage in the two organizations for all parameters' values (whether the equilibrium in the discriminatory auction is separating or semi-separating). While their expression is given is the Appendix, we provide a comparison of both of them in Figures 7 and 8.

Remember the basic tradeoff highlighted in the previous subsection. Depending on the parameters' values (when the reserve price is low enough), the pool provides more coverage at a higher premium when at least one insurer is optimistic about the risk occurrence probability. But it does not offer any coverage when the two insurers provide a pessimistic estimation of the risk occurrence probability. According to Figure 7, for a given κ, the pool generates a larger expected profit if and only if the reserve price is smaller than a threshold. Indeed, when the reserve price is large, insurers taking part to the discriminatory auction can extract higher rents from the insureds, in part because the follower has a larger risk premium than the leader.

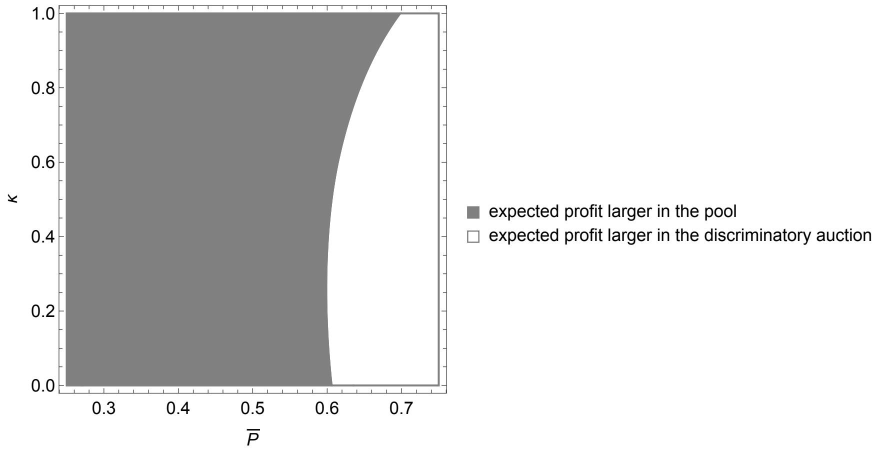


Figure 7: Comparison of the expected profits.

Another criteria that might be of interest for a regulator concerns the expected coverage generated by each of the organization.

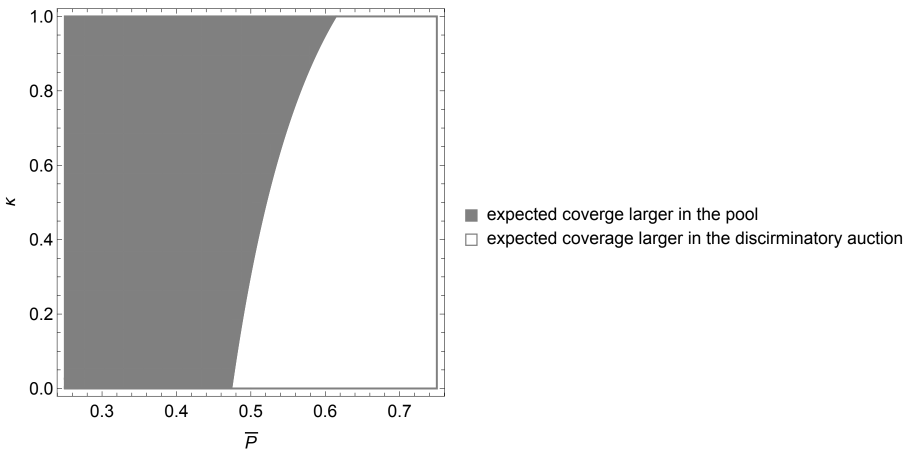


Figure 8: Comparison of the expected coverage.

As expected, we observe in Figure 8 that the expected coverage is larger in the pool when the reserve price is low. However, for a given competition level, when the reserve price increases, observe that the expected coverage becomes larger in the discriminatory auction. As a matter of fact, the region of signals values such that insurance coverage is only provided with the discriminatory auction $(\min(\widehat{\sigma}^D, \overline{\sigma}^D) - \widehat{\sigma}^P)$ increases when $\overline{P}$ increases. On the contrary, for a given $\overline{P}$ high enough, when competition increases, the discriminatory auction offers a smaller expected coverage. Indeed, as κ increases, $\widehat{\sigma}^D$ and $\overline{\sigma}^D$ decreases so that the region in which only the discriminatory auction provides coverage $(\min(\widehat{\sigma}^D, \overline{\sigma}^D) - \widehat{\sigma}^P)$ decreases.

5.3 Conclusion

In this paper, we investigate how insurance companies can coordinate to extend their joint capacity for the coverage of large new risks. We consider that organizing such insurance supply amounts to auction a common value divisible good between capacity constrained insurers where insurers have private information. Two auctions formats, a uniform price auction with a exit/re-entry option (the pool) and a discriminatory auction, are compared with respect to premiums and their ability to offer full and partial coverage.

We show that both auction formats lead to different coverage/premium tradeoffs. Our findings may not entirely negate the existence of pools to cover large unconventional risk categories in the insurance sector. If insurers agree about the ex-post evaluation of risk, the discriminatory auction offers more coverage. On the contrary, if insurers receive opposite evaluations, the exercise of the re-entry option allows the pool to increase insurance coverage. We also provide some results about the ex-ante comparison of the two auctions for the case of new risks (independent insurers' signals). We show that a regulator aiming at maximizing the expected coverage should promote the pool when the reserve price is low enough or when competition is high enough.

Two major differences exist between the two settings. The follower pricing rule (uniform versus heterogenous pricing) and the re-entry option. When re-entry is possible, an insurer takes less risks in its bidding strategy since submitting a bid is not a necessary condition to participate to the auction anymore. Therefore, an insurer reduces the risk of being a leader despite a high signal by reducing the bidding region. Introducing the re-entry option in the discriminatory auction then leads to more conservative strategies. Also, being a follower in the first round leads to higher premiums than re-entering in the second round and being a follower at the leader's premium. Therefore, the re-entry option is less valuable under heterogenous pricing. As a consequence, insurers take advantage of the first round by enlarging the bidding region, compared to the pool. The presence of re-entry option in an auction then yields to more complete market failure but reduces the partial market failure. In the discriminatory auction, re-entry raises premium since re-entering followers benefit from the leader's revenue. When re-entry is possible, the comparison between heterogenous and uniform pricing relies on the same forces than the ones described in Lemma 5. Re-entry also increases premium in the uniform auction. The presence of re-entry option yields to higher premiums both with heterogenous and uniform pricing.

A Insurance demand

In this section, we model the demand side. Assume a risk averse agent faces the risk described in the model. The expected utility with the contract described in the model writes

$$V(P,p;\beta) = pu\left(w - L + \beta L - \beta LP\right) + (1-p)u\left(w - \beta LP\right)$$

where u denotes the increasing and concave utility function and w the agent's initial wealth. βL is the indemnity paid by insurers in case of loss and βLP is the insurance premium paid for this coverage. Let $\overline{P}$ be the maximum premium that the agent is willing to pay for this coverage. At $\overline{P}$, he is indifferent between insuring or not $V(\overline{P},p;\beta) = V(0,p;0)$.

B Determination of the equilibria

B.1 Equilibrium analysis in the pool

We look for an equilibrium strategy such that firm i has an incentive to submit a bid according to its true signal. The per unit of demand profit of firm i that observed a signal s_i and bids a risk premium $P^P(b)$ reads

$$\Pi^P(b,s_i) = \begin{cases} \frac{1}{2-\kappa}\left(1-G(b|s_i)\right)\mathbb{E}\left[P^P(b) - p(s_i,S_j)|S_j > b\right] & \\ \quad + \frac{1-\kappa}{2-\kappa} G(b|s_i)\,\mathbb{E}\left[\left(P^P(S_j) - p(s_i,S_j)\right)_+ |S_j < b\right] & \text{for } b \leq \widehat{\sigma}^P \\ \frac{1-\kappa}{2-\kappa} G\left(\widehat{\sigma}^P|s_i\right)\mathbb{E}\left[\left(P^P(S_j) - p(s_i,S_j)\right)_+ |S_j < \widehat{\sigma}^P\right] & \text{for } b > \widehat{\sigma}^P. \end{cases}$$

Therefore, in order the incentive compatibility constraint to be satisfied, the risk premium satisfies $\forall s_i \leq \widehat{\sigma}^P$

$$\frac{\partial \Pi^P(b,s_i)}{\partial b}|_{b=s_i} = 0, \forall s_i \leq \widehat{\sigma}^P.$$

As a consequence, the equilibrium bid $P^P(s_i)$ satisfies the following differential equation

$$P^{P'}(s_i) = \kappa \frac{g(s_i|s_i)}{1-G(s_i|s_i)}\left(P^P(s_i) - p(s_i,s_i)\right), \forall s_i \leq \widehat{\sigma}^P.$$

It is solved with the boundary condition that $P^P(\widehat{\sigma}^P) = \overline{P}$. Using the method of the parameters' variation, we obtain that

$$P^P(s) = \overline{P}(1 - L(\widehat{\sigma}^P|s)) + \int_s^{\widehat{\sigma}^P} p(x,x)dL(x|s) \;\; \forall \;\; s \leq \widehat{\sigma}^P$$

with

$$L(x|s) = 1 - \exp\left(-\kappa \int_s^x \frac{g(\tau|\tau)}{1 - G(\tau|\tau)} d\tau\right).$$

B.2 Equilibrium analysis in the discriminatory auction

First case: $\kappa \geq \kappa^*$. We look for an equilibrium strategy such that firm i has an incentive to submit a bid according to its true signal. Therefore, at equilibrium, the risk premium satisfies $\forall s_i \leq \widehat{\sigma}^D$

$$\frac{\partial \Pi^D(b, s_i)}{\partial b}|_{b=s_i} = 0, \forall s_i \leq \widehat{\sigma}^D$$

so that the equilibrium bid $P^D(s_i)$ satisfies the following differential equation

$$P^{D'}(s_i) = \frac{\kappa g(s_i|s_i)}{1 - \kappa G(s_i|s_i)} \left(P^D(s_i) - p(s_i, s_i)\right), \forall s_i \leq \widehat{\sigma}^D.$$

It is solved with the boundary condition that $P^D(\widehat{\sigma}^D) = \overline{P}$. Using the method of the parameters' variation, we obtain that

$$P^D(s) = \overline{P}(1 - K(\widehat{\sigma}^D|s)) + \int_s^{\widehat{\sigma}^D} p(x, x) dK(x|s) \;\; \forall \;\; s \leq \widehat{\sigma}^D$$

with

$$K(x|s) = 1 - \exp\left(-\int_s^x \frac{\kappa g(\tau|\tau)}{1 - \kappa G(\tau|\tau)} d\tau\right).$$

Second case: $\kappa < \kappa^*$. We look for an equilibrium strategy such that firm i has an incentive to submit a bid according to its true signal. Therefore, at equilibrium, the risk premium satisfies $\forall s_i \leq \underline{\sigma}^D$

$$\frac{\partial \Pi^D(b, s_i)}{\partial b}|_{b=s_i} = 0, \forall s_i \leq \underline{\sigma}^D$$

so that the equilibrium bid $P^D(s_i)$ satisfies the following differential equation

$$P^{D'}(s_i) = \frac{\kappa g(s_i|s_i)}{1 - \kappa G(s_i|s_i)} \left(P^D(s_i) - p(s_i, s_i)\right), \forall s_i \leq \underline{\sigma}^P.$$

It is solved with the boundary condition that $P^D(\underline{\sigma}^D) = \overline{P}$. Using the method of the parameters' variation, we obtain that

$$P^D(s) = \overline{P}(1 - K(\underline{\sigma}^D|s)) + \int_s^{\underline{\sigma}^D} p(x, x) dK(x|s) \;\; \forall \;\; s \leq \underline{\sigma}^D$$

with

$$K(x|s) = 1 - \exp\left(-\int_s^x \frac{\kappa g(\tau|\tau)}{1 - \kappa G(\tau|\tau)} d\tau\right).$$

When $s \in [\underline{\sigma}^D, \overline{\sigma}^D]$, $P^D(s) = \overline{P}$.

C Proofs

C.1 Proof of Lemma 1

We first prove that $\widehat{\sigma}^P < \widetilde{\sigma}$. Assume by contradiction that $\widehat{\sigma}^P \geq \widetilde{\sigma}$. Then,

$$\begin{aligned}\left(1 - G\left(\widehat{\sigma}^P|\widehat{\sigma}^P\right)\right)\mathbb{E}\left[\overline{P} - p(\widehat{\sigma}^P, S_j)|S_j > \widehat{\sigma}^P\right] &= \int_{\widehat{\sigma}^P}^1 \left(\overline{P} - p(\widehat{\sigma}^P, s_j)\right) g\left(s_j|\widehat{\sigma}^P\right) ds_j \\ &< \left(\overline{P} - p(\widehat{\sigma}^P, \widehat{\sigma}^P)\right)\left(1 - G(\widehat{\sigma}^P|\widehat{\sigma}^P)\right) \\ &\leq 0\end{aligned}$$

where the first inequality comes from the fact that p is strictly increasing in each of its argument, and the second from the fact that $\widehat{\sigma}^P$ is assumed to be greater than or equal to $\widetilde{\sigma}$. This contradicts the definition of $\widehat{\sigma}^P$ (see Equation (9)).

We introduce functions ψ and $\mathcal{L}$ defined by

$$\psi(x) \equiv (1 - G(x|x))\,\mathbb{E}\left[\overline{P} - p(x, S_j)|S_j > x\right] \tag{28}$$

$$\mathcal{L}(s|x) \equiv \frac{1}{g(s|x)}\frac{dg(s|x)}{dx}. \tag{29}$$

We have $\widehat{\sigma}^P$ defined by $\psi(\widehat{\sigma}^P) = 0$. Function $s \mapsto \mathcal{L}(s|x)$ is increasing according to Assumption 2. Let us prove that ψ has a unique zero. To do so, we are going to prove that if a zero exists it must be the case that ψ' evaluated at this point is negative.

$$\begin{aligned}\psi'(x) &= -\left(\overline{P} - p(x,x)\right) g(x|x) - \int_x^1 p_1(x, s_j) g(s_j|x)\, ds_j \\ &\quad + \int_x^1 \left(\overline{P} - p(x, s_j)\right)\mathcal{L}(s_j|x)\, g(s_j|x)\, ds_j.\end{aligned}$$

The first term is negative (since $\widehat{\sigma}^P < \widetilde{\sigma}$) and the second term is also negative since p is increasing in each of its argument. Let us focus on the third one

$$\begin{aligned}\int_x^1 \left(\overline{P} - p(x, s_j)\right)\mathcal{L}(s_j|x)\, g(s_j|x)\, ds_j &= \int_x^{\alpha(x)} \left(\overline{P} - p(x, s_j)\right)\mathcal{L}(s_j|x)\, g(s_j|x)\, ds_j \\ &\quad + \int_{\alpha(x)}^1 \left(\overline{P} - p(x, s_j)\right)\mathcal{L}(s_j|x)\, g(s_j|x)\, ds_j \\ &\leq \mathcal{L}(\alpha(x)|x)\int_x^{\alpha(x)} \left(\overline{P} - p(x, s_j)\right) g(s_j|x)\, ds_j \\ &\quad + \mathcal{L}(\alpha(x)|x)\int_{\alpha(x)}^1 \left(\overline{P} - p(x, s_j)\right) g(s_j|x)\, ds_j \\ &= \mathcal{L}(\alpha(x)|x)\,\psi(x)\end{aligned}$$

where the inequality comes for the fact that $\mathcal{L}$ is increasing in s and that $\overline{P}-p(x,s_j)>0$ if and only if $s_j<\alpha(x)$. Therefore, the derivative of function ψ is negative when ψ equals zero, and the zero of function ψ, if it exists is unique. Assumption 1(i) implies that $\psi(0)>0$. Moreover $\psi(1)=0$. As a consequence, ψ is positive and then negative as x increases and $\widehat{\sigma}^P$ always exists and is unique.

C.2 Proof of Proposition 1

See Subsection B.1.

C.3 Proof of Lemma 2

We have that $s_i \mapsto P^P(s_j)-p(s_i,s_j)$ is a decreasing function. We prove the result by showing that $P^P(s_j)-p(\widehat{\sigma}^P,s_j)>0$, $\forall s_j \leq \widehat{\sigma}^P$.

$$\begin{aligned}
P^P(s_j)-p(\widehat{\sigma}^P,s_j) &= \overline{P}\left(1-L\left(\widehat{\sigma}^P|s_j\right)\right)+\int_{s_j}^{\widehat{\sigma}^P} p(x,x)dL(x|s_j)-p\left(\widehat{\sigma}^P,s_j\right)\\
&= \overline{P}\left(1-L\left(\widehat{\sigma}^P|s_j\right)\right)+L\left(\widehat{\sigma}^P|s_j\right)\int_{s_j}^{\widehat{\sigma}^P} p(x,x)\frac{dL(x|s_j)}{L\left(\widehat{\sigma}^P|s_j\right)}-p\left(\widehat{\sigma}^P,s_j\right)\\
&\geq \overline{P}\left(1-L\left(\widehat{\sigma}^P|s_j\right)\right)+L\left(\widehat{\sigma}^P|s_j\right)p\left(\widehat{\sigma}^P,s_j\right)-p\left(\widehat{\sigma}^P,s_j\right)\\
&= \left(\overline{P}-p\left(\widehat{\sigma}^P,s_j\right)\right)\left(1-L\left(\widehat{\sigma}^P|s_j\right)\right)\\
&> 0
\end{aligned}$$

where the first inequality holds if $\int_{s_j}^{\widehat{\sigma}^P} p(x,x)\frac{dL(x|s_j)}{L\left(\widehat{\sigma}^P|s_j\right)} \geq p\left(\widehat{\sigma}^P,s_j\right)$ and the second inequality holds since $s_j \leq \widehat{\sigma}^P < \widetilde{\sigma}$.

The remaining of the proof consists in proving that $\int_{s_j}^{\widehat{\sigma}^P} p(x,x)\frac{dL(x|s_j)}{L\left(\widehat{\sigma}^P|s_j\right)} \geq p\left(\widehat{\sigma}^P,s_j\right)$, $\forall s_j \leq \widehat{\sigma}^P$. As p is symmetric and supermodular (Assumption 1(iv)), it holds that $p(\widehat{\sigma}^P,\widehat{\sigma}^P)+p(s_j,s_j)\geq 2p(\widehat{\sigma}^P,s_j)$ so that it is sufficient to prove that

$$\int_{s_j}^{\widehat{\sigma}^P} p(x,x)\frac{dL(x|s_j)}{L\left(\widehat{\sigma}^P|s_j\right)} \geq \frac{p(s_j,s_j)+p(\widehat{\sigma}^P,\widehat{\sigma}^P)}{2}\forall s_j \leq \widehat{\sigma}^P. \tag{30}$$

An integration by part implies that

$$\int_{s_j}^{\widehat{\sigma}^P} p(x,x)\frac{dL(x|s_j)}{L\left(\widehat{\sigma}^P|s_j\right)} = p\left(\widehat{\sigma}^P,\widehat{\sigma}^P\right)-\int_{s_j}^{\widehat{\sigma}^P}\frac{d}{dx}p(x,x)\frac{L(x|s_j)}{L\left(\widehat{\sigma}^P|s_j\right)}dx.$$

Therefore, inequality (30) reads

$$p(\widehat{\sigma}^P,\widehat{\sigma}^P)-p(s_j,s_j)-2\int_{s_j}^{\widehat{\sigma}^P}\frac{d}{dx}p(x,x)\frac{L(x|s_j)}{L\left(\widehat{\sigma}^P|s_j\right)}dx \geq 0\ \forall s_j \leq \widehat{\sigma}^P.$$

The derivative of function

$$s_j \mapsto p(\widehat{\sigma}^P, \widehat{\sigma}^P) - p(s_j, s_j) - 2\int_{s_j}^{\widehat{\sigma}^P} \frac{d}{dx}p(x,x)\frac{L(x|s_j)}{L\left(\widehat{\sigma}^P|s_j\right)}dx$$

equals

$$-\frac{d}{dx}p(x,x)|_{x=s_j} - 2\int_{s_j}^{\widehat{\sigma}^P} \frac{d}{dx}p(x,x)\frac{L_2(x|s_j)L\left(\widehat{\sigma}^P|s_j\right) - L(x|s_j)L_2\left(\widehat{\sigma}^P|s_j\right)}{\left(L\left(\widehat{\sigma}^P|s_j\right)\right)^2}dx < 0$$

since $L(s_j|s_j) = 0$ and since $x \mapsto \frac{L_2(x|s)}{L(x|s)}$ is decreasing. Moreover, as

$$p(\widehat{\sigma}^P, \widehat{\sigma}^P) - p(s_j, s_j) - 2\int_{s_j}^{\widehat{\sigma}^P} \frac{d}{dx}p(x,x)\frac{L(x|s_j)}{L\left(\widehat{\sigma}^P|s_j\right)}dx = 0$$

when $s_j = \widehat{\sigma}^P$ it follows that

$$p(\widehat{\sigma}^P, \widehat{\sigma}^P) - p(s_j, s_j) - 2\int_{s_j}^{\widehat{\sigma}^P} \frac{d}{dx}p(x,x)\frac{L(x|s_j)}{L\left(\widehat{\sigma}^P|s_j\right)}dx \geq 0 \; \forall s_j \leq \widehat{\sigma}^P$$

and the result is proved. □

C.4 Proof of Proposition 2

Observe first that $L(x|s)$ is increasing in κ. Remember that

$$P^P(s) = \overline{P}(1 - L(\widehat{\sigma}^P|s)) + \int_s^{\widehat{\sigma}^P} p(x,x)dL(x|s).$$

As $\widehat{\sigma}^P$ is independent from κ, $L(\widehat{\sigma}^P|s)$ is increasing in κ as we just underlined. This also implies that $\int_s^{\widehat{\sigma}^P} p(x,x)dL(x|s)$ is increasing in κ because, for $\kappa' > \kappa$ function L_κ first order stochastic dominates $L_{\kappa'}$.[17]

The boundary between regions the full coverage region and the partial coverage region is defined by $\left\{(s_i, s_j) \in [0,1]^2 | P^P(s_i) = p(s_i, s_j)\right\}$. For a given s_j, if κ increases, in order the equality $P^P(s_i) = p(s_i, s_j)$ to hold it must be the case that s_i decreases, so that the full coverage region shrinks. As a consequence, the partial coverage region expands. □

C.5 Proof of Proposition 3

From the definition of $\widehat{\sigma}^P$ (equation (9)), we obtain that

$$\frac{\partial \widehat{\sigma}^P}{\partial \overline{P}} = -\frac{1 - G\left(\widehat{\sigma}^P|\widehat{\sigma}^P\right)}{\psi'(\widehat{\sigma}^P)}.$$

[17]The subscript "κ" indicates the parameter value, κ.

The denominator being negative, $\frac{\partial \widehat{\sigma}^P}{\partial \overline{P}} > 0$.

As for the equilibrium bidding strategy, observe that

$$\frac{\partial P^P(s)}{\partial \overline{P}} = 1 - L\left(\widehat{\sigma}^P|s\right) - L_1\left(\widehat{\sigma}^P|s\right)\frac{\widehat{\sigma}^P}{\partial \overline{P}}\left(\overline{P} - p\left(\widehat{\sigma}^P, \widehat{\sigma}^P\right)\right).$$

The first term is positive and corresponds to the direct effect, whereas the second term is negative and corresponds to the indirect effect. It is therefore necessary to compute all the terms together. Observing that

$$L_1\left(\widehat{\sigma}^P|s\right) = \kappa\frac{g\left(\widehat{\sigma}^P|\widehat{\sigma}^P\right)}{1 - G\left(\widehat{\sigma}^P|\widehat{\sigma}^P\right)}\left(1 - L\left(\widehat{\sigma}^P|s\right)\right),$$

we have that

$$\frac{\partial P^P(s)}{\partial \overline{P}} = \left(1 - L\left(\widehat{\sigma}^P|s\right)\right)\left(1 - \frac{\widehat{\sigma}^P}{\partial \overline{P}}\left(\overline{P} - p\left(\widehat{\sigma}^P, \widehat{\sigma}^P\right)\right)\kappa\frac{g\left(\widehat{\sigma}^P|\widehat{\sigma}^P\right)}{1 - G\left(\widehat{\sigma}^P|\widehat{\sigma}^P\right)}\right).$$

The term in the parenthesis equals

$$\frac{\psi'(\widehat{\sigma}^P) + \left(\overline{P} - p\left(\widehat{\sigma}^P, \widehat{\sigma}^P\right)\right)\kappa g\left(\widehat{\sigma}^P|\widehat{\sigma}^P\right)}{\psi'(\widehat{\sigma}^P)}$$

Observe that

$$\begin{aligned}\psi'(\widehat{\sigma}^P) + \left(\overline{P} - p\left(\widehat{\sigma}^P, \widehat{\sigma}^P\right)\right)\kappa g\left(\widehat{\sigma}^P|\widehat{\sigma}^P\right) \quad = \quad & -(1-\kappa)\left(\overline{P} - p\left(\widehat{\sigma}^P, \widehat{\sigma}^P\right)\right) g\left(\widehat{\sigma}^P|\widehat{\sigma}^P\right) \\ & - \int_{\widehat{\sigma}^P}^1 p_1\left(\widehat{\sigma}^P, s\right) g\left(s|\widehat{\sigma}^P\right) ds \\ & + \int_{\widehat{\sigma}^P}^1 \left(\overline{P} - p\left(\widehat{\sigma}^P, s\right)\right) g_2\left(s|\widehat{\sigma}^P\right) ds\end{aligned}$$

In the proof of Lemma 1, we have shown that each term of this expression is negative. As $\psi'(\widehat{\sigma}^P)$ is negative, it follows that $P^P(s)$ is an increasing function of $\overline{P}$. □

C.6 Proof of Lemma 3

Introduce functions ϕ and θ defined by

$$\phi(x) \equiv G(x|x)\,\mathbb{E}\left[\overline{P} - p(x, S)|S < x\right] \tag{31}$$

$$\theta(x) \equiv \psi(x) + (1-\kappa)\phi(x) \tag{32}$$

where ψ is defined by Equation (28) in the proof of Lemma 1. $\widehat{\sigma}^D$ (Equation (19)) is defined by $\theta(\widehat{\sigma}^D) = 0$. Observe that the only possibility for Equation (19) to be satisfied is that $\phi(\widehat{\sigma}^D) > 0$ and $\psi(\widehat{\sigma}^D) < 0$.

Introduce in addition functions Φ and Ψ defined by

$$\Phi(x) \equiv \mathbb{E}\left[\overline{P} - p(x,S)|S < x\right] \quad (33)$$

$$\Psi(x) \equiv \mathbb{E}\left[\overline{P} - p(x,S)|S > x\right] \quad (34)$$

so that $\theta(x) = (1 - G(x|x))\Psi(x) + (1-\kappa)G(x|x)\Phi(x)$. Φ and Ψ are decreasing functions (Milgrom and Weber (1982)).

Step 1. Assume first that $\kappa \geq \kappa^*$. We show that a unique threshold $\widehat{\sigma}^D \in [0, \widetilde{\sigma}]$ exists. Observe that $\theta(0) > 0$ and that $\theta(\widetilde{\sigma}) \leq 0$ when $\kappa \geq \kappa^*$. Therefore, function θ has at least one zero. Therefore, $\widehat{\sigma}^D$ exists. As in the proof of Lemma 1, we are going to prove that the derivative of θ is negative at $\widehat{\sigma}^D$.

$$\theta'(\widehat{\sigma}^D) = (1-\kappa)G(\widehat{\sigma}^D|\widehat{\sigma}^D)\Phi'(\widehat{\sigma}^D) + (1-G(x|x))\Psi'(\widehat{\sigma}^D) + \left.\frac{dG(x|x)}{dx}\right|_{x=\widehat{\sigma}^D}\left((1-\kappa)\Phi(\widehat{\sigma}^D) - \Psi(\widehat{\sigma}^D)\right)$$

The first two terms are negative. Since $\phi(\widehat{\sigma}^D) > 0$ and $\psi(\widehat{\sigma}^D) < 0$, it follows that $(1-\kappa)\Phi(\widehat{\sigma}^D) - \Psi(\widehat{\sigma}^D) \geq 0$. Therefore, if $\frac{dG(x|x)}{dx}|_{x=\widehat{\sigma}^D} < 0$, $\theta'(\widehat{\sigma}^D) \leq 0$. To complete this part of the proof, we must treat the case where $\frac{dG(x|x)}{dx}|_{x=\widehat{\sigma}^D} > 0$. To do that, we have to detail the expressions of $\phi'(x)$ and $\psi'(x)$.

$$\begin{aligned}
\theta'(x) &= \psi'(x) + (1-\kappa)\phi'(x) \\
&= -\left(\overline{P} - p(x,x)\right)g(x|x) - \int_x^1 p_1(x,s_j)g(s_j|x)\,ds_j \\
&\quad + \int_x^1 \left(\overline{P} - p(x,s_j)\right)g_2(s_j|x)\,ds_j + (1-\kappa)\left(\overline{P} - p(x,x)\right)g(x|x) \\
&\quad -(1-\kappa)\int_0^x p_1(x,s_j)g(s_j|x)\,ds_j + (1-\kappa)\int_0^x \left(\overline{P} - p(x,s_j)\right)g_2(s_j|x)\,ds_j \\
&= -\int_x^1 p_1(x,s_j)g(s_j|x)\,ds_j - (1-\kappa)\int_0^x p_1(x,s_j)g(s_j|x)\,ds_j - \kappa\left(\overline{P} - p(x,x)\right)g(x|x) \\
&\quad + \int_x^1 \left(\overline{P} - p(x,s_j)\right)g_2(s_j|x)\,ds_j + (1-\kappa)\int_0^x \left(\overline{P} - p(x,s_j)\right)g_2(s_j|x)\,ds_j.
\end{aligned}$$

An integration by part of the last two terms imply that

$$\begin{aligned}
\theta'(x) &= -\int_x^1 p_1(x,s_j)g(s_j|x)\,ds_j - (1-\kappa)\int_0^x p_1(x,s_j)g(s_j|x)\,ds_j - \kappa\left(\overline{P} - p(x,x)\right)g(x|x) \\
&+ \int_x^1 p_2(x,s_j)G_2(s_j|x)\,ds_j + (1-\kappa)\int_0^x p_2(x,s_j)G_2(s_j|x)\,ds_j - \kappa\left(\overline{P} - p(x,x)\right)\frac{dG(x|x)}{dx}.
\end{aligned}$$

The first five terms are negative (remember that affiliation implies that $G_2 < 0$). As for the last term, it is negative when evaluated at $x = \widehat{\sigma}^D$ since $\widehat{\sigma}^D < \widetilde{\sigma}$ and $\frac{dG(x|x)}{dx}|_{x=\widehat{\sigma}^D} > 0$. When θ equals zero, its derivative is negative meaning that $\widehat{\sigma}_D$ is unique.

Step 2. Assume now that $\kappa < \kappa^*$. We show that function θ does not cancel on $[0, \widetilde{\sigma}]$ so that there does not exist $\widehat{\sigma}^D$.
When $\kappa < \kappa^*$, $\theta(\widetilde{\sigma}) > 0$, so that either θ never cancels on $[0, \widetilde{\sigma}]$, or it cancels an even number of times. However, we have shown that when θ cancels on $[0, \widetilde{\sigma}]$, its derivative is negative. Therefore, when $\kappa < \kappa^*$, θ is positive and $\widehat{\theta}^D$ does not exist. □

C.7 Proof of Proposition 4

We prove first that if the separating equilibrium does not exist ($\kappa < \kappa^*$), then the semi-separating equilibrium exists and is unique.

The first part of this proof goes through a series of steps. Let us first introduce function I, J and K

$$I(x,y) \equiv (G(y|x) - G(x|x))\,\mathbb{E}\left[\overline{P} - p(x,S)|x < S < y\right] \tag{35}$$

$$J(x,y) \equiv (G(y|y) - G(x|y))\,\mathbb{E}\left[\overline{P} - p(y,S)|x < S < y\right] \tag{36}$$

$$H(x,y) \equiv \theta(y) + \frac{\kappa}{2} J(x,y) \tag{37}$$

where θ is defined by equation (32). Observe that $\psi(x) = I(x,1)$, $\phi(x) = J(0,x)$ and $\theta(x) = I(x,1) + (1-\kappa)J(0,x)$. Let us also introduce

$$\mathcal{J}(x,y) \equiv \mathbb{E}\left[\overline{P} - p(y,S)|x < S < y\right] \tag{38}$$

$\mathcal{J}$ is decreasing in each of its argument (Milgrom and Weber [21]).
Step 1.
We show that $\alpha(\overline{\sigma}^D) < \underline{\sigma}^D < \widetilde{\sigma} < \alpha(\underline{\sigma}^D) < \overline{\sigma}^D < \widehat{\sigma}^D$.
To prove this step, assume that $(\underline{\sigma}^D, \overline{\sigma}^D)$, with $\underline{\sigma}^D, \overline{\sigma}^D$, is a solution meaning that $I(\underline{\sigma}^D, \overline{\sigma}^D) = 0$ and $H(\underline{\sigma}^D, \overline{\sigma}^D) = 0$. $I(\underline{\sigma}^D, \overline{\sigma}^D) = 0$ implies that

$$\begin{aligned} \overline{P}\left(G(\overline{\sigma}^D|\underline{\sigma}^D) - G(\underline{\sigma}^D|\underline{\sigma}^D)\right) &= \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} p(\underline{\sigma}^D, t) g(t|\underline{\sigma}^D) dt \\ &> p(\underline{\sigma}^D, \underline{\sigma}^D)\left(G(\overline{\sigma}^D|\underline{\sigma}^D) - G(\underline{\sigma}^D|\underline{\sigma}^D)\right). \end{aligned}$$

Therefore, $\overline{P} > p(\underline{\sigma}^D, \underline{\sigma}^D)$ implying that $\underline{\sigma}^D < \widetilde{\sigma}$.

Remember that $t \mapsto \overline{P} - p(\underline{\sigma}^D, t)$ is a decreasing function. In order to have $I(\underline{\sigma}^D, \overline{\sigma}^D) = 0$, it must be the case that $\overline{P} - p(\underline{\sigma}^D, t)$ is first positive and then negative as t increases from $\underline{\sigma}^D$ to $\overline{\sigma}^D$. In particular, we must have that $\overline{P} - p(\underline{\sigma}^D, \overline{\sigma}^D) < 0$. This implies that $\overline{\sigma}^D > \alpha(\underline{\sigma}^D)$ and $\underline{\sigma}^D > \alpha(\overline{\sigma}^D)$ where function α is defined by equation (5). Note that the symmetry of p and the fact that it is increasing with respect to each of its argument imply that $\alpha = \alpha^{-1}$.

To prove that $\overline{\sigma}^D < \widehat{\sigma}^D$, we show that $J(\underline{\sigma}^D, \overline{\sigma}^D) < 0$ so that $\theta(\overline{\sigma}^D) > 0$.

$$\begin{aligned}
J(\underline{\sigma}^D, \overline{\sigma}^D) &= \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P} - p(t, \overline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|\overline{\sigma}^D)}{g(t|\underline{\sigma}^D)} dt \\
&< \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P} - p(t, \underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|\overline{\sigma}^D)}{g(t|\underline{\sigma}^D)} dt \\
&= \int_{\underline{\sigma}^D}^{\alpha(\underline{\sigma}^D)} \left(\overline{P} - p(t, \underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|\overline{\sigma}^D)}{g(t|\underline{\sigma}^D)} dt + \int_{\alpha(\underline{\sigma}^D)}^{\overline{\sigma}^D} \left(\overline{P} - p(t, \underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|\overline{\sigma}^D)}{g(t|\underline{\sigma}^D)} dt \\
&\leq \frac{g(\alpha(\underline{\sigma}^D)|\overline{\sigma}^D)}{g(\alpha(\underline{\sigma}^D)|\underline{\sigma}^D)} I(\underline{\sigma}^D, \overline{\sigma}^D) \\
&= 0.
\end{aligned}$$

The second inequality holds because $t \mapsto \frac{g(t|y)}{g(t|x)}$ is an increasing function $\forall x \leq y$.

It therefore holds that $\alpha(\overline{\sigma}^D) < \underline{\sigma}^D < \widetilde{\sigma} < \alpha(\underline{\sigma}^D) < \overline{\sigma}^D < \widehat{\sigma}^D$. This allows us to define the region $\mathcal{D} \equiv \left\{(x, y) \in [0, 1]^2 | \alpha(y) < x < \widetilde{\sigma} < \alpha(x) < y < \widehat{\sigma}^D\right\}$ to which the solution to the following system should belong to

$$\begin{cases} I(x, y) &= 0 \\ H(x, y) &= 0. \end{cases}$$

Step 2.

We show that $x_I(y)$ defined by $I(x_I(y), y) = 0$ on $\mathcal{D}_y = \left\{\widetilde{\sigma} < y < \widehat{\sigma}^D | \alpha(y) < x_I(y) < \widetilde{\sigma}\right\}$ is a decreasing function.

The implicit function theorem implies that

$$\frac{dx_I(y)}{dy} = -\frac{I_2(x_I(y), y)}{I_1(x_I(y), y)}.$$

$I_2(x_I(y), y) = (\overline{P} - p(x_I(y), y)) g(y|x_I(y)) \leq 0$ since $\alpha(y) < x_I(y)$ (or equivalently $y > \alpha(x_I(y))$).

$$\begin{aligned}
I_1(x_I(y), y) &= -\left(\overline{P} - p(x_I(y), x_I(y))\right) g(x_I(y)|x_I(y)) - \int_{x_I(y)}^{y} p_1(x_I(y), t) g(t|x_I(y)) \\
&\quad + \int_{x_I(y)}^{y} (\overline{P} - p(x_I(y), t)) \mathcal{L}\left(t|x_I(y)\right) g(t|x_I(y)) dt
\end{aligned}$$

The first two terms are negative (the first because $x_I(y) < \widetilde{\sigma}$). As for the third term, using the fact that $t \mapsto \mathcal{L}\left(t|x_I(y)\right)$ is an increasing function and as we did in the proofs of Lemmas 1 and 3

$$\begin{aligned}
&\int_{x_I(y)}^{y} (\overline{P} - p(x_I(y), t)) \mathcal{L}\left(t|x_I(y)\right) g(t|x_I(y)) dt \\
\leq\ & \mathcal{L}\left(\alpha(x_I(y))|x_I(y)\right) \int_{x_I(y)}^{y} (\overline{P} - p(x_I(y), t)) g(t|x_I(y)) dt \\
=\ & 0.
\end{aligned}$$

This implies that $y \mapsto x_I(y)$ is a decreasing function.

Note moreover that $x_I(\widetilde{\sigma}) = \widetilde{\sigma}$ and that $x_I(\widehat{\sigma}^D) > \alpha(\widehat{\sigma}^D)$. To prove this last inequality, observe that

$$\begin{aligned}\int_{x_I(\widehat{\sigma}^D)}^{\widehat{\sigma}^D} \left(\overline{P} - p(\alpha(\widehat{\sigma}^D), t)\right) g(t|x_I(\widehat{\sigma}^D))dt &> \int_{x_I(\widehat{\sigma}^D)}^{\widehat{\sigma}^D} \left(\overline{P} - p(\alpha(\widehat{\sigma}^D), \widehat{\sigma}^D)\right) g(t|x_I(\widehat{\sigma}^D))dt \\ &= 0,\end{aligned}$$

implying that $x_I(\widehat{\sigma}^D) > \alpha(\widehat{\sigma}^D)$.

The last property that remains to be showed for this function x_I is that $y \mapsto x_I(y)$ and $y \mapsto \alpha(y)$ only cross once when $y \in [\widetilde{\sigma}, \widehat{\sigma}^D]$. This is not a priori obvious since the two functions are decreasing. We know that $x_I(\widetilde{\sigma}) = \alpha(\widetilde{\sigma}) = \widetilde{\sigma}$. Assume that there exists $\overline{y} \in (\widetilde{\sigma}, \widehat{\sigma}^D]$ such that $x_I(\overline{y}) = \alpha(\overline{y})$. By definition of x_I, this implies that

$$\int_{\alpha(\overline{y})}^{\overline{y}} \left(\overline{P} - p\left(\alpha(\overline{y}), t\right) g\left(t|\alpha(\overline{y})\right)\right) dt = 0.$$

However, $p\left(\alpha(\overline{y}), t\right) < p\left(\alpha(\overline{y}), \overline{y}\right) = \overline{P}$, $\forall t \in [\alpha(\overline{y}), \overline{y}]$, so that it is not possible that the integral equals 0. Therefore such an $\overline{y}$ does not exist. As a consequence, $y \mapsto x_I(y)$ and $y \mapsto \alpha(y)$ only cross for $y = \widetilde{\sigma}$, and $\forall y \in [\widetilde{\sigma}, \widehat{\sigma}^D]$, $x_I(y) > \alpha(y)$.

Step 3.
We show that $y_H(x)$ defined by $H(x, y_H(x)) = 0$ on $\mathcal{D}_x = \left\{\alpha\left(\widehat{\sigma}^D\right) < x < \widetilde{\sigma} | \alpha(x) < y_H(x) < \widehat{\sigma}^D\right\}$ is an increasing function.
The implicit function theorem implies that

$$\frac{dy_H(x)}{dx} = -\frac{H_1(x, y_H(x))}{H_2(x, y_H(x))}.$$

Remembering that $H(x, y) = \theta(y) + (\kappa/2)J(x, y)$ (where θ is defined by equation (32)), it follows that

$$\begin{aligned}H_1(x, y_H(x)) &= \frac{\kappa}{2} J_1(x, y_H(x)) \\ &= -(\overline{P} - p(x, y_H(x)))g(x|y_H(x))) \\ &> -(\overline{P} - p(x, \alpha(x)))g(x|y_H(x))) \\ &= 0.\end{aligned}$$

In order to prove that $H_2(x, y_H(x))$ is negative, let us first write

$$H(x, y) = (1 - G\left(y|y\right)) \Psi\left(y\right) + (1 - \kappa) G\left(y|y\right) \Phi\left(y\right) + \frac{\kappa}{2} \left(G\left(y|y\right) - G\left(x|y\right)\right) \mathcal{J}\left(x, y\right),$$

so that

$$\begin{aligned}
H_2(x,y) &= -\left(\frac{d}{dy}G(y|y)\right)\Psi(y) + (1-G(y|y))\Psi'(y) + (1-\kappa)\left(\frac{d}{dy}G(y|y)\right)\Phi(y) \\
&\quad + (1-\kappa)\Phi'(y) + \frac{\kappa}{2}\left(\frac{d}{dy}G(y|y) - G_2(x|y)\right)\mathcal{J}(x,y) \\
&\quad + \frac{\kappa}{2}\left(G(y|y) - G(x|y)\right)\mathcal{J}_2(x,y) \\
&= (1-G(y|y))\Psi'(y) + (1-\kappa)\Phi'(y) - \frac{\kappa}{2}G_2(x|y)\mathcal{J}(x,y) \\
&\quad + \frac{\kappa}{2}\left(G(y|y) - G(x|y)\right)\mathcal{J}_2(x,y) \\
&\quad + \frac{d}{dy}G(y|y)\left(-\Psi(y) + (1-\kappa)\Phi(y) + \frac{\kappa}{2}\mathcal{J}(x,y)\right).
\end{aligned}$$

Observe that

$$\begin{aligned}
\frac{\kappa}{2}\mathcal{J}(x, y_H(x)) &= -\frac{\theta(y_H(x))}{G(y_H(x)|y_H(x)) - G(x|y_H(x))} \\
&< 0
\end{aligned}$$

since $y_H(x) < \widehat{\sigma}^D$. Therefore, the first four terms of $H_2(x, y_H(x))$ are negative. Concerning the multiplicative term of $\frac{d}{dy}G(y|y)$, since $H(x, y_H(x)) = 0$, it follows that

$$\begin{aligned}
\Psi(y_H(x)) - \frac{\kappa}{2}G(x|y_H(x))\mathcal{J}(x, y_H(x)) &= G(y_H(x)|y_H(x))\left[-\Psi(y_H(x)) + (1-\kappa)\Phi(y_H(x))\right. \\
&\quad \left. + \frac{\kappa}{2}\mathcal{J}(x, y_H(x))\right].
\end{aligned}$$

Therefore, it is sufficient to focus on the sign of the left hand side of the equation so that we analyze $\Psi(y) - \frac{\kappa}{2}G(x|y)\mathcal{J}(x,y)$. Observe that when x and y are such that $\mathcal{J}(x,y) < 0$, $x \mapsto \Psi(y) - \frac{\kappa}{2}G(x|y)\mathcal{J}(x,y)$ is a decreasing function. When $x \to y$, we have that

$$\begin{aligned}
\left(\Psi(y) - \frac{\kappa}{2}G(x|y)\mathcal{J}(x,y)\right)\Big|_{x=y} &= \frac{1}{1-G(y)}\int_y^1 \left(\overline{P} - p(y,s)\right)g(s|y)ds - \frac{\kappa}{2}G(y|y)\left(\overline{P}p(y,y)\right) \\
&< \left(\overline{P} - p(y,y)\right)\left(1 - \frac{\kappa}{2}G(y|y)\right) \\
&< 0,
\end{aligned}$$

where the equality comes from the fact that

$$\overline{P} - p(y,y) < \mathcal{J}(x,y) < \overline{P} - p(y,x)$$

so that $\lim_{x\to y}\mathcal{J}(x,y) = \overline{P} - p(y,y)$ and the second inequality from the fact that $y > \widetilde{\sigma}$.

Therefore, $\Psi(y) - \frac{\kappa}{2} G(x|y) \mathcal{J}(x,y) < 0 \; \forall x < y$. It follows that

$$\Psi(y) - \frac{\kappa}{2} G(x|y) \mathcal{J}(x,y) < 0.$$

As a consequence, if $\frac{d}{dy} G(y|y) > 0$, then $\frac{d}{dy} G(y|y) \left(-\Psi(y) + (1-\kappa)\Phi(y) + \frac{\kappa}{2}\mathcal{J}(x,y) \right) < 0$, so that $H_2(x, y_H(x)) < 0$.

As for the case where $\frac{d}{dy} G(y|y) < 0$, using $H(x,y) = \theta(y) + (\kappa/2) J(x,y)$:

$$\begin{aligned}
H_2(x, y_H(x)) &= -\left(\overline{P} - p(y_H(x), y_H(x))\right) g(y_H(x)|y_H(x)) - \int_{y_H(x)}^{1} p_1(y_H(x), t)\, g(t|y_H(x))\, dt \\
&\quad + \int_{y_H(x)}^{1} \left(\overline{P} - p(y_H(x), t)\right) G_{12}(t|y_H(x))\, dt \\
&\quad + (1-\kappa)\left(\left(\overline{P} - p(y_H(x), y_H(x))\right) g(y_H(x)|y_H(x)) - \int_{0}^{y_H(x)} p_2(t, y_H(x))\, g(t, |y_H(x))\, dt \right. \\
&\quad \left. + \int_{0}^{y_H(x)} \left(\overline{P} - p(t, y_H(x))\right) G_{12}(t|y_H(x))\, dt \right) \\
&\quad + \frac{\kappa}{2}\left(\left(\overline{P} - p(y_H(x), y_H(x))\right) g(y_H(x), y_H(x)) - \int_{x}^{y_H(x)} p_2(t, y_H(x))\, g(t|y_H(x))\, dt \right. \\
&\quad \left. + \int_{x}^{y_H(x)} \left(\overline{P} - p(t, y_H(x))\right) G_{12}(t|y_H(x))\, dt \right) \\
&= -\int_{y_H(x)}^{1} p_1(y_H(x), t)\, g(t|y_H(x))\, dt - (1-\kappa)\int_{0}^{y_H(x)} p_2(t, y_H(x))\, g(t, |y_H(x))\, dt \\
&\quad - \frac{\kappa}{2}\int_{x}^{y_H(x)} p_2(t, y_H(x))\, g(t|y_H(x))\, dt - \frac{\kappa}{2}\left(\overline{P} - p(y_H(x), y_H(x))\right) g(y_H(x)|y_H(x)) \\
&\quad + \int_{y_H(x)}^{1} \left(\overline{P} - p(y_H(x), t)\right) G_{12}(t|y_H(x))\, dt + \frac{\kappa}{2}\int_{x}^{y_H(x)} \left(\overline{P} - p(t, y_H(x))\right) G_{12}(t|y_H(x))\, dt \\
&\quad + (1-\kappa)\int_{0}^{y_H(x)} \left(\overline{P} - p(t, y_H(x))\right) G_{12}(t|y_H(x))\, dt.
\end{aligned}$$

The first three terms are negative. Let us analyze the last three terms.

$$\begin{aligned}
K(x, y_H(x)) &\equiv -\frac{\kappa}{2}\left(\overline{P} - p(y_H(x), y_H(x))\right) g(y_H(x)|y_H(x)) \\
&\quad + \int_{y_H(x)}^{1} \left(\overline{P} - p(y_H(x), t)\right) g(t|y_H(x))\, \mathcal{L}(t|y_H(x))\, dt \\
&\quad + (1-\kappa)\int_{0}^{y_H(x)} \left(\overline{P} - p(t, y_H(x))\right) g(t|y_H(x))\, \mathcal{L}(t|y_H(x))\, dt \\
&\quad + \frac{\kappa}{2}\int_{x}^{y_H(x)} \left(\overline{P} - p(t, y_H(x))\right) g(t|y_H(x))\, \mathcal{L}(t|y_H(x))\, dt
\end{aligned}$$

An integration by part implies that

$$\begin{aligned}
\int_y^1 \left(\overline{P} - p(y,t)\right) G_{12}(t|y)\, dt &= -\left(\overline{P} - p(y,y)\right) G_2(y|y) + \int_y^1 \left(\overline{P} - p(y,t)\right) G_{12}(t|y)\, dt \\
\int_0^y \left(\overline{P} - p(y,t)\right) G_{12}(t|y)\, dt &= \left(\overline{P} - p(y,y)\right) G_2(y|y) + \int_0^y \left(\overline{P} - p(y,t)\right) G_{12}(t|y)\, dt \\
\int_x^y \left(\overline{P} - p(y,t)\right) G_{12}(t|y)\, dt &= \left(\overline{P} - p(y,y)\right) G_2(y|y) - \left(\overline{P} - p(y,x)\right) G_2(x|y) \\
&\quad + \int_x^y \left(\overline{P} - p(y,t)\right) G_{12}(t|y)\, dt
\end{aligned}$$

This implies that

$$\begin{aligned}
K(x, y_H(x)) &= -\frac{\kappa}{2}\left(\overline{P} - p(y_H(x), y_H(x))\right)\left(G_2(y_H(x)|y_H(x)) + G_1(y_H(x)|y_H(x))\right) \\
&\quad -\frac{\kappa}{2}\left(\overline{P} - p(y_H(x), x)\right) G_2(y_H(x)|y_H(x)) \\
&= -\frac{\kappa}{2}\left(\overline{P} - p(y_H(x), y_H(x))\right)\frac{d}{dy}\left(G(y|y)\right)\Big|_{y=y_H(x)} \\
&\quad -\frac{\kappa}{2}\left(\overline{P} - p(y_H(x), x)\right) G_2(y_H(x)|y_H(x))
\end{aligned}$$

As $y_H(x) > \widetilde{\sigma}$, $\overline{P} - p(y_H(x), y_H(x)) < 0$. Moreover, $G_2 < 0$ and we assumed that $\frac{d}{dy}G(y|y) < 0$. This implies that $H_2(x, y_H(x)) < 0$. We can therefore conclude that y_H is increasing function.

Moreover note that $y_H(\widetilde{\sigma}) \in [\widetilde{\sigma}, \widehat{\sigma}^D]$. Suppose by contradiction that $y_H(\widetilde{\sigma}) > \widehat{\sigma}^D$. In this case, $\theta(y_H(\widetilde{\sigma})) < 0$ and $J(\widetilde{\sigma}), y_H(\widetilde{\sigma}) > 0$. This implies that

$$\begin{aligned}
0 &< \int_{\widetilde{\sigma}}^{y_H(\widetilde{\sigma})} \left(\overline{P} - p(y_H(\widetilde{\sigma}), t)\right) g(t|y_H(\widetilde{\sigma}))\, dt \\
&< \int_{\widetilde{\sigma}}^{y_H(\widetilde{\sigma})} \left(\overline{P} - p(\widetilde{\sigma}, t)\right) g(t|y_H(\widetilde{\sigma}))\, dt \\
&< 0,
\end{aligned}$$

leading to a contradiction. Therefore $y_H(\widetilde{\sigma}) \leq \widehat{\sigma}^D$. The same reasoning implies that $y_H(\widetilde{\sigma}) \geq \widetilde{\sigma}$.

Step 4.

We show that the solution $(\underline{\sigma}^D, \overline{\sigma}^D)$ is unique.

x_I is a decreasing function such that $x_I(\widetilde{\sigma}) = \widetilde{\sigma}$, $x_I(\widehat{\sigma}^D) = \widetilde{\sigma}$ and Y_H is an increasing function such that $y_H(\widetilde{\sigma}) \in [\widetilde{\sigma}, \widehat{\sigma}^D]$. As $x_I(y) > \alpha(y)$, $\forall y \in (\widetilde{\sigma}, \widehat{\sigma}^D]$, the two function cross only once on $\mathcal{D}$. This intersection point that is unique corresponds to the unique solution of the system that is $(\underline{\sigma}^D, \overline{\sigma}^D)$.

The last part of the proof consists in showing that if the semi-separating equilibrium exists, then the separating equilibrium does not exist. We are going to prove that if the separating equilibrium exists then the semi-separating equilibrium does not exist. Assume therefore that $\widehat{\sigma}^P < \widetilde{\sigma}$. We have proven in Step 1 that $\overline{\sigma}^P \leq \widehat{\sigma}^P$, this implies that $\underline{\sigma}^D < \overline{\sigma}^D < \widetilde{\sigma}$. But in this case, $I(\underline{\sigma}^D, \overline{\sigma}^D) > 0$. Therefore, if $\widehat{\sigma}^P < \widetilde{\sigma}$, there do not exist $(\underline{\sigma}^D, \overline{\sigma}^D)$ satisfying $I(\underline{\sigma}^D, \overline{\sigma}^D) = 0$ and $H(\underline{\sigma}^D, \overline{\sigma}^D) = 0$.

It remains to prove that

$$\int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P} - p(s,t)\right) g\left(t|s\right) dt \leq 0 \quad \forall s \in [\underline{\sigma}^D \overline{\sigma}^D].$$

$$\begin{aligned}
\int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P} - p(s,t)\right) g\left(t|s\right) dt &= \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P} - p(t,s)\right) g(t|\underline{\sigma}^D) \frac{g(t|s)}{g(t|\underline{\sigma}^D)} dt \\
&\leq \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P} - p(t,\underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|s)}{g(t|\underline{\sigma}^D)} dt \\
&= \int_{\underline{\sigma}^D}^{\alpha(\underline{\sigma}^D)} \left(\overline{P} - p(t,\underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|s)}{g(t|\underline{\sigma}^D)} dt \\
&\quad + \int_{\alpha(\underline{\sigma}^D)}^{\overline{\sigma}^D} \left(\overline{P} - p(t,\underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|s)}{g(t|\underline{\sigma}^D)} dt \\
&\leq \frac{g(\alpha(\underline{\sigma}^D)|s)}{g(\alpha(\underline{\sigma}^D)|\underline{\sigma}^D)} I(\underline{\sigma}^D, \overline{\sigma}^D) \\
&= 0.
\end{aligned}$$

C.8 Proof of Corollary 1

See Subsection B.2.

C.9 Proof of Lemma 4

$\theta(\widehat{\sigma}^D) = \psi(\widehat{\sigma}^D) + (1-\kappa)\,\phi(\widehat{\sigma}^D) = 0$. As we noted in the proof of Lemma 3, $\phi(\widehat{\sigma}^D) > 0$.

The implicit function theorem implies that

$$\frac{\partial \widehat{\sigma}^D}{\partial \kappa} = \frac{\phi\left(\widehat{\sigma}^D\right)}{\psi'\left(\widehat{\sigma}^D\right) + (1-\kappa)\,\phi'\left(\widehat{\sigma}^D\right)} = \frac{\phi\left(\widehat{\sigma}^D\right)}{\theta'\left(\widehat{\sigma}^D\right)}.$$

In Lemma 3, we have proved that $\theta'(\widehat{\sigma}^D) \leq 0$ so that $\frac{\partial \widehat{\sigma}^D}{\partial \kappa} \leq 0$.

If $\widehat{\sigma}^D > \widetilde{\sigma}$, $\underline{\sigma}^D$ and $\overline{\sigma}^D$ are such that

$$\begin{cases} I(\underline{\sigma}^D, \overline{\sigma}^D) &= 0 \\ H(\underline{\sigma}^D, \overline{\sigma}^D) &= 0. \end{cases}$$

The implicit function theorem implies that

$$\begin{aligned}
\frac{\partial \overline{\sigma}^D}{\partial \kappa} &= \frac{J(0,\overline{\sigma}^D) - \frac{1}{2}J(\underline{\sigma}^D,\overline{\sigma}^D)}{I_1(\overline{\sigma}^D,1) + (1-\kappa)J_1(0,\overline{\sigma}^D) + \frac{\kappa}{2}J_2(\underline{\sigma}^D,\overline{\sigma}^D) - \frac{\kappa}{2}J_1(\underline{\sigma}^D,\overline{\sigma}^D)\frac{I_2(\underline{\sigma}^D,\overline{\sigma}^D)}{I_1(\underline{\sigma}^D,\overline{\sigma}^D)}} \\
\frac{\partial \underline{\sigma}^D}{\partial \kappa} &= -\frac{\partial \overline{\sigma}^D}{\partial \kappa}\frac{I_2(\underline{\sigma}^D,\overline{\sigma}^D)}{I_1(\underline{\sigma}^D,\overline{\sigma}^D)}.
\end{aligned}$$

We have proven in Step 3 of the proof of Proposition 4 that $H_2(\underline{\sigma}^D,\overline{\sigma}^D) = I_1(\overline{\sigma}^D,1) + (1-\kappa)J_1(0,\overline{\sigma}^D) + \frac{\kappa}{2}J_2(\underline{\sigma}^D,\overline{\sigma}^D) \leq 0$ and that $J_1(\underline{\sigma}^D,\overline{\sigma}^D) \geq 0$. In Step 2 of the proof of Proposition 4, we also showed that $\frac{I_1(\underline{\sigma}^D,\overline{\sigma}^D)}{I_2(\underline{\sigma}^D,\overline{\sigma}^D)} \geq 0$. This implies that the denominator of $\frac{\partial \overline{\sigma}^D}{\partial \kappa}$ is negative and that $\frac{\partial \overline{\sigma}^D}{\partial \kappa}$ and $\frac{\partial \underline{\sigma}^D}{\partial \kappa}$ have opposite signs.

$$\begin{aligned}
J(0,\overline{\sigma}^D) - \frac{1}{2}J(\underline{\sigma}^D,\overline{\sigma}^D) &= \frac{1}{\kappa}\Big(I(\overline{\sigma}^D,1) + J(0,\overline{\sigma}^D)\Big). \\
&= \frac{1}{\kappa}\Big(I(\overline{\sigma}^D,1) + (1-\kappa)J(0,\overline{\sigma}^D) + \kappa J(0,\overline{\sigma}^D)\Big).
\end{aligned}$$

In Step 1 of the proof of Proposition 4, we also proved that $J(\underline{\sigma}^D,\overline{\sigma}^D) \leq 0$. Together with $H(\underline{\sigma}^D,\overline{\sigma}^D) = 0$ implies that

$$I(\overline{\sigma}^D,1) + (1-\kappa)J(0,\overline{\sigma}^D) = -\frac{\kappa}{2}J(\underline{\sigma}^D,\overline{\sigma}^D) \geq 0.$$

Moreover, as $I(\overline{\sigma}^D,1) \leq 0$ and $\kappa \in (0,1)$, it must be the case that $J(0,\overline{\sigma}^D) \geq 0$. It follows that $J(0,\overline{\sigma}^D) - \frac{1}{2}J(\underline{\sigma}^D,\overline{\sigma}^D) \geq 0$. As a consequence, $\frac{\partial \overline{\sigma}^D}{\partial \kappa} \leq 0$ and $\frac{\partial \underline{\sigma}^D}{\partial \kappa} \geq 0$.

C.10 Proof of Lemma 5

We proceed as in the proof of Lemma 4. Therefore, the implicit function theorem applied to the definition of $\widehat{\sigma}^D$ implies that

$$\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}} = -\frac{\Big(1 - G(\widehat{\sigma}^D|\widehat{\sigma}^D)\Big) + (1-\kappa)G(\widehat{\sigma}^D|\widehat{\sigma}^D)}{\theta'(\widehat{\sigma}^D)}.$$

The numerator is positive and we already proved taht the denominator is negative. It follows that

$$\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}} \geq 0.$$

The implicit function theorem applied to the definition of $(\underline{\sigma}^D, \overline{\sigma}^D)$ implies that

$$\begin{aligned}
\frac{\partial \overline{\sigma}^D}{\partial \overline{P}} &= \frac{-(1-G(\overline{\sigma}^D|\overline{\sigma}^D)) - (1-\kappa)G(\overline{\sigma}^D|\overline{\sigma}^D) - \frac{\kappa}{2}(G(\overline{\sigma}^D|\overline{\sigma}^D) - G(\underline{\sigma}^D|\overline{\sigma}^D))}{I_1(\overline{\sigma}^D,1) + (1-\kappa)J_1(0,\overline{\sigma}^D) + \frac{\kappa}{2}J_2(\underline{\sigma}^D,\overline{\sigma}^D) - \frac{\kappa}{2}J_1(\underline{\sigma}^D,\overline{\sigma}^D)\frac{I_2(\underline{\sigma}^D,\overline{\sigma}^D)}{I_1(\underline{\sigma}^D,\overline{\sigma}^D)}} \\
&+ \frac{\frac{\kappa}{2}\frac{J_1(\underline{\sigma}^D,\overline{\sigma}^D)}{I_1(\underline{\sigma}^D,\overline{\sigma}^D)}\left(G(\overline{\sigma}^D|\overline{\sigma}^D) - G(\underline{\sigma}^D|\overline{\sigma}^D)\right)}{I_1(\overline{\sigma}^D,1) + (1-\kappa)J_1(0,\overline{\sigma}^D) + \frac{\kappa}{2}J_2(\underline{\sigma}^D,\overline{\sigma}^D) - \frac{\kappa}{2}J_1(\underline{\sigma}^D,\overline{\sigma}^D)\frac{I_2(\underline{\sigma}^D,\overline{\sigma}^D)}{I_1(\underline{\sigma}^D,\overline{\sigma}^D)}} \\
\frac{\partial \underline{\sigma}^D}{\partial \overline{P}} &= -\frac{G(\overline{\sigma}^D|\overline{\sigma}^D) - G(\underline{\sigma}^D|\overline{\sigma}^D)}{I_1\left(\underline{\sigma}^D,\overline{\sigma}^D\right)} - \frac{\partial \overline{\sigma}^D}{\partial \overline{P}}\frac{I_2(\underline{\sigma}^D,\overline{\sigma}^D)}{I_1(\underline{\sigma}^D,\overline{\sigma}^D)}.
\end{aligned}$$

The denominator of $\frac{\partial \overline{\sigma}^D}{\partial \overline{P}}$ is negative as we proved in Lemma 4. The numerator is negative since $J_1(\underline{\sigma}^D,\overline{\sigma}^D) \geq 0$ and $I_1(\underline{\sigma}^D,\overline{\sigma}^D) \leq 0$. It follows that

$$\frac{\partial \overline{\sigma}^D}{\partial \overline{P}} \geq 0.$$

□

C.11 Proof of Lemma 6

Remember that

- $\widehat{\sigma}^P$ is such that $\psi(\widehat{\sigma}^P) = 0$ where ψ is defined in Equation (28),
- $\widehat{\sigma}^D$ is such that $\theta(\widehat{\sigma}^D) = \psi(\widehat{\sigma}^D) + (1-\kappa)\phi(\widehat{\sigma}^D) = 0$ where ϕ and θ are defined in Equations (31) and (32),
- $\underline{\sigma}^D$ and $\overline{\sigma}^D$ are the solution of the system $I(\underline{\sigma}^D, \underline{\sigma}^D) = 0$ and $H(\underline{\sigma}^D, \underline{\sigma}^D) = 0$ where I and J are defined in Equations (36) and (37).

Assume first that $\widehat{\sigma}^D \leq \widetilde{\sigma}$.
We already noted (see the proof of Lemma 3) that $\psi(\widehat{\sigma}^D) < 0$. In addition, we also proved in Lemma 1 that $\psi(x) > 0 \Leftrightarrow x < \widehat{\sigma}^P$. As $\psi(\widehat{\sigma}^P) = 0$, this implies that $\widehat{\sigma}^P \leq \widehat{\sigma}^D$.

Assume now that $\widehat{\sigma}^D > \widetilde{\sigma}$.
$\psi(\underline{\sigma}^D) = I(\underline{\sigma}^D, 1) < I(\underline{\sigma}^D, \underline{\sigma}^D) = 0$. The same reasoning implies that $\underline{\sigma}^P \leq \widehat{\sigma}^D$.

C.12 Proof of Proposition 5

Note first that $P^P(\widehat{\sigma}^P) = \overline{P} > P^D(\widehat{\sigma}^P)$. Second, imagine that the two functions P^P and P^D cross at some point such that $P^P(s) = P^D(s)$ at this point. Using the differential equations satisfied by the two premiums (Equations (10) and (20)), we have that $P^{P'}(s) > P^{D'}(s)$. Therefore, if the two functions cross, it happens only once. If $P^D(0) \leq P^P(0)$, the two curves never crossed.

To prove that the boundary s^P is larger than $\widehat{\sigma}^D$, let us first remark that Lemma 2 could easily apply to bidding strategy of the discriminatory auction. This implies that $P^D(s) - p(s,t) > 0$, $\forall s < t < \widehat{\sigma}^D$, so that if $\overline{s}^D$ is defined by $P^D(s) = p(s, \overline{s}^D)$, it holds that $\overline{s}^D > \widehat{\sigma}^D$. Therefore, if $P^P(s) > P^D(s)$, then $\overline{s}^P(s) > \overline{s}^D(s) > \widehat{\sigma}^D > \widehat{\sigma}^P$. As a result, when $s \le \widehat{\sigma}^P$, the pool offers full coverage for a larger set of the follower's signals than the discriminatory auction. □

D Analysis of the example

As an illustration and to obtain explicit results, we consider the case in which the two insurance companies receive independent signals that are distributed according to a uniform distribution on $[0,1]$. The cost function is moreover assumed to be linear in the two signals

$$p(s_i, s_j) = \frac{s_i + s_j}{2}.$$

In this case, Assumption 2(ii) implies that $\overline{P} \in [1/4, 3/4]$. Note also that $\widetilde{\sigma} = \overline{P}$.

With this specification, the pool is characterized by a threshold $\sigma^P = \frac{4\overline{P}-1}{3}$. The bidding strategy equals

$$P^P(s) = \frac{1+\kappa s}{1+\kappa} - \frac{(3-\kappa)(1-\overline{P})}{3(1+\kappa)} \left(\frac{4}{3}\right)^{\kappa} \left(\frac{1-\overline{P}}{1-s}\right)^{\kappa}, \forall s \in [0, \sigma^P].$$

Proof. Remember that

$$\left(1 - G\left((\widehat{\sigma}^P | (\widehat{\sigma}^P\right)\right) \mathbb{E}\left[\overline{P} - p(\widehat{\sigma}^P, S_j) | S_j > \widehat{\sigma}^P\right] = 0.$$

With the specification, this reads

$$\int_{\widehat{\sigma}^P}^1 \left(\overline{P} - \frac{\widehat{\sigma}^P + s}{2}\right) ds = 0$$

implying that

$$\widehat{\sigma}^P = \frac{4\overline{P} - 1}{3}.$$

As for the equilibrium bidding strategy, Proposition 1 tells us that

$$P^P(s) = \overline{P}(1 - L(\widehat{\sigma}^P | s)) + \int_s^{\widehat{\sigma}^P} p(x,x) dL(x|s) \ \ \forall \ \ s \le \widehat{\sigma}^P$$

with

$$L(x|s) = 1 - \exp\left(-\kappa \int_s^x \frac{g(\tau|\tau)}{1 - G(\tau|\tau)} d\tau\right)$$

In this example,

$$L(x|s) = 1 - \exp\left(-\kappa \int_s^x \frac{1}{1-\tau} d\tau\right) = 1 - \left(\frac{1-x}{1-s}\right)^\kappa,$$

so that

$$\begin{aligned} P^P(s) &= \overline{P}\left(\frac{1-\widehat{\sigma}^P}{1-s}\right)^\kappa + \frac{\kappa}{(1-s)^\kappa}\int_s^{\widehat{\sigma}^P} \tau(1-\tau)^{\kappa-1} d\tau \\ &= \frac{1+\kappa s}{1+\kappa} - \frac{3-\kappa}{3(1+\kappa)}\left(\frac{4}{3}\right)^\kappa \left(\frac{1}{1-s}\right)^\kappa (1-\overline{P})^{\kappa+1}. \end{aligned}$$

□

As for the discriminatory auction,

$$\kappa^*(\overline{P}) = \max\left(\frac{2\overline{P}-1}{\overline{P}^2}, 0\right).$$

In the case where the equilibrium is separating ($\kappa \geq \kappa^*(\overline{P})$),

$$\widehat{\sigma}^D = \frac{1 + 2\kappa\overline{P} - \sqrt{\left(1+2\kappa\overline{P}\right)^2 - 3\kappa(4\overline{P}-1)}}{3\kappa}.$$

The bidding strategy equals

$$P^D(s) = \frac{\overline{P}(1-\kappa\widehat{\sigma}^D)}{1-\kappa s} + \frac{\kappa(\widehat{\sigma}^D - s)(\widehat{\sigma}^D + s)}{2(1-\kappa s)}, \forall s \in [0, \sigma^D].$$

In the case where the equilibrium is semi-separating ($\kappa < \kappa^*(\overline{P})$),

$$\begin{cases} \overline{\sigma}^D = \dfrac{9 + 2\kappa\overline{P} - 3\sqrt{9 + 11\kappa - 20\kappa\overline{P}(2-\kappa\overline{P})}}{11\kappa} \\ \underline{\sigma}^D = \dfrac{4\overline{P} - \overline{\sigma}^D}{3} \end{cases}$$

$$P^D(s) = \begin{cases} \dfrac{\overline{P}(1-\kappa\underline{\sigma}^D)}{1-\kappa s} + \dfrac{\kappa(\underline{\sigma}^D - s)(\underline{\sigma}^D + s)}{2(1-\kappa s)} & \text{for } s \leq \underline{\sigma}^D \\ \overline{P} & \text{for } \underline{\sigma}^D \leq s \leq \overline{\sigma}^D \end{cases}$$

Proof. From the definition of $\kappa^*(P)$ as

$$\max\left(\frac{\mathbb{E}\left[\overline{P} - p(\tilde{\sigma}, S_j)\right]}{G(\tilde{\sigma}|\tilde{\sigma})\,\mathbb{E}\left[\overline{P} - p(\tilde{\sigma}, S_j)|S_j < \tilde{\sigma}\right]}, 0\right).$$

that is

$$\max\left(\frac{\int_0^1\left(\overline{P}-\frac{\overline{P}+s}{2}\right)ds}{\int_0^{\overline{P}}\left(\overline{P}-\frac{\overline{P}+s}{2}\right)ds},0\right).$$

So that

$$\kappa^*(P)=\max\left(\frac{2\overline{P}-1}{\overline{P}^2},0\right).$$

Let us first focus on the case where $\kappa\geq\kappa^*(\overline{P})$. Observe that the constraint is always satisfied when $\overline{P}\in[1/4,1/2]$. $\widehat{\sigma}^D$ is the solution smaller than $\overline{P}$ such that

$$(1-G(x|x))\,\mathbb{E}\left[\overline{P}-p(x,S_j)|S_j>x\right]+(1-\kappa)\,G(x|x)\,\mathbb{E}\left[\overline{P}-p(x,S_j)|S_j<x\right]=0.$$

This implies that $\widehat{\sigma}^D$ is the solution smaller than $\overline{P}$ of

$$\begin{aligned}\int_0^1\left(\overline{P}-\frac{x+s}{2}\right)ds-\kappa\int_0^x\left(\overline{P}-\frac{x+s}{2}\right)ds &= 0\\ \Leftrightarrow \frac{3}{4}\kappa x^2-\left(\frac{1}{2}+\kappa\overline{P}\right)x+\overline{P}-\frac{1}{4} &= 0. \qquad (41)\end{aligned}$$

This quadratic equation has two solutions, one of them larger than 1, the other belonging to $[0,\overline{P}]$.[18] It follows that

$$\widehat{\sigma}^D=\frac{1+2\kappa\overline{P}-\sqrt{(1+2\kappa\overline{P})^2-3\kappa(4\overline{P}-1)}}{3\kappa}.$$

In the analysis of the general case, we proved that, in the separating equilibrium,

$$P^D(s)=\overline{P}(1-K(\widehat{\sigma}^D|s))+\int_s^{\widehat{\sigma}^D}p(x,x)dK(x|s)\ \ \forall s\leq\widehat{\sigma}^D,$$

where

$$K(x|s)=1-\exp\left(-\int_s^x\frac{\kappa g(\tau|\tau)}{1-\kappa G(\tau|\tau)}d\tau\right).$$

With our specification,

$$\begin{aligned}K(x|s) &= 1-\exp\int_s^x\frac{-\kappa}{1-\kappa\tau}d\tau\\ &= 1-\frac{1-\kappa x}{1-\kappa s},\end{aligned}$$

[18]The discriminant $\Delta^x=\left(\frac{1}{2}+\kappa\overline{P}\right)^2-4\left(\overline{P}-\frac{1}{4}\right)\frac{3}{4}\kappa$ is a deceasing function of $\overline{P}$. When $\overline{P}=\frac{3}{4}$, it is positive so that two real solutions exist. As the highest root is greater than 1 (this is straightforward to prove), it follows that $\widehat{\sigma}^D$ is the smallest root.

so that

$$
\begin{aligned}
P^D(s) &= \overline{P}\left(\frac{1-\kappa\widehat{\sigma}^D}{1-\kappa s}\right)+\int_s^{\widehat{\sigma}^D}\frac{\kappa x}{1-\kappa s}dx \\
&= \overline{P}\left(\frac{1-\kappa\widehat{\sigma}^D}{1-\kappa s}\right)+\frac{\kappa(\widehat{\sigma}^D-s)(\widehat{\sigma}^D+s)}{2(1-\kappa s)}.
\end{aligned}
$$

Second, let us analyze the case of the semi-separating equilibrium. As we underlined in the beginning of the proof, the equilibrium is semi-separating iff $\overline{P}\in[1/2,3/4]$ and $\kappa\geq\frac{2\overline{P}-1}{\overline{P}^2}$. $\underline{\sigma}^P$ and $\overline{\sigma}^P$ are the roots (smaller than $\overline{P}$ for $\underline{\sigma}^P$ and greater than $\overline{P}$ for $\overline{\sigma}^P$) of the following system of equations

$$
\begin{cases}
\left(G\left(z|y\right)-G\left(y|y\right)\right)\mathbb{E}\left[\overline{P}-p(y,S_j)|y<S_j<z\right]=0 \\
\left(1-G\left(z|z\right)\right)\mathbb{E}\left[\overline{P}-p(z,S_j)|S_j>z\right]+\left(1-\frac{\kappa}{2}\right)\left(G\left(z|z\right)-G\left(y|z\right)\right)\mathbb{E}\left[\overline{P}-p(z,S_j)|y<S_j<z\right] \\
+\left(1-\kappa\right)G\left(y|z\right)\mathbb{E}\left[\overline{P}-p(z,S_j)|S_j<y\right]=0.
\end{cases}
$$

In our illustration, this reads

$$
\begin{cases}
\int_y^z\left(\overline{P}-\frac{y+s}{2}\right)ds=0 \\
\int_z^1\left(\overline{P}-\frac{z+s}{2}\right)ds+\left(1-\frac{\kappa}{2}\right)\int_y^z\left(\overline{P}-\frac{z+s}{2}\right)ds+(1-\kappa)\int_0^y\left(\overline{P}-\frac{z+s}{2}\right)ds=0.
\end{cases}
$$

implying that

$$
\begin{cases}
\overline{P}-\frac{y}{2}-\frac{1}{4}(z+y)=0 & \text{(44a)} \\
\overline{P}-\frac{z}{2}-\frac{1}{4}-\frac{\kappa}{2}\left(\left(\overline{P}-\frac{z}{2}\right)(z-y)-\frac{1}{4}(z-y)(z+y)\right)-\kappa\left(\left(\overline{P}-\frac{z}{2}\right)-\frac{y^2}{4}\right)=0. & \text{(44b)}
\end{cases}
$$

By eliminating y thanks to (44a), one gets that

$$
\frac{11}{4}\kappa z^2-\left(\frac{9}{2}+\overline{P}\kappa\right)z+9\left(\overline{P}-\frac{1}{4}\right)-4\kappa\overline{P}=0.
$$

This quadratic equation has two solutions, one greater than 1, the other belonging to $[\overline{P},1]$.[19] It follows that

$$
\overline{\sigma}^D=\frac{9+2\overline{P}\kappa-3\sqrt{9+11\kappa-20\kappa\overline{P}(2-\kappa P)}}{11\kappa}.
$$

[19] The discriminant equals $\Delta^z=45\kappa^2\overline{P}^2-90\kappa\overline{P}+\frac{81+99\kappa}{4}$. It is a decreasing function of $\overline{P}$ and is positive for $\overline{P}=\frac{3}{4}$ so that it is positive for all values of $\overline{P}$. To check that the smallest root is smaller than 1, it is necessary to study the function $\overline{P}\mapsto 16\kappa^2\overline{P}^2-4\kappa(9-\kappa)\overline{P}+\kappa(27-11\kappa)$. This is a decreasing function, taking positive values for $\overline{P}=3/4$. It is straightforward to prove that the smallest root is greater than $\overline{P}$.

Using (44a),

$$\underline{\sigma}^D = \frac{4\overline{P} - \overline{\sigma}^D}{3}.$$

Using the computations done for the separating equilibrium, it follows that

$$P^D(s) = \begin{cases} \dfrac{\overline{P}(1-\kappa\underline{\sigma}^D)}{1-\kappa s} + \dfrac{\kappa(\underline{\sigma}^D - s)(\underline{\sigma}^D + s)}{2(1-\kappa s)} & \text{for } s \leq \underline{\sigma}^D \\ \overline{P} & \text{for } \underline{\sigma}^D \leq s \leq \overline{\sigma}^D \end{cases}$$

□

In order to continue the analysis of this example, let us focus on the parameters' values such that the separating equilibrium exist in the discriminatory auction. This implies that we restrict κ to be greater than $\kappa^*(\overline{P})$. Observe that when $\overline{P} \in [1/4, 1/2]$, the equilibrium is separating for any value of κ ($\kappa^*(\overline{P}) = 0$).

Proposition 6 *If $\kappa \geq \kappa^*(\overline{P})$, there exists a unique $\widehat{\overline{P}}$ such that*

- *if $\overline{P} < \widehat{\overline{P}}$, $P^P(s) > P^D(s)$, $\forall s \in [0,1]$*
- *if $\overline{P} \geq \widehat{\overline{P}}$, $P^P(s) < P^D(s)$ and then $P^P(s) > P^D(s)$, when s increases from 0 to 1.*

Proof. We have proven in the general case that P^P and P^D cross at most once and this happens when $P^D(0) > P^P(0)$. The objective of this proof is therefore to determine the sign of $P^P(0) - P^D(0)$. We decide to analyze this difference as a function of $\overline{P}$. Highlighting the dependence with respect to $\overline{P}$, we compute the following

$$\begin{aligned} \frac{\partial P^P(0;\overline{P})}{\partial \overline{P}} &= \frac{(3-\kappa)(\kappa+1)}{3(1+\kappa)} \left(\frac{4}{3}\right)^{\kappa} (1-\overline{P})^{\kappa} > 0 \\ \frac{\partial^2 P^P(0;\overline{P})}{\partial \overline{P}^2} &= -\frac{(3-\kappa)\kappa}{3} \left(\frac{4}{3}\right)^{\kappa} (1-\overline{P})^{\kappa-1} < 0 \\ \frac{\partial^3 P^P(0;\overline{P})}{\partial \overline{P}^3} &= -\frac{(3-\kappa)\kappa(1-\kappa)}{3} \left(\frac{4}{3}\right)^{\kappa} (1-\overline{P})^{\kappa-2} < 0 \end{aligned}$$

$\overline{P} \mapsto \frac{\partial^2 P^P(0;\overline{P})}{\partial \overline{P}^2}$ is therefore a decreasing function. As

$$\frac{\partial^2 P^P(0;\frac{1}{4})}{\partial \overline{P}^2} = -\frac{(3-\kappa)\kappa}{3}\frac{4}{3} < 0,$$

this implies that $\frac{\partial^2 P^P(0;\overline{P})}{\partial \overline{P}^2}$ is negative $\forall \overline{P}$. The same analysis is conducted for P^D

$$\begin{aligned}
\frac{\partial P^D(0;\overline{P})}{\partial \overline{P}} &= 1-\kappa\widehat{\sigma}^D - \kappa\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}}\left(\overline{P}-\widehat{\sigma}^D\right) \\
\frac{\partial^2 P^D(0;\overline{P})}{\partial \overline{P}^2} &= -2\kappa\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}} + \kappa\left(\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}}\right)^2 - \kappa\left(\overline{P}-\widehat{\sigma}^D\right)\frac{\partial^2 \widehat{\sigma}^D}{\partial \overline{P}^2} \\
\frac{\partial^3 P^D(0;\overline{P})}{\partial \overline{P}^3} &= 3\kappa\frac{\partial^2 \widehat{\sigma}^D}{\partial \overline{P}^2}\left(\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}}-1\right) - \kappa\left(\overline{P}-\widehat{\sigma}^D\right)\frac{\partial^3 \widehat{\sigma}^D}{\partial \overline{P}^3}
\end{aligned}$$

In order to compute the partial derivatives of $\widehat{\sigma}^D$, we apply the implicit function theorem to (41).

$$\begin{aligned}
\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}} &= \frac{2\left(1-\kappa\widehat{\sigma}^D\right)}{1+2\kappa\overline{P}-3\kappa\widehat{\sigma}^D} > 0 \\
\frac{\partial^2 \widehat{\sigma}^D}{\partial \overline{P}^2} &= \frac{\partial \widehat{\sigma}^D}{\partial \overline{P}}\kappa\frac{3\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}}-4}{1+2\kappa\overline{P}-3\kappa\widehat{\sigma}^D} \\
&= \frac{\partial \widehat{\sigma}^D}{\partial \overline{P}}2\kappa\frac{1+3\kappa\widehat{\sigma}^D-4\kappa\overline{P}}{\left(1+2\kappa\overline{P}-3\kappa\widehat{\sigma}^D\right)^2} > 0 \\
\frac{\partial^3 \widehat{\sigma}^D}{\partial \overline{P}^3} &= \frac{\partial^2 \widehat{\sigma}^D}{\partial \overline{P}^2}\kappa\frac{-6+9\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}}}{1+2\kappa\overline{P}-3\kappa\widehat{\sigma}^D} \\
&= \frac{\partial^2 \widehat{\sigma}^D}{\partial \overline{P}^2}\kappa\frac{12\left(1-\kappa\overline{P}\right)}{\left(1+2\kappa\overline{P}-3\kappa\widehat{\sigma}^D\right)^2} > 0.
\end{aligned}$$

It follows that

$$\frac{\partial^3 P^D(0;\overline{P})}{\partial \overline{P}^3} = \frac{\partial^2 \widehat{\sigma}^D}{\partial \overline{P}^2}\frac{3\kappa\left(1-4\kappa\overline{P}+4\kappa^2\overline{P}\widehat{\sigma}^D+2\kappa\widehat{\sigma}^D-3\kappa^2\left(\widehat{\sigma}^D\right)^2\right)}{\left(1+2\kappa\overline{P}-3\kappa\widehat{\sigma}^D\right)^2} > 0.$$

$\overline{P} \mapsto \frac{\partial^2 P^D(0;\overline{P})}{\partial \overline{P}^2}$ is therefore an increasing function. As

$$\frac{\partial^2 P^D(0;\frac{1}{4})}{\partial \overline{P}^2} = \frac{\kappa\left((2\kappa-4)^2+8\kappa\right)}{(2+\kappa)^3} > 0,$$

this implies that $\frac{\partial^2 P^D(0;\overline{P})}{\partial \overline{P}^2}$ is positive $\forall \overline{P}$.

This analysis allows us to conclude that $\overline{P} \mapsto \frac{\partial P^P(0;\overline{P})}{\partial \overline{P}} - \frac{\partial P^D(0;\overline{P})}{\partial \overline{P}}$ is a decreasing function.

$$\begin{aligned}
\frac{\partial P^P(0;\frac{1}{4})}{\partial \overline{P}} - \frac{\partial P^D(0;\frac{1}{4})}{\partial \overline{P}} &= \frac{\kappa(1-\kappa)}{3(2+\kappa)} \\
\frac{\partial P^P(0;\frac{3}{4})}{\partial \overline{P}} - \frac{\partial P^D(0;\frac{3}{4})}{\partial \overline{P}} &= \frac{3(3\kappa-2)(3-\kappa)-2\times 4^\kappa}{9\times 4^\kappa(3\kappa-2)} < 0.
\end{aligned}$$

This concludes the proof. □

It follows that $\overline{P} \mapsto P^P(0;\overline{P}) - P^D(0;\overline{P})$ is increasing and then decreasing as $\overline{P}$ increases.

$$\begin{aligned} P^P(0;\frac{1}{4}) - P^D(0;\frac{1}{4}) &= 0 \\ P^P(0;\frac{1}{4}) - P^D(0;\frac{3}{4}) &= \frac{3^\kappa\left(19\kappa - 9\kappa^2 - 8 - 3\kappa(3-\kappa)\right)}{(1+\kappa)36\kappa 3^\kappa} < 0 \end{aligned}$$

Therefore there exists a unique $\hat{\overline{P}}$ such that

- for all $\overline{P} < \hat{\overline{P}}$, $P^P(0;\overline{P}) > P^D(0;\overline{P})$ and
- for all $\overline{P} > \hat{\overline{P}}$, $P^P(0;\overline{P}) < P^D(0;\overline{P})$.

Let us now compute the expected profit in both organizations using expressions (7), (16) and (21). In the pool, computations imply that

$$\begin{aligned} \Pi^P &= \int_0^{\widehat{\sigma}^P} \pi^P(s)ds + \int_{\widehat{\sigma}^P}^{\overline{s}^P(\widehat{\sigma}^P)} \pi^P(s)ds + \int_{\overline{s}^P(\widehat{\sigma}^P)}^1 \pi^P(s)ds \\ &= \int_0^{\widehat{\sigma}^P} \frac{1-s}{2-\kappa}\left(\frac{1+\kappa s}{1+\kappa} - \frac{(3-\kappa)(1-\overline{P})^{\kappa+1}}{3(1+\kappa)(1-\kappa)}\left(\frac{4}{3(1-s)}\right)^\kappa - \frac{1+3s}{4}\right)ds \\ &+ \int_0^{\widehat{\sigma}^P} \frac{1-\kappa}{2-\kappa}\left(\frac{s(4-s(3+\kappa))}{4(1+\kappa)} - \frac{(3-\kappa)(1-\overline{P})^{\kappa+1}}{3(1+\kappa)(1-\kappa)}\left(\frac{4}{3}\right)^\kappa\left(1-(1-s)^{1-\kappa}\right)\right)ds \\ &+ \int_{\widehat{\sigma}^P}^{\overline{s}^P(\widehat{\sigma}^P)} \frac{1-\kappa}{2-\kappa}\frac{\widehat{\sigma}^P\left(4-(1-\kappa)\widehat{\sigma}^P - 2s(1+\kappa)\right)}{4(1+\kappa)}ds \\ &- \int_{\widehat{\sigma}^P}^{\overline{s}^P(\widehat{\sigma}^P)} \frac{1-\kappa}{2-\kappa}\frac{(3-\kappa)(1-\overline{P})^{\kappa+1}}{3(1+\kappa)(1-\kappa)}\left(\frac{4}{3}\right)^\kappa\left(1-(1-\widehat{\sigma}^P)^{1-\kappa}\right)ds \\ &+ \int_{\overline{s}^P(\widehat{\sigma}^P)}^1 \frac{1-\kappa}{2-\kappa}\frac{(\overline{s}^P)^{-1}(s)\left(4-(1-\kappa)(\overline{s}^P)^{-1}(s) - 2s(1+\kappa)\right)}{4(1+\kappa)}ds \\ &- \int_{\overline{s}^P(\widehat{\sigma}^P)}^1 \frac{1-\kappa}{2-\kappa}\frac{(3-\kappa)(1-\overline{P})^{\kappa+1}}{3(1+\kappa)(1-\kappa)}\left(\frac{4}{3}\right)^\kappa\left(1-(1-(\overline{s}^P)^{-1}(s))^{1-\kappa}\right)ds. \end{aligned}$$

As for the discriminatory auction, we have to distinguish the case where $\kappa \geq \kappa^*(\overline{P})$ or $\kappa < \kappa^*(\overline{P})$. In case the separating equilibrium exists ($\kappa \geq \kappa^*(\overline{P})$),

$$\Pi^D = \int_0^{\widehat{\sigma}^D}\left(\frac{2\overline{P}(1-\kappa\widehat{\sigma}^D) + \kappa(\widehat{\sigma}^D - s)(\widehat{\sigma}^D + s) - s(1-\kappa s)}{2(2-\kappa)} - \frac{1-\kappa s^2}{4(2-\kappa)}\right)ds.$$

When $\kappa < \kappa^*(\overline{P})$,

$$\Pi^D = \int_0^{\underline{\sigma}^D} \left(\frac{2\overline{P}(1-\kappa\underline{\sigma}^D) + \kappa(\widehat{\sigma}^D - s)(\underline{\sigma}^D + s) - s(1-\kappa s)}{2(2-\kappa)} - \frac{1-\kappa s^2}{4(2-\kappa)} \right) ds$$
$$+ \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\frac{2\overline{P} - s}{4(2-\kappa)} \left(2 - \kappa\overline{\sigma}^D - \kappa\underline{\sigma}^D\right) - \frac{1}{8(2-\kappa)} \left(2 - \kappa(\overline{\sigma}^D)^2 - \kappa(\underline{\sigma}^D)^2\right) \right) ds.$$

As for the expected coverage denoted $\mathcal{C}$, it holds that

$$\mathcal{C}^P = \frac{2}{2-\kappa}\widehat{\sigma}^P + \frac{2(1-\kappa)}{2-\kappa} \int_0^{\widehat{\sigma}^P} \overline{s}^P(x)dx - (\widehat{\sigma}^P)^2.$$

As for the discriminatory auction, if $\kappa \geq \kappa^*(\overline{P})$, then

$$\mathcal{C}^D = (\widehat{\sigma}^D)^2 - \frac{2}{2-\kappa}(1-\widehat{\sigma}^D)\widehat{\sigma}^D.$$

When $\kappa < \kappa^*(\overline{P})$, then

$$\mathcal{C}^D = (\overline{\sigma}^D)^2 - \frac{2}{2-\kappa}(1-\overline{\sigma}^D)\overline{\sigma}^D.$$

To finish the analysis this example, let us analyze how $\min(\widehat{\sigma}^D, \overline{\sigma}^D) - \widehat{\sigma}^P$ evolves with respect to $\overline{P}$ and κ. Let us first focus on the comparative statics with respect to κ. As $\widehat{\sigma}^P$ is independent of κ and as we proved in Lemma 4 that both $\widehat{\sigma}^D$ and $\overline{\sigma}^D$ are decreasing with κ, this implies that $\min(\widehat{\sigma}^D, \overline{\sigma}^D) - \widehat{\sigma}^P$ decreases when κ increases.

To determine the comparative statics with respect to $\overline{P}$, observe that $\widehat{\sigma}^P$ is linear with respect to $\overline{P}$ and that we proved that $\frac{\partial^2 \widehat{\sigma}^D}{\partial \overline{P}^2} > 0$. Therefore, $\overline{P} \mapsto \frac{\partial \widehat{\sigma}^D}{\partial \overline{P}} - \frac{\partial \widehat{\sigma}^P}{\partial \overline{P}}$ is increasing. Moreover,

$$\left.\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}}\right|_{\overline{P}=\frac{1}{4}} = \frac{4}{2-\kappa}$$
$$\left.\frac{\partial \widehat{\sigma}^P}{\partial \overline{P}}\right|_{\overline{P}=\frac{1}{4}} = \frac{3}{4}.$$

It follows that $\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}} - \frac{\partial \widehat{\sigma}^P}{\partial \overline{P}}$ is positive for all values of $\overline{P}$ and that $\overline{P} \mapsto \widehat{\sigma}^D - \widehat{\sigma}^P$ is an increasing function. Let us now focus on $\overline{\sigma}^D - \widehat{\sigma}^P$.

$$\frac{\partial \overline{\sigma}^D}{\partial \overline{P}} = \frac{2}{11} + \frac{60}{11} \frac{1-\kappa\overline{P}}{\sqrt{9 + 11\kappa - 20\kappa\overline{P}(2-\kappa\overline{P})}}$$
$$\frac{\partial^2 \overline{\sigma}^D}{\partial \overline{P}^2} = \frac{11\kappa(1-\kappa)}{(\sqrt{9 + 11\kappa - 20\kappa\overline{P}(2-\kappa\overline{P})})^3} > 0.$$

Therefore, $\overline{P} \mapsto \frac{\partial \overline{\sigma}^D}{\partial \overline{P}} - \frac{\partial \widehat{\sigma}^P}{\partial \overline{P}}$ is increasing. Moreover,

$$\left.\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}}\right|_{\overline{P}=\frac{1}{2}} = 2.$$

As $\frac{\partial \widehat{\sigma}^P}{\partial \overline{P}} = \frac{3}{4}$, it follows that $\frac{\partial \overline{\sigma}^D}{\partial \overline{P}} - \frac{\partial \overline{\sigma}^P}{\partial \overline{P}}$ is positive for all values of $\overline{P}$ and that $\overline{P} \mapsto \overline{\sigma}^D - \widehat{\sigma}^P$ is an increasing function.

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