Reevaluation of the capital charge in insurance after a large shock: empirical and theoretical views*

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Abstract. Motivated by the recent introduction of regulatory stress tests in the Solvency II framework, we study the impact of the reestimation of the tail risk and of loss absorbing capacities on post-stress solvency ratios. Our contribution is threefold: we build the first stylized model for re-estimated solvency ratio in insurance. This leads us to solve a new theoretical problem in statistics: what is the asymptotic impact of a record on the reestimation of tail quantiles and tail probabilities for classical extreme value estimators? Eventually, we quantify the impact of the reestimation of tail quantiles and of loss absorbing capacities on real-world solvency ratios thanks to regulator data from *Banque de France - ACPR*. In particular, we shed a first light on the role of the loss absorbing capacity and its paramount importance in the Solvency II capital charge computations. Our paper ends with policy recommendations for insurance regulators.

Keywords. Insurance, Extreme Value Theory, Financial Regulation, Solvency II, Solvency Capital Requirement, Loss Absorbing Capacities, Stress Tests, Enterprise Risk Management.

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1 Introduction

Modern financial regulation frameworks are designed to take into account the actual risks faced by financial institutions. This precision in evaluating the risks comes at a cost since improving accuracy tends to be pro-cyclical. As a response to the potential increase of systemic risk, stress tests have increasingly become a common tool for insurance and banking supervision. In a nutshell, supervisors check the consequences of adverse shocks on the solvency, liquidity and stability assessment of undertakings. Since Basel I, financial regulation is based on the assessment of capital requirement and its coverage by undertakings. In this respect, undertakings would typically undergo assets and own fund downfall after the simulation of the shock. Some companies pass the test and still hold enough capital after the stress test while some others do not.

This type of financial stability tests are suited for supervision, on the one hand it helps monitor financial stability on the basis of a horizontal and cross sectional analysis of individual responses and on the other hand it can include a forward looking perspective. Moreover, some supervisors almost only rely on the outcome of such exercises. Even if the use of such tests is more recent in the insurance sector than the banking sector, they become more and more on top of the agenda, see for example NAIC and EIOPA's recommendations arising after such exercises (eg. European Insurance and Occupational Pensions Authority (2014)). Different aspects of stress tests exercises need to be clarified: why stress testing? How should such exercises be organized to optimize supervision efficiency? How should scenario be set-up and at which (quantile) level? How should the framework of the exercises be built up, e.g. which simplifying assumptions should be utilized?

In this study we only focus on the latter aspect with a glimpse on the European insurance stress test since those exercises are part of the more general Solvency II regulatory framework which has become fully applicable since January 2016. Since the CEIOPS quantitative impact studies led in 2011, a consensus emerged in the European Union insurance supervisory community: the absence of Solvency Capital Requirement (SCR) reassessment after shock was regarded as a prudent hypothesis. Indeed, it is often believed that the SCR is very likely to be smaller after the stress test is applied than initially, for example after an adverse shock leading to a decrease in the market value of the portfolio and that keeping the SCR constant corresponds to a cautious strategy.

This rationale seems natural when looking at a shock on stock values: if the stock values are decreased by 40%, say, then applying a second downward 40% shock only corresponds to a 24%-decrease w.r.t. the initial stock value. Besides, some countercyclical measures like the equity dampener[1] may reinforce this phenomenon.

[1]For more explanations on how the equity dampener is set up, see the consulation paper CP-14-058 on "the proposal for draft Implementing Technical Standards on the equity index for the symmetric adjustment of the

However, as far as natural or man made catastrophe in P&C risks ("Cat P&C risks") are concerned, if some extreme scenario occurs, then it is likely that the tail distribution of the corresponding risk is re-evaluated: a 1-every-150 years scenario, if it occurs, can be seen as a 90-year scenario after the event, when the probability distribution is updated, as observed empirically by Mornet et al. (2016) for storm risk in France. This may of course lead to an increase in the SCR.

In addition, the loss absorbing capacities generated by deferred tax or technical provisions is not infinite. After a large event, this capacity may be strongly reduced, which would lead to an increase in the SCR, when recomputed.
In this paper, we aim at explaining these opposite effects and quantify their combined impacts on the SCR in a simplified model and also with regulatory data. Our contribution is threefold: we build the first stylized model for re-estimated solvency ratio in insurance. This leads us to solve a new theoretical problem in statistics: what is the asymptotic impact of a record on the reestimation of tail quantiles and tail probabilities for classical extreme value estimators? Eventually, we quantify the impact of the reestimation of tail quantiles and of loss absorbing capacity on real-world solvency ratios thanks to regulator data from $ACPR$featuring cases where re-computing leads to an increase in the SCR. Another striking outcome of our study is the importance of loss-aborbing capacity on solvency capital ratios.

Our paper is organized as follows. In Section 2, we explain how Solvency Capital Requirement (SCR) is computed in Solvency II. In particular, we describe regulatory stress tests and loss absorbing capacity mechanisms. In Section 3, we present our simplified model for SCR re-estimation. Section 4 quantifies the asymptotic underestimation when one neglects a record with a theoretical extreme value analysis point of view. In Section 5, we provide order of magnitude of the different effects using French stress test data (relevant for the whole European Union). In the conclusion, we give some policy implications and we introduce some future research questions.

equity capital chargehttps://eiopa.europa.eu/Publications/Consultations/EIOPA-CP-14-058_ITS_Equity_dampener.pdf

2 Solvency capital, stress tests and loss absorbing capacity in Solvency II

2.1 Prudential balance sheet of European insurers

In the insurance sector, estimating liabilities can be very tricky since no actual market value exists for in-force businesses. Generally, only model-based valuations are available: producing the balance sheet of an insurer is already a difficult task for life insurers, involving simulations. Technical provisions in the Solvency II framework (EU Parliament and Council (2009)) consist in an actualizing of the projection of cash flows made by the undertaking. The calculation methodologies of the best estimate is defined in the Article 28 of the Delegated RegulationCommission (2015) and is completed in the EIOPA guidelines on Technical Provisions (European Insurance and Occupational Pensions Authority (2015)). The overall shape of the balance sheet is given by 1

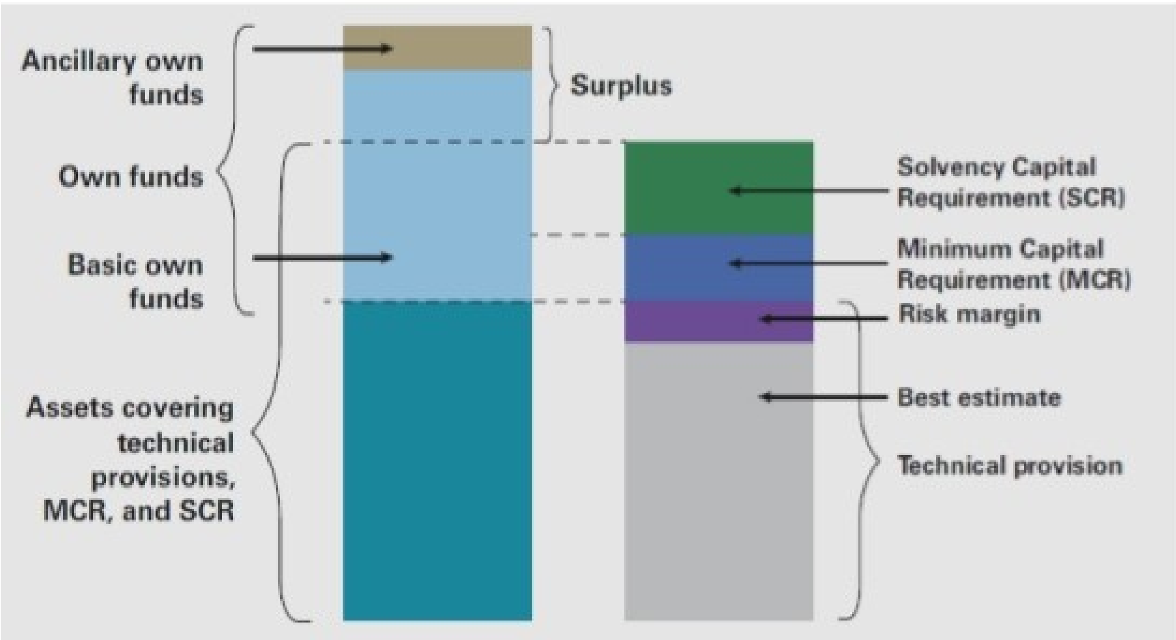

Figure 1: Insurer simplified Balance Sheet (Solvency II) (source: UK actuaries)

Solvency II directive defines the Solvency Capital Requirement ("SCR") as the aggregation of different risk modules. These are besides defined by sub-modules or even sub-sub-modules (see 2).

For each sub-module (or sub-sub-module), a set of risk factors is considered and there exist two cases. Either a formula is used - e.g. the premium risk of the non-life underwriting risk sub-module or the result of a mono factor "stress test". Alternative to formulas for

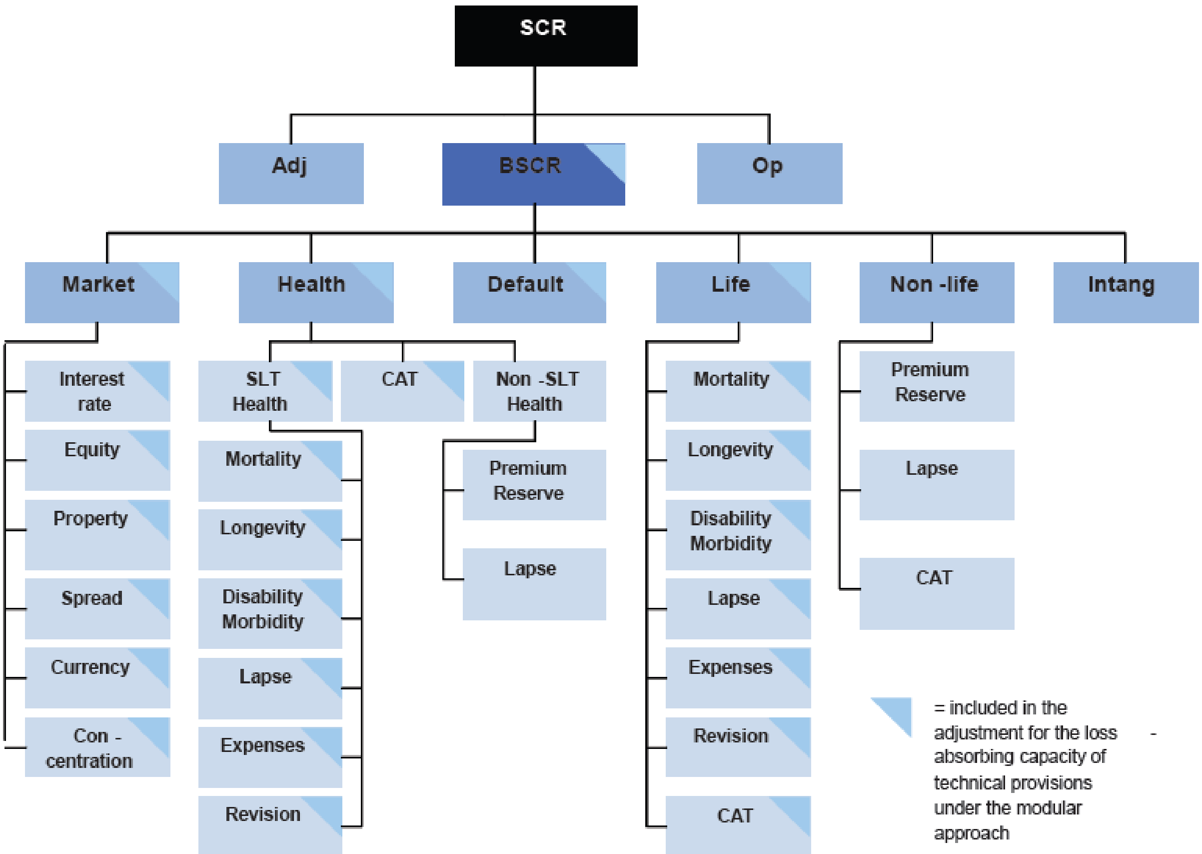

Figure 2: SCR: risk modules breakdown (Source: EIOPA)

sub-(sub-)module is the VaR determination. This alternative method consists in stressing a parameter (interest rates, stock indices, mortality tables, etc.), up to a 99.5% quantile level, and then compute the net-asset value ("NAV") to infer the value of the gross and net sub-module depending on whether the different diversifications **(see "loss-absorbing capacities" after)** are taken into account. The $\Delta-$NAV is actually the difference of Basic Own Funds between the stressed and baseline situations (see 3)

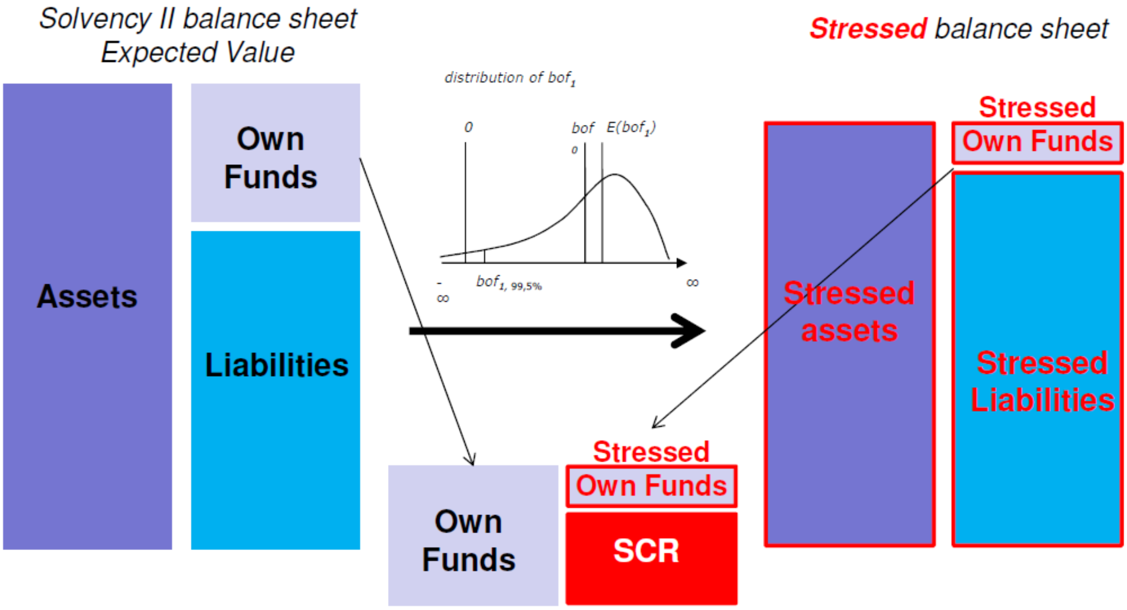

Figure 3: SCR risk sub-modules calculation (Source: ACPR)

In 2014, EIOPA ("European Insurance and Occupational Pensions Authority") led a pan-european insurance stress test. This exercises was composed of a core exercise applied to 167 insurance groups of the EU market[2] which included the 30 largest companies in Europe. Baseline figures revealed that life technical provisions are predominant within this scope. As a consequence, market risk is actually the most important module in the aggregated SCR (see 5 and 4). For this reason and to simplify the calculations, in the following sections we will therefore assume the insurance company only depends on a one dimension risk.

2.2 Loss-absorbing capacities

Before the launch of Solvency II, CEIOPS[3] was responsible for determining which risk-measure should be best suited to insurance industry[4]. Different approaches were tested for

[2]This represents 55% of all gross written premiums. NCA's were allowed to add solo undertakings when unable to reach the 50% threshold with only the groups acting domestically.

[3]Committee of European Insurance and Occupational Pensions Supervisors, the predecessor of the European authority for insurance supervision, "EIOPA"

[4]The results of this analysis, called "QIS" for Quantitative impact studies can be seen on EIOPA website: https://eiopa.europa.eu/publications/qis

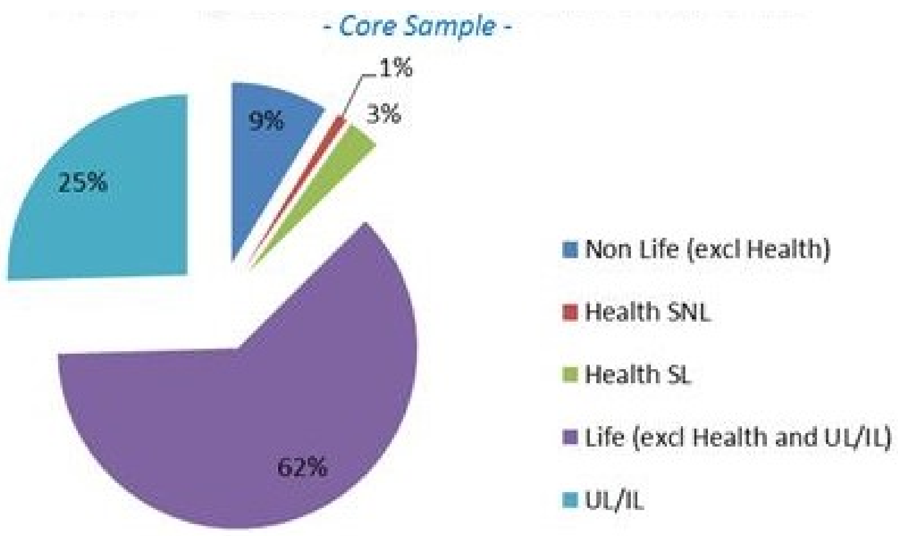

Figure 4: Technical provisions breakdown (source: EIOPA Stress Test 2014)

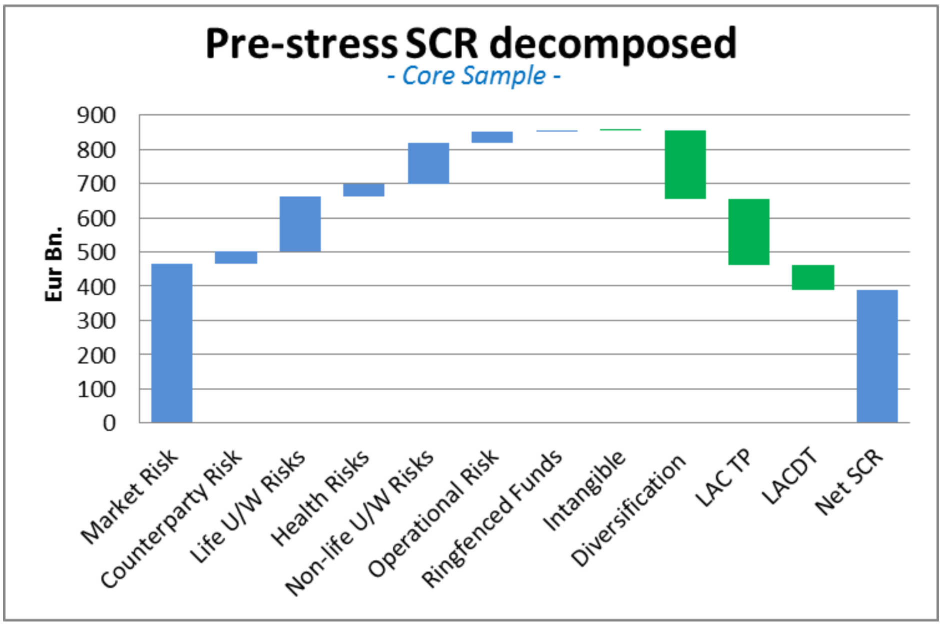

Figure 5: SCR Decomposition (source: EIOPA Stress Test 2014)

the liability valuation[5] and already at this level the impact of the future bonuses seemed to be material. Insurance industry is characterized by risk mitigation and so, Solvency II, being risk based, had to take this feature into account unlike Solvency I, which was based on fixed/all-inclusive calculations. In this regard, CEIOPS progressively introduced the concept of "loss-absorbing capacity": at first in the QIS 2's specifications one could find the "risk absorbing proportion of $TP_{Benefits}$" or the "risk absorption" property of the future profit sharing only related to the discretionary nature of profit-sharing in almost all jurisdiction:

$$RPS = k \cdot TP_{Benefits},$$

assuming a linear relation between the Reduction for Profit-Sharing (RPS)and the technical provisions which relates to the future discretionary profits, and k was the risk-absorbing proportion of those technical provisions. QIS 3 was only mentionning the "loss-absorbing capacity" for the purpose of the valuation of contingent capital but confirmed the key role played by future bonuses granting those mechanisms some "risk absorption" abilities or properties. QIS2's linear relation was still mentionned but a more complex mechanism, called "three step approach" was introduced: for each risk sub-module two calculations should be performed: a net SCR module, noted $nSCR_{Mod}$ and a gross one, noted $gSCR_{Mod}$. The difference, noted KC_{Mod} between those two quantities is the "risk absorption ability" at the risk module level:

$$KC_{Mod} = gSCR_{Mod} - nSCR_{Mod}.$$

Then those KC's needed to be aggregated at the level of the five largest risk modules (Life module, Non-Life, Market, Counterparty and Health) with the same correlation matrices of the sub-risk-modules so as to produce KC_{Life}, KC_{nl}, KC_{Mkt} and so on. In a final step, those coefficients were eventually aggregated with the same correlation matrices as their risk-module counterparts. With this approach, the loss absorbing capacities were not assumed to be directly comparable to a specific balance-sheet element such as the with profits technical provisions. As a consequence, this modular calculation made it unpredictable to any movement in the balance-sheet, were it on the liability or asset side. QIS 4's specifications only refined this approach by defining more precisely what a "loss-absorbing capacities" were, whether it be linked to an asset or a liability element, insisting on the role played by deferred tax (LAC_{DT}) and absorbing capacities by the technical provisions ("LAC_{TP}"). Finally, the Solvency II directive gave legal perspective to this concept in its articles 103 & 108; article 111 let the implementing measures gives more details on how to compute them.

As explained in the previous section, the core of the whole prudential balance-sheet is the best estimate. For any sample path of simulation used for the projection of the liabilities entering in the valuation of the best-estimate, an undertaking might gain or lose some risk absorbing ability. As an illustration, in the life business, depending both on the market

[5] best estimate, 75th, 90th percentile, company view, 60th percentile

conditions (interest-rates, stock prices, etc.) and on the level of the minimum guarantees granted to the insured, the undertaking running the best-estimate simulation might gain or lose some leeway with respect to the discretionary bonuses. In the end, any of the SCR sub-module (netto) whose calculation depend on a best estimate calculation will strongly be affected by these technical provisions' absorbing mechanisms. Finally, all those sub-module loss absorbing capacities coming from technical provision or future discretionary benefits are gathered at the level of the SCR to account for global diversification effect.

How does the mitigation actually work? In QIS1 and 2, the risk-reduction mechanisms were initially designed and thought by all the supervisors and regulators as constant elasticities to with profit participations. In the final version of the regulatory texts, those mechanisms are not straightforward especially for the calculation of a modular risk module (scenario based calculations). At first, the insurance company needs to compute the SCR net of all effects, which means that the amount of the risk-mitigation techniques are taken into account in the different Best Estimate evaluations (baseline and module shock) and can change on a sample path basis. Then on a second round one has to evaluate the Gross SCR. For this purpose, all the computations need to be made while assuming only the cash flows coming from the guaranteed benefits are rediscounted when the relevant scenario affect the interest rate term structure. In the gross calculation phase, the cash flows arising from the future discretionary benefits are supposed to be constant.

Considering the Market risk as an example, the lower the value of the assets, the less risky it is in absolute terms. Besides, after a large financial shock one would expect net SCR sub-modules linked to Market risk to decrease when risk exposure decreases so that any SCR reevaluation after a large shock would benefit the undertaking thanks to a proportionality effect.

However, this one-to-one correspondance is not actually observed in the 2014 Stress test data (see European Insurance and Occupational Pensions Authority (2014)): despite that very few undertakings reassessed their SCR post-stress - less than 30%, the reassessment was optional - a significant share (more than 40%) of the undertakings underwent an increase of their global net SCR at least in one of the market scenarios.

Indeed, taking a closer look to Figure 6, we observe that diversification effects can present some non-linearities, maybe due to the "modular" nature of their estimation. A very naive explanation to this counter-intuitive result could be that the post-stress reduction in the diversification abilities would be more significant than the reduction of risk exposure. Another simple idea would be that the addition of an extreme point changed the global shape of the underlying loss distribution.

An interpretation based on both effects are developped in the following sections.

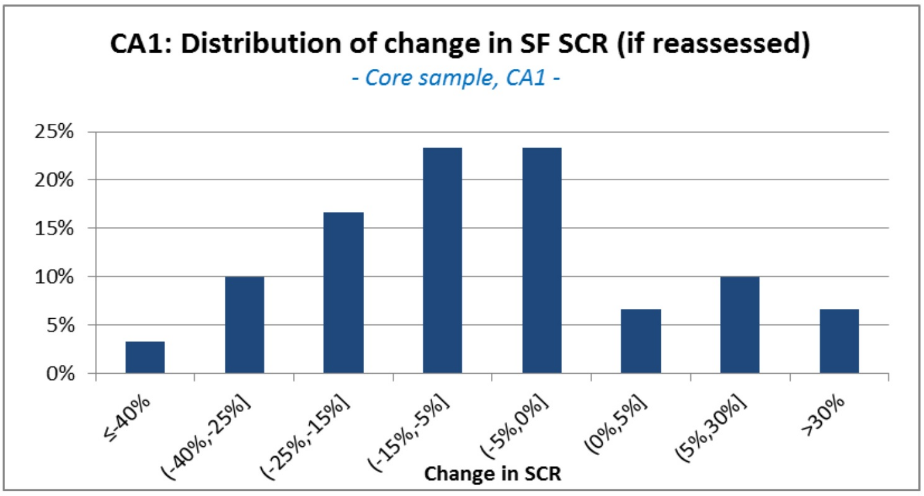

Figure 6: Distribution of reassessed SCR (source: EIOPA Stress Test 2014)

2.3 Stress tests and Solvency II

As explained in the previous part (See 2.1 and 2.2), most of the SCR risk sub-modules for life undertaking are estimated by the mean of stress test on specific risk factor. These are supposed to represent a 1-in-200 year shock. For example, the Interest Rates sub-module is one of the most important as part of the "Market Risk" module. EIOPA used a limited historical data, following 4 datasets to calibrate the different regulatory shocks to apply:

- EUR government zero coupon term structures (1997 to 2009),
- GBP government zero coupon term structures (1979 to 2009),
- and both Euro and GBP LIBOR/swap rates (1997 to 2009).

With this regulatory framework being setup, other risk dimensions or quantiles of different levels are not covered. For this reason, EIOPA can run dedicated exercises complementing the regular Solvency assessment. These exercises allow data collection and analyses to test and measure the resilience and vulnerabilities of the insurance market. In 2014, the EIOPA stress tests run two exercises: a core and a satellite exercise, the first one aiming at testing large groups and the second challenging solos with prolonged low rate environments. In this article we have a closer look at the results of the market shocks of the "Core module". These were two adverse financial market scenarios designed in cooperation with the ESRB which implemented so called "double hit", meaning both an increase in value of the liabilities, in reason of the prolonged low yield environment and a decrease in the assets values for equity but also for sovereigns with a windening of the spreads[6].

[6]For a more detailed view on the scenarios the description of the scenario can be downloaded on the EIOPA website: https://eiopa.europa.eu/Publications/Surveys/Note_on_market_adverse_scenarios_for_the_core_module_in_the_2014_EIOPA_stress_test.pdf

3 A simplified model for post-stress SCR

In this simplified model, we consider that the SCR is given by

$$SCR = [VaR_{99.5\%}(X) - E(X) - b]_+ ,$$

where X is a random variable corresponding to the 1-year random loss the insurer may face. Here, for simplification purposes, we consider only one risk factor, which can be financial or P&C cat. Of course, in the real world, there are many risk factors, aggregated either with the standard formula or by the mean of an internal model. We shall discuss the impact of diversification on our results in the sequel. The parameter b plays an important role: it corresponds to the loss-absorbing capacity, and it is likely to be affected if a large event occurs.

After a shock, b is transformed into b' and X is transformed into $X' = a\tilde{X}$, where a is a factor accounting for the change in the exposure, and $\tilde{X}$ is the revised version of X after taking the last shock into account.

If one considers mass lapse risk or pandemic risk, then the size of portfolio is smaller after the first shock, which corresponds to $a < 1$. Similarly, if stocks go down by 40%, then it is natural to consider $a = 60\% < 1$, even in absence of countercyclical measures. For P&C disasters, the situation is less clear: on the one hand, some buildings might be partly or fully destroyed, which makes the exposure temporarily decrease ($a < 1$) as there is less to be potentially destroyed by a second event. On the other hand, a first event might also cause some frailty and make the consequences of a second event potentially more severe, for example in the case of floodings or earthquakes where some cumulative effect or some replicas may be disastrous ($a > 1$).

If an event like a major, unpreceded earthquake, hurricane or terror attack occurs, then the probability and potential severity of such an event will automatically be re-evaluated by cat models like RMS, EQECAT or AIR or by internal models, following Bayesian techniques. For most events, the impact on high-level Value-at-Risk is very likely to be much more important than the impact on the average. Therefore, we model this as a change from $VaR_{99.5\%}(X)$ to $VaR_{99.5\%}(\tilde{X})$, but for the sake of simplicity we do not update the average, considering that the impact on the average can be neglected: we assume that $E(X) = E(\tilde{X})$.

Of course, this assumption might be inappropriate in some cases, particularly for regime switching models like 3-state Hardy stock model or self-excited processes (in which the best estimate and the volatility tend to move in adverse directions when things go bad) and for mean-reverting models where some mitigation is present when things go bad.
For some other risks like sovereign risk or foreign exchange risk, some shocks may occur as

jumps (CHF/EUR exchange rate in January 2016), which shows that the two types of risks that we consider in this paper (market shocks and large P&C claims) are relevant for our study.

The parameter b, accounting for the loss-absorbing capacity, can be transformed into b' after a large event for several reasons. The loss-absorbing capacity thanks to differed tax and thanks to technical provisions is not infinite, and it may happen that the new loss-absorbing capacity after a large event is much smaller than before, which corresponds to $b' << b$.

Besides, reinsurance might become too costly, reinstatements might be exhausted, or protection from Insurance Linked Securities could be strongly amputed, which would again lead to $b' < b$ for other reasons. Even if we focus here on loss absorbing capacity in the Solvency II framework, the analysis that we develop could be adapted to loss absorbing capacity through risk transfer in a more general Enterprise Risk Management approach.

In opposite, some countercyclical mechanisms like the equity dampener might have a favorable impact by reducing the SCR if a downward shock occurs. This tends to increase the value of b'.

For the sake of simplicity, we have focused on a single risk factor. Nevertheless, due to the complexity of risk aggregation and diversification, a large event might affect the diversification benefit if one risk becomes smaller or larger than other ones. The fact that part of the diversification benefit disappears has of course a negative impact on the SCR (but on the other hand one benefited from mitigation of the initial shock thanks to diversification).

We are in presence of three effects: the ones of a, b as well as the tail quantile reestimation. From a theoretical point of view, the impact of the first two ones is quite straightforward. The tail reestimation effect, however, has not yet been studied in the literature and is a bit more technical. Therefore, in the next section, we quantify the change from $VaR_{99.5\%}(X)$ to $VaR_{99.5\%}(\tilde{X})$ after a record occurs in a P&C framework, in absence of loss absorbing capacity and for $a = 1$. As this is currently not taken into account, we formulate this as the underestimation of high level quantiles when one ignores the record that just occurred.

4 Change in tail estimators after a record: an EVT approach

4.1 Notation and framework

We take a P&C view on the random loss X underlying the SCR calibration. Let $X_1, X_2, \ldots$ be i.i.d. random variables corresponding to observations of X. For simplicity, assume that their common distribution is continuous. Denote the ascending order statistics of $X_1, \ldots, X_n$ by $X_{n:1} < \ldots < X_{n:n}$.

Consider statistics of the type

$$T_n = t_n(X_1, \ldots, X_n),$$

where $t_n : \mathbb{R}^n \to \mathbb{R}$ is a permutation invariant function. Think of T_n as an estimator of some tail-related quantity: a tail quantile, a return level, ... The statistic T_n depends on the data only through the order statistics:

$$T_n = t_n(X_{n:1}, \ldots, X_{n:n}).$$

We want to understand the consequences of not reestimating the risk distribution in a stress test associated to an extreme shock. We focus on the case where the shock is unpreceded: the very recent loss corresponds to a record (like for example the Bar-le-Duc claim in 1976 for motor third party liability or Lothar in 1999 for storm risk in France). In practice, such events might be relevant for different sub-risk-modules of Solvency II (underwriting, cat, ...) and their impact might be diluted with attritional claims during the year. To simplify, we assume here that X corresponds to the random variable whose quantile is used to derive the Solvency Capital Requirement.
We therefore assume that at a given time instant, a record occurs: the new observation is larger than what has been observed before. When should we compute the statistic: right before or right after the record?

First, assume that the record occurs at "time" n, that is, $X_n > X_{n-1:n-1}$, or, in other words, the rank of X_n among $X_1, \ldots, X_n$ is equal to n. At a given sample size, the vector of order statistics is independent of the vector of ranks. We find that

$$[T_n \mid X_n > X_{n-1:n-1}] \stackrel{d}{=} T_n. \tag{4.1}$$

That is, computing the statistic right after a record does not lead to any distortion.

Second, assume that we compute the statistic right before a record occurs. Specifically, suppose that X_{n+1} is a record: $X_{n+1} > X_{n:n}$. How does the occurrence of that event affect the distribution of T_n?

Conditionally on the event that X_{n+1} is a record, the joint distribution of the vector of order statistics $(X_{n:1}, \ldots, X_{n:n})$ is equal to the one of the vector $(X_{n+1:1}, \ldots, X_{n+1:n})$: out of a sample of size $n+1$, we omit the largest variable. Formally,

$$[(X_{n:1}, \ldots, X_{n:n}) \mid X_{n+1} > X_{n:n}] \stackrel{d}{=} (X_{n+1:1}, \ldots, X_{n+1:n}). \tag{4.2}$$

For the uniform distribution on $[0,1]$, identity (4.2) can be proved by Rényi's representation of uniform order statistics. For a general distribution F, identity (4.2) can be proved by applying the quantile function of F to a sample of uniform random variables. (Since F is continuous, its quantile function is strictly increasing.)

Equation (4.2) implies that

$$[T_n \mid X_{n+1} > X_{n:n}] \stackrel{d}{=} t_n(X_{n+1:1}, \ldots, X_{n+1:n}). \tag{4.3}$$

Computing the statistic right before the occurrence of a record has a clear impact on its distribution: compare (4.1) and (4.3).

The size of the effect depends on the function t_n. If T_n is a tail estimator, then the impact of omitting the largest observation could be potentially quite large. We work out two relevant cases for our initial problem in the following subsections.

4.2 Tail probability error estimation

We first investigate the question of tail probability reestimation. After an extreme event, the CEO of an insurance company could ask the cat-modeling team: "What is the return period of yesterday's event?". The cat-modelers could in fact reply: "Well, two days ago I would have answered 200 years (tail probability 1/200), but today I'd rather say 120 years!". One can imagine the reaction of the CEO... This example quantifies the asymptotic change in the tail probability estimation.

Example 1 (Tail probability)**.** Let u be a high level. Aim is to estimate the tail probability $p = 1 - F(u)$. Note that the return level is equal to $1/p$. The simplest possible estimator is the empirical one,

$$T_n = \frac{1}{n} \sum_{i=1}^{n} I(X_i > u).$$

Clearly, the estimator is unbiased:

$$\mathrm{E}[T_n] = p.$$

However, if we ignore the information that at time $n+1$, a new record occurred, then we get

$$\begin{aligned}
\mathrm{E}[T_n \mid X_{n:n} < X_{n+1}] &= \mathrm{E}\left[\frac{1}{n}\sum_{i=1}^{n} I(X_{n+1:i} > u)\right] \\
&= \mathrm{E}\left[\frac{1}{n}\sum_{i=1}^{n+1} I(X_{n+1:i} > u) - \frac{1}{n} I(X_{n+1:n+1} > u)\right] \\
&= \frac{n+1}{n} p - \frac{1}{n}(1-(1-p)^{n+1}).
\end{aligned}$$

The relative error is therefore

$$\frac{1}{p}\,\mathrm{E}[T_n \mid X_{n:n} < X_{n+1}] - 1 = \frac{1}{n} - \frac{1-(1-p)^{n+1}}{np}.$$

If $u = u_n \to \infty$ in such a way that $np = np_n = n\{1 - F(u_n)\} \to \tau \in (0, \infty)$, i.e., if $p \sim \tau/n$, then the relative error is asymptotically

$$\frac{1}{p}\,\mathrm{E}[T_n \mid X_{n:n} < X_{n+1}] - 1 \to -\frac{1-e^{-\tau}}{\tau}, \qquad n \to \infty.$$

The relative error is negative and depends on expected number of exceedances, τ, over the level u.

4.3 Tail-quantile error estimation

The fact that a 200-year event might become a 120-year event implies that the new 200-year event is much more severe after the extreme event. Motivated by the SCR re-estimation question, we now investigate the impact of a record on tail-quantile estimators.

Example 2 (Tail-quantile estimator)**.** Let Q be the quantile function of F. The aim is to estimate a tail quantile, $Q(1-p)$, where the tail probability, $p \in (0,1)$, is small. Assume that F is in the domain of attraction of the Fréchet distribution with shape parameter $\alpha \in (0, \infty)$. We only use here classical tools of extreme value theory. The interested reader may consult for example the book of Beirlant et al. (2006) for a presentation of Fréchet domain of attraction. Let $\gamma = 1/\alpha$ be the extreme-value index. Let $k \in \{1, \ldots, n-1\}$ be such that $p < k/n$. A common estimator is based on the approximation

$$Q(1-p) \approx Q(1-k/n)\,\{(k/n)/p\}^{\gamma}.$$

On a logarithmic scale, the estimator takes the form

$$\log \widehat{Q}_{n,k}(1-p) = \log X_{n:n-k} + \widehat{\gamma}_{n,k} \log\{(k/n)/p\}, \tag{4.4}$$

where $\widehat{\gamma}_{n,k}$ is an estimator of the extreme-value index γ, for instance the Hill estimator

$$\widehat{\gamma}_{n,k} = \frac{1}{k}\sum_{i=1}^{k} \log X_{n:n-i+1} - \log X_{n:n-k}. \tag{4.5}$$

(We implicitly assume that $X_{n:n-k} > 0$.)

Combining (4.4) and (4.5), we find that the tail quantile estimator is linear in the order statistics $Y_{n:n-k} < \ldots < Y_{n:n}$, where $Y_i = \log X_i$. Identity (4.3) then permits in principle to calculate its conditional distribution on the event that $X_{n:n} < X_{n+1}$:

$$\begin{aligned}
&[\log \widehat{Q}_{n,k}(1-p) \mid X_{n:n} < X_{n+1}] \\
&\qquad \stackrel{d}{=} \log X_{n+1:n-k} + \left(\frac{1}{k} \sum_{i=1}^{k} \log X_{n+1:n-i+1} - \log X_{n+1:n-k} \right) \times \log\{(k/n)/p\}.
\end{aligned}$$

To evaluate the impact of ignoring a known record, let us compute the expectation of the estimator under the simplifying assumption that the random variables X_i are iid Pareto with shape parameter α, that is, $F(x) = 1 - x^{-\alpha}$ for $x \geq 1$. Equivalently, the random variables Y_i are iid Exponential with expectation equal to γ. In that case, $\log Q(1-p) = \gamma \log(1/p)$. A well-known representation of the order statistics from an exponential distribution yields

$$\mathrm{E}[Y_{n:n-j+1}] = \gamma \left(\frac{1}{n} + \frac{1}{n-1} + \cdots + \frac{1}{j} \right), \qquad j \in \{1, \ldots, n\}. \tag{4.6}$$

Equation (4.6) yields the following expressions for the expectation of the estimator of the log tail quantile. Unconditionally, we have

$$\mathrm{E}[\log \widehat{Q}_{n,k}(1-p)] = \log Q(1-p) + \gamma \left(\frac{1}{n} + \cdots + \frac{1}{k} - \log(n/k) \right).$$

The second term on the right-hand side converges to zero relatively quickly. In contrast, conditionally on the occurrence of a record on the next day, we have

$$\begin{aligned}
&\mathrm{E}[\log \widehat{Q}_{n,k}(1-p) \mid X_{n:n} < X_{n+1}] \\
&\qquad = (1-a_k) \log Q(1-p) + \gamma \left(\frac{1}{n} + \cdots + \frac{1}{k} - (1-a_k) \log(n/k) \right),
\end{aligned}$$

where

$$a_k = \frac{1}{k} \sum_{j=1}^{k} \frac{1}{j+1}.$$

The sequence a_k tends to zero as k tends to infinity: $a_k \sim \log(k)/k$ as $k \to \infty$. Still, since the relative error occurs on the logarithmic scale, there is potentially a severe under-estimation of the tail quantile: indeed, we have $(1-a_k) \log Q(1-p) = \log[\{Q(1-p)\}^{1-a_k}]$. The relative error is thus given by $\{Q(1-p)\}^{-a_k} = (1/p)^{a_k \gamma}$. The larger the tail index γ and the smaller the tail probability p, the larger the relative error.

In the next section, we investigate the concrete impact of this phenomenon and of two other ones (risk exposure reduction and decrease in diversification elements) on real-world insurance regulatory capitals.

5 Illustration with real-world situations

In this section, we calibrate the 3 effects following two approaches: the first one is related to actual risk levels used in financial regulations, the second one using the 2014 EIOPA stress test data of the French insurance regulator. We first provide order of magnitudes of the re-estimation effect on SCR in the insurance industry, in absence of loss-absorbing capacity effect and for $a = 1$. Then, motivated by the design of the market risk SCR, we investigate the case where $a = 0.6$ and calibrate b and b' from real data. Finally, we study the case where $a > 1$ and identify in practice regions where one effect dominates the other. On top of this empirical illustrations, we highlight the problem of the risk margin valuation which strenghten our main conclusions on the SCR with a view on the whole prudential balance sheet.

5.1 Tail Re-estimation effect

Parameter τ - Tail probability error

In the case of a natural catastrophe, this parameter belongs to a broad range of values. In the case of a stress test, τ is close to 0. It is quite common to consider $\tau = \frac{1}{200}$ which is the typical probability target used in the Solvency II framework.

From the estimation of τ, we see that 10 exceedances already give a 10% misvaluation of the tail-probability. Only 1 exceedance introduces a spread of 63% with the probability estimated without the last record. These amounts clearly highlights the merits of this effect. This effect probability is striking but cannot account for the error on the SCR nonetheless which is homogenous to a quantile. We now consider the quantile error.

Parameter γ - Quantile error

As a first order approximation we can use the formula illustrated in 4.3 for the relative error of the quantile:

$$\delta_{p,k,\gamma} = (1/p)^{a_k \cdot \gamma}$$

with $a_k \approx \frac{log(k)}{k}$.

Regarding the Solvency II context, p should be equal to 0.005. For k, different values are plausible; the natural framework in Solvency II should be $k = 200$, since the current norm set records up to 200 years of magnitude. In a Stress Test context, values of k in the range of 5 to 50 are also admissible.

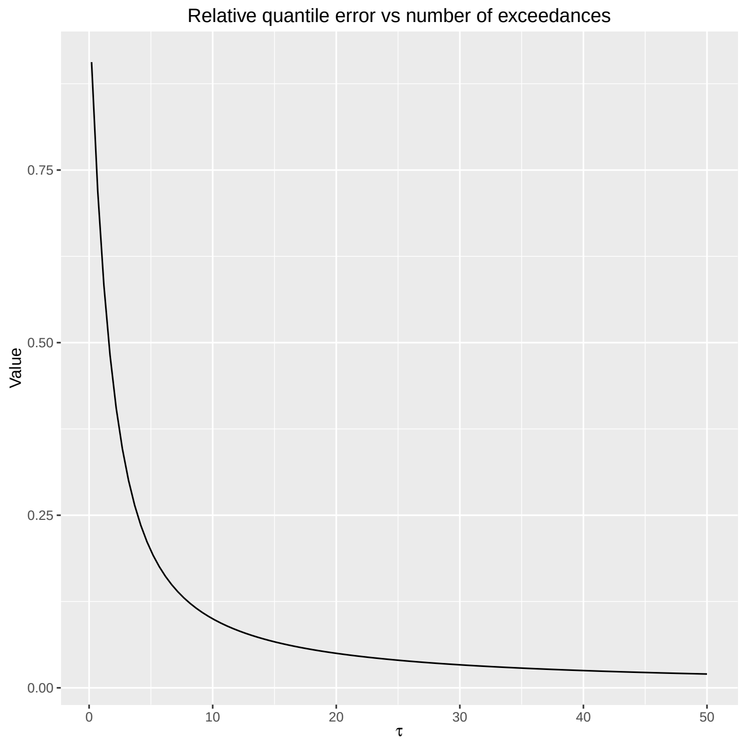

Figure 7: Relative probability error vs number of exceedances

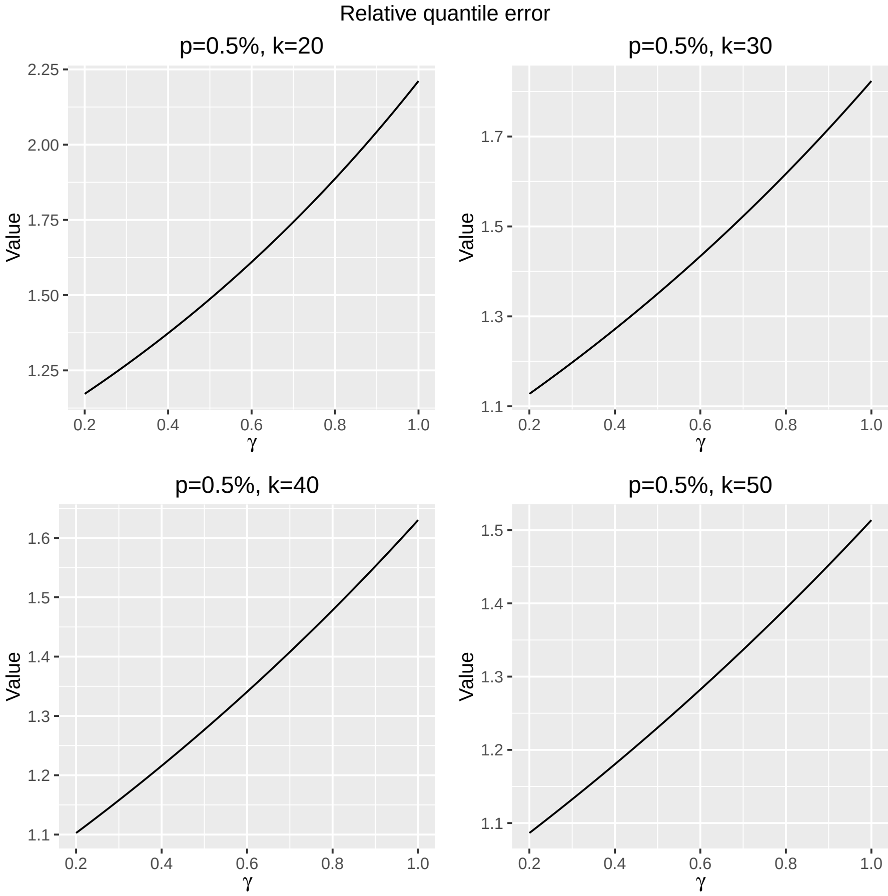

Figure 8: Relative quantile error vs relative expected magnitude as a function of parameter γ for different values of p and k

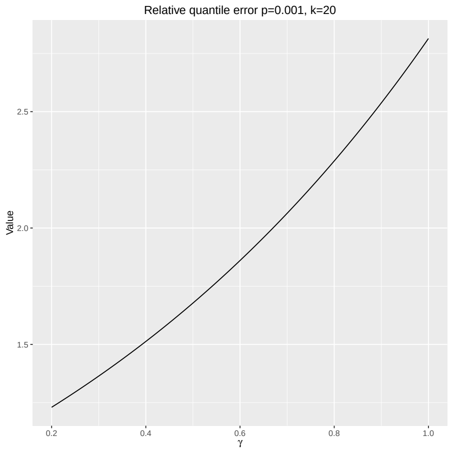

Figure 9: Relative quantile error vs relative expected magnitude as a function of parameter γ (for p=0.001 and k=20)

As illustrated by the graphs in Figure 8, the difference between the actual quantile and its value just after the addition of a shock with magnitude γ times the expectation of the standard shocks can be very significant. For example, even with 200 records, the addition of an event 10 times larger than expected would lead to a quantile more than twice the initial value! Note that we implicitly assimilated the change in the estimated 99.5% VaR to the change in the SCR. This is not true in general as the SCR might be defined in a more complex way. Besides, the Best Estimate of Liabilities would also be impacted. Nevertheless, for reasonable values of n and k, the change in the estimated average of X is small in comparison to the change in the 99.5%-Value-at-Risk level. Therefore, for simplicity, we assume here that the Best Estimate of Liabilities can be neglected in this first study, and we leave it for further research to quantify the change in the best estimate.

In Figure 9, we focus on operational risk for banks, for which banking regulation imposes to compute the 99.9%-quantile of the 1-year loss. Nešlehová et al. (2006) show that for banking operational risk, one cannot exclude that $\gamma > 1$, corresponding to infinite mean models. We therefore consider the impact of quantile reestimation after a record: for finite mean models with γ close to 1, Figure 9 shows that the new result might be as large as 2.8 times the result without re-estimation. This shows that the phenomenon presented here deserves further research regarding banking supervision.

However, this first effect actually accounts only for changes in something equivalent to the gross BSCR (the "quantile error") before diversification (not considered here). Let us now investigate the concrete effect of Loss Absorbing Capacity on the net SCR.

$Liabilities$	$100M€$
$gBSCR$	$7.5M€$
b	$5.25M€$
$Net\,SCR$	$2.23M€$

Table 1: Toy company, pre-stress situation (source: ST 2014 figures)

5.2 The case $a < 1$

The naive model introduced in Section 3 can be calibrated with the 2014 stress test data. An identification of the different terms of the right hand side implies that the $VaR_{99.5\%}(X) - E(X)$ is equal to the gross BSCR (adding Operational risk, denoted by $gBSCR$) and b is the sum of the different diversification effects (in particular Loss-Absorbing Capacity with Technical Provisions and with Deferred Tax).

In absence of quantile re-estimation, after the shock, X becomes $X' = aX$ and the SCR becomes: $SCR(X') = a.gBSCR - b'$. With this simplified setup, it appears very clear why the risk could not depend on the scaling factor a and only on the potential increase of risk of the profit and loss distribution. At this point, we emphasize that the desired quantile is not directly based on the exposure so that there might only exist a tenous link between the risk exposure and the loss distribution.

The gross SCR is multiplied by a when $X' = aX$. Note that this property is very general and remains valid when the Solvency Capital is defined thanks to a Tail-Value-at-Risk as in the Swiss Solvency Test, or when one uses any distortion risk measure for economic capital in Enterprise Risk Management. This positive homogeneity property is also valid in the practical approach adopted during the genesis of Solvency II: practitioners often approximate $VaR_{99.5\%}(X)$ with $E(X) + c\sigma_X$, where σ_X is the standard deviation of X and $2.5 \leq c \leq 5$ is a multiplier close to 3 in the lognormal case and closer to 4 or 5 for loss distributions with heavier tails.

To illustrate this setup, we create a company with $100M€$ total balance sheet representative of the ST2014 data[7].

First remark: the diversification effect represents more than twice the net SCR, which demonstrates how important it is in the Solvency II framework. Another important consequence is that the variance of the profit & loss distribution plays a far greater role than the market risk exposure. Indeed, the a factor does not show up in the final estimation of the SCR. If we make another assumption and assume a perfect correlation between market

[7] The different prudential quantities in the table are computed from the companies which reassessed their SCR post-stress and had a positive increase in at least one of the financial stresses.

in$M€$	$ST\ (a \approx 0.93)$	$a = 0.9$	$a = 0.8$
$Liabilities'$	97.5	96.8	86
$BSCR'$	7.17	6.7	6
b'	4.45	4.02	3.27
$Net\,SCR'$	2.71	2.71	2.71

Table 2: Toy company, post-stress situation (source: ST 2014 figures, authors' calculations)

exposure and the P&L, we would get:

$$SCR(X') = a.gBSCR - b'$$

with[8] $a = 0.6$. In this simple model, the pre-stress net and gross SCR shown in Table 1[9] evolve after the stress as presented in Table 2.

In fact, $a = 0.6$ corresponds to the pure shock for stocks and their spillovers. But given other risk modules and diversification effect it might be more consistent to choose $a = 0.8$ or $a = 0.9$[10]. We also provide numbers for $a = 0.8$ and for $a = 0.9$.

For the completeness of the analysis, the value of b' is deduced with the following equation (for a=0.9):

$$b' = gBSCR' - Net\,SCR' = 7.17 - 2.71M€ = 4.45 = 0.77 \cdot b.$$

As discussed in 2.2 we observe in this simple example that the different diversification effect had to actually decrease much faster than the risk exposure. As a matter of fact, a reassessment of the SCR and at least the different LAC component should be mandatory in any forward looking exercise (ORSA, Stress test, etc.) when it is relevant. More generally, credibility of the different diversification modules should be checked thoroughly and be part of the annual review of risk of any insurance supervisor. It is interesting to note that in the case of the French groups participating to the EIOPA Stress Test 2014 which reassessed their FDB post-stress:

$$\widehat{b'} = 0.26 \cdot b,$$

which empirically validates that this effect is very substantial and our that our model is not too conservative.

[8]This corresponds to a 40% decrease of the value of stock, comparable to the shock of the first scenario of the 2014 Stress test.

[9]The value of the LAC post-stress and BSCR' were not requested in the Stress Test exercise but could be reconstituted.

[10]As an illustration, the value was 0.93 for the French companies used in the ST2014 sample here

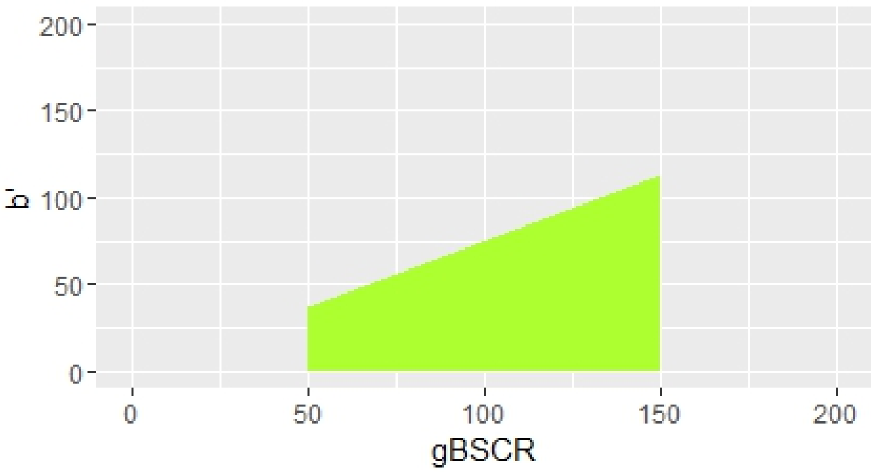

Figure 10: b' value with a positive increase of the net SCR

5.3 The case $a > 1$

The case $a > 1$ corresponds to the situation where the risk exposure increases after the shock: for example after a first earthquake or some floodings, the next event might have more severe consequences if it occurs soon, because some buildings have become more fragile or because the soil is already saturated with water. Another such situation, in the life insurance business, may occur in the case of mass non lapse phenomenon, where remaining policyholders are more numerous than expected (for example if they benefit from a high guaranteed minimum interest rate in a low or negative interest rate context). To illustrate this point, we choose for b a market average and $a = 1.2$. So far, this figure has been provided as a percentage of the aggregate basic solvency capital requirement both for the participants of the 2014 EIOPA ST (See European Insurance and Occupational Pensions Authority (2014)) and their French counterparts (see Borel-Mathurin and Gandolphe (2015)). The absorbing capacity equals to $b = 38\% \cdot \overline{gBSCR}$ (resp. $b = 61\% \cdot \overline{gBSCR}$) for the whole setup of european groups EU participants (resp. the French groups). For values of gross BSCR ranging from 50% to 150% of the market average gross SCR, we plot in Figure 10 the sub-regions of the half-plane (b',gross BSCR) where the re-evaluated SCR is larger than the initial one.

5.4 A potential scissors effect on SCR Coverage ratio

The surge in the post-stress SCR can also have unexpected consequences in solvency capital coverage ratio. A by-product of this SCR increase is the coincident effect on the risk-margin. Indeed, right after the stress, the SCR_{0+} or SCR'_0 is the new basis for the calculation (assuming the cost of capital ("CoC"), generally set to 6%)[11]:

[11] here a simplified version is presented

$$RM' = CoC \sum_{t \geq 0} SCR'(t).$$

To determine $SCR(t)$, one may either project at each timestep "t" and make a complete Best Estimate determination in future time, or, use one of the simplified methodologies, for example the proportional approach based on the Best Estimate[12]. We can then infer the future value of the SCR:

$$SCR(t) = SCR(0) \cdot \frac{BE(t)}{BE(0)}, \quad \text{and} \quad RM = CoC \cdot SCR(0) \sum_{t \geq 0} \frac{BE(t)}{BE(0)}.$$

Assume that the proportions stay constant after stress so that

$$\forall t \geq 0, \quad \frac{BE'(t)}{BE'(0)} = \frac{BE(t)}{BE(0)}.$$

Then, we have

$$RM' = CoC \cdot SCR'(0) \sum_{t \geq 0} \frac{BE'(t)}{BE'(0)} = \frac{SCR'(0)}{SCR(0)} RM.$$

Therefore, the technical provisions would actually increase with even a quasi-constant $BE(0^+)$. In this context, the SCR coverage ratio would deteriorate even more since the SCR would increase and the available own funds would actually decrease as risk margin increases.

6 Conclusion & implication for policy

The Solvency II framework is characterized by the estimation of losses' quantiles based on historical data. This framework allows for diversification and absorbing capacities, which means the possibility to take into account the ability to transfer future risk to the policyholders. In this paper, we studied the implications of the records of large losses on the one hand and, on the other hand, the magnitude of diversification elements of the prudential balance sheet such as loss absorbing capacities using deferred taxes or the technical provisions. We first tried to estimate theoretically the impact of the absence of a large record in terms of the probability distribution and the quantile of the loss functions. We also proposed a

[12]There exist 4 different methods classified by level of simplification defined in Guidelines 61 and 62 in [EIOPA-BoS-14/166]. Here the 4th method is used for illustration.

stylized model to reassess the solvency capital requirement after a large record. The calibration using the data of the French participants to the 2014 EIOPA Stress test confirms our theoretical arguments and showed the very prominent role of the loss absorbing capacities in the Solvency II framework. Based on our data, the decrease in the reassessment is in the range of 23 to 74% as far as our estimations are concerned. One of the regular criticism addressed to the Solvency II framework is the "one year horizon" used for the quantile calculations which could produce a lack of stability in the determination of the solvency capital requirement. In this regard, our work stressed how important the absorbing capacities can be for introducing volatility and as a consequence, emphasizes the importance of the future management actions and other means of diversification and risk mitigation while calculating the Best Estimate of the liabilities.

Implications of our paper could have four facets: research, Enterprise Risk Management, supervision and regulation. As far as research is concerned, one might want to look forward a more advanced framework with a multi-dimensional setup. Actually, insurance companies potentially undergo shocks from different risk factors at the same time. The aggregation step would introduce other effects to model. Another direction could be the use of these ideas in the banking sector, eg the calculation of the capital charge with VaRs such as Market risk in the Basel III framework.

Insurers, reinsurers and captives should take into account the impact of large events on their future ability to continue business. This study shows that re-evaluating the SCR after a shock should be part of a sound Enterprise Risk Management approach of risk measurement, risk controls and risk appetite determination.

The supervision duties should be modified in comparison to what was done in the Solvency I framework. Even in the standard formula, many levers exist and can be used while producing the prudential balance sheet. In this context, supervisory work should integrate the credibility checking of the projection hypotheses. Regarding prospective exercises, be it by the firm (eg ORSA) or the regulation (eg Stress Tests), we strongly recommend to always check the evolution of the solvency capital requirements after shocks since letting them constant cannot always be seen as a conservative assumption. Indeed, the risk exposure reduction does not necessarily decrease the value of the solvency capital requirement as shown in this paper. In this regard we would strongly recommend that future exercises do not only specify the asset side but also the liability side and give guidance on the level of risk transfer to be operated with the technical provision.

Regarding banking supervision, our theoretical analysis and Figure 9 show that the re-estimation of the quantile of the operational loss is a very important question and deserves further research.

Finally, the regulatory bodies might have a closer look to those aspects of Solvency II framework, study them and potentially impose some evolutions to the assessment of those quantities. A plausible response could be regulatory prescriptions on the levels of those absorptions, with a view towards simple multi-period stresses instead of instantaneous ones.

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